PROCEEDINGS

OF THE

ROYAL SOCIETY OF EDINBURGH.

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PROCEEDINGS

OF THE

ROYAL SOCIETY OF EDINBURGH.

VOL. XXX. 1909-10.

I.—Andrews' Measurements of the Compression of Carbon Dioxide and of Mixtures of Carbon Dioxide and Nitrogen. Edited by Professor C. G. Knott.

After the death of the late Professor Tait in 1901, I was asked by the Council of the Royal Society of Edinburgh to carry to completion the task which Professor Tait had undertaken of presenting Professor Andrews' epoch-making work in such a form as to make possible the estimation of the true pressures.

At the meeting of February 20, 1899, Professor Tait communicated the first instalment of a paper on "The Experimental Bases of Professor Andrews' Paper on the Continuity of the Gaseous and Liquid States of Matter (*Phil. Trans.* 1869)." Considerable progress was made during the succeeding months in printing in a form similar to the original the observed numbers and interpolated calculations in Andrews' experimental notebooks; and the galley proofs had undergone a first revision. Among Professor Tait's manuscripts the following draft was found, evidently intended to be the opening paragraphs of the description of these verbatim extracts from the notebooks:—

"In February last there appeared in *Nature* a letter from Professor Tsuruta of Tokyo calling attention to two important questions connected with the accurate reduction of the series of experiments described in Dr Andrews' epoch-making Bakerian Lecture of 1869. Andrews was particularly careful to explain that his own reductions were merely provisional, though sufficient for his immediate purpose.

"The first of these depended on the calculation of true pressures by the air manometer which Andrews was forced to employ and to interpret by

means of Boyle's Law. We are now, thanks to the wonderful measurements of Amagat, enabled to estimate pressures with great precision from the corresponding reduction of volume of air at any given temperatures. The data, so far as this point is concerned, were not quite fully given in Andrews' paper, inasmuch as he had given the estimated compression of air only, the estimate being founded on a law now recognised as not rigorously correct.

"The second depended on a difference between the pressures to which the two gases were simultaneously subjected, arising from the fact that the common mass of compressing mercury stood at different levels in the two tubes. This difference, of course, varied with the stage of compression, being considerably greater throughout the ranges in which the carbonic acid was much more compressed than the air; and less in those regions in which it was entirely liquid, and thus considerably less (further) compressible than air.

"Dr Andrews' notebooks have been sedulously preserved, and on a careful examination I found that they afford the means of almost completely supplying Professor Tsuruta's desiderata. Miss M. K. Andrews has kindly undertaken the laborious task of collecting, copying, and arranging them in the form in which they appear below. She is specially qualified for this work by her familiarity with her father's handwriting, especially in the matter of figures; and I think it barely possible that any mistake due to that cause will be found."

Here the manuscript ends; but I should like to supplement the last sentence by the remark that Miss M. K. Andrews has, in addition to her familiarity with the symbols, a clear knowledge of the whole scope of the enquiry, even to the details of corrections for tube-ends, capillarity, temperature, and so on. Under her guidance I went carefully through all the notebooks covering the experiments on carbon dioxide. Verbatim copies were made of many parts of these, full indexes were prepared, and everything done to facilitate the accurate comprehension of every record. It was at times no easy matter to dig out the meaning of some of the records, but, on the whole, once the clue was found the interpretation of the experimental jottings was not difficult. During the process of elucidation, however, it was borne in upon me that Professor Tait's plan of reproducing the data might, with advantage to all interested, give place to another method. It seemed desirable to present the data in a form at once intelligible to the reader rather than compel him to go through the labour of ferreting out the meaning for himself. Moreover, the form in which the data now appear below is much more economical as regards

space, and shows at a glance the relation of the numbers now tabulated to those given by Andrews himself in his great memoirs.

Professor Tait intended to reproduce only the data of the results given in the first memoir, the Bakerian Lecture of 1869; but the much more compact tabulation now adopted makes it possible to treat similarly the Bakerian Lecture of 1876, and also the posthumous paper in continuation of the same great subject, the manuscript of which was found among Dr Andrews' papers and was transmitted in 1886 by Professor Tait to Sir George Stokes for publication in the *Philosophical Transactions.* On page 52 of that last memoir, reference is made to experiments with a mixture of 6·2 volumes of carbonic acid and 1 volume nitrogen. The data for these experiments, which are fully recorded in the notebooks, have been added for the sake of completeness.

Each series of experiments began with a careful calibration of the tubes used; and to facilitate the calculation of the volumes of the fluids compressed, Dr Andrews prepared very full tables from which the required values were got by mere inspection. The entries from these tables made by Andrews himself have been checked in every case, and once or twice a slight error in the last figure was detected. This was, however, just within the errors of observation.

The tabulation is made on the same principle throughout. There are, in general, ten columns of figures. The first column gives the number attached to the experiment in the notebook, and at the head of the column the corresponding notebook and volume in which it is bound are named. The second column gives the position of the experiment in the corresponding table published in the Bakerian Lectures and the posthumous memoir. The third, fourth, and fifth columns give the temperature, length of the air (or hydrogen) column, and the corresponding volume in cubic centimetres. The seventh, eighth, and ninth columns contain the same quantities for the carbon-dioxide (or the mixture); and the tenth gives the difference of level between the mercury surfaces in the two tubes. The sixth column is reserved for the real pressures to which the carbon dioxide is subjected. To get each true pressure we first estimate the pressure of the manometric gas (air or hydrogen, as the case may be) from the given values of the volume and temperature, and then apply the correction for the difference of level of the mercury surfaces in the two tubes, positive or negative, according as the surface in the tube containing the carbon dioxide stands lower or higher than the other.

On page 580 of the Bakerian Lecture of 1869 (pp. 303–4 of the *Scientific Papers*) Dr Andrews shows how the results were calculated by

working out completely one example. This particular example corresponds to the third entry in the first table given below, the one designated 123 *tris*. It was not included by Andrews in the published Table I. A comparison of what is tabulated here with Dr Andrews' working out will serve to show how the present table has been constructed.

My original intention was to have worked out the true pressures and entered them in the sixth column. After prolonged and careful discussion of Amagat's published results for air, I was reluctantly compelled to come to the conclusion that they were not sufficient for the purpose. In his final "Mémoires sur l'Elasticité et la Dilatabilité des Fluides jusqu'aux très hautes Pressions" (*Annales de Chimie et de Physique*, 1893), Amagat gives three tables of numbers. Tableau No. 10 contains the volumes at pressures ranging from 100 atmospheres to 1000, at intervals of 50; and in Tableaux No. 10 *bis* the volumes are given for pressures ranging from 125 to 475, also at intervals of 50 atmospheres. These are not detailed enough for pressures below 100 to serve for the present purpose. At first sight Tableau No. 11 seems to give results in sufficient detail for pressures from 20 metres pressure to 65 metres pressure, that is from 26·32 to 85·53 atmospheres. I do not find, however, that this last table agrees well with the other two.

For any limited range of pressure and temperature changes we may assume the following empirical formula—

$$p(v-b)=\mathrm{RT}-a/v+c/v^2,$$

where p, v, and T have their usual meanings, and R, a, b, c are constants for any given mass of gas at given temperature and pressure. When $c=ab$we have Van der Waals' formula

$$(p+a/v^2)(v-b)=\mathrm{RT}.$$

On applying this formula to the data of Tableau No. 11, and then to those of Tableau No. 10 and No. 10 *bis*, I did not obtain consistent results. I have accordingly thought it best to leave the pressure column vacant in all cases in which air was the manometric substance.

On the other hand, the approximately linear character of the hydrogen curves, showing the relation between pressure and volume, renders it possible to obtain a formula applicable throughout a wide range of pressures.

The pressures which are entered in the tables of Group C below have been calculated from the formula

$$p(v-0{\cdot}001668)=0{\cdot}00366\ \mathrm{T}-\frac{0{\cdot}0009575}{v}-\frac{0{\cdot}0000008647}{v^2}.$$

Having by means of this expression calculated the pressure corresponding to a given volume v of hydrogen at temperature $T(=t^{\circ}\,C.+273)$ we obtain the pressure to which the mixture was subjected by applying the small correction for the difference of levels of the mercury in the two tubes. The true pressure so calculated is entered in the sixth column of the third set of tables. It will be found that these are all somewhat higher than the values given by Andrews in the posthumous paper of 1887 (*Phil. Trans.*, vol. 178, pp. 45–56; *Scientific Papers*, pp. 457–471).

There are three sets or groups of tables corresponding to the memoirs already mentioned, the first and second Bakerian Lectures of 1869 and 1876, and the posthumous paper of 1887. These are distinguished as Groups A, B, and C.

In following out Professor Tait's desire to have the data of Andrews' classical experiments in as accurate a form as possible, I do not think that any apology is needed. Professor Andrews was a born experimenter who spared no pains to secure absolute accuracy in all his work. The data are now presented in such a form that the pressures can be determined with accuracy. They must of course be studied in connection with Dr Andrews' own original descriptions as contained in the three important memoirs of 1869, 1876, and 1886.

Lengths are in millimetres, volumes in cubic centimetres, temperatures in degrees centigrade.

A positive sign after the Difference of Level in the last column means that the level in the manometer tube was the higher, so that, in order to find the pressure of the carbon dioxide, the corresponding pressure must be added to the pressure as indicated by the manometer; on the other hand, a negative sign means that the correction must be subtracted.

Group A.

(Corresponding to Tables I. to VI. and the Appendix in the Bakerian Lecture of 1869.)

Carbon Dioxide at various temperatures and pressures.

Table I.—CO_2 at 13°·1 C.

Number in Notebook 8, Volume VII.	Place in Table.	Air.			Carbonic Acid.				Difference of Mercury Levels—Air Lower.
		Tempera-ture.	Column Length.	Volume.	Pressure.	Tempera-ture.	Column Length.	Volume.	
		0°.		0·3123	1 atmo.	0°.		0·3095	
123	(1)	10·75	274·7	0·006802	...	13·18	126·1	0·004262	179·1 –
123 *bis*	...	10·75	273·3	6767	...	13·25	125·4	4239	178·4 –
123 *tris*	...	10·76	272·9	6757	...	13·22	124·6	4211	178·8 –
124	(2)	10·86	267·7	6629	...	13·18	119·4	4036	178·8 –
124 *bis* and *tris*	(3)	10·95	267·2	6617	...	13·34	119·2	4029	180·5 –
125	...	11·03	265·0	6563	...	13·35	116·3 liquid visible	3930	179·2 –
126	...	11·20	265·0	6563	...	13·35	116·5 liquid distinct	3937	179·0 –
127	...	12·03	256·8	6361	...	13·38	20·65	0688	266·6 –
	...	,,	265·0	6563	...	,,	116	3920	179·5 –
	...	,,	264·25	6544	...	,,	91	3077	203·7 –
	...	,,	262·25	6495	...	,,	37	1246	255·7 –
	...	,,	261·75	6482	...	,,	32	1075	260·2 –
	...	,,	260·5	6452	...	,,	27	905	264·0 –
	...	,,	260·25	6446	...	,,	26	871	264·7 –
	...	,,	,,	,,	...	,,	24	803	266·7 –
129″	...	12·89	276·9	6857	...	12·79	125·25	4233	182·1 –
130	...	12·85	270·6	6701	...	12·87	119·4	4036	181·7 –
131	...	12·83	270·0	6686	...	12·88	118·1	3991	182·4 –
132	...	,,	270·4	6696	...	12·94	118·6	4008	182·3 –
133	...	12·84	267·1	6614	...	13·0	118·2	3995	181·4 –
134	...	12·86	268·8	6657	...	13·02	115·1	3889	184·2 –
134 *bis*	...	,,	,,	,,	...	,,	115·5	3903	183·8 –
135	...	12·88	,,	,,	...	,,	,,	3903	,,
						liquid just disappearing			
136	...	12·95	268·8	0·006657	...	13·02	117·4	0·003968	181·9 –
						liquid formed—very exact result			
137	(3)	12·95	,,	,,	...	13·09	119	4021	180·3 –
	...	...	268·55	6650	...	...	107	3617	192·1 –
	(4)	...	268·05	6638	...	...	91	3077	207·6 –
	(5)	...	267·55	6625	...	...	68	2299	230·1 –
	(6)	...	267·05	6613	...	...	50	1689	247·6 –
	(7)	...	266·05	6588	...	...	36	1212	260·6 –
	(8)	...	265·05	6564	...	...	28	939	267·6 –
	(9)	...	264·05	6539	...	...	25	837	269·5 –
	(10)	...	261·05	6465	...	...	21	700	270·6 –
	(11)	...	260·3	6447	...	...	20·08	669	270·5
		0°		0·3455		0°		0·3783	
301 Bk. 10	(12)	14·13	267·2	·006618	...	14·40	26·02	0·000871	270·0 –
301 *bis*	(13)	14·13	193·5	4790	...	14·40	25·02	837	197·1 –
301 *tris*	(14)	14·13	162·1	4009	...	14·40	24·52	820	166·2 –

TABLE II.—CO_2 at 21°·5 C.

Number in Notebook 8, Volume VII.	Place in Table.	Air. Tempera-ture.	Air. Column Length.	Air. Volume.	Carbonic Acid. Pressure.	Carbonic Acid. Tempera-ture.	Carbonic Acid. Column Length.	Carbonic Acid. Volume.	Difference of Levels—Air Lower.
		0°.		0·3123	1 atmo.	0°.		0·3095	
139	1	8·58	277·0	0·006860	...	21·50	147·2	0·004974	160·3 –
139 *bis*	...	8·63	277·0	„	...	21·47	147·0	4967	160·5 –
139 *tris*	...	8·73	277·0	„	...	21·44	146·9	4964	160·6 –
140	...	8·73	215·9	5349	...	21·48	liquid just v in subs	isible, but dis equent experi	appeared ments
141	2	8·73	215·9	5349	...	21·46	86·5	2925	159·9 –
						liquid	not visible, b	ut on point of	forming
141 *bis*	2	8·73	215·9	5349	...	21·46	86·2	2915	160·2 –
142	...	„	206·9	5124	...	21·44	22·6	754	214·8 –
	...	„	215·9	5349	...	21·45	86·5	2925	159·9 –
	...	„	215·4	5337	...	„	85·8	2901	160·1 –
	3	„	214·9	5324	...	„	56·3	1902	189·1 –
	4	„	213·9	5299	...	„	40·8	1374	203·6 –
	5	„	211·9	5249	...	„	26·8	897	215·6 –
	6	„	207·9	5149	...	„	22·3	744	216·1 –
	7	„	206·9	5124	...	„	22·1	737	215·5 –
	...	„	215·9	5349	...	„	86·5	2925	159·9 –
	...	„	214·9	5324	...	„	57·5 (L. 9·5)	1943	187·9 –
	...	„	213·9	5299	...	„	42 (L. 14)	1415	202·4 –

TABLE III.—CO_2 at 31°·1 C.

Book 9.									
225″	1	11·59	239·0	0·005922	...	31·17	126·8	0·004286	143·5 –
224″	2	11·59	234·0	5798	...	31·22	122·5	4140	142·7 –
223‴	3	11·58	229·0	5675	...	31·15	117·98	3987	142·5 –
222″	4	11·55	224·0	5551	...	31·19	113·45	3835	141·9 –
221″	5	11·41	219·0	5427	...	31·18	108·82	3678	141·4 –
220″	6	11·40	214·0	5302	...	31·20	104·15	3520	141·3 –
218, 219	7	11·44	209·0	5177	...	31·19	99·0	3347	141·3 –
215 *bis*	8	11·76	204·05	5054	...	31·13	93·1	3148	142·2 –
214	9	11·73	199·05	4929	...	31·19	87·9	2972	142·4 –
213 *bis*	10	11·66	194·05	4804	...	31·18	82·2	2780	143·2 –
„ *tris* and *quat.*	„	11·63	194·05	4804	...	31·13	81·97	2771	143·3 –
212″	11	11·55	189·05	4679	...	31·03	75·85	2565	144·4 –
202″	12	11·40	184·1	4555	...	31·06	68·9	2329	146·3 –
203′	13	11·4	179·1	4430	...	31·12	60·5	2045	149·8 –
203″	...	11·5	179·1	4430	...	31·07	60·35	2040	150·0 –
175	14	13·0	178·7	4420	...	31·08	58·3	1970	151·6 –
204″	15	11·62	174·1	4306	...	31·06	33·0	1109	172·3 –
205	16	11·65	169·1	4182	...	31·06	27·9	935	172·4 –
208″	17	11·16	164·05	4057	...	31·10	26·9	901	168·4 –
209	18	11·23	159·05	3934	...	31·07	26·03	871	164·2 –
210″	19	11·45	154·05	3811	...	31·05	25·40	850	159·9 –

Table IV.—CO_2 at 32°·5 C.

Number in Notebooks 7, 8, 9, 10, Volume VII.	Place in Table.	Air. Tempera-ture.	Air. Column Length.	Air. Volume.	Carbonic Acid. Pressure.	Carbonic Acid. Tempera-ture.	Carbonic Acid. Column Length.	Carbonic Acid. Volume.	Difference of Levels—Air Lower.
Book 8.		0°.	mm.	0·3123	1 atmo.	0°.		0·3095	
153‴	1	12·10	228·6	0·005665	...	32·50	119·4	0·004036	140 2 –
154″	2	12·15	183·9	4550	...	32·34	73·0	2469	142·0 –
155″	3	12·3	178·8	4423	...	32·45	65·75	2223	144·2 –
156″	4	12·3	177·8	4398	...	32·46	64·15	2168	144·6 –
157″	5	12·4	172·7	4271	...	32·38	53·5	1808	150·3 –
158	6	12·5	167·7	4148	...	32·48	33·1	1112	165·7 –
162 (Bk. 9)	7	12·35	165·0	4081	...	32·54	29·4	987	166·7 –
163	8	12·35	155·1	3836	...	32·80	26·7	895	159·5 –

Table V.—CO_2 at 35°·5 C.

Number in Notebooks 7, 8, 9, 10, Volume VII.	Place in Table.	Air. Tempera-ture.	Air. Column Length.	Air. Volume.	Carbonic Acid. Pressure.	Carbonic Acid. Tempera-ture.	Carbonic Acid. Column Length.	Carbonic Acid. Volume.	Difference of Levels—Air Lower.
Book 9.									
230″	1	15·68	233·85	0·005795	...	35·49	125·2	0·004232	140·0 –
231″	2	15·7	223·85	5548	...	35·54	116·5	3937	138·8 –
232″	3	15·66	213·85	5298	...	35·52	107·42	3631	137·8 –
234″	4	15·66	203·85	5049	...	35·51	97·7	3304	137·4 –
235″	5	15·75	193·85	4799	...	35·47	87·45	2958	137·7 –
236 *tris*	6	15·79	183·85	4549	...	35·48	76·65	2592	138·5 –
241″	7	15·52	173·85	4300	...	35·55	64·25	2172	141·0 –
242‴	8	15·61	163·80	4052	...	35·55	45·53	1536	149·6 –
243″	9	15·67	153·80	3805	...	35·48	29·67	995	155·5 –
244″	10	15·67	148·8	3681	...	35·50	27·95	937	152·2 –
245‴	11	15·64	143·8	3557	...	35·61	26·95	902	148·2 –
246‴	12	15·61	133·8	3310	...	35·55	25·45	852	139·7 –
247″	13	15·47	123·8	3061	...	35·53	24·35	814	130·9 –

Table VI.—CO_2 at 48°·1 C.

Number in Notebooks 7, 8, 9, 10, Volume VII.	Place in Table.	Air. Tempera-ture.	Air. Column Length.	Air. Volume.	Carbonic Acid. Pressure.	Carbonic Acid. Tempera-ture.	Carbonic Acid. Column Length.	Carbonic Acid. Volume.	Difference of Levels—Air Lower.
Book 10.		0°.		0·3455		0°.		0·3783	
330–1	1	15·67	235	0·005823	...	47·95	152·5	0·005152	111·8 –
332″	2	15·79	215	5327	...	48·05	132·64	4484	111·6 –
333″	3	15·87	195	4828	...	48·12	112·0	3785	112·3 –
334″	4	15·91	175	4328	...	48·25	90·4	3057	113·9 –
335	5	15·83	155	3834	...	48·13	66·4	2245	118·0 –
337″	6	16·23	135	3339	...	48·25	44·3	1494	119·9 –

APPENDIX.

Number in Notebooks 7, 8, 9, 10, Volume VII.	Place in Table.	Air. Tempera-ture.	Air. Column Length.	Air. Volume.	Carbonic Acid. Pressure.	Carbonic Acid. Tempera-ture.	Carbonic Acid. Column Length.	Carbonic Acid. Volume.	Difference of Levels—Air Lower.
Bks. 7, 8, 9.		0°.		0·3123		0°.		0·3095	
112 Bk. 7	1	12·35	272·7	0·006752	...	15·74	129·5	0·004378	174·3 –
113	...	12 5	272·5	6747	...	15·78	128·9	4357	174·5 –
121	2	11·13	246·3	6102	...	16·45	105·0	3549	172·3 –
151 Bk. 8	3	11·5	275·9	6832	...	31·91	159·7	5395	147·2 –
165–6 Bk. 9	4	13·1	183·9	4550	...	31·65	68·85	2328	146·1 –
168″	5	13·2	178·7	4420	...	31·71	59·82	2028	150·1 –
169″	6	13·2	178·6	4418	...	33·15	65·1	2201	144·0 –
170″	7	12·74	178·7	4420	...	33·58	67·6	2285	142·3 –
190″	8	13·14	178·7	4420	...	35·0	71·55	2420	138·4 –
191″	9	13·21	178·75	4421	...	36·03	74·1	2506	135·7 –
192″	10	13·24	173·7	4296	...	36·00	67·6	2285	137·3 –
193	...	13·3	173·7	4296	...	36·11	67·7	2289	137·2 –
195	11	13·38	168·7	4172	...	36·02	60·5	2045	139·4 –
195 *bis*	...	13·38	168·7	4172	...	36·20	60·8	2055	139·1 –
196″	12	13·40	163·7	4049	...	36·22	52·5	1774	142·5 –
197 *tris*	13	13·45	158·7	3925	...	36·20	41·4	1396	148·5 –
198″	14	13·5	153·7	3802	...	36·08	32·25	1084	152·6 –
199 *tris*	15	13·53	148·7	3678	...	36·18	29·2	980	150·7 –
200″	16	13·55	143·7	3554	...	36·22	27·7	929	147·2 –

Group B.

(Corresponding to Tables I. to IV., IX. to XI. in Second Bakerian Lecture, 1876.)

Table I.—Compressibility of Carbonic Acid Gas—6°·0–6°·9.

Number in Notebook 24, Volume VIII.	Place in Table.	Air. Temperature.	Air. Column Length.	Air. Volume.	Carbonic Acid. Pressure.	Carbonic Acid. Temperature.	Carbonic Acid. Column Length.	Carbonic Acid. Volume.	Difference of Levels.
		0°.		·8706	1 atmo.	0°.		0·7616	
154 *bis* and *tris*	1	6·91	409·4	·07439	...	6·89	311·8	·06033	9·77 +
155 and 155 *bis*	2	6·90	371·2	·06764	...	6·90	281·25	·05440	17·45 +
156 and 156 *bis*	3	6·90	333·6	·06095	...	6·90	250·6	·04847	24·40 +
Mean of 157 & 157 *bis* & 163 & 163 *bis*	4	6·45	285·6	·05242	...	6·44	211·12	·04090	32·86 +
158 and 158 *bis*	5	6·79	241·7	·04450	...	6·79	175·0	·03394	40·70 +
164 and 164 *bis*	6	6·07	217·5	·04008	...	6·05	154·4	·02996	44·27 +
159 and 159 *bis*	7	6·72	195·5	·03604	...	6·73	136·0	·02637	47·90 +
165 and 165 *bis*	8	6·07	174·7	·03222	...	6·05	117·97	·02284	50·69 +
160 and 160 *bis*	9	6·65	156·1	·02878	...	6·62	102·0	·01972	53·30 +
166 and 166 *bis*	10	6·03	140·35	·02585	...	6·02	87·75	·01694	54·8 +

Table II.—Compressibility of Carbonic Acid Gas—63°·6–64°·0.*

Number in Notebook 24, Volume VIII.	Place in Table.	Air. Temperature.	Air. Column Length.	Air. Volume.	Carbonic Acid. Pressure.	Carbonic Acid. Temperature.	Carbonic Acid. Column Length.	Carbonic Acid. Volume.	Difference of Levels.
189 and 189 *bis* 190 and 190 *bis*	1	9·30	280·82	·05156	...	63·86	262·37	·05081	88·75 +
191 and 191 *bis*	2	9·19	241·50	·04447	...	63·76	224·3	·04349	90·00 +
192 and 192 *bis*	3	8·82	217·27	·04004	...	63·79	200·65	·03894	90·57 +
193 & *bis* & *tris*	4	8·82	195·4	·03603	...	63·77	179·07	·03476	90·87 +
194 and 194 *bis*	5	8·85	174·4	·03216	...	63·85	158·2	·03073	91·0 +
195 and 195 *bis*	6	8·85	155·94	·02875	...	63·83	139·65	·02711	90·85 +
196 and 196 *bis*	7	8·85	140·25	·02583	...	63·65	124·15	·02408	91·1 +
197 and 197 *bis*	8	8·75	120·9	·02223	...	63·64	104·85	·02030	91·15 +
Book 25.									
198 and 198 *bis*	9	8·97	105·4	·01936	...	63·68	89·17	·01724	90·95 +
199 *bis* and *tris*	10	8·99	90·4	·01659	...	63·57	73·9	·01425	90·68 +
200 and 200 *bis*	11	9·05	75·6	·01387	...	63·74	58·69	·011275	90·27 +
201 and 201 *bis*	12	9·13	60·57	·01111	...	63·75	42·96	·008250	89·60 +
202 and 202 *bis*	13	9·20	46·00	·00843	...	63·75	26·37	·005062	87·56 +
203 and 203 *bis*	14	9·25	33·80	·00619	...	63·70	14·96	·002874	88·36 +
204 and 205	15	9·22	22·05	·00404	...	63·82	10·96	·002108	96·11 +

Table III.—Compressibility of Carbonic Acid Gas—99°·5–100°·7.*

Number in Notebook 24, Volume VIII.	Place in Table.	Air. Temperature.	Air. Column Length.	Air. Volume.	Carbonic Acid. Pressure.	Carbonic Acid. Temperature.	Carbonic Acid. Column Length.	Carbonic Acid. Volume.	Difference of Levels.
Book 24.									
171 and *bis* 172 and *bis*	1	5·83	280·97	·05159	...	100·39	299·77	·05809	125·9 +
174 and *bis* and 175	2	6·04	241·5	·04447	...	100·37	257·22	·04983	122·87 +
173 & *bis* & 176 & *bis*	3	6·01	217·4	·04006	...	100·41	230·91	·04478	120·60 +
177 and 178	4	6·00	195·45	·036036	...	100·72	206·70	·04013	118·37 +
179 and 180	5	5·83	174·65	·03221	...	100·65	183·80	·03570	116·25 +
181 and *bis*	6	5·88	156·05	·02877	...	100·64	162·90	·03167	113·97 +
182 and *bis*	7	5·92	140·30	·02584	...	100·62	145·25	·02822	112·05 +
183 and *bis*	8	5·94	121·10	·02227	...	100·60	123·90	·02404	109·93 +
184 and *bis*	9	6·31	105·73	·01943	...	100·37	106·60	·02066	107·97 +
185 & *bis* & *tris*	10	6·34	90·43	·01660	...	100·33	89·65	·01734	106·37 +
186 and *bis*	11	6·34	75·72	·01389	...	100·37	73·30	·014145	104·67 +
187 and *bis*	12	6·35	60·66	·01112	...	100·37	56·35	·01083	102·78 +
188 and *bis*	13	6·36	46·04	·00844	...	100·37	39·63	·007614	100·68 +
Book 25.									
212 and *bis*	14	8·73	33·75	·006183	...	99·46	24·77	·004760	98·13 +
213 and *bis*	15	8·72	21·95	·004021	...	99·44	14·24	·002735	99·38 +

* In the higher temperature experiments Dr Andrews applied corrections for the glass expansion.

TABLE IV.—Values of α from 0° to 7°·5—Constant Pressure.

Number in Notebook 26, Volume VIII.	Place in Table.	Air.			Carbonic Acid.				Difference of Levels.
		Tempera-ture.	Column Length.	Volume.	Pressure.	Tempera-ture.	Column Length.	Volume.	
		0°.		0·8517	1 atmo.	0°.			
392 and 393	1	7·53	399·25	·07251	...	7·54	378·35	·07298	37·85 –
408 and *bis*	1′	6·73	398·25	·07233	...	0·00	366·1	·07065	49·10 –
394, 395, 396	2	7·65	293·90	·05383	...	7·64	272·2	·05243	38·70 –
409 and *bis*	2′	6·65	292·05	·05350	...	0·00	261·45	·05031	47·55 –
397, 8	3	7·61	236·05	·04346	...	7·63	214·80	·04108	38·20 –
410 and *bis*	3′	6·64	235·15	·04330	...	0·00	205·70	·03928	46·40 –
399 and 400	4	7·63	190·95	·03522	...	7·64	168·25	·03192	39·65 –
411 and *bis*	4′	6·64	190·25	·03509	...	0·00	160·00	·03031	47·20 –
405 and *bis*	5	7·44	170·85	·03153	...	7·45	146·80	·02775	41·00 –
417 and *bis*	5′	6·47	170·45	·03145	...	0·00	139·15	·02627	48·25 –
401 and 402	6	7·65	152·45	·02813	...	7·65	126·70	·02386	42·70 –
412 and *bis*	6′	6·64	151·95	·02804	...	0·00	118·80	·02234	50·10 –
404 and *bis*	7	7·45	137·35	·025325	...	7·46	109·65	·02058	44·65 –
413 and *bis*	7′	6·64	136·95	·02525	...	0·00	101·5	·01903	52·40 –
TABLE IX.—Values of α' from 0° to 6°·5—Constant Volume.									
300 and *bis*	1	7·27	220·3	·04059	...	0·00	190·2	0·03623	46·9 –
317 and 318	1′	6·08	212·4	·03915	...	6·07	190·15	0·03623	39·1 –
322 and *bis*	2	7·17	182·7	·03370	...	0·00	151·55	0·02867	47·9 –
325 and *bis*	2′	6·51	175·55	·03239	...	6·50	151·55	0·02867	40·9 –
323 and *bis*	3	7·27	141·1	·02602	...	0·00	105·75	0·01983	52·1 –
324 and *bis*	3′	6·51	134·37	·02478	...	6·50	105·75	0·01983	45·45 –
TABLE X.—Values of α' from 0° to 64°—Constant Volume.									
299 *bis* and *tris*	1	7·29	289·5	·05304	...	0·00	258·3	0·04969	48·07 –
335	1′	3·68	218·95	·04035	...	64·00	258·0	0·04968	21·80 +
300 and *bis*	2	7·27	220·3	·04059	...	0·00	190·2	0·03623	46·9 –
336, 337, 338	2′	3·69	163·35	·03014	...	63·80	190·0	0·03624	9·4 +
322 and *bis*	3	7·17	182·7	·03370	...	0·00	151·55	0·02867	47·9 –
339 and 340	3′	3·70	132·75	·02446	...	63·74	151·50	0·02869	1·55 +
302 and 303	4	6·75	155·40	·02867	...	0·00	122·45	0·02304	49·85 –
341 and *bis*	4′	3·27	109·55	·02016	...	63·98	122·30	0·02303	4·5 –
323 and *bis*	5	7·27	141·1	·02602	...	0·00	105·75	0·01983	52·1 –
342 and *bis*	5′	3·36	96·90	·01781	...	63·94	105·8	0·01986	8·4 –
TABLE XI.—Values of α' from 0° to 100°—Constant Volume.									
299 *bis* and *tris*	1	7·29	289·5	·05304	...	0·00	258·3	0·04969	48·07 –
348 and 349	1′	3·70	194·0	·03577	...	100·67	257·9	0·04969	46·5 +
300 and *bis*	2	7·27	220·3	·04059	...	0·00	190·2	0·03623	46·9 –
350 and 351	2′	3·70	143·74	·02651	...	100·67	189·85	0·03622	28·70 +
322 and *bis*	3	7·17	182·7	·03370	...	0·00	151·55	0·02867	47·9 –
352 and 353	3′	3·73	116·04	·02136	...	100·67	151·45	0·02870	18·00 +
302 and 303	4	6·75	155·40	·02867	...	0·00	122·45	0·02304	49·85 –
357 *bis* and *tris*	4′	3·65	95·30	·01753	...	100·54	122·20	0·02303	9·5 +
323 and *bis*	5	7·27	141·1	·02602	...	0·00	105·75	0·01983	52·1 –
358 and *bis*	5′	3·64	83·65	·01537	...	100·48	105·7	0·01986	4·65 +

Group C.*

(Corresponding to Tables I. to IV. in the posthumous paper published 1886.)

Compressibility of a mixture of 3 parts by volume of carbon dioxide and 4 parts by volume of nitrogen.

The tubes used were :—For hydrogen, Tube A, Hydrogen Tube 2, 1st Table, Book 17 ; for N and CO_2, Tube B, Air Tube 1, 2nd Table, Book 17, Volume VIII.

Table I.—Temperature of Mixture, 2°·2 C.

Number in Notebook 16, Volume VIII.	Place in Table.	Hydrogen.			N and CO_2.				Difference in Levels.
		Temperature.	Column Length.	Volume.	Pressure in atmospheres.	Temperature.	Column Length.	Volume.	
		0°.		0·2892	1	0°.		0·2464	
176″	1	7·30	417·3	·007041	43·27	2·32	310·2	·005268	99·8 –
177″	2	7·30	361·45	·006085	50·38	2·34	262·4	·004446	91·75 –
179″	3	7·22	341·1	·005736	53·56	2·08	244·7	·004140	89·0 –
178″	4	7·26	322·25	·005414	56·80	2·38	229·75	·003881	85·18 –
180″	5	7·21	301·75	·005071	60·91	2·06	212·4	·003584	82·05 –
181‴	6	7·21	281·5	·004732	65·94	2·10	195·5	·003292	78·70 –
182″ } 183″ }	7	7·20	262·4	·004411	70·50	2·16	180·12	·003027	75·0 –
185″	8	7·18	222·0	·003731	84·13	2·21	147·27	·002460	67·47 –
186″	9	7·17	200·9	·003380	93·62	2·21	130·2	·002171	63·4 –
187″	10	7·17	181·05	·003051	104·27	2·17	114·25	·001901	59·60 –
188″	11	7·17	161·7	·002731	117·53	2·19*	99·1	·001645	55·38 –
189″	12	7·17	142·1	·002404	135·0	2·25	84·35	·001399	50·55 –

* In printed paper 2°·21, but 2°·19 on notebook.

* Tables published in posthumous paper "On the Properties of Matter in the Gaseous and Liquid States under various conditions of Temperature and Pressure," *Scientific Papers*, p. 457, and *Phil. Trans. of the Royal Society of London*, vol. 178 (1887), pp. 45–56 (read March 18, 1886).

Table II.—Temperature of Mixture, 7°·5 C.

Number in Notebook 16, Volume VIII.	Place in Table.	Hydrogen.			N and CO_2.				Difference in Levels.
		Tempera-ture.	Column Length.	Volume.	Pressure in Atmos.	Tempera-ture.	Column Length.	Volume.	
		0°.		0·2892	1	0°.		0·2464	
145″	1	7·47	442·35	·007465	40·80	7·50	340·7	·005790	94·45 –
148	2	7·48	422·4	·007128	42·80	7·50	323·4	·005496	91·8 –
150″	3	7·42	402·25	·006785	45·0	7·50	305·85	·005194	89·20 –
151	4	7·46	382·2	·006443	47·57	7·53	288·7	·004901	86·65 –
155″	5	7·48	342·5	·005759	53·40	7·51	253·7	·004295	81·6 –
156″	6	7·52*	322·4	·005417	56·93	7·49	236·85	·004004	78·3 –
157″ } 158 }	7	7·54	303·1	·005093	61·36	7·50	220·57	·003723	75·28 –
159″	8	7·50	282·6	·004750	65·31	7·50	203·25	·003426	72·13 –
160‴	9	7·48	262·95	·004420	70·38	7·50	186·5	·003137	69·25 –
165‴	10	7·59	241·83	·004065	77·00	7·50	168·75	·002830	65·83 –
166‴	11	7·65	221·93	·003729	83·92	7·51	152·65	·002553	61·97 –
169″	12	7·55	202·6	·003408	92·79	7·50	136·7	·002281	58·6 –
173″	13	7·80	182·75	·003080	103·51	7·08	119·6	·001992	55·85 –
190″	14	7·54	161·75	·002731	117·70	7·48	102·47	·001702	52·22 –
192‴	15	7·58	121·63	·002063	160·22	7·48*	73·60	·001220	42·09 –
196″	16	7·79	105·1	·001787	188·41	7·50	62·77	·001040	36·97 –
211	17	7·63	100·8	·001714	197·25	7·49	60·0	·000994	35·4 –
212″	18	7·67	81·15	·001378	254·55	7·50	49·3	·000817	26·45 –
213	19	7·58	61·75	·001047	356·80	7·49	40·45	·000670	16·00 –

Table III.—Temperature of Mixture, 31°·3.

221″	1	11·64	422·2	·007123	43·49	31·35	355·0	·006028	62·35 –
223″	2	11·94	362·1	·006098	51·19	31·31	298·1	·005062	59·1 –
227″	3	11·86	321·4	·005400	58·08	31·21	261·0	·004422	55·5 –
230″ } 231″ }	4	12·38	281·37	·004730	66·85	31·40	225·0	·003800	51·49 –
234″	5	12·38	248·5	·004177	76·19	31·14	195·6	·003294	48·00 –
235″	6	12·38	200·8	·003378	95·45	31·06	153·87	·002575	41·97 –
236″	7	11·96	162·4	·002742	119·28	31·36	120·7	·002010	36·8 –
239″	8	11·63	122·15	·002071	162·21	31·35	87·1	·001445	30·05 –
241″	9	11·70	80·1	·001360	263·41	31·30	56·1	·000929	19·3 –

Table IV.—Temperature of Mixture, 48°·4 C.

244″	1	8·39	421·2	·007107	43·16	48·22	383·4	·006498	33·1 –
245″	2	8·42	362·5	·006104	50·78	48·11	324·7	·005517	33·1 –
246″	3	12·06	320·5	·005385	58·33	48·48	280·2	·004754	35·65 –
248″	4	12·08	280·6	·004717	66·98	48·43	242·1	·004095	33·87 –
249″	5	12·18	247·6	·004162	76·44	48·66	211·5	·003568	31·45 –
250″	6	12·36	162·0	·002735	119·82	48·38	132·4	·002209	25·0 –
251″	7	12·36	121·1	·002055	164·13	48·49	96·02	·001593	20·97 –
252″	8	12·40	79·65	·001352	266·09	48·38	63·35	·001050	11·50 –

* Slight mistake in published paper.

In a letter dated January 27, 1892, Professor Tait writes as follows to Miss Andrews:—

"In the course of my work on the compression of gases I have at last come to gaseous mixtures. Now, with the exception of *air*, which has been fully dealt with by Amagat, I know of no mixtures experimented on except those of which your father treated in his posthumous paper (p. 457 of the memorial volume). The data are given in full there for a mixture of 3 vols. CO_2 and 4 vols. N. But (see p. 467, at foot) it is clear that another mixture in more valuable proportions, viz., 3·43 vols. CO_2 and 1 vol. N, was also examined. The details of this, if they have been preserved, would be very valuable; indeed, I wonder that neither Stokes nor myself thought of asking about them at the time. . . ."

In the posthumous paper there is also another mixture referred to, viz., 6·2 vols. CO_2 and 1 vol. N.

The data for both of these sets of experiment are now given in detail. It will be seen that the manometer gas was not hydrogen but air. The volumes in the first table are not given in cubic centimetres; but as a sufficient datum for the calculation of the pressures, the original volume at ordinary temperature and pressure is given in terms of the length of the containing tube.

The following notes, prepared by Miss M. K. Andrews, will make these tables more easily intelligible to the reader. They should also be read in connection with Dr Andrews' own account as given in the posthumous paper of 1886 (see *Scientific Papers*, pp. 457–471).

With the mixture of 6·2 vols. CO_2 and 1 vol. N, Dr Andrews found that at the lower temperatures nitrogen is absorbed in the ordinary way, and the curvature of the liquid surface is preserved, so long as any portion of the gas is visible (Exp. 551–555); but at higher temperatures the liquid surface loses its curvature and is effaced by pressure alone (Exp. 575 *et seq.*).

With the mixture of 3·43 vols. CO_2 and 1 vol. N, the critical temperature was found to be 14° C., and the corresponding pressure about 98 atmospheres. This point was obtained by gradually lowering the temperature till it was just possible to obtain a small trace of liquid by the application of pressure. Experiments were made at lower temperatures than the critical point (14°) to fix the pressures at which for the same temperature the liquid first appeared and was afterwards effaced (Exp. 568–570, 571–573, 932–934).

Dr Andrews was long perplexed with anomalous results, the carbon dioxide sometimes liquefying by the application of pressure above 20°, at

(*Continued on page* 22.)

Mixture of Nitrogen and CO_2: Nit. = 1, CO_2 = 6·2 ; or, Mixture contained 13·9 per cent. Nitrogen.

L. and G. in the sixth column refer to the liquid and gaseous parts of the whole column of the mixed substances.

Number in Notebook 21, Volume VIII.	Air.		Nit. and CO_2.			Difference in Levels.	
	Temperature.	Column Length.	Pressure.	Temperature.	Column Length.		
	7°·2	1390	1 atmo.				[We can find no record of column length of the mixture at 1 atmo.]
525	7·8	37·8	...	6·2	76·9	151·9 +	
526	7·8	49·8	...	6·2	108·5	171·4 +	
527	7·1	51·7	...	6·44	114·0	175·0 +	
528	7·2	49·8	...	6·44	108·8	171·7 +	
530	7·2	37·8	...	6·48	77·9	152·7 +	No liquid.
531	7·4	28·2	...	6·48	51·5	135·9 +	No liquid.
532	7·4	25·1	...	6·48	41·4	128·95 +	·25 liquid persistent.
533	7·4	26·6	...	6·46	46·2	132·2 +	No liquid, but surface mercury flattened.
534	7·4	26·0	...	6·45	44·6	131·2 +	No liquid.
535	7·4	25·6	...	6·48	43·2	130·2 +	No liquid.
536	7·3	25·3	...	6·48	42·3	129·6 +	No liquid.
537	7·3	25·0	...	6·59	40·5	128·1 +	*Liquid.*
539	6·5	17·7	...	6·48	L. 5·1; G. 11·1	111·0 +	
540	6·5	17·8	...	6·44	L. 5·1; G. 11·0	110·8 +	
541	6·5	13·4	...	6·48	L. 7·5; G. 3·2	109·8 +	
542	6·5	13·4	...	6·48	L. 8·3; G. 2·0	109·4 +	Repetition of last; absorption of gas proceeded.
543	6·4	12·9	...	6·49	9·8	109·4 +	Liquid surface effaced.
545	6·4	13·0	...	6·48	9·7	109·2 +	No liquid.
546	6·4	13·1	...	16·92	12·1	111·5 +	(a = 0·0237.)
547	4·0	38·0	...	3·39	77·6	152·2 +	No liquid.
548	4·5	28·4	...	3·50	L. ·15; G. 50·25	134·6 +	Liquid.
549	4·6	21·3	...	3·45	L. 4·0; G. 17·8	113·1 +	
550	4·7	16·8	...	3·45	L. 5·8; G. 8·5	110·1 +	But liquid continued to augment.
551	5·0	13·3	...	3·46	L. 8·1; G. 1·7	109·1 +	Gas diminished after some time.
552	5·0	13·4	...	3·46	L. 9·5	108·7 +	A minute bubble of gas remained, which after some time disappeared.
553	5·05	13·4	...	3·46	L. 9·5	108·7 +	
554	5·0	13·9	...	...	...	...	Pressure reduced to 98·9 before gas reappeared.
554 *bis*	...	13·9	...	3·46	...	...	554 *bis*. Pressure slightly increased—very small bubble apparently persistent.
554 *tris*	5·1	13·8	...	...	...	...	Pressure slightly increased—bubble just visible in N and CO_2 tube—liquid around it curved.

MIXTURE OF NITROGEN AND CO_2: NIT. = 1, CO_2 = 6·2—*continued.*

Number in Notebook 21, Volume VIII.	Air.		Nit. and CO_2.			Difference in Levels.	
	Tempera-ture.	Column Length.	Pressure.	Tempera-ture.	Column Length.		
	7°·2	1390	1 atmo.				
555	5·3	13·7	...	3°·45	L. 10·0	108·5 +	Pressure again increased. CO_2 and N all liquid. Thus we have reached as nearly as possible the pressure at which the whole becomes *apparently* liquid.
556	6·8	36·1	...	14·27	77·4	154·0 +	
557	...	35·9	...	13·99	77·0	153·8 +	
558	7·0	35·8	...	14·25	76·9	153·8 +	
559	7·0	25·8	...	14·42	48·5	135·4 +	
560	7·1	25·8	...	14·15	48·3	135·2 +	
564	6·8	25·8	...	14·70	48·1	135·0 +	
565	7·0	20·1	...	14·64	30·5	123·1 +	No liquid; mercury surface flat in CO_2 and N.
566	7·1	20·1	...	14·37	30·2	122·8 +	No liquid.
567	7·2	20·1	...	14·23	29·8	122·4 +	No liquid.
567 *bis*	8·7	25·6	...	14·12	47·2	134·1 +	
567 *bis*	8·8	25·5	...	14·16	47·3	134·3 +	Repeated.
568	8·9	21·1	...	14·08	33·3	124·7 +	No liquid apparently.
569	9·1	21·1	...	14·16	33·4	124·8 +	No liquid.
570	9·2	20·3	...	13·97	30·0	122·2 +	0·15 liquid.
571	9·3	19·0	...	14·51	L. 1·5; G. 23·2	118·2 +	1·5 mil. liquid.
572	9·4	19·0	...	14·28	L. 1·5; G. 23·2	118·2 +	
573	9·5	17·6	...	14·3	L. 3·0; G. 16·9	114·75 +	Left for some time.
574	9·6	17·7	...	14·14	L. 3·0; G. 16·5	114·3 +	
575	9·6	13·9	...	14·16	12·3	110·9 +	Surface effaced.
576	9·0	14·4	...	14·04	12·4	110·6 +	Liquid not visible in cathetometer, but seen against black ground. It occupied two-thirds of space.
rep. 577	9·1	14·5	...	14·18	12·6	110·6 +	Liquid supposed to be 8 mm. above mercury.
578	9·1	14·1	...	14·51	12·3	110·7 +	Surface effaced.
579	9·3	14·1	...	14·20	12·2	110·6 +	Surface effaced.

(*Continued on next page.*)

Mixture of Nitrogen and CO_2: Nit. = 1, CO_2 = 6·2—*continued.*

Number in Notebook 22, Volume VIII.	Air.		Nit. and CO_2.			Difference in Levels.	
	Tempera-ture.	Column Length.	Pressure.	Tempera-ture.	Column Length.		
	7°·2	1390	1 atmo.				
580	9·4	13·25	...	14°·08	11·3	110·45 +	
581	9·45	13·2	...	14·32	11·3	110·5 +	
582	9·5	11·8	...	14·24	10·1	110·9 +	
583	9·5	11·7	...	14·24	10·1	110·80 +	

584 "Numerous experiments were made to fix the critical point of mixture and the pressure when liquid a little below that point disappears on augmenting pressure. . . . On augmenting pressure and then suddenly diminishing it, no cloud appeared at 21°·2, 20°·8, 20°·57 (two experiments), **20°·39**. At 20·35 cloud appeared ; also at 20·24 and 20·13. The limit was therefore between 20°·35 and 20°·39."

585	10·1	16·6	...	19·73	21·2	117·0 +	Liquid just visible and permanent at 19°·73.
586	10·7	17·2	...	19·93	not given	...	Liquid permanently formed at about 19·93.
587	10·8	16·0	...	19·28	,,	...	Liquid permanently formed at about 19·28 ?
588	10·9	17·5	...	19·05	,,	...	Permanent liquid 19·05.
588 *bis*	10·9	15·5	...	19·05	,,	...	Liquid effaced 19·05.
589	11·1	17·3	...	19·16	...	...	Liquid formed.
589 *bis*	11·3	15·3	...	19·05	...	...	Liquid effaced.
591	11·3	17·0	...	18·95	...	...	Liquid formed.
591 *bis*	11·3	15·3	...	18·84	16·2	...	Liquid effaced.
592	11·4	14·0	...	18·92	13·3	...	
593	...	12·7	...	18·88	11·7	...	
594	11·4	13·2	...	18·86	12·2	...	
595	...	14·3	...	18·84	14·2	...	
596	11·4	15·3	...	18·84	15·6	...	
596 *bis*	11·4	15·0	...	18·84	15·7	...	
596 *tris*	11·4	15·0	...	18·84	15·7	...	
597	11·5	15·0	...	18·40	14·9	...	
597 *bis*	11·5	15·0	...	18·18	14·7	...	
597 *tris*	11·5	15·0	...	18·84	15·2	...	
599	8·0	15·8	...	7·2	{ L. 6·3 G. 6·6 }	...	
599 *bis*	8·5	15·8	,,	25·0	{ L. 0 G. 20·1 }	...	
600	10·6	15·9	,,	9·3	{ L. 6 G. 7·6 }	...	
	10·4	15·9	...	22·0	G. 18·2	...	
601	10·6	15·9	...	20·0	{ L. 6 G. 11·6 }	...	

On allowing it to stand there changed as follows : the whole volume remained same, but liquid gradually passed into [the gaseous] space ; on allowing the whole to stand for two hours the liquid surface at last disappeared ; the liquid having gradually diminished and the volume of whole increased to 18·0, the pressure and temperature were quite steady.

MIXTURE OF NITROGEN AND CO_2: NITROGEN=1, CO_2=3·43; OR, MIXTURE CONTAINED 22·5 PER CENT. NITROGEN.

Number in Notebook 27, Volume VIII.	Air.			N and CO_2.				Difference in Levels.	
	Tempera-ture.	Column Length.	Volume.	Pressure.	Tempera-ture.	Column Length.	Volume.		
496	0°. 10·2	48·0	0·85600 ·00880	1 atmo. ...	14°·0	...	...	...	[Here also original column length not recorded.] The most minute trace of liquid formed at point.
496 rep.	16·3	47·8	·00876	...	13·9	42·9	·007973	573·8+	On repeating experiment when cloud formed, on taking off pressure a trace of liquid formed. Liquid disappeared when measurement made.
497	10·6	49·1	·00900	...	14·0	45·6	·008474	569·8+	A trace of liquid permanent.
498	10·5	47·9	·00878	...	14·0	43·0	·007985	573·6+	498. Repetition till finest trace of liquid formed; afterwards disappeared.
499	10·5	48·7	·00893	...	14·0	44·8	·008327	571·0+	Pressure again changed till faintest trace.
500	10·4	48·7	·00893	...	12·2	42·2 L. 3	·007838	573·6+	500. The apparatus having been left to itself for an hour, temperature of water fell. Pressure in air tube underwent no change. 3 mil. liquid.
501	...	...	...	...	Jan. 22. The apparatus on 19th heated to 30° after pressure was reduced. It was not disturbed in any way since, so that it has had three days to diffuse. Temperature raised to 14°·6; no liquid on making or removing pressure. At 14°·0 no liquid formed in either case. At 13°·8 liquid formed on taking off pressure after first adding it, and in rather larger quantity than at 14° on 19th. At 14° liquid formed on making pressure. This is therefore true point.				
502	9·1	47·7	·008742	...	14·0	43·1	·008004	573·7 +	Liquid disappeared by pressure, and then pressure taken off till cloud formed; faintest trace of liquid possible.

MIXTURE OF NITROGEN AND CO_2: NIT. = 1, CO_2 = 3·43—*continued.*

Number in Notebook 27, Volume VIII.	Air.			N and CO_2.				Difference in Levels.	
	Tempera-ture.	Column Length.	Volume.	Pressure.	Tempera-ture.	Column Length.	Volume.		
					°				
503	9·3	92·7	01702	...	14·0	143·7	·027148	428·0 +	
504	9·3	72·4	·01328	...	14·0	100·0	·018739	492·0 +	
506	9·3	47·0	·00861	...	14·0	42 1	·007818	575·3 +	No liquid, pressure having been only added.
507	9·3	33·7	·00618	...	14·0	27·3	·005057	603·4 +	
508	9·3	28·6	·00524	...	14·0	24·7	·004594	611·1 +	
509	9·1	102·1	·01876	...	7·8	155·9	·029511	406·4 +	
510	9·2	69·3	·01271	...	7·9	83·5	·015609	511·6 +	No liquid.
511	9·2	84·2	·01546	...	...	On adding pressure till a little liquid was formed and then taking off pressure till liquid had entirely disappeared ; then adding pressure, liquid reappeared when Liquid after being formed slowly evaporated ; liquid formed both at bottom and close to mercury. On again forming more liquid by pressure, then taking off pressure till liquid entirely disappeared, liquid formed when			
511 *bis*	9·2	103·2	·01896	...	7·9	...	...	...	
512	9·4	102·1	·01876	...	7·9	154·7	·029278	407·6 +	The liquid formed in last experiment soon disappeared and new pressure readjusted, compare 509.
513	9·5	102·1	·01876	...	14·0	162·2	·030741	400·1 +	Temperature of CO_2 tube now raised to 14°.
514	9·4	92·7	·01702	...	14·0	142·5	·026917	429·2 +	Compare 503.

515	9·9	92·7	·01702	...	14·0	141·6	·026742	430·1 +	515. [The experiment is preceded by following note:] "Liquid formed again in considerably larger quantity than before at 7°."
516	10·2	92·7	·01702	...	14·0	139·3	·026296	432·4 +	516. In this caste the CO_2 tube was cooled to $-12°$ before, and strong pressure applied so as to obtain a maximum of liquid. Compare 503, 514, and 515.
517	10·2	92·7	·01702	...	14·0	140·0	·026432	431·7 +	517. "Gas largely expanded and again brought to same pressure."
518	10·2	92·7	·01702	...	14·0	140·7	·026567	431·0 +	518. "Again pressure taken twice off and time allowed."
519	9·7	105·2	·01933	...	8·4	162·3	·030760	396·9 +	519. Apparatus in expanded condition all night so that diffusion complete.
rep.	9·7	105·2	·01933	...	8·4	162·2	·030741	396·95 +	
520	9·7	105·2	·01933	...	8·4	159·5	·030212	399·7 +	520. CO_2 cooled to $-12°$ under strong pressure so as to liquefy largely the gas; then expanded.
521	10·0	105·2	·01933	...	8·4	159·9	·030289	399·3 +	521. "Gases not changed." [Note after experiment:] "Showing slow return to former condition."
522	9·9	105·2	·01933	...	8·4	160·1	·030329	399·1 +	522. "CO_2 expanded for some time, then brought back." [Note after experiment:] "In half an hour CO_2 expanded 2·5 glass divisions, or 1·2 millimetre; no time to verify air."
523	9·7	105·2	·01933	...	8·4	161·5	·030604	397·7 +	523. An hour and a half later air readjusted.
524	9·7	105·2	·01933	...	8·4	161·3	·030565	397·9 +	524. Apparatus left unaltered since yesterday.
525	10·2	105·2	·01933	...	20·0	175·9	·033420	383·3 +	525. Same mixture heated.
526	10·1	105·2	·01933	...	20·0	175·75	·033391	383·45 +	Repetition of 525.
527	10·2	105·2	·01933	...	20·0	173·5	·032951	385·7 +	527. Strong pressure now applied while end of tube cooled to -10 so as to form liquid; [then expanded].
528	10·0	105·2	·01933	...	20·0	173·9	·033028	385·3 +	528. Repeated in a quarter of an hour.
529	9·6	105·2	·01933	...	20·0	175·9	·033420	383·3 +	529. Repeated in an hour.
rep.	10·1	105·2	·01933	...	20·0	175·7	·033381	383·5 +	Repeated immediately, air being heated in apartment.
530	9·9	105·2	·01933	...	20·0	176·0	·033441	383·15 +	530. In an hour and a half.

Mixture of Nitrogen and CO_2 Nit 1, CO_2 3·43—*continued.*

Number in Notebook 27, Volume VIII.	Air: Temperature	Air: Column Length	Air: Volume	Air: Pressure	N and CO_2: Temperature	N and CO_2: Column Length	N and CO_2: Volume	Difference in Levels	
503	9·3	92·7	·01702		14·0	143·7	·027148	428·0 +	
504	9·3	72·4	·01328		14·0	100·0	·018739	492·0 +	
506	9·3	47·0	·00861		14·0	42·1	·007818	575·3 +	No liquid, pressure having been only added
507	9·3	33·7	·00618		14·0	27·3	·005057	603·4 +	
508	9·3	28·6	·00524		14·0	24·7	·004594	611·1 +	
509	9·1	102·1	·01876		7·8	155·9	·029511	406·4 +	
510	9·2	69·3	·01271		7·9	83·5	·015609	511·6 +	No liquid
									On adding pressure till a little liquid was formed and then taking off pressure till liquid had entirely disappeared, then adding pressure, liquid reappeared when
511	9·2	84·2	·01546						Liquid after being formed slowly evaporated; liquid formed both at bottom and close to mercury. On again forming more liquid by pressure, then taking off pressure till liquid entirely disappeared, liquid formed when
511 *bis*	9·2	103·2	·01896		7·9	..	..		
512	9·4	102·1	·01876		7·9	154·7	·029278	407·6 +	The liquid formed in last experiment soon disappeared and new pressure readjusted, compare 509.
513	9·5	102·1	·01876		14·0	162·2	·030741	400·1 +	Temperature of CO_2 tube now raised to 14
514	9·4	92·7	·01702		14·0	142·5	·026917	429·2 +	Compare 503
515	9·9	92·7	·01702		14·0	141·6	·026742	430·1 +	515. [The experiment is preceded by following note:] "Liquid formed again in considerably larger quantity than before at 7°."
516	10·2	92·7	·01702		14·0	139·3	·026296	432·4 +	516. In this case the CO_2 tube was cooled to −12 before, and strong pressure applied so as to obtain a maximum of liquid. Compare 503, 514, and 515
517	10·2	92·7	·01702		14·0	140·0	·026432	431·7 +	517. "Gas largely expanded and again brought to same pressure."
518	10·2	92·7	·01702		14·0	140·7	·026567	431·0 +	518. "Again pressure taken twice off and time allowed."
519	9·7	105·2	·01933		8·4	162·3	·030760	396·9 +	519. Apparatus in expanded condition all night so that diffusion complete
rep.	9·7	105·2	·01933		8·4	162·2	·030741	396·95 +	
520	9·7	105·2	·01933		8·4	159·5	·030212	399·7 +	520. CO_2 cooled to −12° under strong pressure so as to liquefy largely the gas, then expanded.
521	10·0	105·2	·01933		8·4	159·9	·030289	399·3 +	521. "Gases not changed." [Note after experiment:] "Showing slow return to former condition."
522	9·9	105·2	·01933		8·4	160·1	·030329	399·1 +	522. "CO_2 expanded for some time, then brought back." [Note after experiment:] "In half an hour CO_2 expanded 2·5 glass divisions, or 1·2 millimetre; no time to verify air."
523	9·7	105·2	·01933	..	8·4	161·5	·030604	397·7 +	523. An hour and a half later air readjusted
524	9·7	105·2	·01933	...	8·4	161·3	·030565	397·9 +	524. Apparatus left unaltered since yesterday.
525	10·2	105·2	·01933		20·0	175·9	·033420	383·3 +	525. Same mixture heated.
526	10·1	105·2	·01933		20·0	175·75	·033391	383·45 +	Repetition of 525.
527	10·2	105·2	·01933		20·0	173·5	·032951	385·7 +	527. Strong pressure now applied while end of tube cooled to −10 so as to form liquid; [then expanded]
528	10·0	105·2	·01933		20·0	173·9	·033028	385·3 +	528. Repeated in a quarter of an hour.
529	9·6	105·2	·01933		20·0	175·9	·033420	383·3 +	529. Repeated in an hour.
rep	10·1	105·2	·01933		20·0	175·7	·033381	383·5 +	Repeated immediately, air being heated in apartment.
530	9·9	105·2	·01933		20·0	176·0	·033441	383·15 +	530. In an hour and a half.

MIXTURE OF NITROGEN AND CO_2: NIT.=1, CO_2=3·43—*continued.*

Number in Notebook 27, Volume VIII.	Air.			N and CO_2.				Difference in Levels.	
	Tempera-ture.	Column Length.	Volume.	Pressure.	Tempera-ture.	Column Length.	Volume.		
531	9·8	105·2	·01933	...	20·0°	176·1	·033460	383·1 +	
561	9·8	105·2	·01933	...	20·0	174·2	·033088	385·0 +	561. Exposed to pressure. [561 follows 531 immediately.]
rep.	10·0	105·1	·01931	...	...	174·3	·033107	385·0 +	Repetition in ten minutes.
rep.	9·6	105·1	·01931	...	20·0	175·1	·033265	384·2 +	Repetition in one and a half hours.
562	10·2	102·0	·01874	...	15·9	164·5	·031190	397·9 +	
rep.	10·2	102·0	·01874	...	16·0	164·5	·031190	397·85+	
563	10·2	102·0	·01874	...	16·0	164·6	·031209	397·80+	563. "Strong pressure applied without changing temperature so as to reduce CO_2 and N to from $\frac{1}{3}$ to $\frac{1}{4}$ of its volume. No liquid formed."
564	10·2	102·0	·01874	...	16·0	161·9	·030681	400·5 +	564. It was now cooled to −12° and pressure applied so as to liquefy CO_2. The whole was then placed in water at 16°·0 and pressure afterwards removed. [Note after experiment:] "Very good experiments, everything steady. It is better to apply heat at first and then take off pressure, as liquid is steadier in passing to gaseous state."
565	10·2	102·0	·01874	...	16·0	162·4	·030779	400·0 +	565. Repeated in quarter of an hour.
566	10·1	102·0	·01874	...	16·0	164·1	·031112	398·3 +	Repeated in an hour and a half. Compare 562 and 563; CO_2 has now recovered original volume within ·5.
567	7·2	104·1	·01913	...	7·3	161·0	·030507	399·3 +	The CO_2 examined at temperature of room after an interval of fifteen hours.

567 *bis*	7·2	104·1	·01913	...	7·3	159·7	·030250	400·6 +	The CO_2 compressed at 7°·7 till liquid was formed by pressure, partly below surface of mercury, partly and chiefly at lower end of tube and then re-expanded.
568	7·2	70·4	·01292	...	6·3	83·6	·015628	510·4 +	568. "Pressure applied till faintest trace of liquid next mercury appeared in CO_2 tube." [Pressure was 68·7 atmospheres.] [Note after experiment :] "Cone filled with liquid."
569	7·2	41·4	·00759	...	6·3	31·0	·005745	592·0 +	569. Pressure applied till liquid disappeared.
570	...	42·6	·007805	...	...	...	...	...	570. Pressure augmented and taken off till cloud appeared and liquid separated.
570 rep.	7·2	42·6	·007805	...	6·3	32·1	·005951	589·7 +	570. Repeated; pressure augmented and then taken off till cloud appeared.
571	9·0	62·7	·01150	...	9·9	70·6	·013170	531·1 +	571. Pressure applied till liquid appeared in cone. [Pressure 77·6 atmospheres.]
572	9·0	42·3	·00775	...	9·9	33·0	·006119	589·1 +	572. Pressure increased till liquid disappeared.
573	9·0	45·05 [Mean of 2 obs.]	·00826	...	9·9	35·7	·006627	583·65 +	573. Cloud formed after increasing and taking off pressures. [Pressure 107·8 atmospheres.]
932	11·0	53·5	·00981	...	13·2	13·5	·002513	597·4 +	932. Liquid in cone. [Pressure 91·6 atmospheres.]
933	11·0	45·8	·00839	...	13·2	...	...	...	933. Liquid disappeared by pressure. [CO_2 not noted.]
934	11·0	47·4	·00869	...	13·2	41·2	·007651	575·8 +	934. Cloud followed by liquid surface. [Pressure 103·2 atmospheres.]
935	...	...	...	...	...	...	...	...	935. To fix critical point. 17°·2 no liquid or cloud. 16°·7 " 15°·4 " 14°·9 " 14°·5 " 14°·3 liquid on adding. Correction of −0°·4 probably to be applied throughout.
936	9·8	49·7	·00911	...	13·9	...	...	...	936. "In last case" [then data in table follow—the pressure in this experiment was 98 atmospheres].
937	9·8	49·3	·00904	...	14·0	...	...	...	937. Per liquid formed on taking off pressure. [Pressure 98·9.]
938	10·0	49·35	·00905	...	14·0	47·1	·008750	567·95 +	938. Repetition of 937, pressure added and then taken off; the merest trace of liquid formed but distinct and permanent, cloud hardly discernible. [Pressure 98·81.] [After experiment cones given]. Air cone, 984·4 CO_2 cone, 318·4 −

[NOTE.—Andrews' pressure estimates are given as a guide.]

Mixture of Nitrogen and CO_2 Nit. = 1, CO_2 = 3·43—*continued*

Number in Notebook 27, Volume VIII.	Air. Tempera-ture	Air. Column Length.	Air. Volume.	Pressure.	N and CO_2 Tempera-ture.	N and CO_2 Column Length.	N and CO_2 Volume.	Difference in Levels.	
531	9·8	105·2	·01933		20·0	176·1	·033460	383·1 +	
561	9·8	105·2	·01933		20·0	174·2	·033088	385·0 +	561 Exposed to pressure. [561 follows 531 immediately.]
rep.	10·0	105·1	·01931			174·3	·033107	385·0 +	Repetition in ten minutes.
rep.	9·6	105·1	·01931	. .	20·0	175·1	·033265	384·2 +	Repetition in one and a half hours.
562	10·2	102·0	·01874		15·9	164·5	·031190	397·9 +	
rep.	10·2	102·0	·01874		16·0	164·5	·031190	397·85 +	
563	10·2	102·0	·01874		16·0	164·6	·031209	397·80 +	563. "Strong pressure applied without changing temperature so as to reduce CO_2 and N to from $\frac{1}{2}$ to $\frac{1}{3}$ of its volume. No liquid formed."
564	10·2	102·0	·01874		16·0	161·9	·030681	400·5 +	564. It was now cooled to −12° and pressure applied so as to liquefy CO_2. The whole was then placed in water at 16°·0 and pressure afterwards removed. [Note after experiment.] "Very good experiments, everything steady. It is better to apply heat at first and then take off pressure, as liquid is steadier in passing to gaseous state."
565	10·2	102·0	·01874		16·0	162·4	·030779	400·0 +	565. Repeated in quarter of an hour.
566	10·1	102·0	·01874	..	16·0	164·1	·031112	398·3 +	Repeated in an hour and a half. Compare 562 and 563; CO_2 has now recovered original volume within ·5.
567	7·2	104·1	·01913		7·3	161·0	·030507	399·3 +	The CO_2 examined at temperature of room after an interval of fifteen hours.

567 *bis*	7·2	104·1	·01913	...	7·3	159·7	·030250	400·6 +	The CO_2 compressed at 7°·7 till liquid was formed by pressure, partly below surface of mercury, partly and chiefly at lower end of tube and then re-expanded.
568	7·2	70·4	·01292	...	6·3	83·6	·015628	510·4 +	568. "Pressure applied till faintest trace of liquid next mercury appeared in CO_2 tube." [Pressure was 68·7 atmospheres.] [Note after experiment:] "Cone filled with liquid."
569	7·2	41·4	·00759		6·3	31·0	·005745	592·0 +	569. Pressure applied till liquid disappeared.
570	. .	42·6	·007805		..	. .	. .	...	570. Pressure augmented and taken off till cloud appeared and liquid separated.
570 rep.	7·2	42·6	·007805		6·3	32·1	·005951	589·7 +	570. Repeated; pressure augmented and then taken off till cloud appeared.
571	9·0	62·7	·01150		9·9	70·6	·013170	531·1 +	571. Pressure applied till liquid appeared in cone. [Pressure 77·6 atmospheres.]
572	9·0	42·3	·00775		9·9	33·0	·006119	589·1 +	572. Pressure increased till liquid disappeared.
573	9·0	45·05 [Mean of 2 obs.]	·00826		9·9	35·7	·006627	583·65 +	573. Cloud formed after increasing and taking off pressures. [Pressure 107·8 atmospheres.]
932	11·0	53·5	·00981		13·2	13·5	·002513	597·4 +	932. Liquid in cone. [Pressure 91·6 atmospheres.]
933	11·0	45·8	·00839		13·2	..	...	...	933. Liquid disappeared by pressure. [CO_2 not noted.]
934	11·0	47·4	·00869		13·2	41·2	·007651	575·8 +	934. Cloud followed by liquid surface. [Pressure 103·2 atmospheres.]
935	. .	. .	.	.	. .		...	. .	935. To fix critical point. 17°·2 no liquid or cloud. 16°·7 " 15°·4 " 14°·9 " 14°·5 " 14°·3 liquid on adding. Correction of −0°·4 probably to be applied throughout.
936	9·8	49·7	·00911		13·9	...	.	. .	936. "In last case" [then data in table follow—the pressure in this experiment was 98 atmospheres].
937	9·8	49·3	·00904		14·0			.	937. Per liquid formed on taking off pressure. [Pressure 98·9.]
938	10·0	49·35	·00905	.	14·0	47·1	·008750	567·05 +	938. Repetition of 937, pressure added and then taken off; the merest trace of liquid formed but distinct and permanent, cloud hardly discernible. [Pressure 98·81.] [After experiment cones given]. Air cone, 984·4 CO_2 cone, 318·4 –

[Note.—Andrews' pressure estimates are given as a guide.]

(*Continued from page* 13.)

other times refusing to liquefy at temperatures several degrees lower than this temperature. These irregularities were traced to the gaseous mixture having separated into two portions, one rich and the other poor in carbon dioxide, when the pressure was reduced after liquefaction so as to convert the whole mixture into the gaseous state.

Advantage was taken of this mode of separating the mixture so as to ascertain what change of volume occurs in the diffusion of the mixed gases at high pressures (Exp. 519–523, 525–531, 563–566). It thus appears that when carbon dioxide and nitrogen diffuse into one another at high pressures an increase of volume takes place, and that, on the other hand, when they are separated from one another there is a diminution of volume.

The following table contains measurements of the compressibility of liquid carbon dioxide, up to a pressure of fully 250 atmospheres as measured on the hydrogen manometer.

Compressibility of Liquid Carbon Dioxide, April 15–18, 1870.

Number in Notebook 16, Volume VIII.	Hydrogen.			Carbon Dioxide.				Difference of Levels.
	Tempera-ture.	Column Length.	Volume.	Pressure in Atmo-spheres.	Tempera-ture.	Column Length.	Volume.	
	0°		0·3052 ± ·0012	1				
320″	14·55	358·1	0·006029	55·0	12°·73	46·64	0·0007717	287·0 –
321	14·51	272·5	4580	73·6	12·81	45·09	7465	203·0 –
322	14·51	273·3	4595	73·3	12·73	45·11	7472	203·7 –
323	14·53	206·1	3466	99·0	12·62	43·52	7193	138·1 –
324	14·56	154·0	2602	135·2	12·77	42·24	6987	87·4 –
325″	14·38	116·4	1977	183·6	12·82	40·82	6753	51·3 –
326	14·48	117·6	1996	181·5	12·73	40·94	6777	52·6 –
327	14·48	87·1	1480	256·9	12·75	39·45	6528	23·8 –
328	14·33	88·6	1505	251·8	12·92	39·48	6533	25·4 –
328 *bis*	14·36	87·0	1478	257·4	12·68	39·39	6515	24·0 –
328 *tris*	14·34	87·3	1483	254·9	12·68	39·38	6513	24·3 –

(*Issued separately December* 1, 1909.)

II.—Seismic Radiations. By Professor C. G. Knott.

(Read June 21, 1909. MS. received July 7, 1909.)

Part II.

This second paper is a discussion of the surface displacements associated with plane elastic waves reflected from the boundary of the elastic solid through which they are being propagated. In my first paper on Seismic Radiations (*Proc. Roy. Soc. Edin.*, vol. xxviii. pp. 217–230, 1908) my aim was to find a simple distribution of density and elasticity in terms of which the transmission of seismic disturbances could be described. The result was given in these words (p. 224):—

"The observed facts of seismic radiation can be co-ordinated on the assumption that throughout all but a comparatively thin crust of the earth the elastic waves of highest speed are transmitted with a speed of 12·23 kilometres per second, and that within this crust, of thickness equal to one-tenth the radius, the speed diminishes from value 12·23 kilometres per second at the inner surface to 6 kilometres per second at the outer surface."

Then follows the remark:—"If this wave of highest speed be a compressional wave with longitudinal vibrations, the cosine of the angle of incidence of the ray will give the ratio of the magnitude of the horizontal motion to the whole motion"; and subsequently it is taken for granted, in accordance with the usual custom of seismologists, that the so-called "angle of emergence" of the ray is equal to the supplement of the angle of incidence internally on the boundary.

I am indebted to Professor Schuster for drawing my attention to an inaccuracy lurking in this statement. He remarks: "The sentence would be correct enough if the incident ray only is taken into consideration, but as there must be almost total reflexion at the surface, it seems to me that the purely longitudinal wave would give, taking account of the reflected wave, a purely vertical displacement." The criticism is, to some extent, just, and calls for a revision of the ordinary nomenclature; but inasmuch as there is a reflected distortional wave as well as a reflected longitudinal wave started at the surface, the amendment indicated by Schuster is not

itself complete. I propose to examine the problem here suggested. Its solution follows almost immediately from the equations of vibratory motion which I discussed in my papers on the Reflexion and Refraction of Elastic Waves, first in 1888 in the *Transactions of the Seismological Society of Japan*, and in 1899 somewhat more fully in the *Philosophical Magazine*, vol. xlviii.

The experimental determination of the real angle of emergence depends on the comparison of the vertical displacement of the ground with the simultaneous maximum horizontal displacement. On the simple hypothesis of plane elastic waves impinging internally on the surface of the earth, we know (see *Seismic Radiations*, vol. xxviii. p. 219) that the sine of the angle of incidence is equal to the ratio of the real speed of propagation of the wave to the apparent fictitious speed of propagation of the surface disturbance. Both these speeds can be obtained with fair accuracy by study of the time curves of the first and second preliminary tremors. On the other hand, we have not as yet any instruments capable of measuring the real displacements in a trustworthy manner. The vertical displacement is particularly difficult to measure; and it is very doubtful if the horizontal pendulum, at present the favourite seismographical instrument, gives an even approximate reproduction of the horizontal motion of the earth.

Let us suppose, however, that we have instruments capable of accurately determining the horizontal and vertical displacements, and thus affording the data for calculating the real angle of emergence. The question then arises, what relation will exist between this externally measured angle of emergence and what is generally called this, viz. the angle of incidence of the radiation impinging internally on the surface?

In the earlier discussion my purpose was to show that with small values for the angle of incidence the vertical effect at the outcrop of the seismic ray became the more important, so that a horizontal pendulum at more than 90° arcual distance from the source of the earthquake disturbance would record only a very small part of the whole motion. This conclusion, it will be seen, is not materially affected when the real angle of emergence is considered.

As pointed out in the papers of 1888 and 1899 already referred to, a wave of condensational type passing through an elastic solid and falling on the surface backed by air or water produces not only a reflected wave of the same type in the solid and a refracted wave in the fluid, but also a reflected wave of the distortional type in the solid. Similarly, a ray of distortional type will give rise to a refracted condensational wave and two

reflected waves, one of distortion and one of condensation, so long as the angle of incidence is not greater than a certain value determined by the relation betwen the speeds of propagation of the two rays. For angles of incidence greater than this critical value there will be no reflected condensational wave.

In this paper I confine myself to further consideration of the case in which the elastic solid is backed with air. The case in which water takes the place of air will present similar features. The distribution of the energy among the various types of reflected and refracted rays is shown diagrammatically in the published abstract of an address I gave in 1899 before the Royal Society of Edinburgh (see *Proc. Roy. Soc. Edin.*, vol. xxii.). These diagrams have also been reproduced in my book on the *Physics of Earthquake Phenomena*, p. 182. They show certain of the outstanding phenomena in a way which is more easily understood than the mode in which they were first represented in the original papers already referred to.

The following tables are taken from the paper in the *Philosophical Magazine* (vol. xlviii. p. 87), and show how the energy of the incident ray is distributed among the reflected and refracted rays into which it is divided.

Here A, A_1, A′ give the energies associated with the incident, reflected, and refracted rays of condensational type, θ, θ, θ' being the corresponding angles of incidence, reflexion, and refraction; and B, B_1, B′, with ϕ and ϕ', refer similarly to the rays of distortional type. In the first table the incident ray is condensational; in the second the incident ray is distortional.

In making these calculations, I assumed the density of rock to be 3, the rigidity to be $1{\cdot}5 \times 10^{11}$, the incompressibility to be $2{\cdot}5 \times 10^{11}$, the density of air to be 0·0015, and its incompressibility to be $1{\cdot}5 \times 10^{6}$.

1. Condensational Wave Incident in the Rock.

θ.	A.	A_1.	θ'.	A′.	ϕ.	B.
0	1	0·9999		0·00013		
14° 2′	1	0·828	0°·9	0·00013	8°	0·172
26 34	1	0·464	1 ·7	0·00011	15	0·536
45	1	0·079	2 ·7	0·00009	24	0·921
59 2	1	0·0002	3 ·3	0·00007	30	0·9999
73 18	1	0·003	3 ·7	0·00006	34	0·997
84 17	1	0·091	3 ·8	0·00005	35	0·909

2. Distortional Wave Incident in the Rock.

ϕ.	B.	B_1.	θ.	A_1.	θ'.	A′.
0	1	1				
14° 2′	1	0·534	25°	0·466	1°·6	0·00002
26 34	1	0·025	51	0·975	3	0·00006
33 40	1	0·003	74	0·997	3 ·7	0·00006
35 13	1	1	90	0	3 ·8	0·00000
39 48	1	0·9998	imaginary		4 ·3	0·00019
45	1	0·9998	,,		4 ·7	0·00016
59 2	1	0·9999	,,		5 ·7	0·00014
73 18	1	0·9999	,,		6 ·3	0·00014
84 17	1	0·9999	,,		6 ·6	0·00006

Thus in both cases for angles of incidence in the neighbourhood of 20° to 30° a large part of the energy which has come in the form of one type of wave is reflected in the form of the other. Associated with this process of reflexion there will be surface displacements which I now proceed to calculate.

The equations of motion for plane waves in an elastic solid are given in suitable form for the present discussion in Part III. of the *Philosophical Magazine* paper already cited. A recapitulation seems necessary in order to make the calculations intelligible.

Let the plane wave impinge at a given angle on the plane boundary or interface separating the elastic solid from some other elastic medium, and let the plane perpendicular to the wave-front and to the boundary be taken as the XY plane, the x-axis being perpendicular to the interface, the y-axis in the interface, and the z-axis parallel to the wave-front.

Following the notation of Thomson and Tait's *Natural Philosophy*, we have P, Q, R, S, T, U as the components of stress, k the incompressibility, n the rigidity, ρ the density, and $m=k+n/3$.

Let ξ, η, ζ be the components of the displacement at the point xyz, and let ϕ and ψ be two functions defined by the equations

$$\xi=\frac{\partial\phi}{\partial x}+\frac{\partial\psi}{\partial y},\quad \eta=\frac{\partial\phi}{\partial x}-\frac{\partial\psi}{\partial y}.$$

Then the equations of motion for plane waves are expressible in the form

$$\left.\begin{aligned}(m+n)\nabla^2\phi&=\rho\frac{d^2\phi}{dt^2}\\ n\nabla^2\psi&=\rho\frac{d^2\psi}{dt^2}\\ n\nabla^2\zeta&=\rho\frac{d^2\zeta}{dt^2}\end{aligned}\right\}\quad . \quad . \quad . \quad . \quad \mathrm{I}.$$

The components of stress have the values

$$\left.\begin{aligned} &\mathrm{P}=(m+n)\nabla^2\phi-2n\frac{\partial\eta}{\partial y}, \quad \mathrm{S}=n\frac{\partial\zeta}{\partial y}\\ &\mathrm{Q}=(m+n)\nabla^2\phi-2n\frac{\partial\xi}{\partial x}, \quad \mathrm{T}=n\frac{\partial\zeta}{\partial x}\\ &\mathrm{R}=(m+n)\nabla^2\phi \qquad\qquad \mathrm{U}=n\left(2\frac{\partial^2\phi}{\partial x\partial y}+\frac{\partial^2\psi}{\partial y^2}-\frac{\partial^2\psi}{\partial x^2}\right)\end{aligned}\right\} \quad . \quad . \quad \text{II.}$$

The components of stress on the plane whose direction-cosines are $\lambda\mu\nu$ are

$$\left.\begin{aligned} \mathrm{F}&=\mathrm{P}\lambda+\mathrm{U}\mu+\mathrm{T}\nu\\ \mathrm{G}&=\mathrm{U}\lambda+\mathrm{Q}\mu+\mathrm{S}\nu\\ \mathrm{H}&=\mathrm{T}\lambda+\mathrm{S}\mu+\mathrm{R}\nu\end{aligned}\right\} \quad . \quad . \quad . \quad . \quad \text{III.}$$

The waves of the ϕ-type are condensational-rarefactional waves travelling with a speed equal to $\sqrt{(m+n)/\rho}$. The waves of the ψ-type are purely distortional waves travelling with speed $\sqrt{n/\rho}$, the vibrations being in the plane XY. The ζ displacement belongs also to a purely distortional wave with vibrations perpendicular to the plane of incidence XY.

Here I confine my attention entirely to the case of an elastic solid like rock, with its plane surface in contact with air, or, with what is practically the same thing, vacuum. The distortional wave ζ is simply reflected at the surface as a distortional wave and sent back into the solid medium. It is quite otherwise, however, with the distortional wave represented by ψ. Except under specially critical conditions, the ψ-type of incident wave will produce two reflected waves, one of the ψ-type and the other of the ϕ-type. Similarly, an incident radiation of the ϕ-type will in general produce a reflected wave of type ϕ and another of type ψ. The refracted ray will in each case pass through the air as a condensational wave.

I shall consider each in turn, taking the incident condensational wave first, as involving the simpler analysis.

1. Condensational Wave Incident.

The solution is of the form

$$\left.\begin{aligned} \phi&=\mathrm{A}e^{ib(cx+y+\omega t)}+\mathrm{A}_1e^{ib(-cx+y+\omega t)}\\ \psi&=\qquad\qquad\qquad\quad \mathrm{B}_1e^{ib(-\gamma x+y+\omega t)}\\ \phi'&=\mathrm{A}'e^{ib(c'x+y+\omega t)}\end{aligned}\right\} \quad . \quad . \quad . \quad (1)$$

The quantities c, γ, c' are the cotangents of the angles of incidence, reflexion, and refraction of the various rays; b and ω are connected re-

spectively with the wave-length and the period; i is the ordinary imaginary of analysis.

The equations of motion (I.) in the two media give

$$\left.\begin{aligned}(m+n)(c^2+1)=\rho\omega^2=n(\gamma^2+1)\\ k'(c'^2+1)=m'(c'^2+1)=\rho'\omega^2\end{aligned}\right\} \quad . \quad . \quad . \quad (2)$$

The conditions to be satisfied at the surface are:—

(1) Equality of normal displacement on each side of the interface, $\xi=\xi'$, or

$$\frac{\partial\phi}{\partial x}+\frac{\partial\psi}{\partial y}=\frac{\partial\phi'}{\partial x} \qquad \text{when } x=0.$$

(2) Equality of normal stress on each side of the interface, $P=P'$, or

$$(m+n)\nabla^2\phi-2n\left(\frac{\partial^2\phi}{\partial y^2}-\frac{\partial^2\psi}{\partial x\partial y}\right)=k'\nabla^2\phi' \qquad \text{when } x=0.$$

(3) Equality of tangential stresses on each side of the interface, $U=0$, or

$$2\frac{\partial^2\phi}{\partial x\partial y}+\frac{\partial^2\psi}{\partial y^2}-\frac{\partial^2\psi}{\partial x^2}=0 \qquad \text{when } x=0.$$

These lead to the equations

$$\left.\begin{aligned}B_1+c(A-A_1)&=c'A'\\ -2\gamma B_1+(\gamma^2-1)(A+A_1)&=\frac{\rho'}{\rho}(\gamma^2+1)A'\\ (\gamma^2-1)B_1-2c(A-A_1)&=0\end{aligned}\right\} \quad . \quad . \quad (3)$$

The object of the inquiry is to find the normal and tangential displacements at the surface, *i.e.* the values of ξ and η when $x=0$. These values are, when $x=0$,

$$\left.\begin{aligned}\xi&=\frac{\partial\phi}{\partial x}+\frac{\partial\psi}{\partial y}=\{c(A-A_1)+B_1\}ibe^{ib(y+\omega t)}\\ \eta&=\frac{\partial\phi}{\partial y}-\frac{\partial\psi}{\partial x}=\{A+A_1+\gamma B_1\}ibe^{ib(y+\omega t)}\end{aligned}\right\} \quad . \quad . \quad (4)$$

These give at once

$$\begin{aligned}\frac{\xi}{\eta}&=\frac{c(A-A_1)+B_1}{A+A_1+\gamma B_1}\\ &=\frac{\gamma^2-1}{2\gamma+\frac{\rho'}{\rho}\frac{\gamma}{c'}\left(\gamma+\frac{1}{\gamma}\right)}\end{aligned}$$

by substitution from (3).

For the case of rock and air we may put

$$2000\rho'=\rho,\ 3n=m+n=9\times10^{11},\ k'=1{\cdot}5\times10^6.$$

Also, by formulæ (2),

$$3(c^2+1)=\gamma^2+1$$

$$\rho k'(c'^2+1)=\rho' n(\gamma^2+1),$$

or

$$2000\,(c'^2+1)=200{,}000(\gamma^2+1),$$

or

$$c'^2=100\gamma^2+99.$$

Thus $\dfrac{\rho'}{\rho}\dfrac{\gamma}{c}$ is of the order $\dfrac{1}{20{,}000}$, which is negligible in comparison with 2. Hence we may write

$$\frac{\xi}{\eta}=\frac{\gamma^2-1}{2\gamma}.$$

Thus, so far as the movements of the surface are concerned, the problem is not materially altered when we neglect the air altogether. It is sufficient for our present purpose to work out the simple problem of the reflexion of elastic disturbances at the plane boundary of an elastic solid. The surface conditions then reduce to the second and third given above, namely, the vanishing of the surface stresses. Equations (3) become

$$\left.\begin{aligned} -2\gamma B_1+(\gamma^2-1)(A+A_1)&=0\\ (\gamma^2-1)B_1-2c(A-A_1)&=0\end{aligned}\right\} \quad . \quad . \quad . \quad . \quad (3a)$$

Hence

$$\frac{A+A_1}{A-A_1}=\frac{4\gamma c}{(\gamma^2-1)^2}$$

$$A+A_1=\frac{8\gamma c}{4\gamma c+(\gamma^2-1)^2}A$$

$$A-A_1=\frac{2(\gamma^2-1)^2}{4\gamma c+(\gamma^2-1)^2}A$$

$$B_1=\frac{4c(\gamma^2-1)}{4\gamma c+(\gamma^2-1)^2}A.$$

Substituting in (4) we find

$$\left.\begin{aligned}\xi&=\frac{2c(\gamma^2-1)(\gamma^2+1)}{4c\gamma+(\gamma^2-1)^2}Aibe^{ib(y+\omega t)}\\ \eta&=\frac{4c\gamma(\gamma^2+1)}{4c\gamma+(\gamma^2-1)^2}Aibe^{ib(y+\omega t)}\end{aligned}\right\} \quad . \quad . \quad . \quad (5)$$

Now the ξ and η displacements in the original incident ray $A\,exp\{ib(cx+y+\omega t)\}$ are, for $x=0$,

$$\left.\begin{aligned}\xi_0&=cAibe^{ib(y+\omega t)}\\ \eta_0&=Aibe^{ib(y+\omega t)}\end{aligned}\right\}.$$

Hence

$$\left.\begin{aligned}\frac{\xi}{\xi_0}&=2(\gamma^2-1)\frac{\gamma^2+1}{4c\gamma+(\gamma^2-1)^2}\\ \frac{\eta}{\eta_0}&=\qquad 4c\gamma\frac{\gamma^2+1}{4c\gamma+(\gamma^2-1)^2}\end{aligned}\right\}.$$

These ratios can be readily calculated for various angles of incidence. If now we take the amplitude of the displacement in the incident wave to be unity, ξ_0 and η_0 will be respectively the *cosine* and *sine* of the corresponding angle of incidence, and the corresponding values of the component surface displacements ξ and η will follow at once. The values of these several displacements for different angles of incidence are given in the following table, together with the angle of emergence $\tan^{-1}(\xi/\eta)$:—

Angle of Incidence.	Incident Displacements.		Surface Displacements.		Angle of Emergence $\tan^{-1}\frac{\xi}{\eta}$.	Ratio ξ/η.
	ξ_0.	η_0.	ξ.	η.		
0°	1	0	2	0	90°	∞
10	0·985	0·174	1·964	0·398	77·6	4·935
20	0·94	0·342	1·86	0·78	67·2	2·385
30	0·866	0·5	1·69	1·122	56·4	1·506
40	0·766	0·643	1·476	1·406	45·5	1·05
50	0·643	0·766	1·24	1·616	37·5	0·767
60	0·5	0·866	1·0	1·732	30	0·577
70	0·342	0·94	0·772	1·71	24·1	0·451
80	0·174	0·985	0·528	1·404	20·7	0·376
90	0	1	0	0	19·5	0·354

It is curious to note that, although both the component displacements of the surface vanish at grazing incidence, the ratio does not vanish. What this means is that the angle of emergence never becomes less than 19°·5, however large the angle of incidence may be. It is only at these approximately grazing incidences that the angle of emergence differs markedly from the angle which the incident ray makes with the surface. A comparison of the angles of emergence with the complements of the angles of incidence shows that they never differ by more than a few degrees so long as the angle of incidence is less than 70°. At incidence 60° the values are identical, the direction of the surface displacement being in line with the displacement in the incident ray.

Thus it appears that the general conclusion to which I was led, although expressed not quite accurately in the former paper, is after all not far wrong.

As established by the calculations now made, the conclusion may be thus expressed. When a plane condensational sinusoidal wave falls on the plane boundary of the elastic solid through which it is travelling, every point of the surface is thrown into a rectilinear sinusoidal motion, whose direction, for most incidences, makes, with the direction of the displacement in the incident ray, an angle not exceeding 4°·5, and generally much less.

In the case of earthquake waves, according to the views adopted in the former paper, the angles of incidence become less than 30° at a comparatively short arcual distance from the source of the disturbance. The vertical displacement is therefore distinctly greater than the horizontal displacement, so that a horizontal pendulum, assumed to record only horizontal movement, will respond to a small fraction of the whole.

2. Distortional Wave Incident in the Rock.

I now pass to the case of the incident distortional wave.

As in the previous case, the surface displacements are affected so slightly by the presence of air that we may treat the problem as practically equivalent to reflexion within the solid. Leaving out the condensational wave in air, we have the required solution in the form

$$\left.\begin{aligned}\psi &= \mathrm{B}e^{ib(cx+y+\omega t)} + \mathrm{B}_1 e^{ib(-cx+y+\omega t)} \\ \phi &= \mathrm{A}_1 e^{ib(-\gamma x+y+\omega t)}\end{aligned}\right\} \qquad (1')$$

The equations of motion (I.) give

$$n(c^2+1) = \rho\omega^2 = (m+n)(\gamma^2+1),$$

or

$$(m+n)\gamma^2 = nc^2 - m,$$

showing that when c^2 becomes less than m/n, γ becomes imaginary—there is no reflected condensational wave.

The surface conditions (2) and (3) hold as before, leading to the equations

$$\left.\begin{aligned}(c^2-1)\mathrm{A}_1 + 2c(\mathrm{B}-\mathrm{B}_1) &= 0 \\ 2\gamma\mathrm{A}_1 + (c^2-1)(\mathrm{B}+\mathrm{B}_1) &= 0\end{aligned}\right\} \qquad (3')$$

The component displacements at the surface $x=0$ are

$$\left.\begin{aligned}\xi &= \frac{\partial\phi}{\partial x} + \frac{\partial\psi}{\partial y} = \{-\mathrm{A}_1\gamma + (\mathrm{B}+\mathrm{B}_1)\}ibe^{ib(y+\omega t)} \\ \eta &= \frac{\partial\phi}{\partial y} - \frac{\partial\psi}{\partial x} = \{\mathrm{A}_1 - (\mathrm{B}-\mathrm{B}_1)c\}ibe^{ib(y+\omega t)}\end{aligned}\right\} \qquad (4')$$

From (3′) we find

$$\frac{\mathrm{B}+\mathrm{B}_1}{\mathrm{B}-\mathrm{B}_1} = \frac{4\gamma c}{(c^2-1)^2}$$

$$\mathrm{B}+\mathrm{B}_1 = \frac{8\gamma c}{4\gamma c + (c^2-1)^2}\mathrm{B}$$

$$\mathrm{B}-\mathrm{B}_1 = \frac{2(c^2-1)^2}{4\gamma c + (c^2-1)^2}\mathrm{B}$$

$$\mathrm{A}_1 = -\frac{4c(c^2-1)}{4\gamma c + (c^2-1)^2}\mathrm{B}$$

Substitution in (4′) gives

$$\left.\begin{aligned}\xi &= \frac{4c\gamma(c^2+1)}{4\gamma c+(c^2-1)^2}\mathrm{B}ibe^{ib(y+\omega t)}\\ \eta &= -\frac{2(c^2-1)(c^2+1)}{4\gamma c+(c^2-1)^2}\mathrm{B}ibe^{ib(y+\omega t)}\end{aligned}\right\} \quad . \quad . \quad . \quad (5')$$

The component displacements in the original ray are, for $x=0$,

$$\left.\begin{aligned}\xi_0 &= \mathrm{B}ibe^{ib(y+\omega t)}\\ \eta_0 &= -\mathrm{B}ibce^{ib(y+\omega t)}\end{aligned}\right\},$$

and hence

$$\left.\begin{aligned}\frac{\xi}{\xi_0} &= +\frac{4c\gamma(c^2+1)}{4\gamma c+(c^2-1)^2}\\ \frac{\eta}{\eta_0} &= +\frac{2(c^2-1)(c^2+1)}{4\gamma c+(c^2-1)^2}\end{aligned}\right\}.$$

So long as c^2 does not become less than m/n, these ratios can be calculated as in the former case; but when c^2 is less than m/n, γ becomes imaginary, and the expressions for ξ and η must be modified. For this purpose we write $\gamma=i\epsilon$; and, putting for convenience $b(y+\omega t)=\theta$, we find for the component displacements the values

$$\left.\begin{aligned}\xi &= -\frac{4c\epsilon(c^2+1)}{4i\epsilon c+(c^2-1)^2}\mathrm{B}b\epsilon^{i\theta}\\ \eta &= -\frac{2(c^2-1)(c^2+1)}{4i\epsilon c+(c^2-1)^2}\mathrm{B}bi\epsilon^{i\theta}\end{aligned}\right\} \quad . \quad . \quad . \quad (6)$$

These become

$$\left.\begin{aligned}\xi &= -\frac{4c\epsilon(c^2+1)}{16\epsilon^2c^2+(c^2-1)^4}\mathrm{B}b((c^2-1)^2-4i\epsilon c)(\cos\theta+i\sin\theta)\\ \eta &= -\frac{2(c^2-1)(c^2+1)}{16\epsilon^2c^2+(c^2-1)^4}\mathrm{B}b((c^2-1)^2-4i\epsilon c)(i\cos\theta-\sin\theta)\end{aligned}\right\}$$

the real parts of which are

$$\left.\begin{aligned}\xi &= -\frac{4\epsilon c(c^2+1)}{16\epsilon^2c^2+(c^2-1)^4}\{(c^2-1)^2\cos\theta+4\epsilon c\sin\theta\}\mathrm{B}b\\ \eta &= -\frac{2(c^2-1)(c^2+1)}{16\epsilon^2c^2+(c^2-1)^4}\{4\epsilon c\cos\theta-(c^2-1)^2\sin\theta\}\mathrm{B}b\end{aligned}\right\}.$$

If we put $\tan p=\dfrac{4\epsilon c}{(c^2-1)^2}$ we get the simpler forms

$$\left.\begin{aligned}\xi &= -\mathrm{B}b(c^2+1)\sin p\cos(\theta-p)\\ \eta &= +2\mathrm{B}b\frac{(c^2+1)}{c^2-1}\cos p\sin(\theta-p)\end{aligned}\right\} \quad . \quad . \quad . \quad (7)$$

Thus the original rectilinear simple harmonic motion, whose components are

$$\xi_0 = -\mathrm{B}b \sin\theta = -\mathrm{B}b\cos\left(\theta - \frac{\pi}{2}\right),$$

$$\eta_0 = +\mathrm{B}bc\sin\theta,$$

is, in general, changed at the surface into an elliptic motion. The normal displacement is accelerated in phase by the angle $\frac{\pi}{2} - p$, and the tangential displacement by the angle $-p$, where $\tan p = 4\epsilon c/(c^2-1)^2$.

The principal axes of the ellipse so described by any point of the surface are in the ratio

$$\frac{(c^2-1)\sin p}{2\cos p} = \frac{2\epsilon c}{c^2-1} = \frac{2}{c^2-1}\sqrt{\frac{m-nc^2}{m+n}}.$$

Gathering up the results, we see that when the angle of incidence is not greater than $\cot\text{an}^{-1}(\sqrt{(m/n)})$, there are two reflected waves of different type sent back into the medium, and the associated displacement of each point of the surface is a rectilinear sinusoidal motion. In the distortional wave the displacement is at right angles to the direction of propagation of the wave. Consequently, the displacement in the original incident wave makes with the surface an angle equal to the angle of incidence. Hence, with original amplitude unity, the ξ and η displacements are measured by the *sine* and the *cosine* respectively of the angle of incidence. With the same numerical data as before, we readily calculate the values of the component displacements. These are given in the following table, along with the corresponding angles of emergence:—

Angle of Incidence.	Incident Displacements.		Surface Displacements.		Angle of Emergence $\tan^{-1}\frac{\xi}{\eta}$.	Ratio ξ/η.
	ξ_0.	η_0.	ξ (normal).	η (tangential).		
0°	0	1	0	2	0°	0
10	0·174	0·985	0·396	1·952	10·5	0·203
20	0·342	0·94	0·756	1·821	22·6	0·415
30	0·5	0·866	1	1·732	30	0·577
35	0·572	0·819	0·658	2·987	12·5	0·221
35 16′	0·577	0·816	0	3·462	0	0

The angle of emergence is in this case to be compared with the angle of incidence, not with its complement. The comparison shows that except in the neighbourhood of the critical angle 35° 16′—or more generally

$\cot^{-1}\sqrt{(m/n)}$—the angle of emergence differs very slightly from the angle which the original displacement of the incident wave makes with the surface.

It is interesting to note how rapidly the normal displacement diminishes to zero as the critical angle is approached, while at the same time the tangential displacement grows steadily. Another curious point is the vanishing of the normal displacement when the angle of incidence has this critical value. The absence of the normal displacement is no doubt associated with the vanishing of the condensational wave; but the absence of the condensational wave does not necessarily mean no normal displacement at the surface. For, as has just been proved, when the angle of incidence exceeds the critical value, each point of the surface executes an elliptic motion.

A comparison of the table just given with the table for the condensational incident ray discloses certain resemblances as well as contrasts. For example, for incidences below 20° there is a strong similarity between the two, except that the ξ and η displacements are interchanged. Again, the condensational wave with incident angle of 60° gives rise to exactly the same surface disturbance as the distortional wave with incident angle of 30°. In other respects, however, there is contrast rather than similarity. The manner in which the tangential displacement for the distortional wave passes through a minimum value and increases markedly as the critical angle is reached has no counterpart in the behaviour of either component in the case of the incident condensational wave. The persistence of high values for the tangential displacement in the distortional wave is a striking feature. This fits in well with the theory developed in the former paper. If the second preliminary tremor passes through the mass of the earth as a distortional wave, the magnitude of the tangential surface component at distant stations at which the angle of incidence is small will declare itself by a correspondingly large record on the recording instrument.

The surface displacements for values of the angle of incidence greater than the critical angle may be tabulated in a similar manner, but because of the change of phase and the transformation of rectilinear motion into elliptic motion the meanings of the quantities tabulated are not exactly the same. The displacements in the original incident wave are, as before, components of a rectilinear sinusoidal motion. Their maximum values ξ_0 and η_0 are the resolved parts of the amplitude, and may be represented by the *sine* and *cosine* of the angle of incidence. But the quantities ξ and η are no longer the resolved parts of an amplitude, but are the semi-axes of the ellipse described by each point of the surface.

Angle of Incidence.	Displacements.		Semi-axes of Ellipse.		Phase Retardation.	Ratio ξ/η.
	Normal.	Tangential.	Normal.	Tangential.		
35° 16′	0·577	0·816	0	3·462	0°	0
36	0·588	0·809	1·348	2·322	52·4	0·381
40	0·643	0·766	1·552	0·621	85·2	2·5
45	0·797	0·707	1·414	0	90	∞
50	0·766	0·643	1·305	0·349	87·7	3·74
60	0·866	0·5	1·118	0·866	75·5	1·291
70	0·940	0·342	0·897	1·326	57·4	0·676
80	0·985	0·174	0·528	1·798	31·3	0·294
90	1	0	0	2	0	0

Instead of tabulating in the second last column the quantity $\tan^{-1}(\xi/\eta)$, which in the present instance has no immediate physical significance, I have entered the phase difference $\tan^{-1} p$ in accordance with the notation of equation (2).

For angles of incidence a little greater than the critical value the tangential rectilinear motion opens out into an elliptic motion with quickly changing lengths of axes as the angle of incidence goes on increasing. When the angle of the incidence is 37° the axes are equal and the motion of the surface point is circular. At 45° angle of incidence the motion becomes again rectilinear but this time perpendicular to the surface. The vanishing of the tangential displacement depends upon the presence of the factor (c^2-1), and as this is independent of the particular values of the elastic constants and density, it follows that a distortional wave totally reflected at 45° internally from the plane boundary of any elastic medium will produce a normal motion only of the boundary. At no other angle of incidence does this vanishing of the tangential displacement occur.

As the angle of incidence is further increased beyond 45° the tangential displacement again comes into existence, and into greater and greater prominence as the angle of incidence grows. In short, elliptic motion is once more established. Up to incident angle 63° 42′ the ellipse has its major axis perpendicular to the surface. At this particular angle of incidence the motion is for a second time circular. It is certainly curious that as the grazing incidence is approached the tangential displacement should be by far the more prominent. Such a conclusion could hardly have been expected on general grounds. The simple kinematic theory of total reflexion suggests the existence at the reflecting surface of a normal displacement only; but a complete investigation along the lines of a rigorous elastic theory shows the insufficiency of this view.

As regards application to the problems suggested by seismic phenomena, we are practically concerned only with cases in which the angle of incidence is small. In the immediate vicinity of the epicentre, where the motions are large and very complex, any application of an elastic solid theory is out of the question. With this limitation to small incidences the numbers in the first and second tables show that an incident condensational wave is accompanied by a comparatively small tangential displacement, but that an incident distortional wave is accompanied by a large tangential displacement. An instrument therefore which records horizontal movements of the earth's surface will be more sensitive to the outcrop of a distortional wave than to the outcrop of a condensational wave. If the first preliminary tremors are the result of condensational waves propagated through the earth, horizontal pendulum records at stations far distant from the earthquake source will tend to be retarded in their appearance.

The above investigation deals with the very simplest case of plane waves. But plane waves in earthquake tremors will be the exception. The nature of the original disturbance, and the influence of the heterogeneity of the crust through which the disturbance must pass before it reaches the surface, will combine to produce a great complication of movements. Let us suppose that the horizontal and vertical components of these movements could be accurately measured. If these components happened to be cophasal, we might be able to evaluate the real angle of emergence. If, as is highly probable, they are not cophasal, then the resultant motion will be almost certainly much more complicated than the elliptic motion established above. It is hopeless to expect any results of consequence to be drawn from such indications.

Note added October 1909.

While this paper was being printed I came across a memoir on Earthquake Waves by E. Wiechert and K. Zoeppritz, published in the *Nachrichten von der königlichen Gesellschaft der Wissenschaften zu Göttingen* (1907). The memoir is a long one of 138 pages, and touches upon many points of interest. Fully one-third of it is a discussion of the elastic problems treated in my early papers of 1888 and 1899, with which apparently the authors were unacquainted. Of the other questions investigated one is very similar to the problem discussed in "Seismic Radiation," but their mode of approach is quite different. Broadly speaking, the conclusions are very similar. In a very ingenious manner Wiechert and Zoeppritz work from the measurable surface phenomena of earthquake transmission to the mode of propagation through the heart of the earth, using for this purpose

a very beautiful theorem regarding the curvature of the rays of propagation as they cross the surfaces of equal velocity. They build up the final solution by supposing the earth to consist of a central core of constant velocity surrounded by two spherical shells, within each of which the rays have constant curvature. The facts of observation are then found to be satisfied on the assumption that the speed of propagation of each type of earthquake tremor is nearly constant throughout the interior of the earth to within 1500 kilometres of the surface, falling off according to a definite law from this depth upwards. In other words, and more explicitly, the elastic waves of highest speed are transmitted through a core of radius equal to three-quarters of the earth's radius with a speed of 12·9 kilometres per second, and through the remaining layer of thickness equal to one quarter of the same radius with a speed which falls off from this value at the inner surface to 7·17 kilometres per second at the outer surface. The corresponding quantities for the second type of tremors are 6·75 and 4 kilometres per second, but the radius of the core of constant velocity is somewhat greater. In a recent publication Wiechert has slightly modified these conclusions. My own results are broadly similar to these, although differing in numerical details. Compare the second paragraph on page 23, and the general discussion in the first paper. Wiechert and Zoeppritz also discuss, in an interesting fashion, the commingling of disturbances which have reached a given locality by one continuous path, or by two paths with one reflexion, or by three successive paths with two reflexions, and so on. There is no doubt that such reflexions will take place. In most cases, however, the loss of energy during reflexion within the heterogeneous crust of the earth will tend to make the later coming disturbances which have experienced such reflexions comparatively feeble. Their presence seems to have been recognised in certain seismograms analysed by Wiechert and Zoeppritz.

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III.—A New Experimental Method of investigating Certain Systems of Stress. By G. H. Gulliver, B.Sc., A.M.I.Mech.E., Lecturer in Engineering in the University of Edinburgh. (Plates I.–VI.)

(MS. received June 24, 1909. Read July 12, 1909.)

On several occasions the writer has called attention to the peculiar deformation phenomenon known as Lüders' lines. When a bar of iron or of steel experiences a permanent strain, the deformation takes place, at least mainly, by slidings along certain surfaces within the body of the piece, and the traces of these surfaces upon the external faces of the bar are the lines of Lüders. The surfaces of sliding, as has been stated before, are inclined at an angle of about 50° to the maximum principal tension (or minimum compression), and at 40° to the maximum principal compression (or minimum tension), and they are parallel to the intermediate principal stress. These surfaces are really irregular, but the disturbances due to the crystalline nature of the metal and the random direction of the cleavage planes are neglected in what follows; such disturbances are usually small in a metal of normal structure, and cannot be detected by the naked eye. The general form of a surface of sliding may be a simple plane or a cone, or it may be very complex.

In a number of practical cases in which pieces of rectangular section are employed, it is nearly correct to say that the directions of two of the principal stresses remain, at all points of the body, parallel to one face of the piece, and the third principal stress is zero. Under these conditions a careful examination of the lines of Lüders, found on a plane face after the bar has been deformed, will give the information necessary to determine some of the surfaces of sliding. If the two principal stresses, of which the directions are parallel to the face under consideration, be a tension and a compression respectively, the lines of Lüders will be inclined at 50° to the first and at 40° to the second; but if these stresses are both tensions or both compressions, the lines will be normal to the direction of the (numerically) greater. If one of the two stresses is zero, the lines of Lüders may be either normal to, or inclined to the direction of the other. These results follow directly from the condition that

the third principal stress is zero. Thus, when one face only is considered, the deformations may be inclined at an acute angle to the longitudinal edges of the bar, as *ab* (fig. 1), or they may be normal to these edges, as *de*. Further examination shows that a normal line, *cb* or *de*, of one face becomes a sloping line, *ba* or *ef*, of an adjacent face, and that there are other lines which slope continuously across all four faces.

When a well-developed network of Lüders' lines has been obtained upon a bar subjected to a "laminar" stress system, the general scheme of stress

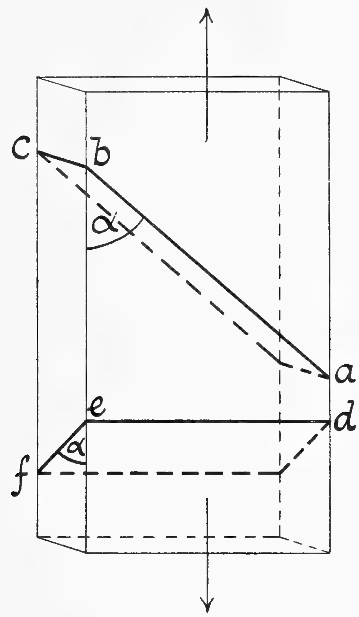

Fig. 1.

direction can be constructed by simple geometry. This method of investigation, due originally to Hartmann, can be applied usefully to problems with which pure mathematics is at present unable to cope; but it suffers from a disadvantage which limits its employment. For some deformation must occur before any Lüders' lines appear, and hence there is some distortion of the original system of stress. This is not very serious if the stress is fairly uniform throughout the piece; but the cases in which the stress variation is considerable are those which particularly require investigation, since they offer so many difficulties to mathematical solution. In these cases only the parts of the body where the stress is greatest will show any lines, unless the deformation is carried so far

that the original stress distribution is much altered. The stress system constructed from such data would be left very incomplete, and might be far from exact.

For some time the writer had suspected that the determination of a system of stress was closely connected with that of hydrodynamic flow under corresponding conditions. The similarity in the mathematical investigation of the problems afforded by elastic solids and by viscous fluids supported this view. The practical value of demonstrating such a connection would be very great. The following comparison of certain systems of "laminar" fluid motion with the corresponding "laminar" stress systems is a first step in this direction.

A stream line is defined to be a line drawn in a fluid so that its direction at any point is the direction of fluid motion at that point. Similarly, a stress line may be defined as a line drawn in a solid so that its direction at any point is the direction of one (the same) of the principal stresses at that point. It is desired to examine whether or not, under similar boundary conditions, the stream line system for a fluid is identical with one of the stress line systems for a solid.

Attention is confined for the present to systems of stress due to the application of simple tensile loads to bars of various shapes. The treatment throughout is necessarily experimental. Test-bars of soft mild steel, $2\frac{1}{2}$ inches wide and $\frac{1}{8}$ inch thick, were employed; these were shaped to various forms as regards their wider dimension, some of which are shown in figs. 2*a* to 6*a*. In all cases the thickness of the bar was left uniform. The tensile load was applied in the ordinary way by gripping the ends of each bar with serrated wedges in a testing machine. The arrows on the figures show the direction of the pull. The loading was carried to a point somewhat above the elastic limit of the piece, so as to obtain well-defined deformations, without at the same time allowing these to progress so far as to cause very great distortion. The bars had a smooth coating of mill scale, the removal of which from the deformed regions allows of good photographic reproduction of the specimens.

The stream line diagrams were obtained by means of the well-known apparatus devised by Professor Hele-Shaw (1). In this apparatus the flow takes place between two pieces of plate-glass, which are kept a small distance apart by a thin template of the required shape. The fluid used is glycerine, of which there are two supplies, one clear and one coloured. The coloured supply is fed through a number of carefully spaced holes in a brass piece inserted in the upper glass plate, and the clear glycerine

occupies the space between the coloured streams. It has been shown by Stokes (2) that the stream lines of a viscous fluid, when in a very thin sheet, are sensibly the same as those of a perfect liquid, except for a distance from the boundaries equal to the thickness of the sheet. It is necessary, therefore, to keep the thickness of the inserted forms small; they were actually cut from the thinnest Bristol board, and had a mean thickness of 0·015 inch. In order to obtain good photographic results the coloured supply of glycerine was tinted with chrysoidine.

The method of investigation was as follows. The steel bars and the cardboard templates were cut, as nearly as possible, to the same dimensions. The bars were loaded, as already explained, until considered satisfactory, taken from the testing machine, and, after the removal of any loosely adhering scale, were photographed. Some of these photographs are reproduced in figs. 2*a* to 6*a*. Next, each cardboard template was inserted in turn in the flow apparatus, and as soon as steady conditions were established the system was photographed, with the results shown in figs. 2*b* to 6*b*. A point—usually the middle point—of each deformation line was transferred from the print of the bar to the corresponding flow diagram, by means of tracing paper. Through this point on the flow diagram a curve was drawn so as to make a constant angle of 50° with all the stream lines which it crossed, and this curve was then transferred back, by the same means, to the photograph of the steel bar, as shown by the fine black lines in figs. 2*c* to 6*c*. The close agreement of most of these curves with the lines of deformation is very evident in the illustrations.

The agreement, though close, is not perfect, for many reasons. There is first the fact, already mentioned, that the bar has been deformed somewhat by the load, and therefore the lines of Lüders are not all due to the same stress distribution; and the lines which appeared first are displaced slightly on account of subsequent deformations elsewhere. Further, the loading is seldom perfectly axial. Then, in spite of all the care taken, there are sure to be some errors in the geometrical construction, and others again in the double transference of the curves. Lastly, as has been pointed out by the writer on a previous occasion (3), the inclination of the lines of Lüders is subject to some variation, and irregularities not infrequently occur, which may be due to an imperfectly axial loading, to an imperfectly homogeneous material, or to other causes unknown. The method of gripping the ends of the test piece (even if the load is perfectly axial), and the incorrect spacing of the apertures of the flow apparatus, give rise to slight errors which are probably negligible in comparison with the above.

If the bars be examined, it is seen that in each case there is an area in the region of the notch where the deformation has taken place in a manner different from that in the more remote parts of the bar. The explanation of this is readily apparent from the stream-line diagram, if it be admitted that it represents, even approximately, the system of maximum principal stress. For in the neighbourhood of the notch the stress lines (stream lines) are convex inwards, and there is, therefore, a tension normal to the system shown. Since the third principal stress—that in a direction perpendicular to the plane of the diagram—is zero, the surfaces of sliding cut the plane of the diagram, within the region considered, in lines which are everywhere normal to the stress lines shown; these surfaces are inclined at 50° to the plane of the diagram. In parts more remote from the notch the stress lines (stream lines) become convex outwards, so that the second principal stress of the face is one of compression. In this case the surfaces of sliding are normal to the plane of the diagram, and their traces on this plane are lines inclined at 50° to the stress system shown. If the bars are watched during the earlier stages of the deformation, it is seen that the first Lüders' lines to appear are those in the region of least width; upon the wide face these have a direction normal to the maximum principal tension, and upon the narrow faces they are inclined at 50°. The lines are very numerous on account of the relatively great intensity of stress here, and therefore most of the scale is removed eventually from this region. The fact that many of the lines of Lüders are normal to the direction of stress has been noticed by Hartmann, but he has considered them as forming a third system of deformations. So far as the writer's observations go, these lines are always traces of surfaces which have the usual slope in another direction.

When the stress lines (stream lines) have no curvature the second principal stress is zero—if the loading is simple—and in regions where this is the case the lines of Luders upon the wide face of the bar may be either normal to, or inclined at 50° to the stress lines. An attempt has been made to determine on the diagrams the points of inflexion of the curved stream lines, by laying a straight-edge along each in turn. In general, it appeared that there was a certain length sensibly straight between the convex and the concave parts—that is to say, the departure from straightness was so slight between two points on each stream line that it could not be detected by the eye. These two points were marked with as much care as possible, and two sets of curves were drawn through them, shown dotted in figs. 2*b* to 6*b*. It should be understood clearly that no great accuracy is claimed for these dotted curves. The method of determination is rudimentary,

though how it could be greatly improved in any simple manner is not easy to see. The readiness with which the departure of a line from straightness is detected depends somewhat upon the scale of the diagram: enlargements of the photographs to nearly three times full size were used for this purpose. Considerable irregularities were found in the positions of the individual points; but when a mean curve was drawn, the corresponding ordinates of each of the four branches, measured from the transverse centre line of the diagram, were found to agree fairly well. The ordinates of the curves plotted are the means of the four separate branches obtained in this way. The inner curves pass upwards or downwards from a point of zero curvature on the edge of the notch, and when near the middle of the face they bend over sharply, and all four branches meet at the centre of the face. The exact form of the curves near the centre point is difficult to determine. The outer curves start from the same or from another point of zero curvature—according to the shape of the notch—and, receding from the transverse centre line, pass to infinity along the longitudinal centre line of the face.

When these dotted curves are transferred to the photograph of a bar, the outer ones are found to form the boundary, as well as could be expected, between the area from which the scale has dropped completely and those parts which show well-developed Lüders' lines. Since the stress lines (stream lines) are sensibly straight in the region between the two sets of dotted curves, the deformations within this area may be either normal to, or inclined to, the direction of stress. In the area enclosed between the inner sets of dotted curves the deformation lines are normal to the stress lines, and beyond the outer dotted curves the deformation lines are inclined at 50° to the stress lines. Thus there would be, in all probability, more loss of the oxide scale between the two sets of curves than elsewhere. The difference at the outer curve is noticeable on all the bars, but that at the inner one is not, although it can be seen in fig. 4. The lack of deformation at the parts of the outer dotted curves near the longitudinal edges of each bar is due to the small stress there. Traces of the boundary indicated by the inner dotted curves are found sometimes upon broken bars. Fig. 7*a* shows an example, somewhat spoilt by a surface flaw near the middle. The curved outlines of the fracture are of the same general form as the inner dotted lines of fig. 7*b*, obtained in the manner described above, but they are by no means identical. It is difficult to reproduce exactly the shape of such a bar upon the flow template. Moreover, it must not be forgotten that fracture is not instantaneous, but proceeds gradually, and that the stress system alters during the process; and

further, that the thickness of the bar does not remain uniform during this period, so that the third principal stress is no longer zero. But the resemblance of the fracture outline to the inner dotted curve is quite noticeable.

In order to show more fully the form of the curves of deformation, a number of fine lines have been drawn upon the upper halves of figs. 2*b* to 6*b*. Passing upwards from the transverse centre line, the first series, which extend to the outer dotted curve, are everywhere normal to the stream lines; these represent the traces of surfaces inclined at 50° to the paper. Above the inner dotted curve there are drawn two series of curves, sloping respectively to the right and to the left, which are inclined at the constant angle of 50° to the stream lines; these represent the traces of deformation surfaces at right angles to the paper. The actual positions of the deformation lines found upon any bar must be considered as largely a matter of chance.

If, as is most convenient, the stream lines are such that the flow is the same for each, then for each stress line the total tension (or compression) is constant. And, as with the stream lines the velocity of flow is inversely proportional to their width, so with the stress lines the intensity of tensile (or compressive) stress is inversely proportional to their width. Thus by careful measurements of the width of the stream lines it should be possible to determine closely the magnitude of the stress at any point of the bar, and to plot lines of constant stress. So far the writer has not attempted this rather tedious operation. For great accuracy, the holes in the brass plate for the coloured glycerine streams would require very careful spacing, in order that the stream system at this part of the template should be identical with the stress system at the corresponding place in the bar. The uniform spacing adopted is correct only for a template extending to infinity on that side of the notch.

Summary.

By experimental methods a close similarity has been shown to exist between certain simple stress systems and the corresponding systems of hydrodynamic flow. How nearly exact this similarity is, and how widely the application of the principle may be extended, are points which remain to be determined.

In a simply loaded bar there are secondary stresses which vary with the shape of the bar; the existence of these secondary stresses explains some apparent anomalies in the character of the lines of deformation.

For the Hele-Shaw apparatus, without which these experiments could not have been carried out, the author is indebted to the Carnegie Trustees.

REFERENCES.

(1) *Brit. Assoc. Report*, 1898, p. 136. (2) *Ibid.*, p. 143.
(3) *Proc. Inst. Mech. Eng.*, 1905, p. 141.

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IV.—On the Illuminating Power of Groups of Pin-hole Burners.

By **R. G. Harris,** Physical Laboratory, Edinburgh University. *Communicated by* Professor MacGregor.

(MS. received August 19, 1909. Read July 12, 1909.)

The object of the present series of experiments is to find the variation of the illuminating power of symmetrical groups of pin-hole burners with the distance of adjacent burners.

The burners were fitted so as to slide on a frame composed of two pieces of a metre-rod fastened together to form a cross, as shown in figs. 1 and 2. The stems of the burners were circular in section and cylindrical in bore, the external and internal diameters being 8 mm. and less than 1 mm. respectively. Their bases were bevelled in such a manner that the burners could be fixed so close as to touch each other; thus their centres could be placed at any distance from the centre of the cross from 4 mm. to 10 cm. The standard of illumination with which they were compared was a similar pin-hole burner. The photometer was of the Bunsen type, but instead of the grease-spot an arrangement described in Stine's *Photometrical Measurements,* p. 69 (1900), was adopted. A square hole with sharply defined edges was cut in a piece of cardboard, and a thin piece of paper, of uniform texture, was firmly attached to each side of it. The screen was placed in a wooden fork, fixed to a large wooden block; it was securely held by the fork without being permanently fixed to it, and could therefore be reversed as desired. The screen was viewed by mirrors placed one on each side of it, and inclined at 60° to it. For convenience in observing, a framework of black cloth was added.

All the burners were connected to water manometers made of bent glass tubes. The manometers were used merely to see that the pressure was constant during any one set of observations. The pressure was always between 5·2 and 7·3 cm. of water above the atmospheric pressure.

In practice the standard burner and the frame carrying the group of burners were fixed, the distance between the centre of the standard burner and the centre of the frame being 150 cm. This was measured by means of metre-rods screwed to the table. The photometer block was slid by hand along these metre-rods until equal contrasts were observed in the mirrors; for the spot of light never disappeared simultaneously on both sides of the screen. It was difficult to judge this exactly, therefore ten observations were made in each case, the photometer screen reversed, and other ten

observations made. The manometers were read before and after, and generally also in the middle of each set of twenty observations, the average pressure of each set being taken, for correction purposes, as the pressure at which each of the twenty observations was made. This involves no appreciable error, since the variations in pressure from minute to minute were gradual, and never more than 1 or 2 mm. of water. In cases where this average pressure was not constant from set to set of twenty observations, a correction was applied, as indicated later.

A plan of the arrangement is shown below in figs. 1* and 2 for the case of four burners and three burners respectively. B is the centre of the framework carrying the group of burners, ϕ is the photometer, and s the standard burner. The distance ϕs was observed by means of an indicator fixed to the photometer block vertically below the centre of the screen. $B\phi$ is

Fig 1

Fig. 2

therefore known, and the illuminating power of the group would be given by $(B\phi/\phi s)^2$ if $B\phi$ were infinite compared to the distance of adjacent burners, since in that case B could be regarded as the position of the group. The above condition is not satisfied in the present case, therefore some point must be selected as the position of the group. This point was chosen such that, if its distance from the screen be R, $(R/\phi s)^2$ would give an illuminating power equal to the sum of the illuminating powers of the individual burners on the assumption that the screen was perpendicular to the incident light. This assumption is approximately true, since all the angles of incidence are less than 3°. Thus if $r_1, r_2, \ldots \ldots r_n$ be the distances of the individual burners from the screen,

$$\frac{1}{R^2} = \frac{1}{n}\left(\frac{1}{r_1^{\,2}} + \frac{1}{r_2^{\,2}} + \quad \ldots \ldots \quad + \frac{1}{r_n^{\,2}}\right).$$

* In the case of four burners, the frame was of course placed slightly squint, so as to allow 3 and 4 to be visible separately at ϕ.

It can be shown * that $R = B\phi + h$, where h is a small quantity with the values given on following page.

* Value of h. Let $\phi B = r$, $\theta =$ angle $1\phi B$, $x =$ distance of adjacent burners, $a =$ distance of the burner referred to from the centre of the cross-piece (B), $r_1 = \phi 1$, $r_3 = \phi 3$, $r_4 = \phi 4$.

(1) Case of two burners. Here $R = r_1$, therefore

$$R = \phi 1 = r \sec \theta = r + r\frac{\theta^2}{2} = r + \frac{x^2}{8r}.$$

(2) Case of three burners:

$$\frac{1}{R^2} = \frac{1}{3}\left(\frac{2}{r_1^2} + \frac{1}{r_3^2}\right) \text{ by definition.}$$

As above, $$r_1 = r + \frac{x^2}{8r}.$$

Also $$r_3 = r + \frac{\sqrt{3}}{2}x.$$

Instead of $\frac{1}{(r+e)^2}$ write $\frac{1}{r^2} - \frac{2e}{r^3} + \frac{3e^2}{r^4}$, where e is small.

$$\therefore \quad \frac{1}{R^2} = \frac{1}{r^2} - \frac{x}{\sqrt{3}r^3} + \frac{7}{12}\frac{x^2}{r^4}$$
$$= \frac{1}{r^2} + k \text{ say,}$$

giving the addition that must be made to $\frac{1}{r^2}$. To find the corresponding addition to r, let $\frac{1}{r^2} = y$. $\therefore r = \frac{1}{\sqrt{y}}$. Let h be the addition to r,

$$\therefore \quad r + h = \frac{1}{\sqrt{(y+k)}} = \frac{1}{\sqrt{y}} - \frac{k}{2}\frac{1}{y^{\frac{3}{2}}} + \frac{3}{8}\frac{k^2}{y^{\frac{5}{2}}}$$
$$= r - \frac{k}{2}r^3 + \frac{3}{8}k^2 r^5$$
$$\therefore \quad h = \frac{x}{2\sqrt{3}} - \frac{x^2}{6r}.$$

(3) Case of four burners:

$$\frac{1}{R^2} = \frac{1}{4}\left\{\frac{2}{r_1^2} + \frac{1}{r_3^2} + \frac{1}{r_4^2}\right\}.$$

Now $$r_1 = r + \frac{(2a)^2}{8r} = r + \frac{a^2}{2r}$$
$$r_3 = r + a$$
$$r_4 = r - a.$$

As before, write $\frac{1}{(r+e)^2} = \frac{1}{r^2} - \frac{2e}{r^3} + \frac{3e^2}{r^4}$—etc.

$$\therefore \quad \frac{1}{R^2} = \frac{1}{r^2} + \frac{1}{4}\left\{4\frac{a^2}{r^4} + \frac{23}{8}\frac{a^4}{r^6}\right\}$$

Neglecting the last term, which is very small, we have

$$\frac{1}{R^2} = \frac{1}{r^2} + \frac{a^2}{r^4}.$$

But if $R = r + h$, we have

$$\frac{1}{R^2} = \frac{1}{r^2} - \frac{2h}{r^3} \text{ approximately.}$$

$$\therefore \quad h = -\frac{a^2}{2r} = -\frac{x^2}{4r}.$$

For a group of 2 burners: $h = +x^2/8r$

" 3 " $h = +\cdot 2887x - \dfrac{x^2}{6r}$

" 4 " $h = -\dfrac{x^2}{4r}$

where x is the distance of the adjacent burners and r the distance $B\phi$.

To correct for small variations in pressure, preliminary experiments were made, from which a curve was obtained giving the variation of the illuminating power throughout a small range of pressure. Again, some observations were spread over several days. Here the additional precaution was taken of repeating the last observation of each day on the next one.

When the burners in the group were moved apart from each other more and more, the illuminating power was found to decrease and to approach a constant value asymptotically. Practically, it was constant for values of x greater than, say, 6 cm., and therefore this constant value was taken as unity in any one group of observations. Of course the constant value will vary from group to group of observations, depending on the pressure selected, the number of burners, atmospheric conditions, etc.

Group of two Burners.

In the experiments with this group, the burners were placed on the rod of the frame parallel to the screen, in positions 1 and 2 in fig. 1. The results of five series of observations are given in the following table, and are plotted in the diagram fig. 3 on page 51. The relative positions of the plotted points suggested an exponential law of the form

$$P = 1 + ae^{-bx}$$

where P is the illuminating power expressed in terms of the constant illuminating power for large distances, x the distance of adjacent burners, and a and b constants. The values of a and b were found by plotting log $(P-1)$ against x, and drawing straight lines among the points obtained. The values of these constants, and the values of the illuminating power calculated by aid of them, are included in Table I.

The possible error in ϕs (fig. 1) was roughly found, by a small number of repetitions of the observations, to be about ·14 cm. when $x = \cdot 8$ cm., and about ·10 cm. when $x = 8\cdot 9$ cm. Hence, by interpolation, the possible error in $\left(\dfrac{\phi B}{\phi s}\right)^2$, which will also be the possible error in $\left(\dfrac{R}{\phi s}\right)^2$, is found to vary from ·6 per cent. to ·9 per cent., corresponding in each case to a possible divergence of ·01 in P. When the pressure is not so constant as usual, the error will, of course, be slightly greater. Series A and B of the observations

TABLE I.

Distance between Adjacent Burners (x).	Illuminating Power.	
	Observed.	Calculated (P).
Series A. Formula: $P=1+\cdot95e^{-1\cdot35x}$.		
·80 cm.	1·33	1·33
·93 ,,	1·27	1·27
1·21 ,,	1·19	1·19
1·44 ,,	1·14	1·14
2·50 ,,	1·04	1·03
8·82 ,,	1·00	1·00
Series B.* $P=1+1\cdot29e^{-1\cdot86x}$.		
·77 cm.	1·315	1·31
1·10 ,,	1·17	1·17
1·46 ,,	1·095	1·08
1·93 ,,	1·035	1·04
4·07 ,,	·99	1·00
7·09 ,,	1·01	1·00
10·2 ,,	1·00	1·00
Series C. $P=1+e^{-1\cdot35x}$.		
·77 cm.	1·33	1·36
·98 ,,	1·30	1·27
1·14 ,,	1·19	1·22
8·53 ,,	1·00	1·00
Series D. $P=1+1\cdot15e^{-1\cdot80x}$.		
·78 cm.	1·27	1·28
·80 ,,	1·27	1·27
1·00 ,,	1·20	1·19
1·46 ,,	1·08	1·08
1·96 ,,	1·01	1·03
3·05 ,,	1·00	1·00
6·13 ,,	1·00	1·00
Series E. $P=1+1\cdot15e^{-1\cdot80x}$.		
·78 cm.	1·28	1·28
·90 ,,	1·23	1·23
1·06 ,,	1·18	1·17
1·26 ,,	1·09	1·12
1·50 ,,	1·04	1·08
2·03 ,,	1·01	1·03
3·00 ,,	1·00	1·00
8·00 ,,	1·00	1·00

* Taken during four different days. Pressure was not quite constant, but the observations were reduced to constant pressure by the pressure calibration curve.

therefore agree with the formulæ quoted within the error limit, while the other series do not agree, though the only excessive divergence in D is ·02, for $x = 1{\cdot}96$. It will be observed, however, that the observed and calculated values agree in each case except C for the higher values of P, which are the more important values.

A more closely approximate formula for the C, D, and E series of observations was found as follows:—Let the ratio of the observed value to the calculated value of $(P-1)$ be w. Graphs were drawn of w against x, which suggested an exponential law of the form

$$w = ae^{-\beta(x-\gamma)^2}.$$

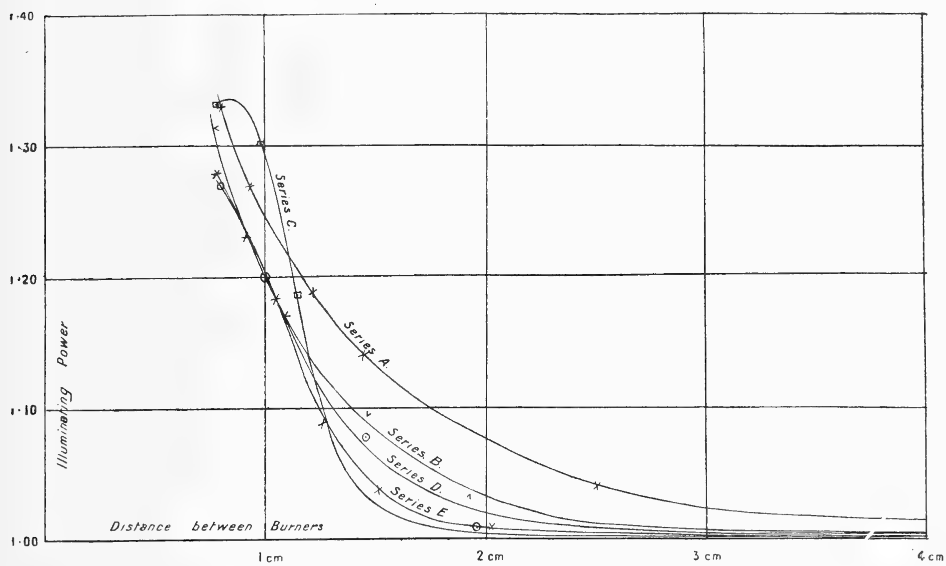

Fig 3

Log w was plotted against x, giving parabolic curves; the maximum point was observed from the graph, giving an approximate value of γ. $(x-\gamma)^2$ was then plotted against log w, and that value of γ was selected which gave the best straight line. From this straight line, a and β were determined. P was then calculated from the formula

$$P = 1 + (ae^{-bx})(ae^{-\beta(x-\gamma)^2}).$$

The curves in fig. 3 are the graphs of these latter formulæ.

The following table shows the correspondence between the observed values of the illuminating power and those calculated according to the above formulæ. It will be seen that all the observations are now in agreement with the empirical formula.

TABLE II.

Distance between Adjacent Burners (x).	Illuminating Power.	
	Observed.	Calculated (P).
Series C. $\begin{cases} w=1{\cdot}12e^{-6\cdot2(x-\cdot94)^2}. \\ P=1+\cdot0048e^{-6\cdot2x^2+10\cdot3x}. \end{cases}$		
·77 cm.	1·33	1·33
·98 „	1·30	1·30
1·14 „	1·19	1·19
8·53 „	1·00	1·00
Series D. $\begin{cases} w=1{\cdot}06e^{-\cdot68(x-1\cdot13)^2}. \\ P=1+\cdot51e^{-\cdot68x^2-\cdot26x}. \end{cases}$		
·78 cm.	1·27	1·275
·80 „	1·27	1·27
1·00 „	1·20	1·20
1·46 „	1·08	1·08
1·96 „	1·01	1·02
3·05 „	1·00	1·00
6·13 „	1·00	1·00
Series E. $\begin{cases} w=1{\cdot}05e^{-2\cdot7(x-\cdot94)^2}. \\ P=1+\cdot12e^{-2\cdot7x^2+3\cdot28x}. \end{cases}$		
·78 cm.	1·28	1·28
·90 „	1·23	1·24
1·06 „	1·18	1·17
1·26 „	1·09	1·09
1·50 „	1·04	1·035
2·03 „	1·01	1·00
3·00 „	1·00	1·00
8·00 „	1·00	1·00

The values of the constant β in the expression for w do not agree with each other at all, and this makes a wide difference in all the constants in the expression for P. This can be accounted for by small errors in P near the point $x=1{\cdot}0$. For, assuming x and all the constants except β to be so well determined as to make da, da, etc., negligible compared to $d\beta$, we have

$$dP=(1-P)(x-\gamma)^2 d\beta.$$

If $x=1{\cdot}0$, $(x-\gamma)^2$ is small, say about $\frac{1}{100}$, while P is 1·2 approximately, so that a small error in observing P would alter the position of the point on the w-x graph in such a way as to give 500 times as large an error in β. Each series of observations includes two readings near $x=1{\cdot}0$ cm., and

thus two of the more important points in each graph are liable to an error which would make a considerable difference in the inclination of the straight line referred to on page 51. β is, of course, the tangent of the angle of inclination.

For the purpose of comparing the results obtained in the various cases—two, three, and four burners—it will be desirable to examine turning points and inflexion points. Writing the equation for P in the form

$$P = 1 + ae^{-bx^2+cx},$$

then the turning point of any of the graphs in fig. 3 is given by $x = \frac{c}{2b}$. This value of x for series C, D, E is respectively $+{\cdot}83$, $-{\cdot}19$, and $+{\cdot}61$. Thus D does not give a turning value for any distance of the flames, while C and E lead us to expect a turning value for a distance about ·7 cm. The burners could not approach so near as this. As in fig. 3, it is consistent with observation to suppose that the A and B curves show curvature only in one direction; but it is also consistent to suppose that they are of the same general form as the other graphs, but with inflection points nearer the origin, and no turning values. Thus series A and B may be classed with series D. Now C, D, E have maximum increase in P (calculated) = 35 per cent., 47 per cent., and 33 per cent. respectively. Since the A and B curves are like the D one, only with more uniform curvature, it may be assumed the maximum increase for A and B is something like 50 per cent. at least. Thus the maximum increase of P for the two burners is probably over 40 per cent. at least on an average.

The inflexion point further to the right of the origin is given by $x = \frac{c}{2b} + \sqrt{\frac{1}{2b}}$. This value of x for the C, D, E observations is 1·11, ·67, and 1·04 cm. respectively. Taking the A and B observations into account, and assuming the inflexion points for these series of observations to occur at a distance not greater than ·7 cm., we get an average distance not greater than ·85 cm.

The simpler formula for P holds very well for the smaller values of x, which are the more important values. Series B, D, E have, roughly, the same formula:

$$P = 1 + 1{\cdot}2e^{-1{\cdot}8x},$$

while A and C are satisfied by

$$P = 1 + e^{-1{\cdot}4x}.$$

Thus a very rough formula to satisfy all the cases at small distances is

$$P = 1 + e^{-\frac{3}{2}x}.$$

The greatest observed increase in illuminating power was in all cases 30 per cent. roughly, 1·31 being the average of the various values obtained.

Groups of three and four Burners.

The procedure was on the same lines as in the case of the two burners, the arrangement being as in figs 1 and 2. The possible error of observation was assumed to be 1 per cent. as before, and no pressure corrections were necessary. The results are given in Tables III. and IV., and are plotted in figs. 4 and 5, pages 54, 55, for the three and four burners respectively.

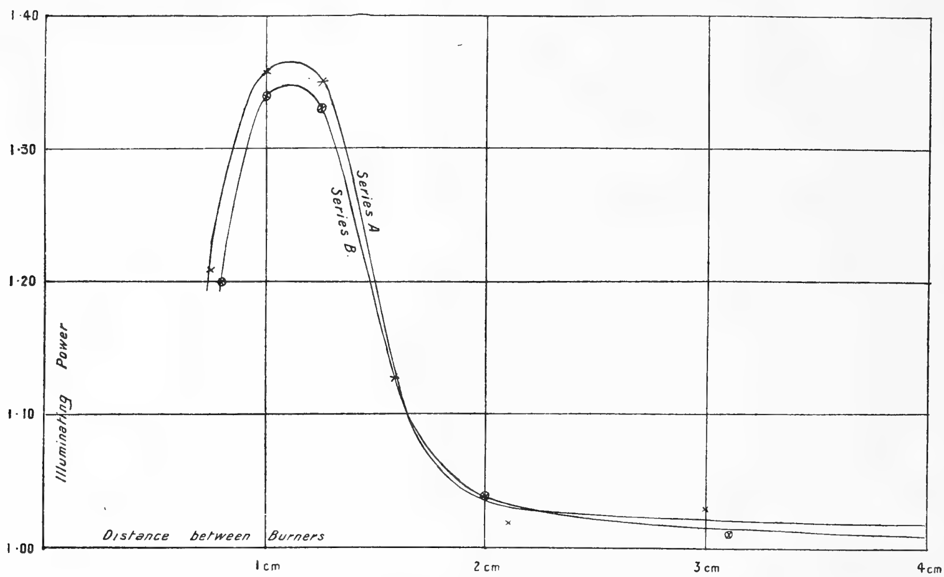

Fig. 4

The graphs in each case suggested a law of the same form as the law connecting w and x given above, namely,

$$P_1 = 1 + ae^{-b(x-c)^2},$$

the constants a, b, c being obtained in the same manner as α, β, γ were obtained above. Tables III. and IV. contain the values of P calculated according to the above formulæ, also the values of the constants. In every case except series C of the four-burner observations, it will be noted that the observed and calculated values agree for the larger values of P, but do not agree for the smaller values.

A more closely approximate formula for P was obtained as follows:— Let the ratio of the observed to the calculated values of $(P-1)$ be w, as

before. The graphs of w against x increased very rapidly after x reached about 2 cm., being nearly the straight line $w=1$ before that point. This suggests the law $w=1+\frac{e^{lx}}{m\times 10^n}$. The values of l were obtained from the ratios of the successive larger values of w, corresponding to the smaller values of P, and then the large constant $m\times 10^n$ calculated directly, so as

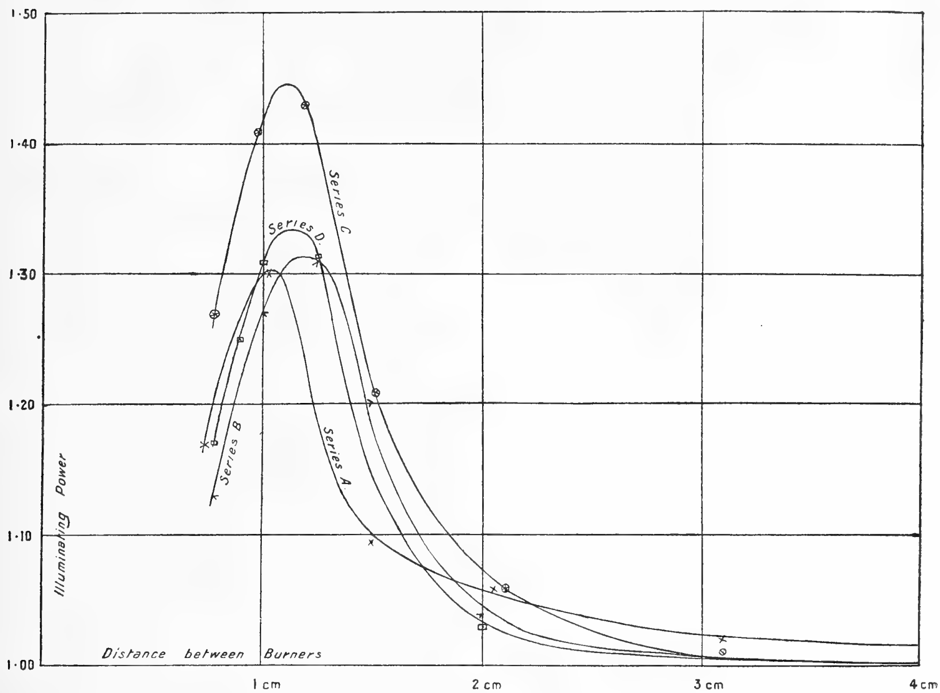

Fig. 5

to reduce w to the values it has on the graphs. Finally, we have the formula

$$P_2=1+ae^{-b(x-c)^2}\left\{1+\frac{e^{lx}}{m\times 10^n}\right\}.$$

It will be seen from the tables that the values of the illuminating power calculated according to this formula agree with those observed, within the error limit.

The curves in figs. 4 and 5 are the graphs of P_2.

TABLE III.

Distance between Adjacent Burners (x).	Illuminating Power.		
	Observed.	Calculated (P_1).	Calculated (P_2).
Series A, 3 burners. $\begin{cases} P_1 = 1 + \cdot 38e^{-4\cdot 9(x-1\cdot 1)^2}. \\ P_2 = 1 + \cdot 38e^{-4\cdot 9(x-1\cdot 1)^2}\left\{1 + \dfrac{e^{13x}}{8 \times 10^{10}}\right\}. \end{cases}$			
·75 cm.	1·21	1·21	1·21
1·00 „	1·36	1·36	1·36
1·25 „	1·35	1·35	1·35
1·58 „	1·13	1·13	1·13
2·10 „	1·02	1·00	1·03
3·00 „	1·03	1·00	1·01
5·00 „	1·00	1·00	1·00
7·92 „	1·00	1·00	1·00
Series B, 3 burners. $\begin{cases} P_1 = 1 + \cdot 36e^{-6\cdot 7(x-1\cdot 1)^2}. \\ P_2 = 1 + \cdot 36e^{-6\cdot 7(x-1\cdot 1)^2}\left\{1 + \dfrac{e^{9x}}{2\cdot 5 \times 10^{6}}\right\}. \end{cases}$			
·80 cm.	1·20	1·20	1·20
1·00 „	1·34	1·34	1·34
1·25 „	1·33	1·32	1·33
2·00 „	1·04	1·00	1·04
3·10 „	1·01	1·00	1·00
7·77 „	1·00	1·00	1·00

TABLE IV.

Distance between Adjacent Burners (x).	Illuminating Power.		
	Observed.	Calculated (P_1).	Calculated (P_2).
Series A, 4 burners. $\begin{cases} P_1 = 1 + \cdot 30e^{-5\cdot 8(x-1\cdot 06)^2}. \\ P_2 = 1 + \cdot 30e^{-5\cdot 8(x-1\cdot 06)^2}\left\{1 + \dfrac{e^{17\cdot 3x}}{9 \times 10^{13}}\right\}. \end{cases}$			
·75 cm.	1·17	1·17	1·17
1·02 „	1·30	1·30	1·30
1·50 „	1·095	1·096	1·096
2·21 „	1·06	1·00	1·06
3·10 „	1·02	1·00	1·02
7·05 „	1·00	1·00	1·00

TABLE IV.—*Continued.*

Distance between Adjacent Burners (x).	Illuminating Power.		
	Observed.	Calculated (P_1).	Calculated (P_2).
Series B, 4 burners. $\begin{cases} P_1=1+{\cdot}32e^{-5{\cdot}15(x-1{\cdot}19)^2}. \\ P_2=1+{\cdot}32e^{-5{\cdot}15(x-1{\cdot}19)^2}\left\{1+\dfrac{e^{11x}}{1{\cdot}5\times10^9}\right\}. \end{cases}$			
·78 cm.	1·13	1·13	1·13
1·00 „	1·27	1·27	1·27
1·25 „	1·31	1·31	1·31
1·49 „	1·20	1·20	1·20
2·00 „	1·04	1·01	1·04
7·10 „	1·00	1·00	1·00
Series C, 4 burners. $\begin{cases} P_1=1+{\cdot}45e^{-5{\cdot}3(x-1{\cdot}1)^2}. \\ P_2=1+{\cdot}45e^{-5{\cdot}3(x-1{\cdot}1)^2}\left\{1+\dfrac{e^{8{\cdot}4x}}{2\times10^6}\right\}. \end{cases}$			
·79 cm.	1·27	1·27	1·27
·98 „	1·41	1·41	1·41
1·19 „	1·43	1·43	1·43
1·52 „	1·21	1·18	1·21
2·06 „	1·06	1·00	1·06
3·10 „	1·01	1·00	1·00
6·11 „	1·00	1·00	1 00
Series D, 4 burners. $\begin{cases} P_1=1+{\cdot}34e^{-5{\cdot}62(x-1{\cdot}13)^2}. \\ P_2=1+{\cdot}34e^{-5{\cdot}62(x-1{\cdot}13)^2}\left\{1+\dfrac{e^{9x}}{1{\cdot}2\times10^7}\right\}. \end{cases}$			
·78 cm.	1·17	1·17	1·17
·90 „	1·25	1·25	1·25
1·00 „	1·31	1·31	1·31
1·25 „	1·31	1·31	1·315
2·00 „	1·03	1·005	1·03
6·00 „	1·00	1·00	1·00

Although the four-burner curves agree with each other in general features, the value of P corresponding to the lowest value of x is much greater in the case of series C than in the other cases. This is probably due to the large experimental errors that are introduced when small distances are worked with. When the distance between the burners was very small, it was extremely difficult to get the four flames exactly symmetrical, for the least inclination of the flames to the vertical made two or more flames coalesce while the remainder were separate. Errors in the vertical, being

due to slight inclinations of the burners themselves, were reduced as far as possible by wedges inserted under the burners. Again, when the mutual distance of the burners was less than 1·5 cm., the flames became smoky at the top, and the light emitted assumed a ruddy hue. This ruddiness, of course, made it difficult to use the photometer accurately, since the standard burner emitted white light. When the burners were very close together—say less than 1 cm. apart—the smokiness increased so much as to make the atmosphere quite stifling before the observations were completed. Thus the atmospheric conditions gradually changed during the course of an experiment. The only series of observations in which the small distances were worked with last was series A of the four-burner observations. Here, it will be observed, there is the smallest maximum value of P, and it occurs at a smaller value of x than in the other sets.

Writing the general formula

$$P = 1 + ae^{-b(x-c)^2}\left\{1 + \frac{e^{lx}}{m10^n}\right\},$$

the values of a, b, c, l, m, n are collected in the following table:—

TABLE V.

	a.	b.	c.	l.	m.	n
Series A, 3 burners	·38	4·9	1·1	13	8	10
" B, "	·36	6·7	1·1	9	2·5	6
Average	·37	5·8	1·1	11	5	8
Series A, 4 burners	·30	5·8	1·06	17·3	9	13
" B, "	·32	5·15	1·19	11	1·5	9
" C, "	·45	5·3	1·10	8·4	2	6
" D, "	·34	5·6	1·13	9	1·2	7
Averages for 4 burners	·35	5·45	1·12	11	3·7	9
Averages for B, C, D	·37	5·35	1·14	9·5	1·6	7

So far as turning values and inflexion points are concerned, the simpler formula $P = 1 + ae^{-b(x-c)^2}$ will suffice, for the remaining factor in the full formula is unity until x is greater than 2 cm. Hence column a in Table V. gives the maximum increase in illuminating power, while column c gives the corresponding distance of adjacent burners in centimetres.

The inflexion point further to the right of the origin is given by

$$x = c + \sqrt{\frac{1}{2b}}.$$

These values are tabulated below:—

Table VI.

Values of x at Inflexion Points.	
3 Burners.	4 Burners.
Series A, 1·42 cm. " B, 1·37 "	Series A, 1·35 cm. " B, 1·50 " " C, 1·41 " " D, 1·47 "
Average = 1·40 cm.	1·43 cm.

General Discussion.

Although the graphs for the two, three, and four burners are alike in form, there are considerable numerical differences. This is brought out in the data for turning points and inflexion points. The following table of turning values is collected from page 53 and Table V., and shows that the maximum increase for the two burners is decidedly greater than for three or four.

Table VII.

No. of Burners in Group.	Max. Increase in Illuminating Power.	Corresponding Distance of Adjacent Burners.
2	Say 40 per cent.	Say ·4 or ·5 cm.
3	37 "	1·10 cm.
4	35 "	1·14 cm.

Thus the maximum value of P, and the corresponding distance of adjacent burners, are nearly the same for the three and four burners. Also, the more burners the smaller is the maximum increase in P, and the greater is the distance required to give the maximum.

Comparing the inflexion values on page 53 with those in Table VI., we see that the greater the number of burners the greater is the value of the x-co-ordinate of the inflexion point. Thus in all these

numerical data there is a regular gradation as we pass from the two to the four burners.

Starting with a group of several burners a large distance apart from each other, the illuminating power in all cases slowly increases, then begins for x = about 2 cm. to increase more rapidly, passes through an inflexion point, comes to a maximum, and then decreases. This suggests that there are two main factors acting, one tending to increase the illuminating power, the other tending to decrease it. Suppose there are burners 1, 2, 3, 4 present, and consider 1. 2, 3, 4 will increase the heat and the draught, and therefore cause 1 to burn better, by increasing the supply of oxygen and the temperature. But 2, 3, 4 will also diminish the supply of oxygen available for 1, since they also need oxygen to burn. The first factor seems to be the important one for the larger values of x, and always preponderates (since there is never a decrease in illuminating power), but begins to lose its relative importance when the distance becomes very small. Taking such factors into account, the curves show that when the flames are very close together the second factor is rapidly approaching the first one in importance, but whether it ever overtakes the first one is not evident from the graph. For instance, the completed curves (see figs. 4 and 5) might very well cut the P axis below $P = 1 \cdot 00$ rather than above.

The data for turning values are consistent with the two causes suggested. Starting with the burners far apart from each other, we should expect the needs of the other flames to come into relative importance sooner with the three and four burners than with the two burners, and hence the turning points to come sooner (*i.e.* further from the origin). And since the draught and the heating effect of the other flames, the predominating factors, decrease with the distance from the origin, the maximum increase should be less for the three and four than for the two burners. Again, the two do not outline a space, while the three and four do; therefore, for *small* distances, the three and four burners should suffer more from the second cause than the two burners would be expected to do. For those parts of the streams of gas that are being supplied with oxygen from inside the volume outlined will burn less readily than those parts outside the volume. For the three and four burners (see figs. 1 and 2) the fractions of the streams are $\frac{60}{360}$ and $\frac{90}{360}$ respectively, or $\frac{1}{6}$ and $\frac{1}{4}$. This fraction approaches a limiting value $\frac{1}{2}$ when the number of burners is increased indefinitely; therefore, on this reasoning, we should expect the second cause to increase asymptotically to its limit. If parts of the gas are not being properly supplied with oxygen, we should expect the flames to be smoky. The two-burner flames were not very smoky, though for small values of x the light showed a tendency in this direction, being

ruddy, while the three- and four-burner flames were decidedly smoky at small distances apart. The two burners would not be deprived, for any very appreciable section of their flames, of the streams of air coming from the surrounding space, but the tops of the flames would suffer from the currents of CO_2 and other combustion products formed at the lower parts, and hence the tops should show smokiness to a slight extent. This applies much more to the three and four flames, since all these products formed inside the space would, owing to the draught, be confined inside the space throughout the whole length of the flames. An increased heat and draught, due to increased number of flames, would tend to make the flames longer, while an increase in the confined space would conduce to smokiness as above. It was observed that the four flames were both much longer and much more smoky than any of the others, for small distances apart.

A puzzling phenomenon occurred with the two flames at small distances. They increased in size without being smoky, but showed no tendency to coalesce. Instead they vibrated rapidly at their tips, say several times per second, and in the same phase, in planes perpendicular to the common plane of the flames. Persistence of vision made the appearence of the flames, viewed in their common plane, very like that of a fish-tail.

A very rough estimate of the distances for which the second set of causes becomes fairly important, or at least attains the same relative importance in the various cases, may be got by examining the inflexion points. As on page 59 the values of x for the inflexion points rise as we increase the number of burners, so that the diminishing cause, whatever it is, comes into prominence sooner and sooner as we increase the number of burners. This is what we would expect if the cause is the twofold one mentioned above—the sharing of the total oxygen supply, and the presence of the products of combustion.

Both for maximum values and inflexion values the figures for the two burners differ more from those of the three burners than those of the three burners differ from those of the four. This might correspond to the big difference between the volume outlined by the two burners and that outlined by the three and four.

The results of the above series of experiments are therefore as follows:—

(1) As the distance of adjacent burners of a symmetrical group is decreased, the illuminating power of the group increases to a maximum of from 35 per cent. to 40 per cent., and then decreases rapidly and is still decreasing when the burners (·8 cm. external diameter) touch. The maximum points and inflexion points are well marked for the cases of three and four burners, but only suggested for the two burners.

(2) The distances between adjacent burners that correspond to the maximum illuminating power, and also the distances corresponding to inflexion points, increase with the number of burners in the group; also, the maximum illuminating power decreases as the number of burners increases (Tables VI. and VII.).

(3) Quantitatively, the three- and four-burner groups behave in a very similar manner, while the two-burner group behaves differently, suggesting a radical difference between a two-burner group and any other symmetrical group.

(4) The effects can be explained qualitatively by assuming the operation of two factors, one tending to increase, the other tending to decrease, the luminosity; the increasing factor is always in excess of the decreasing one (for distances $>\cdot 8$ cm.).

(5) Suggested factors, which also explain (3), are:—

Increasing { (*a*) draught due to neighbouring flames.
(*b*) heating effect of neighbouring flames.

Decreasing { (*a*) decrease of oxygen supply due to neighbouring flames.
(*b*) combustion products from neighbouring flames.

The expenses of these experiments were defrayed from the Tait Memorial Fund.

(*Issued separately December* 23, 1909.)

V.—A New Hydrate of Orthophosphoric Acid. By Alexander Smith and Alan W. C. Menzies.

[Abstract.]

Our purpose was, by a systematic study of the solubilities concerned, to ascertain whether any hydrates of orthophosphoric acid, other than a semi-hydrate, $2H_3PO_4,H_2O$ (Joly's hydrate), exist. The solubilities of

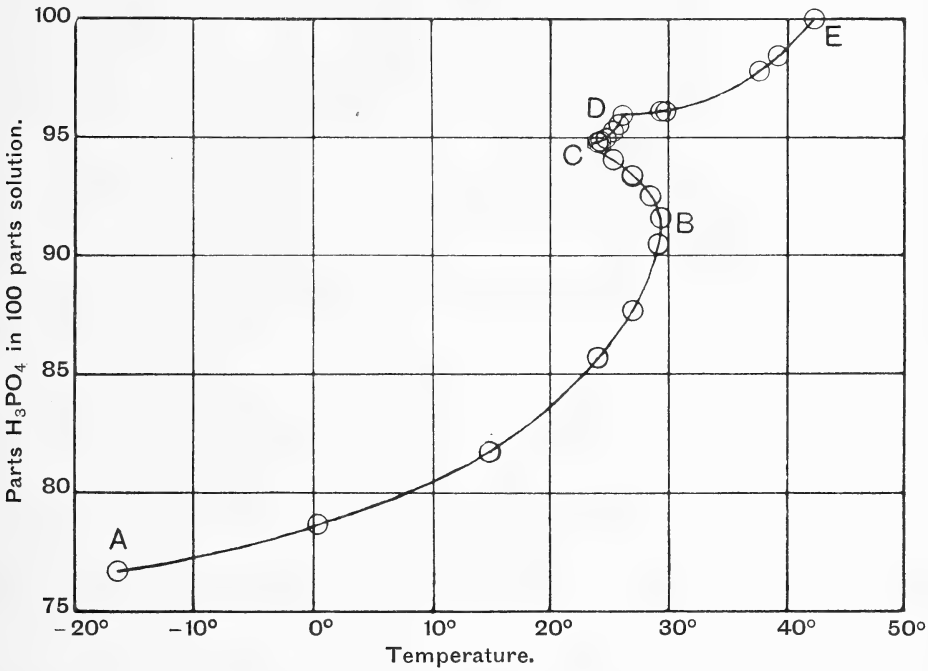

Fig. 1.

phosphoric acid and Joly's hydrate at various temperatures had not previously been measured. Crystalline, anhydrous phosphoric acid and Joly's hydrate were prepared in pure form. The solid was stirred in a thermostat with water until equilibrium was reached at each temperature, and the concentration of the solution was determined. The diagram (fig. 1) shows the results. AB is the curve of solubilities of Joly's hydrate from −16° to 29·35°, its melting point. BC contains the melting points of Joly's hydrate, depressed by phosphoric acid. CD is the solubility curve of a new hydrate, $10H_3PO_4,H_2O$. DE is the solubility curve of anhydrous phosphoric acid, and E is the melting point of the latter (42·30°).

The new deci-hydrate * was obtained in quantity by concentrating ortho-phosphoric acid to 96 per cent. and keeping it at 24·38°. Mechanical stirring for a few hours brought about crystallisation. The crystals are large, transparent prisms, similar in appearance to those of Joly's hydrate and of anhydrous phosphoric acid. They were very hygroscopic, and were wiped dry and transferred to weighing tubes for the analysis in an atmosphere dried with phosphoric anhydride.

Per cent. phosphoric acid found, 98·10 ; calc., 98·195.

The specific electrical conductivity * of solutions of phosphoric acid in water was determined at concentrations ranging from 89·7 to 98·8 per cent. When the conductivity is plotted against the concentration, the curve is perfectly regular, and shows no points of flexure to indicate the presence of hydrates in solution. It will be recalled that, in the case of sulphuric acid, the corresponding curve shows a marked minimum at a concentration corresponding to the composition of the monohydrate, $H_2SO_4 H_2O$.

THE UNIVERSITY OF CHICAGO,
August 1909.

* *Journn. Am. Chem. Soc.*, **31**, 1183–1194.

(*Issued separately December* 23, 1909.)

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Cells, Liver,—Intra-cellular Canaliculi in.
E. A. Schäfer. Proc. Roy. Soc. Edin., vol. , 1902, pp. .

Liver,—Injection within Cells of.
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[*Continued on page* iii *of Cover.*

VI.—A Further Contribution to a Comparative Study of the dominant Phanerogamic and Higher Cryptogamic Flora of Aquatic Habit in Scottish Lakes. By George West. (With Sixty-two Plates.)

SCOTTISH LAKE SURVEY.

Under the direction of Sir John Murray, K.C.B., F.R.S., D.Sc., LL.D., etc., and Laurence Pullar, F.R.S.E.

(MS. received June 1, 1909. Read July 12, 1909.)

CONTENTS.

PART I.—INTRODUCTION.

This contribution is a continuation of a former paper * on the same subject, references to which in this paper will, for the sake of brevity, be referred to as *ante*, p. . . . , or *ante*, fig. . . . The present pages deal especially with the following districts of Scotland:—

1. North-west Kirkcudbrightshire.
2. South-east Kirkcudbrightshire.
3. Wigtownshire.
4. Fife and Kinross.

1. **The Lochs of Kirkcudbrightshire** may be advantageously divided into two Areas by a line passing from Gatehouse of Fleet across the country in a north-easterly direction towards Thornhill in Dumfriesshire. All the

* "A Comparative Study of the dominant Phanerogamic and Higher Cryptogamic Flora of Aquatic Habit in Three Lake Areas of Scotland," *Proc. Roy. Soc. Edin.*, session 1904–5, vol. xxv., part xi., pp. 967–1023, and 110 illustrations.

lochs in Kirkcudbrightshire north-west of this line, including those on the border of Ayrshire, in the neighbourhood of Loch Doon, are mostly of the nature of highland lochs. This district is characterised by mountain and moor; indeed, some of the highest mountains in Scotland, south of Perthshire, are found here. The lonely grandeur of its wild scenery, notwithstanding the paucity of purple heather, gives it rank amongst the foremost of Scotland's charms. Owing to the lack of good roads, the absence of footpaths, and the exceedingly rough and often impassable nature of the ground, the tourist seldom penetrates to the remote fastnesses where lies the wildest of the fascinating scenery. The mountains, for the greater part rounded and reduced by intense glaciation, are, where the rock is not altogether bare of soil, covered with grass-like associations of plants which afford pasturage to enormous numbers of sheep. The predominance of grass-like associations over the mountains and moors, instead of heather, has a great and important influence upon the flora of the lochs. Not only that, but the pastoral life induced thereby stamps the inhabitants with characteristics different from those of the people living in localities that are chiefly devoted to sport, and engenders a higher type of ethics and a superior social organisation amongst the rural folk.

In the district of New Galloway, and in fact throughout the country for miles around, there are a few abundant plants that form a characteristic feature of this neighbourhood. They are as follows:—Jasione montana and Lepidium heterophyllum in dry places; Carum verticillatum and Œnanthe crocata in damp and wet places.

In continuity with a former contribution (*ante*, p. 971), I here call this district Area IV. (pp. 100–127, figs. 1–45).

2. **South-east of the line** from Gatehouse to Thornhill, the lochs of Kirkcudbrightshire are chiefly of a lowland type, and the district is almost wholly agricultural. The land is frequently very rich, and the farmers are prosperous and noted for their wealth. There are comparatively few lochs, and the undulating and often well-wooded country is frequently beautiful. This district I term Area V. (pp. 127–136, figs. 46–58).

3. **Wigtownshire** is remarkable for its great tracts of treeless, monotonous and dreary peat moor. In comparison with the adjoining district of north-west Kirkcudbrightshire, almost the whole county appears flat and tame. The relaxing and enervating atmosphere of south-east Kirkcudbrightshire is here in many places intensified. Agriculture is the dominant industry, particularly dairy-farming, and beyond the intractable moss-hags the land is frequently very rich. Those sheets of water that are situated on the open moors resemble highland lochs in their general features. Those lakes

that are within the zone of active agriculture are decidedly of the lowland type. Wigtownshire I term Area VI. (pp. 136–147, figs. 59–82).

The geological structure of Kirkcudbrightshire and Wigtownshire is not very diversified. A line drawn from Glenluce in a north-easterly direction, passing through Dalry and thence to Moniaive, would roughly divide these counties into two geological sections—Lower Silurian N.W. of the line, and Upper Silurian S.E. of it. Three large areas of intrusive rocks, chiefly granite, occur (1) in the Lower Silurian, extending from the neighbourhood of Loch Doon to Loch Dee, and embracing the range of mountains of which Merrick (2674 feet) is the highest point; (2) in the Upper Silurian, forming the range of hills of which Cairnsmore of Fleet (2331 feet) is the highest, and embracing Loch Grennoch; (3) and again in the same section, at and to the S.E. of Dalbeattie, the culminating point of this group being Criffel (1886 feet). Other formations, such as Carboniferous, Triassic, and Recent, also occur, but only in few places, and over comparatively restricted areas.

4. **In Fife and Kinross** a few lochs of a semi-highland character may be found on the higher hills. The greater number of the lochs in this district, however, are distinctly of a lowland type, and many of them have a very rich flora, comparatively rare plants often occurring in great abundance. The central and western portion of Fife is renowned as a coal-producing district; and whilst thousands of the inhabitants enrich themselves by bringing mineral wealth from the bowels of the earth, others, nearly everywhere, are actively engaged in agricultural operations. The rich soil readily responds to the methods of modern farming, and even many of the less favourable spots are, under the stimulus of scientific treatment, made to grow valuable crops, instead of being relegated to the unproductive realms of sport. Besides this, numerous manufacturing industries are carried on upon a large scale in many places, and the great extent of seacoast gives occupation to a considerable number of fisher folk. This Area is therefore, in comparison with the others, a densely populated one, and the greater number of its lochs have had their natural features considerably altered by the hand of man. Suitably situated lakes have been converted into reservoirs for providing the larger towns and villages with water. In some parts, especially in East Fife, the public water supply presents a serious problem that has not always been satisfactorily solved, owing to the comparatively small rainfall and the absence of suitable water in the form of lochs or streams. As an example of this difficulty, it may be mentioned that the water supply for the Newport district is brought across Strath More, the Sidlaw Hills, and the Firth of Tay, from Lintrathen in Forfarshire. In some parts new lochs have been created by the construction of

dams, etc. In other places, shallow sheets of water that could be put to no useful purpose have been drained and the sites utilised for agriculture, whilst in a few cases lochs are used as receptacles for sewage. The only lochs of this Area that retain their natural conditions are the smaller ones on the Cleish Hills.

The Carboniferous and Old Red Sandstone series of rocks, which largely prevail, have in a former epoch suffered considerable contortion from volcanic activities, and large areas are covered with lavas, tuffs, and dolerite sills. Suffice it to mention Burntisland Bin, Largo Law, May Island, and Norman's Law as eloquent monuments of that period. The country is hilly, but not mountainous; yet in many parts the scenery is beautiful: instance the undulating country to the west of Loch Leven and of the Howe of Fife, or the charming scenic effect produced by the rapid alternation of hill and dale in the neighbourhood of Aberdour, Burntisland, Newburgh, and Newport. Contrast the weird monotony of the flat links of Tents Muir with the bold perpendicular craigs of the Isle of May. Contemplate the picturesque grandeur of the Firth of Tay, equalled, but not surpassed, by the vaster expanse of beauty afforded by the lower reaches of the Firth of Forth. Turn from the grimy atmosphere of the sordid mining villages, and from odoriferous Kirkcaldy, to the wilder portions of the Lomond, Cleish, and the eastern slopes of the Ochil Hills, where one is forcibly reminded that Philistia has not yet completely triumphed over the rural glories and natural monuments of Fife and Kinross.

It is more convenient to treat these counties as one district than to divide them into natural drainage areas; I therefore make this region Area VII. (pp. 147–177, figs. 83–124).

In this investigation an endeavour has been made to record the dominant and abundant plants of the lakes visited, together with some observations upon the environmental conditions under which they grow.* These papers may therefore be considered as being floristic rather than ecological; they also present a number of necessary although brief remarks on the topography and physiography of the districts, as well as some notes from an ecological standpoint. The main idea running through this presentation is that of laying before the reader a plain account, from a botanical aspect, of the lochs, and the plants that grow in them and on their shores, keeping in view the fact that the work is a part of the Survey of the Fresh-water Lochs of Scotland. These papers may further be regarded as a preliminary step towards future fieldwork of a distinctly ecological character which

* A summary of such conditions occurring in the Ness Area may be found in the *Geographical Journal*, January 1908, pp. 67–72.

may possibly be carried on by the writer. It is hoped, however, that others may be led by the perusal of these pages to some of the many sites enumerated, where Nature seems ready, nay, anxious, to reveal some secret to the earnest investigator.

Although a good many additions have been made to the vice-counties—73, 74, 75, 85, 96, and 97 in *Topographical Botany*—still, no special search has been instituted for the purpose of adding records to a county list; at the same time it is frequently shown that comparatively rare species occur at many of the lochs in great abundance. As a general rule, I have only been able to visit each loch once, sometimes working over three or four small ones in a day; at other times spending two, three, or more days at one large loch. Thus have I not only been obliged to ignore seasonal changes, but have conducted the work almost irrespective of weather, although wind or rain seriously handicaps one.* Under such circumstances some plants have probably been overlooked. It will therefore be readily understood that I am not prepared to say that certain species do not exist in certain places, or to refute in any way the records of other botanists. Rather the opposite, for I am quite ready to admit the occurrence of plants not found in my list in the lochs that have been examined, especially in the case of plants that live entirely submersed, even though such may be abundant. One has also to remember the comparatively rapid changes that may take place in the flora of a loch, particularly amongst the plants that seldom occur very plentifully. Again, when the large districts that I have gone over in one season are held in contemplation, the most energetic of botanists will admit the impossibility of examining with microscopic vision, in so short a time, every nook and corner of the margin as well as the bottom of every lake visited. Neither is such minute examination a necessary concomitant to the wider purposes of the investigation, as expressed by the title. Further, the omission of a plant from the list does not by any means necessarily imply that it is absent in the Area, but merely that I did not find it in abundance at any of the lochs. I have simply recorded exactly what I have observed.

My friend Mr James M'Andrew, of Edinburgh, who resided many years in Kirkcudbrightshire, and whose discoveries in the geographical distribution of plants have so greatly enriched the written records of the flora of this and the adjoining counties, especially amongst the Cryptogams, has rendered me many services. Naturally he has had opportunities of observation that

* Those who have not had experience in such or similar boating operations during windy weather can scarcely appreciate the difficulty of carrying on the work, even under an ordinary stiff breeze.

have been denied to me; the records that he has kindly furnished, where my own were deficient, are acknowledged by being placed in brackets, with his initials.

The observational work for this paper was done in the field chiefly during the summer and autumn of 1905. The preparation for the press has been considerably retarded by other duties; hence the delay in the publication of the paper, for which I apologise.

Since the first part of this contribution was written a number of changes in nomenclature have been suggested. As it would appear to savour of inconsistency, besides adding an element of confusion, to give the same plant a different name in the second part of a publication to that which it bore in the first part, I have acknowledged the change in most cases by adding the new name in italics enclosed by brackets, after the old name, in the Systematic List of Plants.

In order to facilitate comparison, I have included in the succeeding Systematic List of Plants the numerals that represent the Areas discussed in a former contribution (*ante*, p. 971); a few plants, however, are enumerated there that are not mentioned in this paper. The whole of the Areas at present examined are therefore included in the following list, and the occurrence of the plants in each of the seven Areas investigated is indicated by the numerals after the authority, viz.—

I. The Ness Area.
II. Island of Lismore.
III. Lochs of Nairn.
IV. N.W. Kirkcudbrightshire.
V. S.E. Kirkcudbrightshire.
VI. Wigtownshire.
VII. Fife and Kinross.

The remarks after the numerals in the following list, unless especially indicated, refer only to Areas IV. to VII. inclusive. I hope to deal in a future contribution with the internal and external morphology of some of the plants; such matters are therefore excluded from the present pages.

PART II.—SYSTEMATIC LIST OF THE PLANTS.

RANUNCULACEÆ.

Ranunculus aquatilis, *L.* IV., V., VI., VII. Many forms of the aquatic Batrachian Ranunculi occur in the lowland non-peaty lochs of the districts now under discussion. Occasionally it was quite impossible to determine the exact species met with, owing to the absence of flowers and fruit, or other data by which the numerous forms are distinguished. The species enumerated below I have been able to collate with published descriptions. In the local lists contained in this paper, when R. aquatilis is given, it will be understood that it has not been possible to distinguish the exact species, for the reasons just mentioned. All forms are very scarce in Areas I. to III.

Ranunculus Drouetii, *F. Schultz.* V., VI., VII. Not very general; occurring chiefly in small lowland lochs.

Ranunculus Baudotii, *Godr.* VII. Not common, but occasionally abundant.

Ranunculus circinatus, *Sibth.* "II. Very scarce." VI., VII. Not general, but occasionally very abundant: at Kilconquhar Loch, for example, it covers a large area of the water, and, with the preceding species, presents a magnificent spectacle when in flower (fig. 92).

Ranunculus peltatus, *Schrank.* V., VI., VII. Widely distributed, and although not a common loch plant, is occasionally very abundant. Sometimes a terrestrial form overgrows mud at the margin of a loch, as at the west end of Kinghorn Loch. At L. Camilla, Fife, there is a curious form with dark spots and blotches on its lobed floating leaves, similar to those on the leaves of R. hederaceus; the latter is also abundant at L. Camilla.

Ranunculus heterophyllus, *Weber.* IV., V. Occasionally very abundant. In Loch Ken, for instance, it overgrows considerable tracts of shallow water at the margin of the loch, and when in flower is extremely picturesque (figs. 37 and 38).

Ranunculus Lenormandi, *F. Schultz.* V. On mud at the margin of lakes, but very scarce. [VI. In ditches S. of Lows Warren.—J. M'A.]

Ranunculus hederaceus, *L.* "I. Very scarce." VII. Frequent on the muddy shores of lochs. In Areas IV., V., and VI. it is frequent about streams, etc., but is seldom seen at the lochs.

Ranunculus sceleratus, *L.* VII. On muddy shores, but very scarce.

Ranunculus Lingua, *L.* V., VII. On marshy ground about lowland lochs; restricted in distribution, but abundant where it does occur.

Ranunculus Flammula, *L.* "I., II., III. Abundant." IV., V., VI., VII. Normal forms are abundant nearly everywhere below 1000 feet above sea level.

Ranunculus scoticus, Marsh. "I. Abundant about the shores of lochs over 1000 feet above sea." IV. Occurs at some of the hill lochs.

Ranunculus Flammula, *L.* A small prostrate form rooting profusely at the nodes, similar to the var. pseudo-reptans but larger, is sometimes found upon the stony or sandy shores of lochs in all the Areas, but is especially abundant at Loch Ken (figs. 39 and 40).

Ranunculus Flammula, *L.*, var. pseudo-reptans, *Syme.* VII. A very small prostrate plant, somewhat resembling the true R. reptans. Occasionally found on exposed shores.

Ranunculus reptans, L. VII. Perhaps another district form of R. Flammula. It occurs on flat, exposed, sandy places that are either bare or covered with short herbage in various spots all around Loch Leven. It is also abundant at Carriston Reservoir (fig. 94), and occurs sparingly at the reservoirs on the Lomond Hills. The filiform stem usually has from 3 to 5 vascular bundles in the lowest internode, but occasionally as many as 12 may be found. The smallest forms of the var. pseudo-reptans have thicker stems containing more vascular bundles which are also better developed. The dwarf prostrate growth assumed by this plant is also exhibited by other plants on the exposed shores of Loch Leven,—for example, Juncus acutiflorus, J. bufonius, Equisetum arvense, E. palustre, Carex hirta, etc.

Ranunculus Flammula, *L.*, var. natans, *Pers.* IV., VII. I have found two distinct forms of this interesting variety. (1) In Area VII., a floating form which is abundant at the margin of peaty pools about Morton Lochs, Tents Muir. This is a rather strong, wiry plant 2 or 3 feet long, having leaves somewhat thinner in texture than those of the terrestrial type, but by no means flaccid; it otherwise resembles the weak flaccid form described below. This is probably the exact form described by Persoon * and Lamarck. † (2) In Area IV., a sub-

* "Ranunculus . . . . . .
2. *Flammula* . . . . . .
. . . . γ. *natans*, fol. inferiorib. ovatis integris, superiorib. linearibus. In aquis prope Montmorency et in Barbaria. Vid. Lam. Enc. bot. 6. p. 98-99." (*Synopsis Plantarum*, ... C. H. Persoon, vol. ii. (1807), p. 102.)

† "On trouve dans l'étang de Montmorency une varieté très-voisine de celle-là,

mersed plant growing at the margin of lochs and in slow streams, in water 6 to 24 inches deep. The floating stem, 12 to 30 inches long, has at every node elongated roots and a fascicle of leaves. The radical leaves are few, and have a slender petiole 3 to 8 inches long, with a small entire ovate or elliptical lamina ½ inch to 1 inch long. The stem leaves are similar to the radical ones, but smaller, more linear, and float more or less upon the surface. No specimens were found in either flower or fruit, perhaps owing to the lateness of the season (September). The whole plant is weak and flaccid when withdrawn from the water. It is very abundant in the neighbourhood of Lochs Recar, Ballochling, etc. It cannot, therefore, be treated as a mere "sport," but rather as an aquatic form of R. Flammula. From this aspect it is most interesting, as it presents a case of an aquatic plant derived from a common semi-aquatic or almost terrestrial progenitor, and the form from Tents Muir may be considered as an intermediate stage in the evolutionary process.

Caltha palustris, *L.* "I., II., III.," IV., V., VI., VII. An abundant plant about lowland lochs, especially in Area VII.

Caltha palustris, *L.*, var. minor, *Syme.* "I.," IV. About the hill lochs, see remarks *ante*, p. 971.

NYMPHÆACEÆ.

Castalia speciosa, *Salisb.* (= *C. alba, Wood*). "I., II.," IV., V., VI., VII. Very common and abundant, especially where the water is not very peaty (figs. 21, 61, 86, 87, etc.).

Castalia speciosa, *Salisb.*, var. minor (*DC.*). "I.," IV. Less abundant than in Area I.; see remarks *ante*, p. 971.

Nymphæa lutea, *L.* "II.," IV., V., VI., VII. Common and abundant, often overgrowing large areas, but seldom seen in the hill lochs (figs. 31, 87, etc.).

Nymphæa lutea, *L.*, var. intermedia (*Ledeb.*). IV., VII. Grows with the larger form, and sometimes alone, particularly in the lower portion of L. Ken, where it is very abundant. Rather rare in Area VII.

Nymphæa pumila, *Hoffm.* "I.," IV. Not common; chiefly in L. Ken and L. Stroan.

plus grande, nageant à la surface de l'eau, dont toutes les feuilles sont entières, les inférieures ovales, obtuses, portées sur de très-longs pétioles ; les supérieures étroites, linéaires, aiguës ; les pédoncules presqu'uniflores" (*Lam. Encyc.*, vol. vi. (1804), pp. 98–99).

CRUCIFERÆ.

Radicula officinalis, *Groves* (=*R. Nasturtium-aquaticum, R. and B.*). "II., III.," V., VI., VII. Seldom abundant at the lakes.

Radicula palustris, *Moench.* V., VI., VII. Occurring sporadically about the shores of lowland lakes.

Radicula pinnata, *Moench* (=*R. sylvestris, Druce*). V. Distribution very restricted.

Cardamine pratensis, *L.* "I., II., III.," IV., V., VI., VII. Almost ubiquitous, but generally sparse. A form which multiplies vegetatively by buds, that arise from the base of the leaflets, occurs at Loch Gelly (p. 160).

Subularia aquatica, *L.* "I." A few plants occasionally observed. IV., V., VI. Often very abundant.

VIOLACEÆ.

Viola palustris, *L.* "I., II., III." Only as scattered specimens upon the shores of lakes. IV., V., VI., VII. Frequent in lowland situations.

ELATINACEÆ.

Elatine hexandra, *DC.* VI. Very abundant in some places. At Loch Magillie and White Loch, Castle-Kennedy, for example, this plant carpets the bottom in large patches from the margin to a depth of 6 feet. At other lochs it sometimes covers an exposed muddy shore. When submersed the plants are of a delicate texture, pale green, with elongated leaves, and seldom produce flowers. When exposed on mud or sand they are much more robust, dark-reddish green, with short leaves, and flower profusely. In the latter state the specimens much resemble small examples of Peplis Portula in both form and colour.

CARYOPHYLLACEÆ.

Sagina nodosa, *Fenzl.* VII. In matted growth on sandy or stony shores; scarce.

Stellaria uliginosa, *Murr.* "I., II., III.," IV., V., VI., VII. Widely distributed, but seldom abundant.

Stellaria palustris, *Retz.* V. Scarce.

HYPERICACEÆ.

Hypericum humifusum, L. V. Wet sandy and gravelly shores, not common, and usually a straggler from an adjoining heath.

Hypericum elodes, *L.* IV., [V.—J. M'A.], VI. Sometimes very abundant, but always in peaty water (fig. 26).

ROSACEÆ.

Spiræa Ulmaria, *L.* "I., II., III.," IV., V., VI., VII. Widely distributed and frequently very abundant, but chiefly about lowland lakes (fig. 98).

Comarum palustre, *L.* (=*Potentilla palustris, Scop.*). "I., II., III.," IV., V., VI., VII. Remarks the same as to the last-mentioned plant.

LYTHRACEÆ.

Peplis Portula, *L.* "I.," IV., V., VI., VII. Aquatic and terrestrial forms are common about the shores of lochs, but chiefly lowland. The ordinary fragile form is mostly met with, but at Loch Barhapple a very large and stout terrestrial form grows abundantly in large, flat, spreading patches over the muddy shore; many of these patches, consisting of but one plant, were more than a foot in diameter. Opposed to the latter, at the south end of Loch Doon, an entirely submersed form was very abundant, growing to a depth of 3 feet. The leaves of these were much larger, thin, and semi-pellucid, stems weak and considerably elongated.

Lythrum Salicaria, *L.* "II.," IV., V., VI., VII. Frequently very abundant on the shores of lochs, chiefly lowland, but rare in Area VII. (fig. 74).

ONAGRACEÆ.

Epilobium angustifolium, *L.* "I," VI. Very scarce at the lochs.

Epilobium palustre, *L.* "II.," VI., VII. Usually with other herbage in marshy places on the shores of lochs.

Epilobium tetragonum, *L.* IV., VII. Remarks same as to the last mentioned, but this is a less frequent species, and is generally scarce.

Epilobium hirsutum, *L.* "I," VII. Seldom abundant, but occasionally dominant over a small area of marshy shore. Of common occurrence in ditches and by rivers.

HALORAGACEÆ.

Hippuris vulgaris, *L.* "I., II., III.," [V., VI.—J. M'A.], VII. Occasionally very abundant, but always in lowland lochs (fig. 90).

Myriophyllum alterniflorum, *DC.* "I., III.," IV., V., VI., VII. Generally very abundant, but it usually seems to require water that

is more or less peaty. In lowland non-peaty lochs that receive the drainage from cultivated land, M. spicatum as a rule takes its place. It is exceptional to find the two species in the same water.

Myriophyllum spicatum, *L.* "II.," V., VI., VII. Abundant where the water is not peaty; see remarks on the preceding species.

CALLITRICHACEÆ.

Callitriche vernalis, *Koch* (= *C. palustris, L.*). VI., VII. Rare at the lochs, but it sometimes occurs in sheltered bays or in shore pools.

Callitriche stagnalis, *Scop.* "I., III.," IV., V., VI., VII. Terrestrial and aquatic forms are rather common in shallow places and pools about the shores of non-peaty lowland lochs. When growing on a muddy or sandy shore the plants are rather sturdy, and form sward-like, spreading tufts. When submersed they are weaker, with leaves and internodes elongated.

Callitriche hamulata, *Kütz.* (= *C. intermedia, Hoffm.*). "I., III.," IV., V., VI., VII. Widely distributed in peaty lochs, but nowhere so generally abundant as in Area I. It is usually found without the floating rosettes, but in a few places, Loch Stroan for example, the two forms occur. When rosettes are present, the floating, spathulate, apical leaves gradually become transformed downwards into the linear type, and from about 3 inches below the apex all the lower leaves are linear and emarginate, as in the form without the floating rosettes.

Callitriche autumnalis, *L.* V., VI., VII. This fine species is widely distributed in non-peaty lowland lochs, and is frequently very abundant. At Soulseat Loch, for example, this plant and Ranunculus circinatus are the only submerged Phanerogams that are plentiful. Again, at Carlingwark and Kilconquhar Lochs, notwithstanding strenuous competition by more robust rivals, this plant maintains a dominance over certain portions of the bottom. This species varies somewhat in the form of leaf and fruit.

PORTULACEÆ.

Montia fontana, *L.*, and its aquatic form, var. rivularis, *Gemel.* "I., II.," V., VI., VII. A very common plant about the shores of some of the less peaty lochs in both aquatic and terrestrial forms.

SAXIFRAGACEÆ.

Parnassia palustris, *L.* "I., II.," IV., [V., VI.—J. M'A.], VII. Occasionally represented on boggy shores.

UMBELLIFERÆ.

Hydrocotyle vulgaris, *L.* "I., II., III.," IV., V., VI., VII. The ordinary form abounds nearly everywhere on the shores of lochs (*ante*, fig. 36). At Barlockhart Loch there occurred a floating form having stems from 30 to 50 inches long, with leaves only about ½ inch in diameter and very thin.

Apium nodiflorum, *H. G. Reichb.* V., VII. Scarce, seldom seen as a constituent of a loch flora.

Apium inundatum, *H. G. Reichb.* "I.," IV., V., VI., VII. Sometimes very abundant, but always in water that is not very peaty. It usually occurs from the margin to 3 feet and occasionally even to 6 feet deep, reaching the surface from even the greatest depth. In places where the water has retreated, the seedlings sometimes grow so thick as to cover the mud with a sward, but their further development in an aerial environment is restrained.

Carum verticillatum, *Koch.* IV., V., VI. This is one of the characteristic plants of the lowland parts of Galloway in wet meadows, moors, and about the shores of lochs (fig. 27).

Cicuta virosa, *L.* V., VII. As a member of a loch flora, I have only observed this plant at Carlingwark Loch, where it is abundant, and at Otterston Loch (fig. 56).

Sium angustifolium, *L.* (=*S. erectum, Huds.*). [VI.—J. M'A.], VII. Always scarce.

Œnanthe crocata, *L.* "I.," IV., V., VI. In the lowland Areas this is a common plant on the marshy shores of lochs. It occurs in Area VII., but not at the lochs.

Angelica sylvestris, *L.* IV., VII. Occasionally on marshy shores.

RUBIACEÆ.

Galium palustre, *L.* "I.," IV., V., VI., VII. Frequent on the marshy shores of lochs (fig. 30).

VALERIANACEÆ.

Valeriana officinalis, *L.* IV., V., VI., VII. Sometimes found at the marshy shores of lowland lochs. In Area I. it is very rare at the lochs.

COMPOSITÆ.

Eupatorium cannabinum, *L.* "II.," VI. Only observed about the lochs of the Mochrum district (p. 139). [Often found in damp places by the seashore of Wigtownshire and Kirkcudbrightshire.—J. M'A.]

Gnaphalium uliginosum, *L.* VI., VII. Sometimes it forms a loose sward on damp sandy-muddy shores.

Bidens cernua, *L.* V., VI. Distribution restricted, and plants usually scarce (fig. 78).

Senecio aquaticus, *Hill.* "I., II.," IV., V., VI., VII. Frequent about the shores of lowland lakes, but scarce in Area VII.

Serratula tinctoria, *L.* IV. This southern plant is well established in dry bushy places about the west shores of Loch Ken.

Cnicus palustris, *Willd.* VII. In this Area it is frequently very abundant about marshy shores. In the other Areas, although a common plant, I have not seen it in any abundance on the shores of the lakes.

CAMPANULACEÆ.

Lobelia Dortmanna, *L.* "I., III.," IV., V., VI., VII. Frequently very abundant, but only in lochs that are more or less peaty. Rare in Area VII.

GENTIANACEÆ.

Menyanthes trifoliata, *L.* "I., II., III.," IV., V., VI., VII. This species is ubiquitous, and thrives under all kinds of environmental conditions.

BORAGINACEÆ.

Myosotis palustris, *With.*, including M. scorpioides, M. strigulosa, M. repens, and M. cæspitosa. "I., II., III.," IV., V., VI., VII. The characters distinguishing these are so interwoven that it is frequently almost impossible to decide definitely upon the specimen in hand. Although common, they are of but small importance as a constituent of a loch flora; I have therefore included the whole in the aggregate M. palustris. They occur chiefly about lowland non-peaty lakes.

Symphytum officinale, *L.* Seldom found upon the shores of lochs, but it does rarely so occur in Area VII.

SCROPHULARIACEÆ.

Scrophularia aquatica, *L.* "I.," IV., V. [VI.—J. M'A.]. Always scarce.

Scrophularia nodosa, *L.* IV. Abundant about the shores of Loch Ken and a few other places. A few plants occur sporadically about the lochs of Area I.

Mimulus Langsdorffii, *Donn.* V., VII. Well established on the muddy, marshy shores of several lakes.

Pedicularis palustris, *L.* "I., II., III.," IV., V., VI., VII. Common, and widely distributed.

Veronica scutellata, *L.* V., VI., VII. Seldom abundant.

Veronica Anagallis, *L.* "II.," V. Scarce.

Veronica Beccabunga, *L.* "I.," V., VI., VII. Sometimes very abundant in sheltered positions about the shallow margins of lowland lakes, and frequently overgrowing the shore.

LABIATÆ.

Mentha aquatica, *L.* IV., V., VI., VII. Often abundant on the marshy shores of lowland lochs. It also occurs in Areas I. and III., but sparsely.

Mentha sativa, *L.* "I., II., III.," IV., V., VI., VII. Abundant in lowland districts about marshy shores. The var. rubra, *Huds.*, also occurs, but less frequently.

Mentha arvensis, *L.* "I., III.," VI., VII. Sometimes on the dry, gravelly shores of lowland lakes this plant is represented by either the type or one of the varieties.

Scutellaria galericulata, *L.* "I.," V., VII. Scarce (*ante*, fig. 23).

Stachys palustris, *L.* IV., V., VII. Sporadic upon the shores of lowland lakes.

Lycopus europæus, *L.* V., VI., VII. Remarks appended to the last species apply to this also.

LENTIBULARIACEÆ.

Utricularia vulgaris, *L.* "I., II., III.," IV., V., VI., VII. Less abundant in the southern than in the northern Areas.

Utricularia intermedia, *Hayne.* "I.," IV., VI. This is also less abundant in the southern lochs. It occurs in pools in Area VII., but not in the lochs. I have never seen any Utricularia flowering in a loch; they appear to be continually reproduced by hybernacula.

PRIMULACEÆ.

Lysimachia nemorum, *L.* "I.," IV., V., VI., VII. Occasionally on the shores of lowland lakes, never abundant.

Lysimachia Nummularia, *L.* "I.," IV., V., VI., VII. Remarks appended to the last species apply to this also.

Lysimachia vulgaris, *L.* "I.," V., VI., VII. Restricted to a very few places regarding the lochs.

Anagallis tenella, *Murr.* V. On the shores of a very few lowland lochs, but seldom abundant (fig. 49).

PLANTAGINACEÆ.

Littorella lacustris, *L.* (=*L. uniflora, Aschers.*). "I., II., III.," IV., V., VI., VII. Abundant everywhere (*ante*, fig. 67).

Plantago lanceolata, *L.* Often conspicuous by its abundance on the stony shores of lochs in agricultural districts, especially in Area V.

POLYGONACEÆ.

Rumex Hydrolapathum, *Huds.* V. Scarce. The water-docks are very seldom seen at the lakes under consideration.

Polygonum amphibium, *L.* "I., II.," IV., V., VI., VII. Aquatic and terrestrial forms are frequently very abundant, but chiefly in lowland places (figs. 47, 84, 91, 101, etc.).

Polygonum Hydropiper, *L.* "I.," IV., V., VI., VII. Chiefly about lowland lochs; common, but seldom in abundance.

Polygonum Persicaria, *L.* VI., VII. Sometimes abundant on the shores of lowland lochs.

Polygonum aviculare, *L.* VI., VII. Sometimes this plant overgrows the drier parts of the shores of lowland lochs.

CERATOPHYLLACEÆ.

Ceratophyllum demersum, *L.* VII. Otterston Loch is the only record for the waters under consideration, and it grows there in such extraordinary abundance that in many places a boat can only be rowed through it with difficulty. Its presence doubtless excludes many plants that would otherwise thrive in that loch.

AMENTIFERÆ.

Alnus glutinosa, *Gært.* (=*A. rotundifolia, Mill.*). "I., II., III.," IV., V., VI., VII. Frequent.

Betula glutinosa, *Fries* (=*B. tomentosa, Reith and Abel*). "I., II. III.," IV., V., VI., VII. Frequent.

Myrica Gale, *L.* "I.," IV., V., VI. Frequent, sometimes 5 feet high in sheltered places (fig. 34).

Salix aurita, *L.* "I., II., III.," IV., V., VI., VII. Frequent.

The above four species are the most dominant trees or shrubs that

occur naturally in wet places about the shores of lakes. Many other species and genera occur, especially about the lowland lakes, but mostly on drier ground. These and the damp-loving species of Salix, Populus, etc., that are found, generally bear evidence of having been planted for ornamental or other purposes.

HYDROCHARIDACEÆ.

Anacharis Alsinastrum, *Bab.* (=*Elodea canadensis, Michx.*). VII. Particularly abundant in Loch Leven, but less so than formerly on account of the raids made against it by the angling authorities [VI. Monreith Loch.—J. M'A.].

IRIDACEÆ.

Iris Pseud-acorus, *L.* "I., II., III.," V., VI., VII. Frequent, often overgrowing a considerable patch of littoral marsh, but rarely seen at the hill lochs (figs. 97 and 102).

ALISMACEÆ.

Alisma Plantago, *L.* (=*A. Plantago-aquatica, L.*). "I., II.," V., VI., VII. Abundant in many places. A curious submersed form occurs at Loch Gelly (p. 161).

Alisma ranunculoides, *L.* "III.," IV., V., VI., VII. Widely distributed, but seldom abundant. Dwarf forms about 1½ inches high occur on the exposed sandy shores of Loch Leven and other places. An entirely submersed form about 3 inches high, with quite subulate leaves, is occasionally found, and is abundant at Loch Corsock. There it flowers under water to a depth of 3 feet; without the flower-stalk these submersed forms look extremely like Isoetes lacustris.

JUNCAGINACEÆ.

Triglochin palustre, *L.* "I., II.," IV., VI. Scarce, and sporadically scattered about the shores of lochs.

MELANTHACEÆ.

Narthecium ossifragum, *Huds.* "I.," IV., V., VI. On peaty shores, but seldom abundant.

JUNCACEÆ.

Juncus effusus, *L.* "I., II., III.," IV., V., VI., VII. Abundant everywhere, and often covering large areas of ground (figs. 66, 75, 79, etc.).

Juncus conglomeratus, *L.* "I., III.," IV., V., VI., VII. On dryer ground than J. effusus, and very much less abundant.

Juncus glaucus, *Ehrh.* (=*J. inflexus, L.*). VII. Not uncommon in some of the other Areas, but I have only observed it on the shores of lochs in Area VII.

Juncus bufonius, *L.* "I.," IV., VI., VII. Frequent about the shores of many lowland lochs, but less common at the hills. Its var. fasciculatus, *Bert.*, is sometimes found growing in dense prostrate tufts on exposed sandy shores.

Juncus lamprocarpus, *Ehrh.* (=*J. articulatus, L.*). I., II., III.," IV., V., VI., VII. Abundant on the shores of lochs, especially where the water is more or less peaty.

Juncus acutiflorus, *Ehrh.* (=*J. sylvaticus, Reichb.*). "I., II., III.," IV., V., VI., VII. Abundant on the shores of lochs, but it is perhaps more plentiful about the non-peaty lowland lochs than J. lamprocarpus.

When, as sometimes happens, it could not be readily determined to which of the last two species a specimen was referable, I have called the plant J. articulatus, with the old aggregate meaning. They both vary greatly, but I think most of the reduced forms so frequently met with are from the acutiflorus group (figs. 18, 51. etc.).

Juncus supinus, *Moench* (=*J. bulbosus, L.*) "I.," IV., V., VI., VII. A more protean species than the last mentioned. In a former contribution (*ante*, p. 976), I gave some descriptive matter regarding the forms of this plant; all of these variations occur also in the Areas now under consideration. In the districts now being discussed, the terrestrial forms are more abundant than in the Ness Area, whilst the aquatic form var. fluitans (*Juncus fluitans, Lam.*) is less abundant than in the Ness Area. The last-mentioned form, however, is plentiful in many of the peaty lochs of Areas IV. and VI., but in V. and VII. it is scarce.

TYPHACEÆ.

Typha latifolia, *L.* "III.," V., VI., VII. Chiefly on the shores of lowland lochs, but not common, nor is it often abundant (figs. 73 and 74). The only place at which I have ever seen this plant in a peaty lochan on an exposed peat moor is at L. Wayoch, Drumdow Moss, Wigtownshire. The appearance of this and other plants such as Eupatorium cannabinum, in the midst of so peaty an area as the Mochrum district, seems to indicate the presence of some agent (*e.g.* calcium or potassium) counteracting the influence of the humic acids

from the peat, and derived from the disintegration of the underlying rock.

Typha angustifolia, *L.* V., VII. Of more restricted distribution than the last, but where it does occur it is usually in greater abundance and covers a considerable area of ground (figs. 84 and 85).

Sparganium ramosum, *Huds.* (=*S. erectum, L.*). "I., II., III.," V., VI., VII. More abundant in Area VII. than elsewhere, chiefly on the rich boggy margins of lowland lochs. Dwarf varieties occur as well as the large normal form.

Sparganium simplex, *Huds.* V., VII. In similar situations to the last species, but usually much less abundant. Weak forms with elongated floating leaves also occur, usually in a foot or so of water. The var. longissimum, *Fries,* occurs sparingly at Loch Fitty.

Sparganium natans, *L.* (=*S. affine, Schnizl.*). "I.," IV., V., VI., VII. Rather frequent in IV., scarce elsewhere. Chiefly in peaty lochs of moderate elevation (*ante,* fig. 34).

Sparganium minimum, *Fries.* "I.," IV., VI. Generally scarce, and mostly confined to the hill lochs. I have previously remarked (*ante,* p. 977) upon the variable nature of this genus. It seems to me that the most degenerate form of S. minimum and the most robust condition of S. ramosum are connected by numerous intermediates. Perhaps experimental culture on the right lines with S. ramosum would produce all the other forms.

LEMNACEÆ.

Lemna trisulca, *L.* VII. Rarely found in the lochs, and only in those having a luxuriant marginal vegetation and non-peaty water, well sheltered by trees. It is very abundant in Kilconquhar Loch.

Lemna minor, *L.* VII. Remarks same as to the preceding species, but more frequently met with in the lochs. It is a common plant in ditches, etc., in all the Areas below about 500 feet elevation.

POTAMOGETONACEÆ.

Zannichellia palustris, *L.,* var. brachystemon (*Gay*). VII. Rare in the lochs as a rule, but extremely abundant in Kilconquhar Loch.

Potamogeton natans, *L.* "I., II., III.," IV., V., VI., VII. Abundant everywhere. It seems to me that this plant and its congener P. polygonifolius run into one another somewhat, although the two species can usually be easily distinguished by the characters exhibited by leaf, fruit, etc. The typical P. natans is most often

found in non-peaty lowland areas. The typical P. polygonifolius is frequently met with in peaty areas, and often in mountainous districts, whilst the two are sometimes seen growing together in all kinds of situations.

Potamogeton polygonifolius, *Pourr.* "I.," IV., V., VI., VII. The typical form is certainly less abundant in non-peaty lochs than the form approaching P. natans in size and texture of leaf, etc.

Potamogeton polygonifolius, *Pourr.*, var. pseudo-fluitans, *Syme.* IV. Abundant in Loch Recar and others in the same district, as well as in the streams. None fertile (*i.e.* in September). A very distinct variety with elongated, rather pellucid leaves that are beautifully netted near the midrib.

Potamogeton rufescens, *Schrad.* (=*P. alpinus, Balb.*). V., VI., VII. Sometimes abundant, but not a very common species. The variety spathulifolius, *Fischer*, occurs in Black Loch (p. 168).

Potamogeton heterophyllus, *Schreb.* "III.," V., VII. In lowland lakes, but not a very common plant in these Areas. The var. terrestris, *Schlecht.*, occurs at Loch Leven.

Potamogeton lucens, *L.* "I., II.," IV., V., VI., VII. Frequently abundant, and variable in form. The very large forms usually occur in non-peaty lochs.

Potamogeton Zizii, *Koch* (=*P. angustifolius, Bercht. and Presl.*). IV., VI., VII. Sometimes very abundant, especially in Area VII.

Potamogeton crispus, *L.* "I.," V., VI., VII. Not a very common species, nor often in abundance, excepting in Area VII. At White Loch, Castle-Kennedy, and in other lochs near it, a beautiful form occurs, having wide, bright red midribs to the leaves. The var. serratus (*Huds.*), which has greener, flatter leaves than the type, sometimes occurs.

Potamogeton perfoliatus, *L.* "I., II.," IV., V., VI., VII. Frequent, and sometimes abundant.

Potamogeton prælongus, *Wulf.* "I.," IV., V., VI., VII. A deep-water species (6–20 feet). Rather frequent, but seldom very abundant.

Potamogeton Friesii, *Rupr.* V. Only in Carlingwark Loch, where it is very plentiful.

Potamogeton pusillus, *L.* "I., II.," IV., V., VI., VII. Frequent. The narrow-leaved form — var. tenuissimus, *Mert.* and *Koch* — occurs occasionally in rather deeper water than the type.

Potamogeton obtusifolius, *Mert.* and *Koch.* II., V., VI., VII. Sometimes abundant. At Barlockhart Loch a dwarf bushy form 6–8

inches long occurred in the shallow water, normal forms being abundant in the deeper water. The var. fluvialis, *Lange* and *Mort.*, occurs in deep water at Burntisland Reservoir, Lismore, etc.

Potamogeton filiformis, *Pers.* (=*P. marinus, L.* ?). "II.," VII. Frequently abundant, particularly so at Loch Fitty, where in some places the ripe fruits of this plant, with those of P. pusillus and P. pectinatus, washed up on the shore, formed a considerable stratum. The var. alpina, *Blytt*, is plentiful in Loch Gelly.

Potamogeton flabellatus, *Bab.* (=*P. interruptus, Kit.*). VII. Very abundant in Town Loch, Dunfermline.

Potamogeton pectinatus, *L.* VII. Scarce, but plentiful in Lochs Gelly, Fitty, Kilconquhar, etc. [VI. Rare. Ravenstone Loch, near Sorbie.—J. M'A.] It is abundant in Duddingston Loch, near Edinburgh.

Mr A. Bennett kindly examined a number of difficult forms of Potamogeton for me.

CYPERACEÆ.

Schœnus nigricans, *L.* VI. About the peaty shores of lochs, not common, nor often abundant. [Frequent in damp places along the seashore of Wigtownshire.—J. M'A.]

Cladium Mariscus, *Br.* VI. Very abundant about lochs in the Mochrum district. On Anabaglish Moss several small lochans are entirely surrounded by it (figs. 62–64).

Rhynchospora alba, *Vahl.* IV., VI. Sometimes abundant on shores of boggy peat; a very common moorland plant in these Areas.

Heleocharis palustris, *Br.* "I., II., III.," IV., V., VI., VII. Ubiquitous and variable; sometimes it grows 3 feet high, as at White Loch, Castle-Kennedy, at other places less sheltered from wind it only grows a few inches high. One such dwarf form, with short, stout, very scaly rhizomes, and few flowering stems, which were about 4 inches high, was overgrowing an exposed sandy shore at Loch Grennoch. H. uniglumis, *Schultes*, occurs at the Isle of May in pools (figs. 98, 121, etc.).

Heleocharis multicaulis, *Sm.* IV., V., VI. Sometimes abundant in the more or less peaty lochs, where the leaves often float on the surface, and are occasionally viviparous at the extremities (fig. 68).

Heleocharis acicularis, *Br.* IV., VI., VII. Not a common plant, but occasionally abundant. It forms a sward on the wet sandy or muddy shores of lochs, and enters the water to a depth of about a foot;

sometimes, however, to a depth of 3 or 4 feet, in which case the plants are much elongated and flowerless.

Scirpus fluitans, *L.* IV., VI. Common in the peaty lochs of these Areas, and sometimes very abundant (fig. 25).

Scirpus setaceus, *L.* VII. On sandy shores, scarce.

Scirpus lacustris, *L.* "I., II., III.," IV., V., VI., VII. Widely distributed and abundant in both peaty and non-peaty lochs (figs. 23, 32, 33, 89, etc.).

Eriophorum vaginatum, *L.* "I.," IV., VI., VII. Sometimes found upon the peaty shores of hill lochs.

Eriophorum polystachion, *L.* (=*E. angustifolium, Roth.*). "I., II.," IV., V., VI., VII. More frequent than the last mentioned, especially upon the more or less peaty shores of lowland lochs; less abundant amongst the mountains.

Carex canescens, *L.* VII. Rare at the lochs.

Carex disticha, *Huds.* VII. On sandy-muddy shores, scarce at the lochs. [In meadows at the head of Loch Ken and near Carlingwark Loch.—J. M'A.]

Carex paniculata, *L.* VII. Forms large tussocks, and occupies a considerable area of deep bog at the west end of Otterston Loch (fig. 103). [VI. Dowalton Loch.—J. M'A.]

Carex aquatilis, *Wahl.* "I., II.," IV., VII. Both the small and the large lowland forms (elatior, *Bab.*) are sometimes very abundant; at the head of Loch Ken specimens over 6 feet high were observed. The appearance of this plant in Fife has been queried. It was first recorded there by Professor J. H. Balfour at Loch Fitty in 1862. There it still exists in abundance, especially at the west end of the loch, where it grows 3 feet high in the water and 1 foot high on the boggy shore. It has also spread to Burntisland Reservoir, but is likely to be destroyed there when the proposed alterations are carried out (fig. 104).

Carex Goodenovii, *Gay.* "I., II.," IV., V., VI., VII. Widely distributed and often very abundant about the shores of both lowland and subalpine lochs; on drier parts of the shores of the latter, very dwarf forms are frequent. By accidentally overlooking a sheet of notes, this plant was omitted from Areas I. and II. (*Proc. Roy. Soc. Edin.*, vol. xxv., pt. xi.). It is very plentiful, particularly in Area I.

Carex flava, *L.* "I., III.," IV., V., VI., VII. The type and its varieties are frequent upon the shores of lochs, but seldom in abundance.

Carex flacca, *Schreb.* "II.," IV., VII. Not uncommon, thriving chiefly

about lowland lochs, and occasionally abundant, as at Burntisland Reservoir, where it overgrows areas of marshy ground; at the same loch stragglers may also be found in drier situations, even on the adjacent roadside. The specimens from the last-mentioned site are much dwarfed, and resemble the var. stictocarpa, *Druce.*

Carex flacca, *Schreb.*, var. stictocarpa, *Druce.* "III.," IV., V., VI., VII. Scattered specimens are frequent, but it seldom occurs in abundance.

Carex binervis, *Sm.* "I.," IV., VI. On boggy, peaty shores usually scarce.

Carex filiformis, *L.* (= *C. lasiocarpa, Ehrh.*). "I.," IV., V., VI. Frequent and abundant at the margins of alpine and sub-alpine lochs, where it is sometimes the only dominant species of this genus (fig. 19).

Carex hirta, *L.* VII. Not general about lochs, but it grows in abundance on the exposed sandy shores of Loch Leven, where, in common with several other plants, it assumes a dwarf habit, growing only from 4 to 8 inches out of the sand. It grows there much after the manner of Carex arenaria on the sandy seashore, and, like it, binds the sand with its long scaly rhizomes.

Carex rostrata, *Stokes* (= *C. inflata, Huds.*). "I., II., III.," IV., V., VI., VII. Probably the most abundant and dominant plant of all. It grows at the margins of both peaty and non-peaty lochs of any elevation, from the shore out into water 1 or 2 feet deep (figs. 22, 48, 96, etc.).

Carex vesicaria, *L.* "I.," IV. Occasionally very abundant; at the head of L. Ken, for example, there are large associations of it. Frequently the beak of the nut is curiously looped within the perigynium; this results from the long style not being allowed to pass out of the mouth of the perigynium as the nut develops.

Carex acutiformis, *Ehrh.* V. As a constituent of a loch flora I have only observed this plant at Carlingwark Loch.

GRAMINEÆ.

Phalaris arundinacea, *L.* "I.," IV., V., VI., VII. Rather common at loch margins, but principally about lowland lochs (fig. 71).

Phragmites communis, *Trin.* "I., II., III.," IV., V., VI., VII. A very dominant and abundant species in lowland or highland, peaty or non-peaty lochs. On the rich muddy shores of wind-sheltered lochs it attains great luxuriance, often being 8 or 10 feet high in such situations, and overgrowing large areas (figs. 44, 65, 90, 95, and p. 168).

Deschampsia cæspitosa, *Beauv.* "I.," IV., V., VI., VII. A very usual member of the shore flora of lochs, but generally in small quantities.

Glyceria fluitans, *Br.* "I., II., III.," IV., V., VI., VII. Widely distributed, but seldom occurring in great abundance (*ante,* fig. 95).

Glyceria aquatica, *Wahlb.* V., VII. Seldom seen at the lochs, but occasionally it occurs in great abundance: instance Carlingwark Loch, Lindores Loch, and others (figs. 54, 83, etc.).

Alopecurus geniculatus, *L.*, is very abundant on the sandy-muddy shore of Town Loch, Dunfermline.

Agrostis vulgaris, *With.*, is very abundant at the same place as the last mentioned, and also at Lindores Loch. Small patches or isolated specimens of these two grasses, with other terrestrial species, are frequently found mixed with the semi-aquatic vegetation of the littoral. Frequently, too, the grass sward of a moor or meadow adjoining a loch enters the water, and, contrariwise, aquatics such as Littorella lacustris leave the water, and compete for *terra firma* against the land plants. Aquatic and terrestrial zones of vegetation in such cases are undeterminable.

EQUISETACEÆ.

Equisetum limosum, *L.* "I., II., III.," IV., V., VI., VII. This is another very abundant plant; large associations occur at the margins of lochs of all descriptions. The var. fluviatile (*L.*) is sometimes found.

Equisetum arvense, *L.* Occasionally found overgrowing sandy or stony shores, in which case, unless sheltered by other vegetation, it is always prostrate and dwarfed.

Equisetum palustre, *L.* Occasionally found amongst the marsh vegetation of lowland lochs, and sometimes on sandy-muddy shores. In the last habitat it is always dwarfed and frequently prostrate. Sometimes this and the preceding plant grow together in a dwarfed and semi-prostrate condition as at Loch Leven, in which case the two species are difficult to distinguish from one another. These two species of Equisetum usually occur so sparsely as to be scarcely worth mentioning as constituents of a loch flora.

MARSILEACEÆ.

Pilularia globulifera, *L.* VI. Rare, but occasionally very abundant, at Loch Dernaglar for example (fig. 69).

LYCOPODIACEÆ.

Isoetes lacustris, *L.* "I.," IV., V., VI. Very general in peaty lochs, but neither so abundant nor so variable in form as in Area I.

CHARACEÆ.

Messrs H. and J. Groves kindly named a number of difficult forms. On the whole, these plants are less abundant in Areas IV., V., and VI. than in the three former Areas, but they are very abundant in Area VII.

Nitella opaca, *Ag.* "I.," IV., V., VI., VII. Generally distributed in peaty lochs. I have never dredged it from a greater depth than about 16 feet in the present Areas, nor, indeed, have I found any of the more highly organised plants at a much greater depth in Areas IV.–VII. Reasons for this will be given on subsequent pages.

Nitella translucens, *Ag.* "I.," VI. Only observed in the Mochrum district. A very large form in a barren state extremely like N. translucens was referred by Messrs Groves to N. flexilis or N. opaca. It was abundant in Loch Ken and Woodhall Loch at depths of from 6 to 8 feet.

Tolypella glomerata, *Leonh.* VII. In Loch Leven, but apparently very scarce.

Chara aspera, *Willd.* "II., III.," V., VII. Sometimes slightly incrusted with lime, and frequently abundant in lowland non-peaty lochs, or in lochs that receive the drainage of villages. This plant sometimes grows in prodigious quantity, as in Loch Leven, where hundreds of acres of the bottom are covered by it. In that loch it occurs at depths of from 1 to 15 feet, but is most abundant at 4 to 8 feet deep. Some of the varieties of this species are also common, and frequently grow with the type, vars. subinermis, *Kuetz.*, and desmacantha, *H.* and *J. G.*, being the most usual, but var. capillata, *Braun.*, is abundant in Area VII.

Chara fragilis, *Desv.* "I., II.," IV., V., VI., VII. This species thrives in both peaty and non-peaty waters. In the former case it is usually free of lime, in the latter it is frequently more or less incrusted with that substance. The common form in peaty lochs is the var. delicatula, *Braun.* Both forms are occasionally very abundant. The var. fulcrata, *Gant.*, occurs in Area VII.

Chara vulgaris, *L.* VII. This does not appear to be a common plant in the lochs, but it occurs in Loch Leven as well as in some other lochs of Area VII. where the var. papillata, *Wallr.*, also grows.

Chara hispida, *L.*, var. rudis, *Braun.* "II," VII. Very plentiful in Lochmill Loch, where it is only slightly incrusted with lime.

Chara contraria, *Braun.* IV., VII. Scarce in the first Area, but more

abundant in VII. Regarding a dwarf slightly incrusted form from shallow water in Loch Whinyeon, Messrs Groves write:—"A very interesting little plant intermediate between C. contraria and C. vulgaris, but we think it best placed under the former." The var. hispidula, *Braun.*, occurs in Area VII.

The Bryophyta often form a conspicuous portion of the shore flora of the lochs in Areas IV. and VI., very much more so than in any of the other districts. By instituting a careful search for these plants, a very long list might be arranged, particularly in Area IV., which is veritably a bryologist's paradise.

SPHAGNACEÆ.

Sphagnum intermedium, *Hoffm.* IV., VI., VII. On boggy shores of peaty lochs.

Sphagnum cymbifolium, *Ehrh.* "I.," IV., VI., VII. On boggy shores of peaty lochs.

Sphagnum acutifolium, *Ehrh.* "I.," IV., V., VI., VII. Universal in one or other of its numerous forms on the boggy shores of peaty lochs.

Sphagnum cuspidatum, *Ehrh.* "I.," IV., VI. Frequent in the water in bays, etc. of small peaty lochs, as well as on their boggy shores. The variety plumosum, *Nees.* and *Hornsch.*, is less frequent.

Sphagnum subsecundum, *Nees.* "I." IV., VI. In similar habitats to the last, but less frequent. Sometimes it occurs in rather deep water: instance, Loch Grennoch, where it grows from 2 to 8 feet deep, mixed with Littorella, etc. (p. 113).

POLYTRICHACEÆ.

Polytrichum commune, *L.* This common moorland moss is occasionally very plentiful on peaty shores in all districts.

Catharinea undulata, *Web.* and *Mohr.* IV. A common moss, but it rarely occurs in abundance at the lochs, as it is more or less a terrestrial plant. At Loch Minnoch, however, submersed rocks were abundantly clothed with an aquatic form of it (p. 126).

DICRANACEÆ.

Dichodontium pellucidum, *Schp.* IV., VI. Common on rocky shores of lochs in hilly districts.

Blindia acuta, *B.* and *S.* "I.," IV., V., VII. On wet or submersed rocks; common about the shores of hill lochs.

Dicranella squarrosa, *Schp.* IV. Often abundant on the wet boggy shores of hill lochs. D. heteromalla, *Schp.*, is also frequent, but on drier ground.

Dicranum fuscescens, *Turn.* IV. Common on rocks of the shores of hill lochs.

Dicranum Scottianum, *Turn.* IV. Habitat as above, but less common.

FISSIDENTACEÆ.

Fissidens adiantoides, *Hedw.* "I.," IV. Often abundant in wet places on rocky shores.

GRIMMIACEÆ.

Grimmia apocarpa, *Hedw.* IV., VI. Common on the dry parts of littoral rocks.

Grimmia apocarpa, *Hedw.*, var. rivularis, *Web.* and *Mohr.* IV., VI. Frequently abundant on wet and submersed rocks, occasionally down to 5 or 6 feet deep. In the other Areas it is scarce at the lochs.

Rhacomitrium lanuginosum, *Brid.* IV. Often a conspicuous object upon rocks of the shores.

Rhacomitrium heterostichum, *Brid.* IV. Various forms of it are common in similar situations to the last.

Rhacomitrium aciculare, *Brid.* "I.," IV., V., VII. Frequent upon wet and submersed rocks, but sometimes only near the embouchure of a burn.

Rhacomitrium protensum, *Braun.* IV. Sometimes abundant on wet rocks. When luxuriant, as at Loch Dungeon, it has a very pleasing appearance.

Hedwigia ciliata, *Ehrh.* IV. The type and the var. leucophæa, *B.* and *S.*, frequently cover the syenite rocks that abound upon the shores, or crop out of the water as little islands.

TORTULACEÆ.

Trichostomum tortuosum, *Dixon.* IV. Occasionally abundant on the littoral rocks of hill lochs.

ORTHOTRICHACEÆ.

Orthotrichum rupestre, *Schleich.* IV., VI. A common moss on rocks about the shores of lochs in hilly districts.

MEESIACEÆ.

Aulacomnium palustre, *Schwæg.* In wet places about shores, or even in the water; frequent in all the Areas.

BARTRAMIACEÆ.

Philonotis fontana, *Brid.* "I.," IV., VII. On wet shores or at the margin of the water; common in IV., and occurring in all the other Areas sparingly. P. calcarea, *Schp.*, occurs on the boggy shore at the S.E. end of L. Leven, where there are probably seams of lime.

Breutelia arcuata, *Schp.* IV. On rocks, generally scarce at the loch shores, but it grows in magnificent luxuriance at Loch Dungeon.

BRYACEÆ.

Several species of Bryum occur, but seldom in abundance. B. alpinum, *Huds.*, is sometimes noticeable on rocks of the hill lochs in Area IV. B. bimum, *Schreb.*, occurs in scattered patches on wet or boggy shores here and there in all the Areas; similarly does B. pallens, *Swartz.* At Burntisland Reservoir in July, a curious form of the last mentioned was very abundant on the clayey bottom in water to a depth of about 18 inches. Returning in October for more of it for Mr H. N. Dixon, not a trace could be found, owing to the water level of the reservoir having fallen greatly through drought. Respecting this plant Mr Dixon wrote as follows:—"Mr Nicholson, to whom I sent the Burntisland Reservoir Bryum, agrees with me that while, at first sight, it resembles B. neodamense, *Itzigs.*, it is really an altered form of Bryum pallens, *Swartz.*"

Mnium punctatum, *L.* IV. Frequently abundant on wet, somewhat sandy shores and in bogs; also in the other Areas, but not plentiful at the lochs.

FONTINALACEÆ.

Fontinalis antipyretica, *L.* "I., II.," IV., V., VI., VII. Abundant in many places. I have not dredged it from a greater depth than 10 feet in any of the four last Areas. In V. and VII. it is less common than elsewhere.

Fontinalis squamosa, *L.* IV., VI. Sometimes plentiful, but not a common moss in the lochs.

HOOKERIACEÆ.

Pterygophyllum lucens, *Brid.* IV. Abundant about the wet shores of the hill lochs, mostly on the soil.

LEUCODONTACEÆ.

Pterogonium gracile, *Swartz.* IV. On shore rocks, frequent and abundant at lochs amongst the hills.

LESKEACEÆ.

Heterocladium heteropterum, *B.* and *S.* IV. This delicate species is seldom found at the lochs. It was, however, very abundant as a submersed aquatic at Loch Grennoch, even to a depth of 10 feet. Mr H. N. Dixon, to whom specimens were submitted, replied as follows:—"It is certainly Heterocladium heteropterum, *B.* and *S.*, and it is a remarkable situation for that moss, which is by no means an aquatic species, though accustomed to wet and even dripping rocks and moist earth."

HYPNACEÆ.

Climacium dendroides, *Web.* and *Mohr.* V., VII. Occasionally abundant on boggy shores.

Hyocomium flagellare, *B.* and *S.* IV. Frequent on shore rocks.

Brachythecium rivulare, *B.* and *S.* "I.," IV., VI., VII. On dry and submersed rocks, frequent.

Eurhynchium rusciforme, *Milde.* "I.," IV., and in all the other Areas, but scarce at the lochs. At the south end of Loch Doon it was very abundant at depths of from 3 to 5 feet, occurring there with Fontinalis antipyretica.

Hypnum riparium, *L.* VII. Not common at lochs, but it is abundant on stones at Burntisland Reservoir.

Hypnum stellatum, *Schreb.* IV., V., VI., VII. Often abundant on boggy shores.

Hypnum fluitans, *L.* "I.," IV., VI., VII. Forms of this plant are common about the hill lochs.

Hypnum uncinatum, *Hedw.* "I.," IV. On rocks of the shores of hill lochs.

Hypnum revolvens, *Swartz.* IV., VII. On wet shores, chiefly of the hill lochs.

Hypnum commutatum, *Hedw.* IV., VII. Remarks same as to the last.

Hypnum falcatum, *Brid.* "I.," IV., VII. Remarks same as to the two preceding species.

Hypnum vernicosum, *Lindb.* IV. Common at L. Minnoch.

Hypnum cupressiforme, *L.* IV. Forms of this common moss frequently cover rocks on the shores in this Area. In the other Areas it is much less abundant at the lochs.

Hypnum scorpioides, *L.* "I., II.," IV., VI., VII. Common on wet shores; less frequent in Area VII. Only the ordinary forms were noticed (*ante*, p. 983).

Hypnum stramineum, *Dicks.* "I.," VII. Only observed in abundance at Loch Fitty [IV., V.—J. M'A.].

Hypnum cuspidatum, *L.* "I.," IV., V., VI., VII. Common in marshy places about lochs, sometimes in great abundance, and frequently growing in the water.

Hylocomium squarrosum, *B.* and *S.* I., II., III., IV., V., VI., VII. A common and often abundant moss on boggy or wet shores, chiefly in lowland districts, particularly in Areas IV. and VI.

JUBULEÆ.

Frullania tamarisci, *Dum.* IV. Frequently covering damp rocks by the shores of the higher lochs.

PORELLEÆ.

Pleurozia cochleariformis, *Dum.* IV. Occasionally conspicuous on boggy shores of the higher lochs.

PTILIDEÆ.

Anthelia julacea, *Dum.* IV. Occasionally its gray-green, tufted patches form a noticeable object on the wet sandy or peaty shores of the mountain lochs.

SCAPANIOIDEÆ.

Scapania undulata, *Dum.* "I.," IV., VI. Abundant on rocks and stones at the margins of hill lochs, either submersed or out of the water. VII. On the shores of the hill lochs, scarce.

Diplophyllum albicans, *Dum.* IV. Sometimes covering the rocks of the shores of the hill lochs.

EPIGONIANTHEÆ.

Chiloscyphus polyanthos, *Dum.* V. Occasionally abundant on wet shores.

Mylia Taylori, *Gr.* and *Benn.* IV. Large patches of this hepatic occasionally occur on wet rocks or wet sandy shores of the hill lochs.

Nardia compressa, *Gr.* and *Benn.* "I.," IV. Abundant on wet and submersed rocks, chiefly at the higher lochs. Sometimes a considerable area of submersed rocks and stones to a depth of 3 feet is covered with a dense carpet of this hepatic: instance Lochs Enoch, Dungeon, etc.

Nardia emarginata, *Gr.* and *Benn.* (= *Marsupella emarginata, Dum.*). "I.," IV. On wet rocks and shores of hill lochs, and often in the water; rather less abundant than the last mentioned.

Nardia scalaris, *Gr.* and *Benn.* IV. On wet sandy-peaty shores of hill lochs, often abundant.

The following species are frequently found covering wet places about lochs, particularly on shady banks, under rocks, etc., in all the Areas, but especially in IV.:—Pellia calycina (*Tayl.*), P. epiphylla, *Lindb.*, Conocephalus conicus, *Dum.*, and Marchantia polymorpha, *L.* The last mentioned is often found in the water of bogs as well as in drier places.

Several species of Jungermannia are met with on the shores of the more elevated lochs, but always in small quantities.

Amongst the mountains of Area IV. many lochs are bordered more or less by rocks which are frequently covered in a most prolific manner with masses of lichens. A description of the flora of these lochs would be incomplete were it not, under such circumstances, to notice the most predominant and conspicuous species of these lichens. They are as follows:—Platysma glaucum, *Nyl.*, Cetraria aculeata, *Fr.*, C. muricata, *Ach.*, Parmelia lanata, *Wallr.*, P. omphalodes, *L.*, Alectoria jubata, *Nyl.*, Lecanora tartarea, *Ach.*, Sphærophoron coralloides, *Pers.*, Lecidea geographica, *Schær.* (p. 101).

ALGÆ.

A few very abundant species that were noticed:—

Batrachospermum moniliforme, *Roth.* "I.," IV. Abundant; very scarce in the other Areas.

Batrachospermum vagum, *Roth.* I. Scarce. IV., VI. Frequent; very scarce in the other Areas.

Enteromorpha intestinalis, *Link.* "III.," V., VII. Particularly plentiful in Kilconquhar Loch and Loch Fitty.

Ulothrix æqualis, *Kütz.*, and its var. catæniformis, *Rabenh.* In all the Areas, but particularly in IV.

Cladophora fracta, *Kütz.* "II.," VII. Occasionally very abundant.

Cladophora crispata, *Roth.* VII. Occasionally abundant.

Cladophora flavescens, *Ag.* V., VII. Sometimes very abundant in lochs that receive the drainage of villages.

Cladophora canalicularis, *Roth.* VI., VII. Abundant; less so in the other Areas. It covers stones and rocks about the shores of lochs that receive the drainage of villages, farms, and cultivated land. It is often covered with a prodigious quantity of diatoms, chiefly

of the genera Cocconeis, Gomphonema, Diatoma, etc. A handful of this Cladophora was taken from Loch Fitty and floated at the top of a tall vessel of water. In the course of several days a considerable deposit of pure diatom frustules had fallen from the Cladophora and collected upon the bottom of the vessel.

Cladophora glomerata, *Kütz.* Very abundant in some lochs of Areas IV. and VI., covering stones and rocks from the margin to 7 feet deep.

Mougeotia sp. Sometimes very abundant in I. and IV.

Zygogonium ericetorum, *De Bary.* "I.," IV. Often very abundant in water near the shores of the hill lochs.

Zygnema Vaucherii, *Ag.* "I., II." It is also very frequent in lochs of Areas IV. and VI.

Porphyridium cruentum, *Näg.* VI. Wet mud at Barhapple Loch, exposed through drought, was in places coloured red by this organism.

Glœotrichia Pisum, *Thur.* VI. Occurred in such extreme abundance as a plankton organism in Soulseat L. that the water in some of the little creeks was of the consistency of liquid mud.

Anabæna circinalis, *Rabenh.* VII. The water of Kinghorn Loch in places had the appearance of pale green paint, due to the vast quantity of this organism. It is frequently common in lowland lochs. The relationship between the presence of this plant and the death of certain fish, particularly perch, requires further elucidation. I found numbers of dead perch around Kinghorn Loch.

Melosira granulata, *Ralfs.* VI. Occurs as a plankton organism in White Loch, Castle-Kennedy, in such abundance that the discoloration of the water (p. 143) is in part due to it.

Dickieia and similar gelatinous Diatomaceæ. "I.," IV. Sometimes abundant at the margins of the hill lochs. Other diatoms, of course, abound everywhere.

I have frequently found submersed plants of the higher orders injured by the luxuriant growth of filamentous Algæ. In Lochs Skerrow and Grennoch, for example, quantities of Scapania undulata were in a defunct condition through being overgrown with Ulothrix æqualis, Batrachospermum vagum, Binuclearia tatrana, etc.

The following comparative table has been arranged in order to show at a glance the most conspicuous and abundant plants (*i.e.* those forming more or less definite associations) of peaty and non-peaty lochs, together with the positions they usually occupy therein. The plants have been divided into seven groups, and those in each group are so arranged that the species

inhabiting the driest ground or the shallowest water are placed first, and those occupying the deepest water last,—the whole table being kept as nearly as possible in the same order. The figures following the species indicate in feet the average depths at which they occur, a medium between the two extremes being the most usual habitat.

COMPARATIVE TABLE.

Peaty Moorland Lochs with Clear Water.	Non-peaty Lowland Lochs with Clear Water.
1. The Drier Marsh Species.	
Bryophytes on shore rocks often abundant.	Bryophytes on shore rocks usually scarce.
Deschampsia cæspitosa.	Gnaphalium uliginosum.
Juncus effusus.	Deschampsia cæspitosa.
Juncus bufonius.	Carex hirta.
Mentha sativa.	Juncus conglomeratus.
Eriophorum polystachion.	Spiræa Ulmaria.
Eriophorum vaginatum.	Juncus effusus.
Phalaris arundinacea.	Juncus bufonious.
Juncus supinus.	Mentha sativa.
Ranunculus Flammula.	Phalaris arundinacea.
	Carex disticha.
	Ranunculus Flammula.
2. Marsh Species with their Bases usually in Semi-Liquid Mud, or even in Water.	
Various bog mosses, including species of Sphagnum and Polytrichum.	Various bog mosses, excluding species of Sphagnum and Polytrichum.
Caltha palustris.	Caltha palustris.
Carex Goodenovii.	Carex Goodenovii.
Carex vesicaria.	Mentha aquatica.
Carex aquatilis (dwarf form).	Iris Pseud-acorus.
Juncus acutiflorus, shore–1.	Ranunculus Lingua.
Juncus lamprocarpus, shore–1.	Alisma Plantago.
Comarum palustre, shore–1.	Sparganium ramosum, shore–1.
Hypericum elodes, shore–1.	Juncus acutiflorus, shore–1.
	Juncus lamprocarpus, shore–1.
	Carex paniculata, shore–1.
	Typha latifolia, shore–1.
	Glyceria aquatica, shore–1.
	Comarum palustre, shore–1.
	Sparganium simplex, shore–1.
	Epilobium hirsutum, shore–1.

PEATY MOORLAND LOCHS—*contd.* | NON-PEATY LOWLAND LOCHS—*contd.*

3. *Semi-Aquatic Species that send up from the Water a strong Aerial Flowering Stem, but without special Submersed or Floating Leaves.*

Menyanthes trifoliata, shore–1.
Heleocharis palustris, shore–1½.
Carex rostrata, shore–2.
Carex filiformis, shore–2.
Carex elata, shore–2.
Cladium Mariscus, shore–2.
Phragmites communis, shore–3.
Equisetum limosum, 1–5.

Menyanthes trifoliata, shore–1.
Heleocharis palustris, shore–1½.
Typha angustifolia, shore–1½.
Carex flacca, shore–1½.
Carex rostrata, shore–2.
Carex acutiformis, shore–2.
Carex aquatilis (elatior), shore–2.
Phragmites communis, shore–3.
Equisetum limosum, 1–5.

4. *Semi-Aquatic Species that send up from the Water a strong Aerial Flowering Stem, having also Submerged or Floating Leaves that often differ from the Aerial ones.*

Glyceria fluitans, ½–1½.
Heleocharis multicaulis, ½–1½.
Sparganium natans, 1–3.
Scirpus lacustris, 2–7.

Glyceria fluitans, ½–1½.
Hippuris vulgaris, 1–6.
Scirpus lacustris, 2–6.

5. *Aquatic Species that grow up into the Water from the bottom more than a foot in height, and usually with at least some Leaves, which reach the surface and float there.*

Species of the following genera of Algæ are characteristic of peaty lochs; they cover rocks, other submersed plants, etc., or, more rarely, float in the water of bays, etc.:—Ulothrix, Mougeotia, Zygogonium, Zygnema, and Batrachospermum.

Polygonum amphibium, shore–4.
Apium inundatum, 1–4.
Potamogeton rufescens, 2–10.
Castalia speciosa, 2–10.
Nymphæa lutea, 2–12.
Nymphæa intermedia, 2–12.
Potamogeton polygonifolius, 2–12.
Potamogeton natans, 2–20.
Nymphæa pumila, 7–15.

The following genera of Algæ are characteristic of lowland lochs, and masses of them often float at the surface:—Enteromorpha, Cladophora, Spirogyra, and Œdogonium.

Polygonum amphibium, shore–4.
Apium inundatum, 1–4.
Ranunculus peltatus, etc., 1–7.
Potamogeton heterophyllus, 1–8.
Potamogeton rufescens, 2–10.
Castalia speciosa, 2–10.
Nymphæa lutea, 2–12.
Potamogeton polygonifolius, 2–12.
Potamogeton natans, 2–20.

6. *Aquatic Species that grow up into the Water from the bottom usually more than a foot in height, but never or rarely producing leaves that float on the surface.*

PEATY MOORLAND LOCHS—*contd.*

Bryophytes on submerged rocks often abundant, especially the following species: Blindia acuta, Eurhynchium rusciforme, Fontinalis squamosa, F. antipyretica, Scapania undulata, Nardia emarginata, N. compressa, etc.

Utricularia intermedia, $\frac{1}{2}$–3.
Scirpus fluitans, 1–5.
Callitriche hamulata, 1–10.
Juncus fluitans, 1–10.
Myriophyllum alterniflorum, 1–10.
Potamogeton pusillus, 2–10.
Utricularia vulgaris, 1–15.
Potamogeton Zizii, 3–15.
Potamogeton crispus, 2–20.
Potamogeton lucens, 3–20.
Potamogeton prælongus, 5–20.
Potamogeton perfoliatus, 5–20.
Chara fragilis, v. delicatula, 2–25.
Nitella opaca, 3–30.
Fontinalis antipyretica, shore–35.

NON-PEATY LOWLAND LOCHS—*contd.*

Bryophytes on submerged rocks almost absent, but the following species sometimes occur:—Blindia acuta, Eurhynchium rusciforme, Fontinalis antipyretica, etc.

Ranunculus circinatus, etc., 2–8.
Zannichellia palustris, 2–8.
Potamogeton flabellatus, 2–8.
Potamogeton filiformis, 2–8.
Myriophyllum alterniflorum, 1–10.
Callitriche autumnalis, 2–10.
Potamogeton Friesii, 2–10.
Myriophyllum spicatum, 2–10.
Ceratophyllum demersum, 2–10.
Nitella translucens, 2–10.
Anacharis Alsinastrum, 2–10.
Potamogeton obtusifolius, 2–10.
Potamogeton pectinatus, 2–12.
Utricularia vulgaris, 1–15.
Chara fragilis, 1–15.
Potamogeton pusillus, 2–20.
Potamogeton crispus, 2–20.
Potamogeton Zizii, 3–20.
Potamogeton lucens, 3–20.
Potamogeton prælongus, 5–20.
Potamogeton perfoliatus, 5–20.
Chara fragilis, v. delicatula, 2–25.
Chara aspera, 2–25.
Nitella opaca, 3–30.
Chara hispida, v. rudis, 10–35.
Fontinalis antipyretica, shore–40.

7. *Aquatic bottom-carpeting Species, usually but a few inches high (excluding flowering stalks).*

Heleocharis acicularis, shore–4.
Subularia aquatica, shore–4.
Littorella lacustris, shore–5.
Pilularia globulifera, $\frac{1}{2}$–3.
Lobelia Dortmanna, $\frac{1}{2}$–6.
Isoetes lacustris, 2–20.

Heleocharis acicularis, shore–4.
Littorella lacustris, shore–5.
Elatine hexandra, shore–8.

It should be mentioned that the plants in the local lists hereafter are not arranged in systematic order, but more or less in accordance with the position they occupy in the loch, usually beginning with the bottom-carpeting species.

PART III.—THE LAKES.

I.—Area IV.

We may begin the examination of the lochs of Area IV. (p. 65) at Loch Doon; from there we proceed to those lochs on the hills to the W. and S.W. of Loch Doon; thence by way of Lochs Dee, Trool, Grennoch, Whinyeon, Ken, Lochinvar, etc. to Loch Dungeon, and finish our tour on the eastern slopes of the Rhinns of Kells.

Loch Doon is the largest sheet of fresh water in Scotland south of Loch Lomond. It is 5½ miles long by 1 mile wide in the widest part; the surface is 673 feet above sea level, and the maximum depth is 100 feet, but generally it is not over 50 feet deep. Its water is rather peaty, and its shores rocky or stony, with an occasional sandy bay. Its surroundings are almost treeless, although fig. 1 appears to belie this statement; at that place, however, (Portmark) there are a few trees near a shepherd's house. Everywhere the loch is surrounded by mountains and moors, the greater part of which are covered by grass-like associations of plants. The population of the district is extremely scanty; the only houses are those of shepherds or small farmers, and the total number of these will scarcely exceed a dozen throughout its whole drainage area. The scenery, as may be gathered from the foregoing remarks, is wild and lonely; yet the broad outlines of the loch, flanked by mountains picturesquely silhouetted, give it a grandeur peculiarly its own.

The shores and margins of this loch are, for the major part, entirely bare of aquatic vegetation. Indeed, the erosive power of the waves on the rocky margins allows no opportunity for the development of aquatic plants; and in the sandy bays that occur here and there, the same power, acting on the shifting sand, prevents any considerable growth of vegetation even in such places. Occasionally in pools situated on large rocks or in sheltered creeks a few specimens of Carex Goodenovii or Phalaris arundinacea may be seen. A few of the same species, with scattered specimens of Juncus lamprocarpus, J. acutiflorus, and J. supinus, occur on wet sandy places; and between the rocks, here and there, patches of Sphagnum cymbifolium or Fontinalis antipyretica may be observed, but always in small quantity. Nor do the littoral rocks bear any wealth of Bryophytes. Of the few that do occur, Eurhynchium rusciforme, Bryum alpinum, Blinda acuta, Orthotrichum rupestre, Grimmia apocarpa, Brachythecium rivulare, and Rhacomitrium aciculare are the most common. On the whole, it may broadly be stated that Loch Doon is destitute of either an aquatic or semi-aquatic marginal flora.

At a few feet above the normal water level quantities of lichens clothe the rocks and give the littoral a distinctive character. The most abundant of these lichens are Lecidea geographica, Parmelia omphalodes, and Sphærophoron coralloides. The first mentioned is so plentiful, and so completely does it overgrow the rocks, that many parts of the shore are for considerable distances coloured bright yellow, and the zone to which its abundance is restricted presents a remarkable appearance. This rocky zone is in reality at the ancient water level of the loch previous to a reduction of its level by about 7 feet some 150 years ago. This lowering of the level was brought about by the construction of a tunnel for the effluent instead of the natural outflow, for the double purpose of reclaiming land at the margin and admitting salmon to the loch. Why the Lecidea should be so abundant at the old water level I am unable to explain.

I dredged this loch in a great many places from end to end, but beyond an average depth of about 7 feet no living plants could be obtained from the bottom. Yet in suitable places the bottom from 2 to 7 feet deep often bears an abundant vegetation, which occasionally may be continued into the shallower water: Littorella lacustris, for example, is plentiful in a few sheltered sandy creeks. The extinction of the submersed Phanerogams at so shallow a depth as 7 feet is distinctly remarkable because it is not brought about, as in some cases (*ante*, p. 1015), by the discoloration of the water. Here the bottom can be seen at a depth of 7 feet when looking over the side of a boat, without the use of any apparatus beyond shading the eyes with one's hat in order to shut out the light reflected from the surface of the water. Reasoning, therefore, from similar cases of translucency, some vegetation should extend to a depth of 28 feet or more.*

It has already been indicated (p. 66) that the grass-like associations of plants which cover the moors and mountains have an influence upon the flora of the lochs in this Area. At the first consideration one would imagine that the influence exercised upon the bottom flora of a loch by the substitution of grass-like plants over the moors, instead of associations of Ericaceæ, would be that of less peat-extract getting into the water. Such, however, is scarcely the case, because the moors have an abundance of ancient peat below the grass, formed there previous to the development of the sheep-rearing industry. The influence is caused in a way one would little expect. In winter the dead leaves of the grass-like plants covering the moors, chiefly Molinia cærulea, which grows very luxuriantly here, but also Nardus stricta, Scirpus cæspitosus, etc., are blown or washed into the burns and drains, and are thence carried into the lochs. There, owing to

* *The Geographical Journal*, January 1908, p. 68.

the antiseptic action of the peaty water, these remains do not readily decay, but accumulate from year to year and become spread out over the loch-bottom in enormous quantity, and, of course, this stratum of dead grass, wherever it lies, prevents the growth of a bottom flora. The depth to which its influence extends varies somewhat in different lochs, and even in any one particular loch. In Loch Doon, at the south end, where the loch receives its principal supply of this grass from the rivers Gala Lane and Carrick Lane, it spreads over the bottom to within 5 feet of the surface of the water, and at other parts of the loch to about 7 feet below the surface. From these depths it is spread over the whole loch-bottom more or less. Even at a depth of 50 feet the dredge came up choked with this deposit, which in such deep water is almost black, but not of a particularly evil odour. The deposit of this substance in the loch must be the result of the accumulation of many years, through the process of decomposition being so slow in the peaty water. At Loch Stroan (p. 115) a large amount of such dead grass is washed upon the east shore by the winter floods (fig 24). Loch Stroan is a small, shallow loch, and in flood-time there is a very considerable current passing through it from the River Dee, so that a portion of the grass must be carried down the river into Loch Ken besides that which is deposited high upon its own shore. Yet, notwithstanding these losses, Loch Stroan has an abundant supply of this material on its bottom. In the neighbourhood of Loch Trool there is much less grass available, and the bottom flora of that loch extends to a depth of 16 feet (p. 112).

The submersed aquatics that flourish in the available zone about the margins of Loch Doon are as follows:—Subularia aquatica, Littorella lacustris, Isoetes lacustris, Heleocharis acicularis, Nitella opaca, Fontinalis antipyretica, Scirpus fluitans, Juncus fluitans. All the foregoing species are abundant with the exception of the last mentioned, which, although plentiful in the Gala Lane, is somewhat scarce in Loch Doon. Peplis Portula, a curious submersed form growing to a depth of 3 feet (p. 75), and Eurhynchium rusciforme, at a depth of 3 to 5 feet (p. 93), were both very abundant at the south end of the loch. The following were much less abundant:—Lobelia Dortmanna, Myriophyllum alterniflorum, Sparganium natans, Chara fragilis, var. delicatula, Batrachospermum moniliforme, and B. vagum. Bryophyta and Algæ other than those mentioned were extremely scanty, and the paucity of plants in the marginal zone has already been referred to. Figs. 1 to 3, with their respective descriptions, will afford additional information regarding the general features of this loch. The historic remains on the island in fig. 2 will perhaps not be without interest to some readers. The builders of this castle having constructed it centuries before the artificial

lowering of the surface of the loch, were under the necessity of providing its base with masonry capable of withstanding the waves in time of storm, and the superior construction of the lower portion is still well exhibited, although the foundations are partially destroyed.

On the south-west of Loch Doon there is a large and somewhat circular, elevated, treeless moor 3 or 4 miles in diameter, surrounded by mountains on every side and presenting the aspect of some huge amphitheatre in utter ruin. Rugged rock and deep bog vie with one another for possession of the space. Here a gurgling burn divides the combatants; there a broad lane * dashes over its rocky bed with foaming impetuosity; whilst ever and anon a slow, deep, sinuous river winds its labyrinthine course through some level stretch of moss, scarcely more stable than the river itself. Numerous lochs, characterised by stretches of coarse white sand intercalated here and there on their otherwise rocky or peaty shores, are sprinkled over this lonely and wild moor. In some of these lochs, flourish in great abundance two interesting aquatic plants that I have met with nowhere else in Scotland, namely, a truly aquatic form of Ranunculus Flammula (var. natans) and Potamogeton polygonifolius, var. pseudo-fluitans, already mentioned on pp. 72 and 84 respectively.

I shall first describe the general features of these lochs, and then give a list of the plants common to most of them. I was unable to obtain the use of a boat at any of the lochs hereabout, and am therefore not able to give any account of the bottom flora beyond the marginal zone, excepting what could be gleaned from an examination of the remains of such plants found upon the shores.

Loch Recar is one of the largest of the lochs on the above-mentioned moor. It is about a mile across in either direction, and has a very irregular outline. The water is somewhat peaty, but, considering the moorland situation, remarkably clear and bright. The shores are rocky or peaty, but, on the east side particularly, large bays are filled with coarse white sand, which results from the disintegration of the syenitic granite in which this loch as well as neighbouring ones is set. This sand is found chiefly on the eastern shores, in consequence of the erosive power of the waves caused by the prevailing westerly winds. The somewhat scanty vegetation is much more abundant on the western than on the eastern shores, saving that aquatic plants are much more plentiful in the long and narrow neck of water leading to the effluent on the east side than elsewhere in the loch.

Loch Macaterick is a mile south of the last-mentioned loch, and is about the same size; the outline also is very irregular. This loch is almost

* A stream is often termed a lane in this part of Scotland.

cut in twain by two promontories which jut out from opposite sides of the shore near its middle. Like Loch Recar, it has a long narrow arm leading to the effluent on the east side; there are also several small islands. The hill Macaterick rises boldly from the south shore; similarly, but less boldly, Maccallum rises from the south of Loch Recar. In the shores, water, and vegetation, this loch also resembles Loch Recar.

Loch Slochy is half a mile S.W. of Loch Recar, into which it drains. It is of some considerable area, but very shallow, and consequently is almost entirely overgrown with associations of marsh plants, some of which spread over the adjoining boggy moor, so that in many places one has difficulty in discovering where the water ends or where the shore begins. This loch is well on the way towards the formation of another of those deep bogs with which the district already abounds. The most abundant and dominant plants of the deeper marsh are Equisetum limosum and Phragmites communis, with which are mixed a few groups of Scirpus lacustris. Carex rostrata, Heleocharis palustris, Juncus lamprocarpus, and J. acutiflorus dominate the shallow margin, where Juncus effusus also occurs, but less plentifully. Littorella lacustris and Juncus fluitans appear to be the most abundant submersed plants.

Loch Ballochling is a small sheet of water having the same general features as Loch Recar, and is situated a mile north-east of it. It illustrates well the difference between east and west shores, caused by the prevailing westerly winds; the west side has an abundance of plants, whilst the east side consists chiefly of sandy bays almost without vegetation. Carex rostrata, C. filiformis, Potamogeton polygonifolius and its var. pseudo-fluitans are particularly abundant on the west side. Ranunculus Flammula, var. natans, is also very plentiful, especially in the affluent. Two hepatics not generally common on the shores of lakes grew luxuriantly—Pleurozia cochleariformis and Jungermannia inflata.

Loch Goosie is a small loch about a mile west of the last mentioned, and similar to it in general features. Its dominant plants are Phragmites communis and Potamogeton polygonifolius; the beautiful moss Pterogonium gracile was abundant on the dry rocks of the shore.

Loch Brecbowie is a small sheet of water about a mile N.W. of the last mentioned, and being about 1200 feet above sea level, is at a greater elevation than any of the foregoing lochs. It is prettily situated amongst hills, in a pass leading from Loch Goosie to Loch Bradan. Waterhead Hill rises immediately from its east side. The margin is sinuous, and the very narrow zone of shore between the water and the moor is rocky, stony, or sandy; its general features are otherwise similar to those of the preceding

lochs. Carex rostrata, Equisetum limosum, Phragmites communis, Lobelia Dortmanna, and Littorella lacustris are its most abundant plants.

Descending northwards from Loch Brecbowie for about a mile, one comes to Lochs Bradan and Lure, which are connected together by a narrow channel. Loch Bradan is nearly a mile long, with a maximum breadth of a quarter of a mile. It is very shallow, the greatest depth being 8 feet. Loch Lure is about a third the length of its neighbour, with a maximum depth of 7 feet. The elevation of these lochs is nearly 1000 feet above sea level. Except for a plantation of conifers on the south of Loch Lure, about the ruins of Craiglure Lodge, these lochs are entirely surrounded by a treeless, grassy moor. Their shores are rocky or stony, and the water is slightly peaty. On an island in Loch Bradan are the ruins of a small castle, but there is now little more to be seen than what is presented by a stone sheep-enclosure. Both lochs have an area of marsh at the west end, but the vegetation is dwarfed, and, like that in the water, consists of the same species as grow in the lochs previously mentioned.

Passing over the hill by way of the little pool, Loch Duh, which contains nothing of particular interest, one crosses the Girvan Water and enters the desolate moor in which are situated Derclach Loch and Loch Finlas, which are connected by a short and narrow channel, and together form the source of the water supply for Ayr. Derclach Loch is a very narrow sheet of water about half a mile long and not more than 12 feet deep. Loch Finlas is also very narrow and shallow throughout the greater part of its length, which is $1\frac{1}{2}$ miles. It widens at each end, and is therefore dumb-bell-shaped. Its surface is 830 feet above sea level, being 7 feet less than Derclach Loch. These lochs have a narrow shore, which is either peaty, stony, or rocky. Their water is clear, but slightly peaty. They have a scanty vegetation, and present nothing of botanical interest beyond a number of plants common to the preceding lochs.

The plants more or less common to all the foregoing lochs, excluding Loch Doon, are as follows:—Lobelia Dortmanna, Isoetes lacustris, Littorella lacustris, Subularia aquatica, Chara fragilis, var. delicatula, Nitella opaca, Sparganium natans, Scirpus fluitans, Ranunculus Flammula, var. natans, Castalia speciosa and its var. minor, Juncus fluitans, Myriophyllum alterniflorum, Potamogeton natans, P. polygonifolius and its var. pseudo-fluitans, Carex Goodenovii, C. filiformis, C. rostrata, Menyanthes trifoliata, Hydrocotyle vulgaris, Equisetum limosum, Scirpus lacustris, Juncus lamprocarpus, J. acutiflorus, J. effusus, Heleocharis palustris, H. multicaulis, Phragmites communis, Caltha palustris and its var. minor, Eriophorum polystachion, Ranunculus Flammula and its var. scoticus, Cardamine pratensis, Batracho-

spermum moniliforme, B. vagum, Ulothrix æqualis, Zygogonium ericetorum. Numerous Bryophytes were also frequently abundant on the peaty shores, or clothing the rocks at the margins.

Proceeding from the head of Loch Doon towards Loch Enoch by way of the glen drained by the Gala Lane, and lying between the two mountain ranges, of which Merrick, on the west, and Corserine, on the east, are the highest points, one passes over the site from which Loch Doon obtains its chief supply of Molinia cærulea. Here is a stupendous bog 5 miles long by a mile or so wide, almost everywhere treacherous to walk upon, and in some places quite impassable.* A characteristic feature of this bog is the luxuriant growth of Molinia cærulea, which is often about 18 inches high. The same grass also dominates the sides of the hills, but there it is much shorter.

After receiving numerous tributary streams, the Gala Lane for the last three miles of its course is of some considerable size, and only in a few places can it be crossed dry-shod by jumping with alacrity from rock to rock across its bed. Sometimes it passes swiftly down a rocky incline; generally, however, it meanders its tortuous course, slow, deep, and wide. In such places flourishes a vegetation abundant in quantity but poor in variety; or its bottom may be covered with dead grass like that of Loch Doon, in which case no living vegetation occurs. Carex rostrata forms a marginal zone of varying width, and in the water Potamogeton natans, P. polygonifolius, Castalia speciosa, and Juncus fluitans are the ruling, and in fact almost the only, species. The last particularly is so abundant that the slow, deep river appears in places full of it, yet in Loch Doon it is scarce.

Looking from the monotonous and treacherous Molinia-covered glen, the scenery is indeed unique. On the east, Carlin's Cairn, Meikle Craigtarson, Millfire, and Meikle Millyea thrust their grassy flanks, with here and there a steep gray scree of Lower Silurian rock, into the strath below. On the west, Mullwharchar, Dungeon Hill, Craignaw, and Craiglee plunge their rocky and precipitous shoulders of syenitic granite boldly into the insidious bog of the glen. This untamed grandeur is further enhanced by numberless erratic boulders perched on the western sky-line in fantastic variety (fig. 4). On the east, the great ice-sheet has completed its work of rounding off the mountain tops and giving their flanks a symmetrical slope. On the west, glaciation has but half completed its task because of the harder igneous rock, and here one sees to perfection the battle-ground whence one of the mightiest of nature's gladiators has been driven before completing his

* After passing the watershed at the Dry Loch of the Dungeon, the glen continues for another 5 miles, down to Loch Dee.

victory—simply by the softening of the evening breeze. From the strath one climbs to Loch Enoch by way of Pulskaig Burn, nor can I pass this without a word. Imagine some cyclopean staircase,

> "Piled by the hands of giants
> For godlike kings of old,"

lifted into the hypothetical ether, and dropped pellmell into a gully on the face of a rocky and precipitous mountain, and you have some idea of Pulskaig Burn. But it appeals to a botanist by reason of the luxuriant splendour of its rare Bryophytes, as well as by the pleasing scenic effect. Here is a riser of the gigantic stair 20 feet high, now tilted to an angle of 45 degrees, over which the pellucid water swiftly glides in rippling dance to music of its own begetting. Scoured and ground by the rush of gravel in time of spate, the face of the granite is perfectly clean and sparkles in the snnlight. But look to right and left! From that ruined balustrade hang masses of yellow-green or purple Mylia Taylori, interrupted with cushions of Trichostomum tortuosum or Rhacomitrium sudeticum, and, sheltering in the dampness below, the delicate Cephalozia bicuspidata, Heterocladium heteropterum, or Jungermannia Floërkii cover the granite with rare luxuriance. Yon purple-black rock, dripping with water, owes its colour to Andreæa alpina, and above it the bright green capital of a half-exposed column is due to the exquisite Metzgeria hamata, whilst the plinth is wreathed in a purple mass of Pleurozia cochleariformis. Look again! This ruined corbel still supporting the floor of a balconette, from which beads of water drip into the current below, is almost hidden by the wealth of Hypnum molluscum and Breutelia arcuata, whilst the floor itself is covered with a domed mass of the beautiful Sphagnum rigidum. Scrambling upwards over a few steps that have scarcely been misplaced, we come to a semicircular excavation like a reversed ambo. The floor has been hollowed into a deep cavity, and the process of carving is still continued by the whole body of the burn dropping into it in foaming cataract from 8 or 9 feet above. The spray-splashed margins of this agitated pool are carpeted with masses of Blindia acuta, Hypnum scorpioides, Marsupella aquatica, and Campylopus atrovirens, whilst the surrounding chinks and crannies are gray with Anthelia julacea or green with Philonotis fontana. The surrounding drapery is of richest form. Such Diplophyllum albicans! Such Nardia emarginata! But chiefest of all are the glorious masses of Nardia compressa and Scapania undulata that cover the dripping rocks and hang in festoons below. What a list of Bryophytes a botanist might write after a day or two spent here! But clambering on, we pass here a foaming cascade,

there a gentle pool, reflecting as a silver mirror the colours of the marginal vegetation—a thing of exquisite beauty—and ever and anon a black, deep hole that almost induces a shudder as the eye catches the faint light glimmering from cavernous recess of horrid rock. Pushing upwards, we reach the top, and here what a sight awaits! Great masses of ancient peat, worn into deep gullies by the storms of ages, almost bare of vegetation, black and grim; and lying beyond, Loch Enoch, with its shores of silver sand, and its clear, sparkling water reflecting the adjacent mountains like a speculum.

Loch Enoch is 1617 feet above sea level, and is the most elevated of a series of unique alpine lochs situated in a singularly rugged mountain district. It occupies a very wind-exposed position, which probably accounts for the sand of its shores being finer than that of other lochs in the district. Its outline is very irregular; and there are several small islands, the largest of which has upon it a small pool, hence the name Loch-in-loch, of which its better-known name—Loch Enoch—is said to be a contraction. There are several bays that have a shore of beautiful white sand produced from the disintegrated syenitic granite of which these mountains are largely composed. Nearly all the lochs of this district possess similar shores of white sand, but that of Loch Enoch is finer than any other, and is prized above all by shepherds, far and near, for the purpose of sharpening their scythes, although those living in this district frequently use that from the nearest loch for the same purpose. The scythe is used for the purpose of cutting the uncultivated Molinia cærulea, called "bog-hay" or "blow-grass" of the moors, for feeding the sheep and cattle during winter, the rough or boggy nature of the ground excluding the use of the modern mowing-machine for this purpose. To sharpen a scythe, a strip of wood about 18 inches long by 3 inches wide is smeared with butter, which is then sprinkled with sand, and used in a similar way to the ordinary whetstone. Others stick the sand to the wood with glue; the latter, however, has to be purchased, whilst the former is a home product of no monetary value.

The shores of Loch Enoch, with the exception of the sandy bays, are rocky, and the water is exceptionally clear and sparkling, although slightly peaty. The flora is very poor in species. On the west side Sparganium natans is abundant in bays; there are also several small associations of Carex rostrata in bays on the west and north sides. Isoetes lacustris, Lobelia Dortmanna, and Littorella lacustris carpet the bottom in places. Juncus fluitans is very abundant, whilst Myriophyllum alterniflorum is scarce; Batrachospermum vagum is abundant, and in some places the submersed rocks are thickly covered with Zygogonium ericetorum or Nardia compressa. The littoral Phanerogams, besides those already mentioned,

are extremely scanty, there being merely a few plants of common species here and there. Amongst numerous Bryophytes that were abundant on the littoral the following may be mentioned:—Mnium hornum, Trichostomum tortuosum, Rhacomitrium gracilescens, Jungermannia ventricosa, J. Floërkii, Anthelia julacea, Nardia emarginata, N. compressa, Pellia epiphylla, Diplophyllum albicans, Hypnum scorpioides, etc. Figs. 5 and 6 represent this loch, with the adjacent mountains Merrick and Mullwharchar.

Loch Neldricken.—Proceeding a few hundreds of yards to the south-east of Loch Enoch, one comes to a narrow ridge of rugged rock connecting Dungeon Hill with Craignaw (fig. 9). From this spot, called the Nick of the Dungeon, an excellent bird's-eye view is obtained of Lochs Neldricken and Valley (fig. 7). These lochs are similar in general features to Loch Enoch,—they have clear, brilliant, slightly peaty water, white sandy bays, shores otherwise rocky, and very irregular outlines. The vegetation is also similar, and usually scanty. Loch Neldricken differs from Loch Enoch by having considerable associations of Equisetum limosum, more Myriophyllum alterniflorum, and a few specimens of Glyceria fluitans. On the N.W. side there is a very regularly shaped "murder-hole" (*ante*, p. 1014, figs. 84 and 109), formed in a somewhat circular bay or arm of the loch, the shallow margin of which affords a suitable situation for sedge-like plants. The bottom, I presume, sinks suddenly and regularly like a basin, at some distance from the shore, to a greater depth than these plants can accommodate themselves to; consequently they end abruptly, and present an even circular outline at the place where the water is too deep for further advance (fig. 8). The plants surrounding this "murder-hole" are in three well-marked zones as follows:—Adjoining the shore, Carex rostrata, then a zone of Equisetum limosum, followed by a narrow zone of a plant which, from distant examination with a telescope, was apparently a large form of Carex rostrata, but as specimens could not be obtained, it was impossible to exactly identify the species. Beyond those enumerated, the marginal Phanerogams are very sparse. In many places the sandy shores are covered with patches of Nardia scalaris and Anthelia julacea, and the littoral rocks also are frequently overgrown with Bryophytes common to the district. Fig. 9 gives a representation of a portion of this fine loch from the south shore.

Loch Valley, as already indicated, is adjacent to the last mentioned and receives its outfall. The physical and botanical features are similar to those of the adjacent lochs already described, but it has in addition Carex filiformis and Menyanthes trifoliata. There are several associations of Carex rostrata, and Hypnum fluitans is very abundant on a boggy portion of the shore, whilst Anthelia julacea and Pleurozia cochleariformis

are features of other parts. Submersed rocks are frequently covered with such Algæ as Batrachospermum vagum, Ulothrix, Zygogonium, etc. Figs. 7 and 10 illustrate this loch and its surroundings.

Loch Narroch is quite a small circular sheet of water at the east end of Loch Valley. In general features it closely resembles the neighbouring lochs. The rocks of the margin were covered with remarkable quantities of Algæ, chiefly of the genera Batrachospermum, Ulothrix, Zygogonium, Zygnema, and Mougeotia.

Round Loch and Long Loch of Glenhead are both to the south of Loch Valley; they are, however, at a lower elevation and smaller. These lochs are very bare of plants, and are otherwise similar to those recently described. They are not of sufficient botanical interest, so far as I could glean without a boat, to merit further discussion. Fig. 11 illustrates these lochs, and the treeless, wild mountains around them.

Loch Dee, which is $1\frac{1}{4}$ miles long by $\frac{3}{4}$ mile wide, and 36 feet deep, is the largest of this series of lochs. The outline is irregular, a peninsula from the south shore and another from the north jutting out so as almost to divide the loch. It is situated at an elevation of 739 feet above sea level, amidst wild and lonely scenery, about 5 miles south of Loch Enoch. Although at a lower elevation, it is similar in general features to that and the neighbouring lochs, excepting that the sand of its shores is not white but of a brownish tinge; the water also differs in being somewhat more peaty. Away from the sandy bays the shores are mostly rocky. The flora is extremely poor, and being composed of the same species as occur in the previously mentioned lochs, need not be specially described. A boat which is kept here was out of repair during my visit, but careful attention to plants washed up on the shore revealed nothing uncommon to the district. Bryophytes abound on the shores and on the exposed rocks. Very conspicuous also are the lichens which cover the numerous rocks by the shore; the most plentiful of these are—Platysma glaucum, Cetraria muricata, Parmelia lanata, P. omphalodes, Alectoria jubata, Sphærophoron coralloides, and Lecanora tartarea. Fig. 12 affords a view of the loch, chiefly its S.W. portion.

Dry Loch, Round Loch, and Long Loch of the Dungeon.—These are small sheets of water, each a few hundreds of yards long, and they are all connected by a stream which first flows out of the Dry Loch, that being the highest of the three; this stream ultimately becomes the River Dee. Their shores are stony or peaty, and their water is slightly peaty but clear. These lochs are situated at the highest and wildest part of the glen (p. 106), between Dungeon Hill and Craignaw, and the scenery around is extremely

fine. They have no feature of botanical interest beyond a number of such plants as are contained in the other lochs of the neighbourhood, and such need not, therefore, be independently described.

The plants that flourish in and about this series of lochs, from Loch Enoch to the Dungeon Lochs, are as follows:—Lobelia Dortmanna, Littorella lacustris, Isoetes lacustris, Juncus fluitans, Myriophyllum alterniflorum, Sparganium natans, and Potamogeton polygonifolius, all abundant; Subularia aquatica, Callitriche hamulata, Utricularia vulgaris, U. intermedia, Scirpus fluitans, Sparganium minimum, and Castalia speciosa, var. minor, all more or less scarce. Characeæ are apparently scarce. Batrachospermum, Ulothrix, Mougeotia, Zygnema, Dickieia, etc., are frequently very abundant. Heleocharis palustris, Eriophorum vaginatum, E. polystachion, Carex rostrata, C. Goodenovii, C. filiformis, and Equisetum limosum are all abundant. Menyanthes trifoliata, Juncus effusus, J. lamprocarpus, J. acutiflorus, J. supinus, Carex binervis, C. flacca, var. stictocarpa, and dwarf forms of Ranunculus Flammula, Caltha palustris, Cardamine pratensis, and Hydrocotyle vulgaris are all more or less scarce. The following are some of the most conspicuous Bryophytes that occur, either in the water or clothing the rocks of the shores. A number are distinctly terrestrial forms, yet they constitute such a feature of the shores and are so inextricably associated with the loch that an enumeration of the flora would be incomplete were they excluded:—Sphagnum sp. abundant on boggy shores, Blindia acuta, Grimmia apocarpa, var. rivularis, Rhacomitrium aciculare, Fontinalis antipyretica, Aulacomnium palustre, Pterygophyllum lucens, Philonotis fontana, Brachythecium rivulare, Hypnum fluitans, H. revolvens, H. falcatum, H. scorpioides, H. commutatum, H. cuspidatum, H. uncinatum, Dicranella squarrosa, Scapania undulata, S. sub-alpina, Nardia compressa, N. emarginata, and N. scalaris. The foregoing species occur more or less in water, and the following in drier situations:—Dichodontium pellucidum, Dicranella heteromalla, Dicranum fuscescens, D. Scottianum, Grimmia apocarpa, Rhacomitrium lanuginosum, R. heterostichum, Hedwigia ciliata, Trichostomum tortuosum, Orthotrichum rupestre, Bryum alpinum, Mnium punctatum, Heterocladium heteropterum, Pterogonium gracile, Plagiothecium undulatum, P. elegans, Thuidium tamariscinum, Hypnum cupressiforme, Frullania tamarisci, Pleurozia cochleariformis, Anthelia julacea, Diplophyllum albicans, Mylia Taylori, Pellia calycina, P. epiphylla, etc.

Loch Trool is 246 feet above sea level, and is $1\frac{1}{2}$ miles long by $\frac{1}{4}$ mile wide, with a maximum depth of 55 feet. It is approached from Loch Dee through a narrow, rugged, and trackless pass about 3 miles long, This loch affords a splendid piece of highland scenery, which is probably

unequalled south of Perthshire. Mountains rise from the shores almost throughout its whole length, their lower slopes being clothed with either coniferous (fig. 15) or deciduous-leaved trees. This loch somewhat resembles Loch Oich (*ante*, fig. 40), but is smaller. The water is clear, but slightly peaty. At the west end the margin is formed chiefly by peaty banks; elsewhere, except at the east end, which is flat and boggy, the shores are stony, rocky or sandy, or the steep hillside enters the water directly without the intervention of a shore. The upper portions of the adjacent hills, above the tree zone, are, where the rock is not bare of plants, mostly clothed with bracken and grass associations (fig. 15). The rank growth of the latter is here, however, much restricted, so that, in comparison with Loch Doon there is but a small quantity of dead vegetation available for covering the loch-bottom. Having noticed the relative scarcity of rank Molinia about the neighbourhood of this loch, I was anxious to discover to what depth aquatic plants flourished at its bottom. Careful dredging revealed the fact that the living vegetation here extends to a depth of 16 feet, but at greater depths the dead remains of Molinia, Carex, etc. cover the bottom, and no plants flourish within this zone. The flora of the loch is poor in the number of species, but some of them occur in great abundance. About the west end, at which is the effluent, the loch is narrow, shallow, and bears a considerable marsh vegetation (fig. 14). Beds of Carex rostrata are abundant, and on drier parts of the boggy shore these are gradually or suddenly exchanged for common bog plants. At the east end the affluent passes through an extensive delta, which is overgrown with marsh plants common to the district (fig. 13). Along the shores, Equisetum limosum is abundant, and here and there occur belts of Phragmites communis and Menyanthes trifoliata, whilst Juncus acutiflorus and J. effusus are both well represented. The shore rocks, which are not a particular feature of this loch, bear a number of common Bryophytes. The submerged plants are as follows:—Littorella lacustris, Lobelia Dortmanna, and Isoetes lacustris, all very abundant, and forming a dense bottom-carpet. The last mentioned extends to a depth of 16 feet. Utricularia vulgaris is abundant to a depth of 8 feet. Potamogeton pusillus and P. obtusifolius are abundant to 10 feet deep. Potamogeton polygonifolius, a few plants only. Juncus fluitans is extremely abundant, and Batrachospermum moniliforme grows on stones by the shore. Beyond the plants mentioned, the flora is extremely sparse, consisting merely of a few specimens of common species.

Loch Grennoch, by Cairnsmore of Fleet (not to be confounded with Loch Grenoch or Woodhall Loch near Laurieston), is a fine sheet of water 2 miles long by $\frac{2}{3}$ mile wide, at an elevation of 690 feet above sea

level. It is 68 feet deep, and is situated in a somewhat open and wind-exposed position among grass-covered mountains. Its surroundings are treeless, except for the plantation about a small fishing lodge at the south-west end, below Craigronald, which rises immediately from the shore of the loch. The water is very clear, and but slightly peaty. The shore is rocky, with the exception of numerous bays of white syenitic sand. The exposed littoral rocks bear a number of common Bryophytes and lichens, but to no great extent, the most abundant being Hyocomium flagellare, Hedwigia ciliata, Grimmia apocarpa, and Rhacomitrium aciculare. The aquatic flora is very poor in species, and the semi-aquatic plants of the shore are also poor in species and in numbers, the greater portion of the shore being almost devoid of such plants. At the north end there are associations of Phragmites communis, Juncus lamprocarpus, and J. acutiflorus; a little bay on the east side has also a quantity of the first mentioned. On the north-west margin there are associations of Carex rostrata and Equisetum limosum. Groups of Heleocharis palustris and dwarf specimens of Ranunculus Flammula occur here and there all around the loch. Juncus alpinus and a dwarf form of Heleocharis palustris (p. 85) grow upon the drier parts of some of the sandy bays. In certain of these bays the copious sand is blown up into miniature dunes capped with Calluna, etc., resembling those of a sandy seashore on a small scale. The bottom is for the most part very rocky, but there are considerable areas of sand or gravel extending from the margin to a depth of 8 or 10 feet. These areas are usually more or less carpeted with Littorella lacustris, Lobelia Dortmanna, and Isoetes lacustris, most of which are more or less overgrown with Algæ, chiefly Batrachospermum vagum, Ulothrix æqualis, Binuclearia tatrana, etc. (p. 96). These plants, however, bear no external evidence of injury by the Algæ, although Nardia emarginata and Scapania undulata, both of which grow abundantly on submersed rocks, were much injured by the dense growth of such epiphytes upon them. Fontinalis squamosa and F. antipyretica occur in abundance upon the submersed rocks of the margin from the surface to 3 or 4 feet deep. In many places Juncus fluitans is extremely abundant from 2 to 5 feet deep. In some parts, particularly at the south end, Sphagnum subsecundum (p. 90) and Heterocladium heteropterum (p. 93), mixed with Scapania undulata, were abundant at the bottom from 2 to 8 feet deep, an uncommon situation for such plants. They may have been brought into the loch by one of the burns in time of spate, and then become adapted to the submersed environment.

I could obtain no living plants in this loch beyond a depth of about 10 feet, not because of the presence of vegetable detritus, nor of the opacity of

the water, but because in deeper and often in shallower water too, the bottom is very rocky. I have noticed in many lochs that a rocky bottom is nearly always destitute of the higher plants, that is, when the bottom could be seen or felt with a pole having teeth at the end, or with a weight attached to a line. By bumping such instruments over the bottom of a loch the vibrations carried to one's hand up the wood or cord give an indication of the constitution of the bottom—mud, sand, gravel, rock, etc. Dredging over a rocky bottom is, of course, impossible, leaving out of the question the certainty of losing the apparatus. The reasons for a rocky bottom within the photic zone being either devoid of plants or supporting very few are probably—(1) want of a suitable substratum in which the plants may root; (2) because of the scarcity of plants, the refuse-eating organisms at the bottom are able to deal with all the organic remains that reach them, so that nothing is left but the excrement of such creatures; this, in turn, is attacked by bacteria, and by these means the rocky bottom is kept clean instead of being covered with mud. A general view of this loch is given in fig. 16.

Loch Fleet is a somewhat oval sheet of water situated about a mile east of Loch Grennoch, and surrounded by treeless hills, excepting on the south-east. It is about $\frac{1}{3}$ mile long by $\frac{1}{4}$ mile wide, 56 feet deep, and 1113 feet above sea level. The margin is rocky, and there is very little shore suitable for the development of littoral Phanerogams. The water is clear and but slightly peaty. The scanty flora is restricted to the common types found at Loch Grennoch.

Loch Skerrow is situated amongst wild, rocky, moorland scenery, 4 miles east of Loch Grennoch at an elevation of 414 feet above sea level. It is a shallow, somewhat triangular loch $\frac{3}{4}$ mile long with a very rocky shore (fig. 18), and clear, slightly peaty water. Its maximum depth is 33 feet, and the bottom is mostly covered with rocks which frequently rise above the surface of the water. The larger of these island-rocks are capped with vegetation of the moorland type, such as Calluna vulgaris, Vaccinium Myrtillus, etc. (fig. 17). More numerous are the rocks which rise to just below the surface of the water. These necessitate caution in navigating a boat, and obviously such a rocky bed greatly hinders dredging operations. Sandy portions of the bottom to a depth of 12 feet bore an abundant vegetation, but of a limited variety; otherwise there was little to be noted, excepting at the margins and in shallow, sheltered bays. The submersed plants were—Littorella lacustris, Lobelia Dortmanna, Isoetes lacustris, Subularia aquatica, Juncus fluitans, Myriophyllum alterniflorum, Nitella opaca, Chara fragilis, var. delicatula, Fontinalis

antipyretica, F. squamosa, and Potamogeton polygonifolius, all abundant; particularly so at the S.W. side were Potamogeton polygonifolius, Castalia speciosa, and Nymphæa lutea. The semi-aquatic plants of the littoral zone were—Carex filiformis, very abundant, and replacing C. rostrata to a considerable extent (fig. 19), Carex Goodenovii, Heleocharis palustris, Hydrocotyle vulgaris, Ranunculus Flammula, Juncus lamprocarpus, J. effusus, and Caltha palustris, all very abundant, whilst the following were less abundant—Carex rostrata, C. flava, var. lepidocarpa, Phragmites communis, Menyanthes trifoliata, Equisetum limosum, Galium palustre, and Stachys palustris. The following Algæ were very abundant—Batrachospernum vagum, species of Zygnema, Spirogyra, and Ulothrix, particularly Ulothrix æqualis, var. catæniformis, with which submersed Phanerogams, etc. were thickly covered. The water of some of the little creeks was tinged with Oscillatoria. Masses of gelatinous diatoms such as Dickieia were abundant at the S.W. side, where also Sphagnum cuspidatum was plentiful. Many of the shore rocks were covered with a luxuriant growth of lichens, particularly with Parmelia omphalodes. Noteworthy also was the abundance of Bryophytes upon the littoral rocks and in damp places. The most important of these were—Blindia acuta, Fontinalis antipyretica, F. squamosa, Dicranella squarrosa, D. heteromalla, Rhacomitrium aciculare, Hyocomium flagellare, Pterygophyllum lucens, Heterocladium heteropterum, Sphagnum cuspidatum, and other species, Hedwigia ciliata, Dicranum Scottianum, D. fuscescens, Minium hornum, M. punctatum, Scapania undulata, S. purpurescens, Marsupella emarginata, Pellia calycina, P. epiphylla, Kantia trichomanes, var. aquatica, Chiloscyphus polyanthos, Diplophyllum albicans, Cephalozia bicuspidata, etc.

Loch Stroan.—The Airie Burn, which flows from the north-east corner of Loch Skerrow, joins the River Dee after flowing northwards for about 2 miles (figs. 20–22). Thence the Dee, impetuous nearer its source, slowly meanders eastwards, deep and wide, through an alluvial flat for about a mile and a half, and then it flows into Loch Stroan.

This loch is about 230 feet above sea level, and its north-west shore consists chiefly of sandy or muddy flats, the result of detrital matter brought into it by the River Dee; this is continuous with the extensive alluvial flat through which the river flows before entering the loch, and is overgrown near the water with Carex rostrata, etc. Farther away from the loch the drier portions are covered with moorland herbage of the grass-like type—Molinia cærulea, Scirpus cæspitosus, Deschampsia cæspitosa, etc. Elsewhere the shores of the loch are stony or rocky, with a gentle inclination, and merge gradually into grassy or heathery moor (figs. 23, 24). Although

slightly peaty, the water is clear and bright, so that vegetation at the bottom may be observed through a depth of 10 feet. In many places the stones, etc. from the bottom were thickly incrusted with the green sponge, Spongilla fluviatilis, and some of the sandy areas were abundantly strewn with the large mussel Anodonta cygnea. The Dee is, indeed, a particularly favourable habitat for this mollusc, and some of the country folk add a few pounds annually to their incomes by the sale of the pearls which they frequently find in them. Subsequently, when visiting Carlingwark Loch at Castle-Douglas, I found the sandy-muddy bottom covered in places with enormous quantities of this mussel, some specimens being 7 inches long. Directing the attention of my boatman to this fact, he determined to take the first opportunity for a pearl hunt, having learned the method of search from a resident in the neighbourhood of Loch Stroan, and incited also by the knowledge that the latter had sold a few pearls for £6. His chance soon arrived, as I had perforce to give him a holiday, because I became *hors de combat* from the stings of insects, chiefly clegs, which caused such swelling to my face that I was almost blinded for two or three days. Repairing to the loch, hundreds of mussels were brought ashore by this "worthy," but, to his utter disgust, after a whole day's labour, not a single pearl was found. It may be that the different conditions prevailing in Carlingwark Loch do not readily induce the formation of pearls in the bivalves.

The bottom of Loch Stroan is to a great extent sandy or muddy, but no living vegetation occurs at a greater depth than 20 feet, as one might expect would be the case from a consideration of the clearness of the water. The reason for this is that, beyond depths of from 15 to 20 feet, the loch-bottom is covered with the remains of grass-like moorland and marsh vegetation, chiefly those of Molinia, Carex, and Scirpus, brought into the loch by winter floods, as at Loch Doon (p. 101). The dead remains do not come so near the surface here as at Loch Doon because of the scour caused by the River Dee in flood-time. In this case the bulk of such material is derived from the flat, marshy ground, extending, as already mentioned, from the west shore of the loch. Fig. 24 shows a great bank of this dead substance deposited by winter floods high above the normal water level.

The principal plants at this loch are as follows:—Littorella lacustris, Lobelia Dortmanna, Subularia aquatica, Isoetes lacustris, Apium inundatum, Myriophyllum alterniflorum, Scirpus fluitans (fig. 25), Potamogeton polygonifolius, P. natans, Sparganium natans, Glyceria fluitans, Fontinalis antipyretica, Phragmites communis, Heleocharis palustris, Equisetum limosum, Juncus effusus, Carex rostrata, C. Goodenovii, C. flacca, and Ranunculus Flammula, all the foregoing being abundant. Callitriche

hamulata is abundant, sometimes with floating rosettes of leaves. Nuphar pumila is abundant in 8 to 10 feet of water on the south-west side, its semi-transparent submersed leaves being copiously produced at such depths. Juncus fluitans is extremely abundant in the shallow water at the north-west side. Scirpus lacustris (fig. 23) is very abundant, producing at depths of from 3 to 5 feet the grass-like submersed leaves in great luxuriance. Carum verticillatum, a characteristic marsh plant of this district, is abundant (fig. 27). Hypericum elodes (fig. 26) is scarce; Hydrocotyle vulgaris, Epilobium tetragonum, and Galium palustre occur on marshy ground. Nitella opaca, Chara fragilis, var. delicatula, and Ranunculus aquatilis occur, but none of them is abundant. Besides Batrachospermum, the filamentous Algæ were scarce; neither were the littoral rocks and banks conspicuous with Bryophytes, a few of the common forms only being observed to be abundant. [On boulders on the east side of the loch the rare moss Grimmia commutata may be found.—J. M'A.].—Juncus fluitans and Scirpus fluitans, both so plentiful here, are much alike in the barren state and easily confounded, but frequently they can be distinguished at a glance when growing, because the Juncus is usually slightly reddish, whilst the Scirpus is always bright green, without a trace of red.

Loch Whinyeon is somewhat circular in outline, and about ½ mile in diameter. It occupies an exposed position over 700 feet above sea level, 3 miles north from Gatehouse-of-Fleet. The water, which has a maximum depth of 33 feet, is clear, and but very slightly peaty. The shore, which is everywhere stony or rocky, consists chiefly of broken shale (Upper Silurian), the beds of which are frequently very highly inclined. The flora of the shore as well as of the water is extremely poor. The most interesting plants noticed were Alisma ranunculoides, in small quantity, but luxuriant, and a very dwarf form of Chara contraria (p. 90), of which the Messrs Groves, to whom specimens were submitted, write—"We have no specimens exactly like it." Rhynchospora alba grows on boggy patches of the shore, to which it has evidently strayed from the adjoining moor, where, in places, it is the dominant plant. The following Bryophytes were abundant upon the shore:—Sphagnum subsecundum and its var. contortum, Mnium punctatum, Dichodontium pellucidum, Bryum alpinum, B. bimum, Dicranella squarrosa, Trichostomum tortuosum, Hypnum commutatum, H. revolvens, H. scorpioides, H. cuspidatum, H. cordifolium, H. cupressiforme, Grimmia apocarpa, Rhacomitrium aciculare, R. lanuginosum, Fissidens adiantoides, Jungermannia bantriensis, J. pumila, Scapania undulata, Nardia emarginata and its var. aquatica. The other plants noticed were not in dense, wide-spreading associations as frequently happens, but more or

less scattered; they are as follows:—Littorella lacustris, Lobelia Dortmanna, Myriophyllum alterniflorum, Juncus fluitans, Potamogeton lucens, P. natans, Glyceria fluitans, Heleocharis multicaulis, H. palustris, Equisetum limosum, Carex rostrata, C. Goodenovii, C. flacca, var. stictocarpa, C. flava, var. minor, Juncus effusus, J. conglomeratus, J. lamprocarpus, Ranunculus Flammula, Mentha sativa, Eriophorum polystachion, and Hydrocotyle vulgaris.

Lochenbreck Loch has a rhomboidal outline, each side being about $\frac{1}{4}$ mile long. It is situated at an elevation of 651 feet above sea level, amongst the hills, about 7 miles N.N.E. from Gatehouse-of-Fleet, and has the characteristic features of a bare highland loch, modified by a plantation of coniferous trees on its eastern shore. The shores are stony, and the water, which has a maximum depth of 15 feet, is clear, but slightly peaty. The flora is of the ordinary type, excepting an abundance of Heleocharis multicaulis, some of which, growing in water 6 to 12 inches deep, had floating leaves. The western shore has a thin zone of Phragmites communis and associations of Carex rostrata, whilst the following grow not only there, but some of them at other parts of the loch as well:—Lobelia Dortmanna, Littorella lacustris, Isoetes lacustris, Juncus fluitans, Heleocharis palustris, Castalia speciosa, Juncus lamprocarpus, J. bufonius, Ranunculus Flammula, Juncus supinus (erect form 6 inches high), Caltha palustris, Sparganium natans, and Potamogeton polygonifolius. A number of common Bryophytes occur upon the shores, Sphagnum acutifolium being particularly abundant in some of the wet places.

Woodhall Loch, or Loch Grenoch, is 2 miles N.E. of the last mentioned. It is nearly 2 miles long by $\frac{1}{4}$ mile broad, at an elevation of 173 feet above sea level. Being somewhat wind-sheltered by low hills, and surrounded by meadow, grassy moor, or deciduous wood, it presents the general features of a lowland loch, saving that its water, which has a maximum depth of 49 feet, is slightly peaty. Here and there a gravelly bay occurs, but frequently the moor or meadow land abuts upon the water without the intervention of a shore. Where a strip of shore does occur, it is narrow, stony, and frequently covered with Juncus lamprocarpus and J. acutiflorus. Being provided with a wide but shallow outflow, and fed only by small streams, the level of this loch has but little rise and fall, because in wet weather the water readily escapes, and in a dry season the level is maintained by the shallow effluent. The west side has a reedy or sedgy margin, almost continuous throughout its length, but on the east side the reeds are mostly restricted to the bays. At either end there are large associations of Equisetum limosum, and at the north end the specimens of this plant are very large, rising 3 or 4 feet out of water 6 feet

deep. The floor of this loch, from a depth of about 8 feet to the deepest part, is covered with the dead remains of vegetation, which prevents the growth of plants upon that portion of the bottom, as at some other lochs previously mentioned. From this zone of dead material to the margin, the bottom in many places is carpeted with Lobelia Dortmanna and Littorella lacustris. The following plants also occur here abundantly:—Nymphæa lutea, Castalia speciosa, Potamogeton lucens, P. natans, P. polygonifolius, a large Nitella at a depth of from 6 to 8 feet, respecting which the Messrs Groves, to whom specimens were submitted, write: "A large barren form of either N. opaca or N. flexilis." Menyanthes trifoliata, Scirpus lacustris, Phragmites communis, Heleocharis palustris, Carex rostrata, C. filiformis, Juncus effusus, J. acutiflorus, J. lamprocarpus, Ranunculus Flammula, Mentha aquatica, M. sativa and its var. rubra, Spiræa Ulmaria, Comarum palustre, Carum verticillatum, Galium palustre (fig. 30), and Eriophorum polystachion. The following are less abundant:—Heleocharis acicularis, Myriophyllum alterniflorum, Potamogeton prælongus, Juncus fluitans, J. conglomeratus, Iris Pseud-acorus, Deschampsia cæspitosa, Carex Goodenovii, and Lythrum Salicaria.

At some parts of this loch the following successive zones of plant associations were observed, starting from the shore:—(1) Juncus effusus, J. lamprocarpus, J. acutiflorus, and Ranunculus Flammula, all more or less mixed. (2) Carex rostrata or C. filiformis. (3) Heleocharis palustris. (4) Phragmites communis. (5) Equisetum limosum. (6) Scirpus lacustris. (7) Potamogeton natans, P. polygonifolius, and P. lucens, mixed. (8) Nymphæa lutea, Castalia speciosa, and Potamogeton natans, mixed. (9) Carpeting the bottom below these zones, wherever there was space, Lobelia Dortmanna and Littorella lacustris.

From a study of the foregoing details it will be observed that this loch forms a somewhat transitional stage between a typical peaty highland loch and a typical lowland one: figs. 28 and 29 represent some of its features.

Blates Mill Loch is a small circular pool within a few hundreds of yards of the east shore of Woodhall Loch. It is surrounded by a zone of Carex rostrata and Equisetum limosum, the former being next the shore. There are also quantities of Nymphæa lutea, Castalia speciosa, and some other plants common to the district.

Mossdale Loch is a peaty pool ½ mile from New Galloway railway station. It contains a few plants common to the neighbourhood, but, like the last mentioned, it appears to be of no further botanical interest. On the moor east of Mossdale Loch there occurs a particularly fine example of the destruction of forest by wind.

Loch Ken is one of the largest lochs in this part of Scotland, being only exceeded in size by Loch Doon. It is 142 feet above sea level, and has a maximum depth of 62 feet. The loch proper is generally considered to lie between the grounds of Kenmure Castle and the Boat-of-Rhone railway viaduct; this portion is $4\frac{1}{2}$ miles long by $\frac{1}{2}$ mile wide in the widest part. The River Dee joins the loch a little below the viaduct, and thence the combined waters are continued as a narrow lake, in some places, however, $\frac{1}{2}$ mile wide, considerably to the south of Crossmichael. This lake-like portion extends the loch a further distance of over 4 miles, and is usually recognised as a part of the River Dee, although to the uninitiated it belongs to Loch Ken, and must be considered so from a botanical point of view. This sheet of water is thus about $8\frac{3}{4}$ miles long: it varies in width from 100 yards to $\frac{1}{2}$ mile, and has a drainage area of nearly 300 square miles. Like Woodhall Loch, Loch Ken presents a mixture of the highland and lowland types, not only in its flora and physical conditions, but also in scenic effect, as a comparison of figs. 31 to 40 readily shows. Endowed with charming and picturesque surroundings, which are further enriched by interesting historical associations, it seems strange that this inviting country should attract so small a flow of tourists. In many places the shore consists of stones and rocks, which are usually angular or but slightly waterworn and afford support to a very scanty flora; a narrow strip of such shore usually passes at once into moor, meadow, or wood (figs. 33, 36, 37). In other places the loch is bordered by bog, which makes it difficult to distinguish any line of demarcation between land and water (fig. 35). More rarely, the shore may be sandy, or the water may be bordered by a bank without the intervention of a shore. In that portion of the loch above the railway viaduct vegetation seldom occurs at a greater depth than 6 or 7 feet; beyond that depth, clay, mud, or vegetable detritus covers the bottom, to the exclusion of living plants. In the lower portion, about Parton and Crossmichael, there is in places a bottom flora down to a depth of 12 feet. This fact is accounted for by the great body of peaty water from the River Dee scouring the bottom, and washing the vegetable detritus either down stream or into deeper places.

Near the head of the loch the slight peatiness of the clear water is somewhat modified by the drainage received from the villages and cultivated areas through which the Water of Ken flows, and at that part the variety and luxuriance of the marsh vegetation affords evidence of a greater abundance of food-salts than is usual in peaty lochs. It is also interesting to note that Isoetes lacustris, a plant very impatient of water rich in normal plant food-salts, was not found nearer the head of the loch than the

vicinity of Burned Island, whence it occurred, but quite sparingly, down to the viaduct. After the loch had received the peaty water of the River Dee, Isoetes became abundant, and continued so down to below Crossmichael. At the embouchure of the Water of Ken and the Knocknairling Burn, at the head of the loch, there is a very considerable area of alluvium, consisting of gravel, sand, or mud, in a more or less marshy condition. This alluvial flat is covered with a very luxuriant vegetation, as previously mentioned, the dominant plants being Carex rostrata, C. aquatilis, var. elatior, C. vesicaria, Phalaris arundinacea, Deschampsia cæspitosa, Equisetum limosum, Heleocharis palustris, Juncus effusus, J. lamprocarpus, J. acutiflorus, J. bufonius, Plantago lanceolata, Galium palustre, and Ranunculus Flammula. A dense jungle is formed by the colonies of Phalaris, Deschampsia, and Carex elatior, the two former attaining a height of from 4 to 5 feet, and the latter a height of from 4 to 6 feet. The masses of Carex rostrata could be distinguished at a considerable distance, when blown by the wind, by their glaucous leaves; the colonies of C. vesicaria, which attain a similar height, by their green leaves; and the associations of C. elatior by their superior height, and broad, green, flowing leaves, waving in the breeze like a luxuriant field of grain. Although in many places the last mentioned were over 6 feet high, yet the general height of the level top when blown by the wind was 4 feet. The marsh vegetation of the littoral zone at other parts of the loch often grows luxuriantly: the low bushes in figs. 32 and 33 are chiefly Myrica Gale, with a background of Salix aurita, Alnus glutinosa, etc., the Myrica frequently being 5 feet high (fig. 34). At other places a strip of bog, often wide and deep, intervenes between the water and *terra firma*; in such places Eriophorum polystachion and other bog plants thrive (fig. 35). Occasionally a dry stony shore is overgrown with a dwarf prostrate form of Ranunculus Flammula, which roots copiously at the nodes; this resembles the var. pseudo-reptans of Syme, but is rather larger (p. 72). Fig. 39 shows the extent of this plant upon a stony shore, and fig. 40 affords a nearer view of the same. Scirpus lacustris grows very luxuriantly throughout the whole area of the loch (fig. 32). In some places this species was flourishing upon the dry shore, and there growing to a height of 3 or 4 feet (fig. 33). Nymphæa lutea is very abundant in some parts of the loch, particularly near the head, where the surface of the water is covered for hundreds of yards along the margin by its leaves and flowers (fig. 31). In certain situations, particularly near the viaduct, where shelter from wind is provided by adjoining woods, and where the narrowness of the loch prevents the formation of waves, Ranunculus heterophyllus covers the surface of the water with its white flowers and floating leaves, and

forms one of the characteristic features of this portion of the lake (figs. 37 and 38).

From the viaduct to below Crossmichael the general features are somewhat similar, although the water is a little more peaty, but, being somewhat remote from hills and moors, the lowland type becomes quite assertive, and the gently inclined shores quickly merge into meadow-land or bog. Here, again, Scirpus lacustris occupies large areas of the margin; there are also numerous large associations of Phragmites communis, Equisetum limosum, Heleocharis palustris, Carex aquatilis, C. rostrata, C. vesicaria, C. Goodenovii, etc. A large barren form of either Nitella opaca or N. flexilis occurs abundantly about and below the embouchure of the Dee: this is the same variety as that found in Woodhall Loch, previously mentioned, and probably it was transported from there by water, as the effluent of Woodhall Loch flows into the Dee near New Galloway railway station.

The submersed plants of Loch Ken are as follows:—Littorella lacustris, Lobelia Dortmanna, Isoetes lacustris, Juncus fluitans, Apium inundatum, Myriophyllum alterniflorum, Fontinalis antipyretica, F. squamosa, Callitriche hamulata, Chara fragilis, var. delicatula, Nitella opaca, which occurs throughout the loch (near Burned Island there are vast beds of it, although it does not extend beyond a depth of 7 feet), the large Nitella previously mentioned, Castalia speciosa, Nymphæa lutea and its var. intermedia, N. pumila, Ranunculus peltatus, R. heterophyllus, Glyceria fluitans, Sparganium natans, Potamogeton natans, and P. polygonifolius. The preceding were all abundant, whilst the following were more or less scarce:—Scirpus fluitans, Subularia aquatica, Potamogeton prælongus, P. lucens, P. obtusifolius. Filamentous Algæ were scarce, but occasionally Cladophora glomerata was abundant on stones down to 6 or 7 feet deep. The plants of the littoral zone are—Scirpus lacustris, Equisetum limosum, Carex aquatilis, C. rostrata, C. vesicaria, C. Goodenovii, Heleocharis palustris, Juncus acutiflorus, J. lamprocarpus, J. effusus, J. bufonius, Menyanthes trifoliata, Comarum palustre, Spiræa Ulmaria, Caltha palustris, Phalaris arundinacea, Deschampsia cæspitosa, Cardamine pratensis, Mentha aquatica, M. sativa, Scrophularia nodosa, S. aquatica, scarce, Equisetum palustre, scarce, Hydrocotyle vulgaris, Ranunculus Flammula and a dwarf prostrate form of it, Carum verticillatum, Eriophorum polystachion, Pedicularis palustris, P. sylvatica, Polygonum Hydropiper, Galium boreale, G. palustre, Plantago lanceolata, Valeriana officinalis, Œnanthe crocata, Angelica sylvestris, Senecio aquaticus, Hypericum elodes, Lythrum Salicaria, Serratula tinctoria, Myosotis palustris, etc. [In the lagoons at Kenmure Holms, which are connected with Loch Ken, are Carex elongata, Lysimachia vulgaris, Lythrum Salicaria,

Potamogeton obtusifolius, Alisma Plantago, Callitriche stagnalis, Ranunculus Lenormandi, Scirpus sylvaticus, and Thalictrum flexuosum.—J. M'A.] The Bryophytes are scarce or altogether absent on many parts of the shore, excepting in boggy places, where the usual forms, such as species of Sphagnum, etc., abound. The upper portion of the loch is more favourable for the growth of these plants, but even there they do not form a conspicuous feature of the shore. The following were observed in fair abundance:—Grimmia apocarpa and its var. rivularis, Rhacomitrium aciculare, Bryum alpinum, Amblystegium filicinum, Bartramia ithyphylla, Camptothecium sericeum, Hypnum cupressiforme, Neckera crispa, Philonotis fontana, Blasia pusilla, Jungermannia bantriensis, Pellia epiphylla, etc. [Bryum filiforme, Hypnum sarmentosum, H. Patientiæ, Cryphæa heteromalla, Grimmia subsquarrosa, G. Hartmani, and Orthotrichum rivulare grow on stones by the shore, whilst several species of Sphagnum are abundant. Leskea polycarpa, Helicodontium pulvinatum, and Scleropodium cæspitosum flourish on tree trunks in Kenmure Holms.—J. M'A.]

[Near the embouchure of the Shirmers Burn, which is about 2 miles from the head of Loch Ken, on the east side, there is a bank in the loch over which the water is quite shallow. On this raised portion of the bottom submersed plants can be easily seen when sailing over it in a boat. Between Ken Bridge and Loch Ken there is a great extent of alluvial ground upon either side of the River Ken, yielding a large quantity of excellent meadow hay, and that without manure, to the people of New Galloway and the farmers on the Kenmure estate. In time of flood all this extensive flat is covered with water by the overflowing of the River Ken and by the damming back of the water of Loch Ken, caused by the peculiar way in which the River Dee enters Loch Ken immediately below the railway viaduct. This river enters the loch at such an angle that its powerful current is directed against the more gentle downward flow of the loch, and this causes the damming back of the water of Loch Ken as far north as the Kenmure Holms, which are thus enriched by a valuable deposit of mud every time that a flood occurs. This accounts for the luxuriant vegetation at the head of the loch referred to on p. 121.—J. M'A.]

Barscobe Loch is about 3 miles N.E. of New Galloway. It is about $\frac{1}{4}$ mile long, and is situated in the midst of a treeless, hilly grass moor, which everywhere, excepting where bog occurs, meets the water, so that there is no shore. The water is quite clear and scarcely peaty. On the east side there are thin beds of Carex rostrata, and on the west side associations of Scirpus lacustris and Carex rostrata. On grassy bogs which occur here and there at the margin the usual marsh plants are found. The

flora consists of plants common to the district, and need not be especially enumerated, nothing of particular interest being noticed.

Loch Brack is a mile N.E. of Barscobe Loch, and is similar to it in general features, but smaller. Between the grass moor and the water a narrow zone of stony shore overgrown with Juncus acutiflorus intervenes more or less all around the loch; at the base of these plants quantities of Scapania sub-alpina find a congenial habitat. There are also a number of commoner Bryophytes upon the shores, and an average number of common Phanerogams occur, but nothing of special merit was observed.

Loch Howie is 2 miles N.E. of Barscobe Loch, and is larger than it, being $\frac{3}{4}$ mile long by $\frac{1}{8}$ mile wide, and lying S.W. and N.E. The surface is 757 feet above sea level, and the maximum depth is 39 feet. In general features it, again, resembles Barscobe Loch. At the S.W. end there are a few plants of Phragmites communis; these also occur, but much more abundantly, at the N.E. end, but they are all small specimens, none standing more than 3 feet above the water. There is also a small association of Scirpus lacustris at the N.E. end. Carex filiformis is very abundant at this loch, occupying situations that are usually taken up by Carex rostrata. Besides a number of common plants, no other features of interest were noted here.

Loch Skae is a small oval loch about $\frac{1}{4}$ mile long, situated $\frac{1}{2}$ mile E. of Loch Howie, at an elevation of 864 feet above sea level. The maximum depth is 35 feet. The general flora is similar to that of the three lochs just mentioned, but the physical features are different. The surrounding moors have more heather and peat; the scenery, particularly on the east, is rocky and wild, the hill rising steeply above the loch; the water is a little more peaty, and the east shore is rocky or stony. Isoetes lacustris, Utricularia intermedia, and Potamogeton polygonifolius appear to be more abundant here than at the other three lochs. The rocks on the east shore are overgrown with mosses, chiefly Hypnum cupressiforme and Rhacomitrium aciculare. There are associations of Phragmites communis, Carex rostrata, and C. filiformis.

The chief plants more or less common to the four last-mentioned lochs are—Littorella lacustris, Lobelia Dortmanna, Isoetes lacustris, Nitella opaca, Juncus fluitans, Myriophyllum alterniflorum, Utricularia intermedia, Potamogeton lucens, P. natans, P. polygonifolius, Castalia speciosa, Glyceria fluitans, Scirpus lacustris, Equisetum limosum, Phragmites communis, Heleocharis palustris, Menyanthes trifoliata, Carex filiformis, C. rostrata, C. Goodenovii, Juncus lamprocarpus, J. acutiflorus, J. effusus, Hydrocotyle vulgaris, Ranunculus Flammula, Carum verticillatum, Mentha sativa,

Galium palustre, Pedicularis palustris, Philonotis fontana, Aulacomnium palustre, Hypnum falcatum, H. scorpioides, H. fluitans, H. cuspidatum, H. cupressiforme, Rhacomitrium aciculare, Scapania sub-alpina, various species of Sphagnum, etc. Filamentous Algæ are scarce everywhere.

Lochinvar is 3 miles N.E. of Dalry. It is about $\frac{1}{2}$ mile long by $\frac{1}{3}$ mile wide, and is situated at an elevation of 736 feet above sea level, in a depression of a hilly, grass and heather moor. The scenery around is bare, desolate, and, with the exception of a few conifers about the gamekeeper's house, treeless. The water, which is the source of the public supply for Dalry, is slightly peaty but very clear, and its maximum depth is 10 feet. Nearly everywhere the moorland vegetation approaches to the water's edge, so that there is practically no shore, unless it be a narrow zone of stones or rocks (fig. 41). The bottom is rocky, save some patches of sand, and, with the exception of a few species which are abundant in isolated places, the flora is extremely poor. There are no associations of marsh plants about the shores; such of these plants as do occur are either as scattered specimens, or in a few very small groups,—Carex rostrata, Juncus lamprocarpus, J. effusus, Heleocharis palustris, and Ranunculus Flammula being the chief species. On an island, there is a quantity of Phalaris arundinacea overgrowing the fallen remains of a small castle. The submersed plants are more interesting. Here and there Lobelia Dortmanna or Littorella lacustris scantily carpets the bottom, particularly at the east end, where there is a little sand. Utricularia vulgaris, Myriophyllum alterniflorum, and Fontinalis antipyretica are abundant, whilst Juncus fluitans is not abundant. Strange to say the following Potamogetons were all plentiful:—P. pusillus, P. perfoliatus, P. Zizii, and a very long-leaved form of P. lucens. No other plants worth noting were found here.

Loch Dungeon is 7 miles N.W. of Dalry, at an elevation of over 1000 feet above sea level, and near the cliff at the west side it attains a depth of 94 feet. Beautifully situated at the base of rocky and precipitous mountains, it forms a magnificent, although treeless, piece of highland scenery, wild in the extreme, particularly on the south and west, where the mountains rise almost perpendicularly from the water's edge (figs. 42 and 44). This loch is irregularly shaped, being almost cut in twain at one part by a rocky promontory from the south shore, and by gravel from a moraine, washed into the loch by the Hawse Burn, forming a peninsula or alluvial cone from the north-west shore (fig. 43). The loch is about $\frac{3}{4}$ mile long by $\frac{1}{4}$ mile broad. Its water is extremely brilliant and clear, although of a steely-gray colour, and its shores are mostly rocky or stony. Excepting associations of Equisetum limosum and Phragmites

communis (fig. 44) in some of the bays, the littoral flora is very scanty. Potamogeton polygonifolius and Juncus fluitans were the only submerged Phanerogams occurring in abundance near the margin, but as there was no boat available, I am unable to state what the result of dredging might have been. The submersed rocks near the shores, as well as those exposed, frequently exhibited a wealth of Bryophytes: Scapania undulata, Nardia compressa, and N. emarginata were particularly luxuriant on many submersed rocks, as well as upon those dripping with water from the cliffs above. By their charm of colour and exuberant growth, the following were very conspicuous at the western margin of the loch:—Breutelia arcuata, Trichostomum tortuosum, Rhacomitrium protensum, R. lanuginosum, Hyocomium flagellare, Anthelia julacea, and various species of Sphagnum, particularly S. cymbifolium. Lichens were also abundant on the exposed rocks, especially a species of Collema. In some places submersed rocks were covered with a felty, mat-like growth of Algæ, which, upon careful examination, proved to be a mixture of three Myxophyceæ—Scytonema mirabile, Stigonema ocellatum, and Dichothrix Nordstedtii.

Loch Minnoch is a mile N. of Loch Dungeon. It is only ¼ mile long, and is beautifully situated amidst rugged hills. The water is very clear, being, in fact, that of Loch Dungeon, which flows into it by the Hawse Burn. This burn, which enters the loch on its south side, has brought in a large amount of detrital matter, causing a shallow area and a considerable bog on that side of the loch (fig. 45). This shallow part is overgrown with Equisetum limosum, etc., whilst the bog, which is covered with appropriate vegetation, merges imperceptibly into moor. The west shore is peaty, and it, together with the south shore, forms a suitable habitat for a considerable number of plants, such as associations of Scirpus lacustris, Phragmites communis, Equisetum limosum, Carex rostrata, Heleocharis palustris, Eriophorum polystachion, E. vaginatum, as well as mixed groups of common but less dominant plants. The north and east shores are rocky and bear a very scanty vegetation. Rhacomitrium aciculare and Blindia acuta are abundant on submerged rocks; so also is an aquatic form of Catharinea undulata, which covers submerged rocks to a depth of a foot or more. Dicranella squarrosa, Hypnum vernicosum, H. scorpioides, and others are common on the shores.

Loch Harrow is rather larger than the last mentioned, and about ½ mile north of it. The surface is 812 feet above sea level, and the maximum depth is 29 feet. The shores are more stony and there are fewer associations of littoral plants, otherwise it is similar to Loch Minnoch. The moor about the three last-mentioned lochs is mostly covered with

grass-like associations, Molinia cærulea being the most abundant. The dominant plants of these lochs are as follows:—Subularia aquatica, Lobelia Dortmanna, Littorella lacustris, Isoetes lacustris, Scirpus fluitans, Juncus fluitans, Glyceria fluitans, Fontinalis antipyretica, Potamogeton polygonifolius, P. natans, Scirpus lacustris, Phragmites communis, Equisetum limosum, Heleocharis palustris, Carex rostrata, C. flava, Juncus acutiflorus, J. lamprocarpus, J. effusus, Menyanthes trifoliata, Eriophorum polystachion, E. vaginatum, Ranunculus Flammula, and the Bryophytes and lower Cryptogams already mentioned.

In the paucity of species which comprise their flora, the three last-mentioned lochs agree with those on the Merrick range a few miles to the west. The scarcity of water-birds about these and other mountain lochs is probably a factor to be considered when forming a theory to account for the poverty of species in their flora. Doubtless mountain lochs offer an inhospitable asylum to the majority of our water-fowl. That such birds are active agents in the distribution of aquatic plants is beyond doubt. They are also great destroyers of the less robust vegetation, especially in shallow water, and are frequently the cause of the sudden disappearance of an association of small plants from some particular part of a shore. To cite examples, I have known Scirpus setaceus quite obliterated from a sandy shore in one season, probably by black-headed gulls. On the other hand, I have observed new additions to the flora of a loch which were probably introduced there by birds. Such changes amongst the minor plants of a loch are no doubt constantly occurring (p. 152).

II.—Area V.

Having now passed, by means of a circuitous and zigzag route, over the majority of the lochs situated in N.W. Kirkcudbrightshire, where the highland type predominates, let us leave this "Land of the mountain and the flood!" and beginning at Loch Corsock, examine S.E. Kirkcudbrightshire (p. 66), where many of the lakes are lowland in character.

Loch Corsock is a somewhat triangular sheet of water, about $\frac{1}{3}$ mile long, situated in an upland district, whose moorland character has been modified by cultivation. It lies about 6 miles north of Crossmichael, at an elevation of 540 feet above sea level, and the water is somewhat peaty. The western shores are flat and muddy or peaty, and have an extensive vegetation, whilst the eastern shores are rocky and stony, with only a few plants. On the south-west side there is an extensive marsh, now partially drained (fig. 46). The west, north, and north-east sides are

clothed with coniferous wood, and there is also a small plantation of the same kind on the south side. The loch is therefore wind-sheltered to a considerable extent although open to the south-west (fig. 46). The water of this loch was low owing to drought at the time of my visit, and the margin being gently inclined, a considerable area normally under water was consequently exposed. In many places such exposed portions consisted of bare peat or mud, but at other parts where the plants, normally submerged, readily adapt themselves to terrestrial circumstances—for instance, Littorella lacustris, Polygonium amphibium, etc.—the exposed mud was thickly covered by them (fig. 47). Here, as in other instances that I have frequently observed, sheep were grazing upon the exposed Littorella. The shepherds do not approve of this kind of food for their sheep, having an empirical belief that "this grass" is liable to cause liver-fluke in the animals. When one calls to mind the life-history of Distomum hepaticum, it seems quite likely that it would very readily occur in the encysted stage upon exposed Littorella, and would thus freely gain access to the bile-ducts of sheep that had fed upon that plant. Alisma ranunculoides was abundant; many specimens of it were flowering at a depth of 3 feet below the surface (p. 81); it was also flowering freely in the normal terrestrial condition around the margins. The following plants were more or less abundant:—Littorella lacustris, Lobelia Dortmanna, Isoetes lacustris, Myriophyllum alterniflorum, Apium inundatum, Nitella opaca, Chara fragilis, var. delicatula, Juncus fluitans, Ranunculus heterophyllus, Sparganium natans, Utricularia vulgaris, Castalia speciosa, Nymphæa lutea, Potamogeton polygonifolius, P. natans, Fontinalis antipyretica, Menyanthes trifoliata, Polygonum amphibium, Comarum palustre, Alisma ranunculoides, Carex rostrata, Carex flava, Heleocharis palustris, Peplis Portula, Montia fontana, the last two in both aquatic and terrestrial forms; Polygonum Hydropiper, Myosotis palustris, Juncus conglomeratus, J. acutiflorus, J. effusus, Ranunculus Flammula, Mentha sativa, Hydrocotyle vulgaris, Caltha palustris, Senecio aquaticus, Galium palustre, Carum verticillatum, Spiræa Ulmaria, Scutellaria galericulata, Phalaris arundinacea, and Plantago lanceolata. Hypnum fluitans, H. scorpioides, H. cuspidatum, and Fontinalis antipyretica were abundant in the water or in wet places, whilst Grimmia apocarpa and Hypnum cupressiforme covered the rocks on the east shore; otherwise Bryophytes were scarce.

Loch Roan is a small, somewhat triangular sheet of water, 2 miles north of Crossmichael. The west, north, and east margins are clothed with wood, chiefly coniferous, to the water's edge, whilst the south shore abuts upon meadow-land. Where the shores are gravelly or sandy there is little vegetation, but where boggy the usual marsh plants occur (fig. 48).

This is the reservoir for the water supply of Castle-Douglas, and presents little of botanical interest beyond a few common plants, such as associations of Carex rostrata and Equisetum limosum upon the south shore. Hypericum humifusum in dry places and Anagallis tenella on wet sand (fig. 49) were abundant on the west shore, both being unusual members of a shore flora. The latter was especially noticeable because it grew in pure patches, instead of straggling amongst other vegetation as is its usual habit, a fact due to the paucity of competing species. Although this is the reservoir for Castle-Douglas, yet cattle have free access to the water from the grazing grounds to the south, and in many places the shore was filthy with the excrement of these animals. This is a feature too common with the water supply of small towns, which, in the interest of public health, should be safeguarded against.

Loch Erncrogo is about a mile north-east of Crossmichael. It is a small loch of the lowland type about $\frac{1}{3}$ mile long, and being more or less surrounded by marsh, there is little shore. Outside the zone of bog, rich agricultural land prevails, excepting on the west side, where there is a plantation of conifers. The chief features here are the great associations of Carex rostrata (fig. 50), beyond which the shallower areas of the loch, particularly at the north end, are overgrown with Castalia speciosa, Nymphæa lutea, and Equisetum limosum. As the water was more or less unapproachable by reason of the bog, and as no boat was available, I am not able to indicate all the submerged plants. Those of the marginal zone are chiefly as follows:—Littorella lacustris, Nymphæa lutea, Castalia speciosa, Potamogeton natans, Scirpus lacustris, Equisetum limosum, Heleocharis palustris, Menyanthes trifoliata, Comarum palustre, Sparganium ramosum, Iris Pseud-acorus, Carex rostrata, Œnanthe crocata, Spiræa Ulmaria, Ranunculus Flammula, Lythrum Salicaria, Phalaris arundinacea, Myosotis palustris, Veronica Beccabunga, Mentha sativa, M. aquatica, Stachys palustris, Carum verticillatum, Eriophorum polystachion, Caltha palustris, Galium palustre, Juncus effusus, J. acutiflorus, J. lamprocarpus, Plantago lanceolata, etc. These plants were more or less intermingled, and not in definite associations of one kind, excepting in the case of Carex rostrata and Equisetum limosum. This, I suppose, is due to the gentle inclination of the boggy shore towards the water, and to the general conditions being equally agreeable to many species without being particularly favourable to a few only.

Loch Dornell is also a small loch, and occupies a somewhat exposed situation in an agricultural and moorland district 2 miles west of Crossmichael. The water is very clear, the shores are stony, and, besides associa-

tions of Carex rostrata and Phragmites communis in the bays, there is no great development of the littoral flora. Nearly everywhere the stony shore has a thin, narrow zone of Juncus articulatus, often mixed with Ranunculus Flammula at the margin of the water (fig. 51). Besides those already mentioned, the chief species of this loch are as follows:—Littorella lacustris, Lobelia Dortmanna, Isoetes lacustris, Juncus fluitans, Potamogeton prælongus, P. polygonifolius, P. lucens, Nymphæa lutea, Castalia speciosa, Equisetum limosum, Iris Pseud-acorus, Mentha aquatica, Myosotis palustris, Polygonum Hydropiper, Veronica scutellata, Hydrocotyle vulgaris, etc.

Meikle Dornell Loch is a small circular pool ½ mile west of the last mentioned, and connected with it by a burn. This little loch is surrounded by low hills, and the water is bordered by peaty banks, so that no shore intervenes between it and the moor. It is almost surrounded by a belt of Phragmites communis. There are also a few plants of Scirpus lacustris. A number of species common to the last-mentioned loch abound here also.

Loch Glentoo is 4 miles west of Castle-Douglas. It lies in a hollow of the moor, and appears to have occupied a much larger area at one time, if one may judge by the extent of low, marshy ground around it. The margins of this loch are treeless and its water is rather peaty. From the north and west shores outwards it is half overgrown with great beds of Phragmites communis mixed with Scirpus lacustris, and about the shore Carex rostrata and C. filiformis abound (fig. 52). At the south-west end the growth of marsh vegetation is very dense, and merges gradually into moor through an area of bog. Occasionally the shore is stony, but more frequently only a peaty bank divides the water from the moor. The flora resembles that of the next loch.

Loch Bargatton occupies an open position on the moor ½ mile south-west of the last mentioned. It is somewhat circular in outline, and the water is peaty. The eastern shore is stony and rocky, and comparatively bare of plants. The western side is overgrown with dwarf Phragmites communis, which also occurs in bays at other parts of the loch.

This loch and Loch Glentoo, although at an elevation of only about 200 feet above sea level, resemble lochs of a highland type in their floras, because of their exposed position on the open moor and their peaty water. Shore rocks were sometimes freely coated with Grimmia apocarpa, Hypnum cupressiforme, Orthotrichum rupestre, etc., whilst common bog mosses, particularly various species of Sphagnum, were occasionally abundant. In tiny pools upon the shores grew Chiloscyphus polyanthos, var. rivularis, but otherwise Hepatics were scarce. The other plants more or less common to

these two lochs are as follows:—Subularia aquatica, Lobelia Dortmanna, Littorella lacustris, Isoetes lacustris, Myriophyllum alterniflorum, Juncus fluitans, Chara fragilis, var. delicatula, Potamogeton prælongus, P. polygonifolius, P. lucens, P. crispus, P. perfoliatus, Nymphæa lutea, Castalia speciosa, Phragmites communis, Scirpus lacustris, Equisetum limosum, Carex rostrata, C. filiformis, C. flava, var. lepidocarpa, C. Goodenovii, Heleocharis palustris, Menyanthes trifoliata, Comarum palustre, Juncus articulatus, J. effusus, Caltha palustris, Galium palustre, Hydrocotyle vulgaris, Ranunculus Flammula, Scutellaria galericulata, Narthecium ossifragum, Lycopus europæus, Lysimachia vulgaris, Rhynchospora alba, Cardamine pratensis, Spiræa Ulmaria, Pedicularis palustris, Mentha sativa, Eriophorum polystachion, etc.

Carlingwark Loch forms a pleasing addition to the prosperous little town of Castle-Douglas (fig. 53), imparting an impression of repose to the clean and well-ordered streets. It is $\frac{3}{4}$ mile long by $\frac{1}{3}$ mile wide, and has a maximum depth of 17 feet, but over a considerable portion of its area the depth is less than 8 feet. The surface is 143 feet above sea level. The loch is connected with the River Dee by a narrow canal which is about $1\frac{1}{2}$ miles long. This canal was cut for the transport of marl up the River Dee, even as far as the Glenkins. Marl was discovered in abundance in and about the loch, and was formerly in great demand by agriculturists for fertilising their land, instead of lime. There are several islands, wooded with poplars, willows, alders, etc., which add to the picturesque appearance of the loch. An unpleasing feature is that the sewage of the town is drained into the loch, which, although about 105 acres in extent, is very shallow except at the sites of the old marl-pits, so that in hot, dry summers the residents of the town are inconvenienced by unpleasant odours and the risk of disease. The water at the south end is fairly clear and bright, but at the north end it is somewhat turbid and dead-looking, which is probably the result of the drainage from the town. The vegetation also has doubtless been affected thereby, for the semi-aquatic flora is composed of a large number of species, most of which grow in great luxuriance (figs. 54–56), whilst the submerged aquatics, although extremely abundant, are restricted in variety, possibly because the abnormal abundance of food-salts in the water, combined with the general shallowness of the loch, has favoured the excessive increase of a few species to the exclusion of others. I have, in fact, seen few lakes with such exuberant vegetation. The margin is frequently marshy and overgrown with a dense growth of reed or sedge, particularly in the south portion of the loch (figs. 55 and 56). At other places, especially at the northern end, the flat shore is either stony or of

muddy sand, and nearly everywhere such shores are covered near the water with Cladophora flavescens, mixed with Œdogonium, Spirogyra, etc., and the same species float on the surface of the loch, occupying large areas in sheltered bays. Such floating Algæ are a constant feature in lowland lochs where the water is polluted with sewage. Many submersed plants had a deposit of calcium carbonate upon their leaves, particularly Myriophyllum spicatum. Reference has already been made to the large quantities of the fresh-water mussel found in various parts of this loch (p. 116). The roots and rhizomes of numerous plants, especially Glyceria aquatica, were frequently found covered with the young of this mollusc. The shallower portions at the south end of the loch are being rapidly encroached upon by the marsh vegetation, if one may judge by the wide area of bog, which, in turn, is being converted into meadow-land by the accumulation of the remains of plants that grow there. It would be very instructive to have a series of exact measurements from various lochs, extending over a number of years, in order to show the rate of encroachment upon the water, together with the rate of conversion of the bog behind into *terra firma*. The submerged plants of this loch are as follows:—Littorella lacustris, forming a bottom carpet in shallow places as usual; Callitriche autumnalis, covering large areas of the bottom; Potamogeton Friesii in vast quantity, some parts of the loch being choked with it; Myriophyllum spicatum, Potamogeton prælongus, and Nymphæa lutea were all very abundant, as well as the Algæ previously mentioned; Potamogeton lucens was less abundant, while P. natans, Castalia speciosa, and Ranunculus aquatilis were scarce. The littoral flora is more varied, and is composed of—Phragmites communis, Equisetum limosum, Glyceria aquatica (fig. 54), Typha latifolia, Carex rostrata, C. Goodenovii, Phalaris arundinacea, Menyanthes trifoliata, Polygonium amphibium, Heleocharis palustris, Deschampsia cæspitosa, Juncus lamprocarpus, J. effusus, Sparganium ramosum, all of which form more or less pure associations on many parts of the shore. In other places the bog is covered with an association in which any of the above may occur, more or less, as subordinate members, mixed with some of the following:—Carex acutiformis, Bidens cernua, Rumex Hydrolapathum, Cicuta virosa (fig. 56), Œnanthe crocata, Carum verticillatum, Apium nodiflorum, Radicula palustris, R. pinnata, Valeriana officinalis, Senecio aquaticus, Plantago lanceolata, Ranunculus Lingua (fig. 55), R. Flammula, Stellaria palustris, Myosotis palustris, Comarum palustre, Caltha palustris, Mentha sativa, M. aquatica, Equisetum palustre, Spiræa Ulmaria, Galium palustre, etc. [Carex disticha and C. teretiuscula occur in marshy ground to the south of the loch.—J. M'A.] Figs. 53 to 56 represent some of the features of this loch.

Auchenreoch Loch is 6 miles north of Dalbeattie, at an elevation of 345 feet above sea level, and it is surrounded by agricultural land. It is about a mile long, and varies in width from 600 yards at the south-west end to 100 yards at the north-east end, the greatest depth being 34 feet. The water is clear, and not peaty. The main road from Dumfries to Castle-Douglas adjoins the east shore of the loch throughout its length. At the north-east end there are associations of Scirpus lacustris standing out in the loch; nearer the shore a large area is covered with Phragmites communis, behind which there is a marsh, with the usual plants. These conditions extend for some distance down the loch towards the south-west end. At other places there is a narrow strip of stony shore, with meadow beyond, or there is scarcely any shore, grass-land coming down quite to the water.

Milton Loch is situated about a mile east of the last mentioned, at an elevation of 410 feet above sea level. It is over a mile long by ½ mile wide, and has a maximum depth of 15 feet. The outline of the loch is very irregular, and it is surrounded by agricultural land. The water is clear, and not peaty. The shores are flat and stony, and merge imperceptibly into meadow or arable land, except where bordered by trees or public roads. There are no associations of marsh plants entering the loch; such as occur are merely a few species as stragglers over the stony shore, Alisma ranunculoides being one of the most abundant. Chara fragilis, var. delicatula, and Chara aspera, var. subinermis, must be very abundant, as considerable quantities of both species were washed up on the shore.

Lochrutton Loch is situated at an elevation of 305 feet above sea level, 3 miles east of Loch Milton. It is a little smaller than that loch, but has a maximum depth of 58 feet. This loch is the reservoir for the water supply of Dumfries, and the non-peaty water is clear. An extensive marsh at the south end, which has been shut off from the loch by a dam, is overgrown with common plants, the chief of which are Scirpus lacustris, Phragmites communis, Phalaris arundinacea, Spiræa Ulmaria, Carex rostrata, Alisma Plantago, and in the water, Potamogeton natans and P. heterophyllus. The shores of the loch are mostly stony, and it is surrounded by cultivated land. The flora of the littoral zone is scanty, and presents nothing of particular interest (fig. 57).

The three last-mentioned lochs occupy somewhat bleak, wind-exposed situations in an area of active agriculture, and the scenery around is tame and uninteresting. I am unable to give a full account of the submerged flora of any of these lochs, because during the period of my visit the continuous storms of wind made the use of a boat for my purpose impossible. From an examination of the refuse washed upon the shores, I imagine the

aquatic flora is of the ordinary type, and not particularly abundant. The chief plants more or less common to these three lochs, so far as I could find, are as follows:—Littorella lacustris, Chara aspera, var. subinermis, C. fragilis, var. delicatula, Potamogeton natans, P. lucens, P. heterophyllus, P. crispus, Sparganium natans, Ranunculus peltatus, Fontinalis antipyretica, Scirpus lacustris, Phragmites communis, Equisetum limosum, Heleocharis palustris, Sparganium simplex, Carex rostrata, C. Goodenovii, Phalaris arundinacea, Juncus acutiflorus, Spiræa Ulmaria, Alisma Plantago, A. ranunculoides, Ranunculus Flammula, R. Lenormandi, Myosotis palustris, Mentha sativa, Caltha palustris, Lythrum Salicaria, etc.

Lochaber Loch is picturesquely situated, 8 miles north-east from Dalbeattie. This loch, which is 298 feet above sea level, has a somewhat triangular outline; it is ½ mile long by ¼ mile wide, and has a maximum depth of 55 feet. It is surrounded by low hills, the lower slopes of which are wooded, chiefly with coniferous trees, to the water's edge (fig. 58), excepting on the west where the country is open and agricultural land prevails. The water is slightly peaty, and the marginal flora is poor in variety. No boat being available, I am unable to indicate the bottom flora. At the south-east end there are associations of Scirpus lacustris, Equisetum limosum, and Carex rostrata, none of which grows so tall and luxuriant as might be expected from the lowland situation. This is probably the result of wind, combined with a poor supply of food-salts. At the west side, where the shore is boggy, there are associations of Phragmites communis, but the specimens are dwarfed; also of Carex rostrata, Equisetum limosum, Castalia speciosa, and Menyanthes trifoliata; otherwise the somewhat flat and stony shores are either bare of vegetation, or sparsely clothed with a few common plants. Lobelia Dortmanna occurs abundantly here and there in the marginal zone, along with Littorella lacustris, but other submerged aquatics appear to be scarce.

Auchenhill Loch, which is 4 miles south of Dalbeattie, is the smallest of a group of four lochs. It is about ¼ mile long by 100 yards wide, and is a typical lowland pool, situated amidst pleasant pastoral scenery. There are no trees at its margin, but it is more or less surrounded by a zone of Phragmites communis, behind which there is a border of marsh, overgrown with plants common to such a habitat, and merging imperceptibly into meadow. In front of the Phragmites, a belt of Castalia speciosa almost encircles the loch. The water cannot be approached on account of the surrounding bog; and no boat being available, I am unable to give an account of the submerged flora.

Barean Loch is about ½ mile east of the last mentioned, but it is

considerably larger, and has an irregular outline. Its water is rather peaty. It is picturesquely surrounded by low hills, some portions of which are cultivated, while the remainder consists either of moor or wood; the margin of the loch is also well wooded. It is more or less surrounded by a sedge or reed marsh, composed chiefly of the following species:—Scirpus lacustris, Phragmites communis, Equisetum limosum, and Carex rostrata. In the water beyond this zone, as well as mixed with it, Potamogeton natans, P. perfoliatus, Sparganium natans, Castalia speciosa, and Apium inundatum are the dominant plants. The last-mentioned species is particularly abundant, and grows luxuriantly in water 6 or 7 feet deep, reaching the surface from that depth, although not fruiting freely in such deep water. A number of common plants also occur, but less abundantly.

Clonyard Loch is ¼ mile south-west of the last mentioned. It is smaller than Barean Loch, but the features are somewhat similar. It is surrounded by a sedge or reed swamp, composed chiefly of Scirpus lacustris and Carex rostrata; there is also an association of Typha latifolia, as well as minor colonies of Phragmites communis and Equisetum limosum. In the water outside the swamp zone there is a broad belt of Castalia speciosa. Other abundant plants are Iris Pseud-acorus, Sparganium ramosum, and Lythrum Salicaria.

White Loch is the largest of this group, being about ½ mile long by ¼ mile broad. It is ½ mile south-east of the last mentioned, and the public road from Douglas Hall to Dalbeattie adjoins its western shore. The neighbouring district is a mixture of moor, cultivated land, and plantation, and the water is rather peaty. Where not marshy, the shores are sandy or stony, with a few syenitic rocks. It is little more than 100 feet above the level of the sea, which is about a mile distant; and although distinctly lowland in general aspect, yet some plants usually associated with peaty highland lochs flourish here alongside those commonly found in lowland lakes. This is probably because the loch has not been interfered with, whilst the surrounding moor has been brought under partial cultivation. Phragmites communis forms a belt around a considerable portion of the loch, especially on the east. On the west side there is a large association of Typha angustifolia, as well as smaller groups of the same at other parts of the loch. In the water, beyond the Phragmites and Typha, associations of Scirpus lacustris occur, whilst Carex rostrata and Equisetum limosum occupy other sites. Minor plants of the marsh formation are—Heleocharis multicaulis, Comarum palustre, Alisma Plantago, Juncus acutiflorus, J. lamprocarpus, J. effusus, Lythrum Salicaria, Ranunculus Flammula, Mentha sativa, Pedicularis palustris, Spiræa Ulmaria, Scutellaria

galericulata, Hydrocotyle vulgaris, Hypnum cuspidatum, etc. The bottom is carpeted in many places with Littorella lacustris, Lobelia Dortmanna, Isoetes lacustris, Nitella opaca, and Fontinalis antipyretica, while the other submersed plants are Castalia speciosa, Potamogeton natans, P. pusillus, P. perfoliatus, P. lucens, P. crispus, P. rufescens, the last with floating coriaceous leaves, and Ranunculus Drouetii.

Photographs of these lochs could not be obtained, owing to the very unpropitious weather that occurred during the time allotted for their inspection.

At none of the lochs of this Area (V.) was there any particular abundance of Bryophytes; hence their general absence from the lists of plants.

III.—Area VI.

We now proceed to examine some of the lochs of Wigtownshire (p. 66), where both lowland and highland types may be found, although none of those that I have visited are at a greater elevation above sea level than 400 feet. Stormy weather considerably hindered work, so that during the time at my disposal I was unable to visit some of the lochs situated in outlying places, and difficult of access under any conditions. In particular, I regret having to omit those in the north of the county, within and without the Ayrshire border, because these are less likely to have undergone alteration by human agencies.

Black Loch is the smallest of a series of three, and is situated 5 miles north-west of Kirkcowan. It is about $\frac{1}{4}$ mile long, is surrounded by a treeless moor, and its water is rather peaty. The only strip of shore is at the east end; elsewhere a bank of peat separates the water from the moor. Rocks at the east end are overgrown with Grimmia apocarpa, Orthotrichum rupestre, Rhacomitrium lanuginosum, etc. Bryophytes are otherwise very scarce. The aquatic vegetation is chiefly at the west end of the loch, and bottom-carpeting plants such as Littorella lacustris are scarce. At the margin there are associations of the following species:—Phragmites communis, Scirpus lacustris, Equisetum limosum, Carex rostrata, C. filiformis, and Castalia speciosa, and the submerged aquatics are fairly represented.

Loch Heron is a somewhat rectangular sheet of water, nearly as large again as the last mentioned, and situated $\frac{1}{2}$ mile to the south-west of it. There is a plantation of conifers upon the south and east shores, otherwise it is surrounded by cultivated land or moor. The water is clear, and slightly peaty. The shores are stony, or in some places there is a peat bank entering the water without the intervention of a shore. There are associations of

the following plants about the margins:—Phragmites communis, Carex rostrata, Scirpus lacustris, and a few sparse patches of Equisetum limosum. Littorella lacustris and Lobelia Dortmanna carpet the bottom in places, and there is a fair number of other submerged aquatics.

Loch Ronald is close to the last mentioned, and is about a mile long. There is a plantation of conifers on the east side, otherwise it is surrounded by agricultural land or moor. The water is very clear, and the shores are stony, flat, and, from a botanical aspect, almost featureless (fig. 59), much resembling Loch Ashie in Inverness-shire (*ante,* p. 1009, fig. 73). Here and there a bank of peat 8 or 10 feet high dips into the water without the intervention of a shore. There are two small associations of Equisetum limosum and one of Scirpus lacustris, all at the south-west end, and groups of Carex rostrata in the effluent. I was not able to obtain the use of a boat because it had been previously engaged, but a close examination of the barren shore for the remains of submersed plants suggested a scarcity of vegetation in the water.

The plants occurring at these lochs, but more particularly at Lochs Black and Heron, are as follows:—Littorella lacustris, Lobelia Dortmanna, Isoetes lacustris, Myriophyllum alterniflorum, Juncus fluitans, Scirpus fluitans, Potamogeton polygonifolius, P. lucens, P. rufescens, P. pusillus, P. obtusifolius, Fontinalis antipyretica, F. squamosa, Castalia speciosa, Sparganium natans, Scirpus lacustris, Phragmites communis, Equisetum limosum, Heleocharis palustris, Carex rostrata, C. filiformis, C. flava, var. lepidocarpa, Juncus lamprocarpus, J. acutiflorus, J. effusus, Ranunculus Flammula, Hydrocotyle vulgaris, Eriophorum polystachion, Comarum palustre, Polygonum Hydropiper, Mentha sativa, and Senecio aquaticus. There are a few specimens of Lythrum Salicaria at Black Loch, but this species will probably not succeed in getting well established there.

Clugston Loch is a small sheet of water, 3 miles south of Kirkcowan, with slightly peaty water, and surrounded by moor. The shores are rocky or peaty, and beyond colonies of Carex Goodenovii, C. rostrata, and Equisetum limosum there are no large associations of semi-aquatic plants, although the following are more or less abundant:—Littorella lacustris, Lobelia Dortmanna, Juncus fluitans, Apium inundatum, Scirpus fluitans, Fontinalis antipyretica, Potamogeton polygonifolius, P. lucens, Castalia speciosa, Menyanthes trifoliata, Utricularia intermedia, Equisetum limosum, Carex rostrata, C. Goodenovii, C. flava, Juncus lamprocarpus, J. effusus, Caltha palustris, Viola palustris, Ranunculus Flammula, Polygonum Hydropiper, Hydrocotyle vulgaris, Lythrum Salicaria, Eriophorum polystachion, etc.

Loch Wayoch is the most northerly of a group of lochs situated on a

dreary, boggy moor many miles in extent. The last-mentioned loch is, indeed, upon the same moor, but at its outskirts, where the ground is less boggy, whilst there the scenery is enlivened eastwards by the adjacent area of cultivation. An old resident informed me that during his life the view of the county beyond the moor (*i.e.* looking from Anabaglish southwards) had been considerably curtailed, owing to the gradual elevation of the intervening moss.* Exact measurements of such development over a long period would not be without interest.

This loch is 4 miles south-west of Kirkcowan, and is a somewhat circular pool, 200 yards across. There is no shore, the water being surrounded by deep bog, differing only from the moor in being more ready to engulf the unwary. I succeeded in getting within a few feet of the water, and was surprised to find it was beautifully clear, and apparently not peaty. Another interesting fact was the presence of an association of Typha latifolia, a plant usually associated with the evil-smelling mud of lowland lakes rather than with that of a lochan in the midst of a peat moor. Other uncommon members of the marginal flora were Cladium Mariscus and Hypericum elodes, while the more usual species, such as Carex rostrata, Juncus effusus, J. bufonius, Menyanthes trifoliata, Narthecium ossifragum, Eriophorum polystachion, Hydrocotyle vulgaris, etc., formed the bulk of the phanerogamic vegetation. The following Bryophytes were also abundant:—Sphagnum cuspidatum, var. falcatum, S. cymbifolium, S. subsecundum, Polytrichum commune, Aulacomnium palustre, Hypnum Schreberi, H. cupressiforme, var. ericetorum, Cephalozia Sphagni, etc. On the drier parts of the bog Calluna and Myrica have spread from the adjacent moor, where Cladonia uncialis occurs in extraordinary abundance. I was not able to discover what plants, if any, grew in the water.

Fell Loch is larger than Wayoch, and $\frac{1}{2}$ mile south-east of it. The water is peaty, and the bottom is of peat. The chief plants are Lobelia Dortmanna, Castalia speciosa, Potamogeton polygonifolius, Juncus fluitans, Heleocharis multicaulis, Cladium Mariscus, Carex rostrata, Phragmites communis, Equisetum limosum, Menyanthes trifoliata, Hydrocotyle vulgaris, etc.

Black Loch is close to the last mentioned, and similar to it, but the water is not so peaty, and there is less vegetation. Cladium Mariscus and Carex filiformis are abundant, as well as other commoner plants.

Mochrum Loch is $\frac{1}{2}$ mile south of the last mentioned, but is much larger, being about $1\frac{1}{2}$ miles long by $\frac{1}{3}$ mile broad, and the elevation above sea level is 248 feet. This loch is very shallow, the average depth

* A wet moor, with much Sphagnum, etc., is frequently called a moss.

being only 7 or 8 feet, with a maximum of 13 feet. The outline is irregular and there is very little shore, merely a narrow strip of rocks or stones intervening between the water and the moor or wood; neither are there any sandy bays, but occasionally there is a stretch of peaty shore. The north and north-east sides are wooded, chiefly with coniferous trees, and on the east there is some cultivated land, otherwise the surrounding district consists of spongy moor. There are numerous islands scattered over the loch, some of which are wooded, and these, with the plantation at the northern end, form a pleasing feature in the otherwise bare scenery. In many places the bottom is rocky, and at these areas there is no vegetation; but where sand or mud obtains, there is an abundance of plants. The water is scarcely peaty, and is so clear that the bottom can be seen through a depth of 7 feet, even in dull weather. This pellucidity, not only of Mochrum Loch but of neighbouring ones as well, is not easily explained, because, from their situation in the midst of a spongy peat moor, one would expect the water to be quite peaty. The chief feeder is the burn from the adjacent Castle Loch; and as no stream of considerable size enters either loch, presumably they are fed partly by springs, which may, of course, have no connection with the water of the moor. It is probable, however, that some constituent, such as an alkali of the underlying rock, may neutralise the peat extract, thus rendering the water clear; the presence of certain calciphilous plants, *e.g.* Eupatorium Cannabinum, suggests lime also. Geologists might find this matter of some interest. A narrow cylindrical form of the fresh-water sponge is very abundant at some parts of this loch, the dredge occasionally coming up loaded with it.

The plants that flourish here are as follows:—Littorella lacustris, from the margin to 8 feet deep; Subularia aquatica, from the margin to 6 feet deep; Elatine hexandra, in small patches, from 2 to 8 feet deep; Isoetes lacustris, from 6 to 10 feet deep; Callitriche hamulata, from 4 to 6 feet deep; Potamogeton pusillus, and a very slender variety of it, extremely abundant at from 4 to 10 feet deep; P. obtusifolius, P. crispus, P. perfoliatus, P. natans, Myriophyllum alterniflorum, Fontinalis antipyretica, and F. squamosa. All the foregoing were very plentiful, while the following were less so:—Lobelia Dortmanna, Chara fragilis, var. delicatula, Nitella translucens, Juncus fluitans, Utricularia intermedia, Ranunculus aquatilis, Castalia speciosa, Nymphæa lutea, and Sparganium minimum. The marginal flora was rather scanty: there were fairly large associations of Phragmites communis, Carex rostrata, and Equisetum limosum, while the other species were either in small groups or more or less scattered, the chief being—Cladium Mariscus, Phalaris arundinacea, Sparganium simplex, Carex filiformis, Ranunculus Flammula,

Juncus lamprocarpus, J. acutiflorus, J. effusus, Caltha palustris, Heleocharis palustris, H. multicaulis, Menyanthes trifoliata, Comarum palustre, Lysimachia vulgaris, Œnanthe crocata, Lythrum Salicaria, Lycopus europæus, Spiræa Ulmaria, Schœnus nigricans, Radicula officinalis, Hydrocotyle vulgaris, Scutellaria galericulata, and Polygonum Hydropiper. Eupatorium cannabinum occurred chiefly at the islands, on some of which Osmunda regalis was abundant. Bryophytes on the shore, with the exception of Hypnum cupressiforme, which covers rocks, and Sphagnum sp. in peaty places, were not abundant.

Castle Loch is $\frac{1}{2}$ mile west of the last mentioned, which it much resembles in size and general features. It is 16 feet higher than Mochrum Loch, into which its north-eastern effluent flows. There are a few trees at the north end and on one of the islands, which also has upon it the remains of a small castle. The surrounding country is bare, open moor. This loch is studded with numerous bare, rocky islands, the largest being occupied by hundreds of cormorants, which breed there. The shores are rocky or stony, and the bottom is rocky nearly everywhere. The water is clear, like that of Mochrum Loch, and the average depth is from 6 to 8 feet, with a maximum of 11 feet. I dredged a dozen or more of the less rocky places and examined many other parts of the bottom, but could obtain no plants from the water save Fontinalis antipyretica and F. squamosa, which abound on the rocks. The bottom appears to be quite destitute of plants, excepting the two species just enumerated. This is remarkable, especially when the adjoining Mochrum Loch has such an abundant aquatic flora. Mr David M'Dowall, the keeper, informed me that he had never seen any plants upon the net when netting this loch. The water was remarkably free of plankton organisms, the tow-net gathering extremely little (end of August), but Mr M'Dowall told me that in early summer the water is thick and green with some organism that dies away towards the end of July. Perhaps the presence of this organism in the spring accounts for the absence of plants in the water. The scanty vegetation of the rocky shores was of no particular interest, being similar to that of Mochrum Loch, but less abundant. Lythrum Salicaria and Phalaris arundinacea were the most plentiful species. Fig. 60 illustrates this loch, with Mochrum Loch in the distance.

On Anabaglish Moss, to the north-west of Castle Loch, there are a number of small lochans of some interest because of the abundance of their vegetation, which includes some unusual species. Figs. 61 to 64 illustrate four of these tarns; the legends appended to the illustrations afford sufficient description.

[**Monreith Lake**, near Port William, is entirely surrounded by wood

affording shelter to many rare species of water-fowl. In addition to the usual marsh and aquatic plants, which grow here very luxuriantly, this lake is becoming choked up with Anacharis Alsinastrum.—J. M'A.]

[**Dowalton Loch**, near Sorbie, was once an extensive sheet of water, but about sixty years ago it was almost emptied by cutting a deep outlet at its eastern end. Since then it has become overgrown with a dense growth of marsh plants, but cannot yet be said to be of much use agriculturally.—J. M'A.]

[**South of Whithorn** are numerous small lochs, becoming gradually overgrown with vegetation, amongst which several uncommon species of Carex may be found. Further south, and to the west of the Isle of Whithorn, there are several small lochs, in which grows the beautiful Chara polyacantha.—J. M'A.]

Barhapple Loch is 4 miles east of Glenluce, on an extension of the same moor as Castle Loch, from which it is distant also about 4 miles. It is a circular loch, about $\frac{1}{4}$ mile across, with dirty, peaty water. The north side is bordered by a dense association of Phragmites communis (fig. 65), whilst the same plant occurs scattered over the peaty and muddy south shore. On the west side there is a considerable extent of marsh, dominated by Carex rostrata, C. filiformis, etc. On the east the shore is peaty or gravelly, and is bordered by a bank of peat from 4 to 6 feet high. Large tussocks of Molinia cærulea extend over the peaty portion of this shore, but where gravelly it is encroached upon by large tussocks of Juncus effusus (fig. 66). Drainage from the farm on the south appears to gain access to the loch, and the exposed mud on that side is very foul, large patches of it being coloured red by Porphyridium cruentum. Juncus supinus, var. subverticillatus, with all the flowers viviparous, was very abundant on this mud, growing in large, flat tussocks. At the same place a very robust form of Peplis Portula, growing in prostrate patches, was plentiful (p. 75). On peaty portions of the east shore a short, erect, cæspitose form of Juncus supinus was common, and dwarf prostrate forms of Juncus bufonius were also abundant at the same place. There were very few mosses and no hepatics about the shores of this loch. No boat being available, the bottom could not be examined, but, so far as I could tell, submerged plants were scarce. Besides the above mentioned, the following species were observed here:—Littorella lacustris, Callitriche hamulata, Comarum palustre, Montia fontana, Mentha aquatica, M. sativa, Spiræa Ulmaria, Juncus lamprocarpus, J. acutiflorus, Ranunculus Flammula, R. hederaceus, Myosotis palustris, Hydrocotyle vulgaris, Viola palustris, Galium palustre, Veronica scutellata, and Alisma Plantago.

Loch Dernaglar, $\frac{1}{2}$ mile south of the last mentioned, is somewhat circular in outline, and about $\frac{1}{3}$ mile across. The moor around is flat and treeless, and the water is peaty. Banks of peat usually separate the water from the moor, but occasionally the shore is stony, or is formed of flat rock, this being particularly the case on the east side, which consequently is rather bare of littoral vegetation. The western margin is marshy, especially near the affluent (fig. 67), and supports a considerable vegetation, associations of Scirpus lacustris, Equisetum limosum, Phragmites communis, Carex rostrata, C. filiformis, and C. Goodenovii being dominant. Juncus lamprocarpus and J. acutiflorus grow together in abundance. Heleocharis multicaulis was common in water about a foot deep, many of its leaves floating on the surface, whilst the flowering stems were erect (fig. 68); viviparous forms of it were also plentiful. Pilularia globulifera was extremely abundant in shallow parts on the eastern side (fig. 69). The other plants observed at this loch were—Littorella lacustris, Lobelia Dortmanna, Subularia aquatica, Isoetes lacustris, Juncus fluitans, Myriophyllum alterniflorum, Chara fragilis, var. delicatula, Castalia speciosa (fig. 67), Potamogeton polygonifolius, P. natans, P. rufescens, P. lucens, Sparganium natans, Menyanthes trifoliata, Heleocharis palustris, Juncus effusus, Eriophorum polystachion, Ranunculus Flammula, Carex flava, var. minor, Hydrocotyle vulgaris, etc.

Whitefield Loch is 3 miles south-east of Glenluce, at an elevation of 191 feet above sea level. It has an angular outline, is about $\frac{1}{2}$ mile long by $\frac{1}{4}$ mile wide, and is a good deal enclosed by trees with cultivated land or moor beyond. The water, which has a maximum depth of 14 feet, is slightly peaty, the shores are stony, and for the greater part bare of vegetation. The most noticeable feature of the shore flora is the abundance of Lythrum Salicaria. Besides a number of plants usual to the district, there is nothing here of particular interest.

Barlockhart Loch is a small circular pool, with non-peaty water, about a mile south-east of Glenluce. It is surrounded, excepting on the west, by low hills, the land being either pasture or arable. This loch is enclosed by a zone of Phragmites communis, beyond which, in the water, there is an association of Castalia speciosa and Nymphæa lutea also extending around the loch; but at the east end Equisetum limosum is interposed outside the Phragmites. Behind the last mentioned there is a strip of marsh with a number of the usual bog plants, as well as Salix aurita and Alnus glutinosa in places (fig. 70). A curious floating form of Hydrocotyle vulgaris occurred here (p. 77). Dwarf forms of Potamogeton obtusifolius in shallow water (p. 84), as well as the normal form in deeper water, were abundant.

The plants more or less common to this and the last-mentioned loch are as follows:—Littorella lacustris, Lobelia Dortmanna, Elatine hexandra, Nitella opaca, Chara fragilis, var. delicatula, Juncus fluitans, Myriophyllum alterniflorum, Apium inundatum, Fontinalis antipyretica, Callitriche stagnalis, aquatic and terrestrial forms; C. hamulata, Ranunculus Drouetii, Potamogeton natans, P. polygonifolius, P. obtusifolius, P. rufescens, P. Zizii, P. crispus, P. lucens, Castalia speciosa, Nymphæa lutea, Equisetum limosum, Phragmites communis, Sparganium natans, S. simplex, S. ramosum, Carex rostrata, C. filiformis, C. Goodenovii, C. flava, Polygonum amphibium, P. aviculare, Iris Pseud-acorus, Alisma Plantago, A. ranunculoides, Juncus acutiflorus, J. lamprocarpus, J. effusus, J. bufonius, J. conglomeratus, Comarum palustre, Ranunculus Flammula, Mentha aquatica, M. sativa, M. arvensis, Lythrum Salicaria, Myosotis palustris, Triglochin palustre, Epilobium palustre, Senecio aquaticus, Caltha palustris, Viola palustris, Veronica Beccabunga, Pedicularis palustris, P. sylvatica, Galium palustre, Carum verticillatum, Eriophorum polystachion, Hydrocotyle vulgaris, Stellaria uliginosa, Gnaphalium uliginosum, Sphagnum sp., and a few of the common marsh mosses.

White Loch is about 1 mile long by $\frac{1}{3}$ mile broad, with a maximum depth of 38 feet, and is one of the largest of a group situated about 3 miles east of Stranraer. This and the adjoining Black Loch are within the private grounds of Castle-Kennedy, the seat of the Earl of Stair, and are ornamental waters to Lochinch Castle. Although left as far as possible in a natural condition, these lakes are surrounded by lawns or meadows, which are furnished with groups of decorative trees; there are also wooded islands (figs. 71 and 72). There is no extent of shore anywhere about White Loch, neither is there any considerable development of marsh vegetation, but here and there narrow zones of marsh plants, 1 to 10 feet wide, intervene between the water and the grassy banks. The water, which has the same elevation above sea level as that of Black Loch, viz. 54 feet, is not peaty, but is so turbid and greenish-coloured that the bottom cannot be seen at a greater depth than 18 inches when looking over the side of a boat (*i.e.* in August). Plankton organisms are the cause of this turbidity, more especially the diatom Melosira granulata. There is neither affluent nor effluent to this loch save a shallow boat-canal connecting it with the adjoining Black Loch, the water of which is dark and peaty (presumably these facts guided the nomenclator of the lochs). The water is therefore more or less stagnant, a condition favouring the increase of certain plankton organisms. A feature of both this and Black Loch is the narrow border of Heleocharis palustris that prevails nearly everywhere, growing luxuri-

antly to a height of 3 feet, with very large inflorescences. With these, Radicula palustris, Myosotis palustris, Polygonum Hydropiper, P. Persicaria, P. amphibium, Phalaris arundinacea (fig. 71), Mentha aquatica, Juncus acutiflorus, Caltha palustris, and Ranunculus Flammula are the chief plants of the margin. Elatine hexandra grows exposed upon the shore, also in the water to a depth of 2 feet. Myriophyllum alterniflorum abounds to a depth of 6 or 7 feet; M. spicatum is abundant from 4 to 7 feet deep. Chara fragilis, var. delicatula, occurs sparingly from the margin to a depth of 6 feet. The following species of Potamogeton are all abundant, especially at the north-west end:—P. crispus, a beautiful form, with broad leaves, having a wide red midrib; the same form is also found in other lochs of this neighbourhood; P. Zizii, P. perfoliatus, P. prælongus, P. obtusifolius, P. pusillus, and a very large form of P. lucens. Callitriche autumnalis and Ranunculus peltatus occur sparingly. Bryophytes are scarce.

Black Loch, as already explained, adjoins the last mentioned. It is about $1\frac{1}{3}$ miles long, and has a maximum breadth of $\frac{1}{3}$ mile, but it narrows considerably towards the north-west end. The surroundings are similar to those of White Loch, but the water, which has a maximum depth of 50 feet, is brown and peaty; and although plankton organisms abound, the bottom can be seen to a depth of 3 feet when looking over the side of a boat. The shore is similar to that of White Loch, but the littoral flora is more varied. Usually water from 7 to 10 feet deep, or even deeper, occurs within a few feet of the shore. To a depth of about 7 feet a few of the usual submersed plants may be gathered, but they are by no means abundant as the bottom is generally stony. At greater depths than 7 feet no living plants can be found, but an abundance of dead vegetable remains, as at other shallow peaty lochs with no current to scour them. Cladophora glomerata covers stones abundantly from 2 to 7 feet deep. Fontinalis antipyretica occurs sparingly in similar positions. Myriophyllum alterniflorum grows from the margin to 7 feet deep, but is not very abundant. Littorella lacustris is found in places about the shores, but appears to be neither general nor plentiful, nor does it enter the water beyond a depth of a few inches, probably because of the constancy of the water level. Nymphæa lutea is abundant, and Potamogeton Zizii is scarce. No other submersed plants were observed in the main body of the loch. At the north-west end there is a somewhat circular basin, connected with the loch by a narrow channel. This is almost surrounded, excepting on the south-west side, by a narrow border of Phragmites communis, Typha latifolia, and Scirpus lacustris, whilst the surface is largely overgrown with Nymphæa lutea. On the south-west side of this basin there are small associations of marsh

plants, composed chiefly of Carex rostrata, Heleocharis palustris, Lythrum Salicaria (fig. 74), Spiræa Ulmaria, and the recently mentioned species as well. At the south-east end of the loch there is a marsh, with the usual common plants (fig. 72). The most important plants about the shores of this loch are as follows:—Scirpus lacustris, Equisetum limosum, Glyceria fluitans, Heleocharis palustris, Phragmites communis, Typha latifolia (figs. 73, 74), Carex rostrata, Lythrum Salicaria, Juncus acutiflorus, J. effusus, Phalaris arundinacea, Spiræa Ulmaria, Œnanthe crocata, Deschampsia cæspitosa, Mentha aquatica, etc.

In the canal between the two lochs, Littorella lacustris, Alisma ranunculoides, Potamogeton Zizii, Myriophyllum alterniflorum, Callitriche autumnalis, and C. vernalis are the dominant species. Bryophytes are everywhere scarce.

Cults Loch is ½ mile east of the last mentioned. It is a small, somewhat circular loch, with non-peaty water, surrounded by meadow-land. This loch has no visible effluent, and near its centre the remains of a lake dwelling or crannog are to be seen. At the north-west and south-east sides there are small bogs; at other places a narrow zone of marsh, chiefly occupied by Juncus effusus, intervenes between the water and the pasture (fig. 75). No other features of interest were noticed here beyond a number of plants which need not be especially enumerated.

Loch Magillie is about a mile south-west of White Loch. It is a small oval lake 43 feet above sea level, having clear, non-peaty water, and no visible affluent or effluent. This loch is situated in a hollow, and the meadow-land, which surrounds it on three sides, runs down almost to the water's edge, a narrow strip of stony shore intervening. The shore is chiefly occupied by Juncus effusus, with which a few other plants are mingled, but there is no marsh. At the south-west side there is a plantation between the water and the adjacent road. The average depth is from 6 to 8 feet, and the floor of the loch is almost entirely covered with vegetation. Littorella lacustris carpets the bottom to a depth of 3 feet, and, creeping up the shore, mingles with the grass of the meadow. Lobelia Dortmanna is abundant to a depth of 5 feet, whence long peduncles elevate the flowers above the surface. Isoetes lacustris is abundant from 4 to 8 feet deep. Elatine hexandra occurs in patches very plentifully from the margin to 6 or 7 feet deep, and also on the shore (p. 74). Nitella opaca is very abundant from 4 to 9 feet deep. Fontinalis antipyretica and Potamogeton obtusifolius are scarce. Besides the above there are a few of the usual plants.

Soulseat Loch, which has a very irregular outline, is close to the above,

but is not visibly connected with it. It is 39 feet above sea level, is about ½ mile long and ¼ mile broad, with a maximum depth of 42 feet. The surrounding features are similar to those of Loch Magillie, as also is the margin. The west shore has a zone of Heleocharis palustris, as at White Loch, behind which, in some places, there is a narrow strip of marsh with the usual variety of plants. At other parts a narrow border of stones intervenes between the water and the meadow, this shore, as at Loch Magillie, being occupied by Juncus effusus. Nowhere is there any broad zone of marsh. The stones from the margin to a depth of from 2 to 3 feet are often thickly overgrown with Cladophora canaliculata, etc. A marked feature of this loch is the vast quantity of plankton organisms, which render the water quite turbid; in addition to which, there are such enormous numbers of Glœotrichia Pisum that in some parts the water resembles pale green paint. No doubt the turbidity of the water of this loch accounts in some measure for the poor bottom flora. The Rev. Mr Paton, whose manse is pleasantly situated on a peninsula jutting into the loch, informed me that in the winter the turbidity disappears, and then it is possible to see the bottom to a depth of 6 feet. Obviously the clearness of the water in winter has no effect upon the extension of a bottom flora of Phanerogams. No plants occur at a greater depth than about 6 feet, because in deeper water there is a deposit of vegetable detritus lying upon mud. Ranunculus circinatus and Callitriche autumnalis are the only dominant submerged Phanerogams, and both are extremely abundant. Potamogeton perfoliatus abounds in a few spots; Littorella lacustris and Nitella opaca occur, but not plentifully. These few species were the only submerged plants I could find. The marginal flora beyond what has been mentioned is of little interest.

There are three small lochs lying close to the railway, about a mile west of Castle-Kennedy station. The easternmost one was dry at the time of my visit, the site being covered with Juncus effusus and other marsh plants. The others are entirely overgrown with aquatic vegetation, and are so surrounded with extensive marsh that the water cannot be approached. No boat being available, I am unable to give a proper account of their floras, but some of the features are illustrated in figs. 76 to 78.

The plants observed at the lochs described after Black Loch, but not hitherto mentioned, as they form no prominent features at these lochs, are as follows:—Iris Pseud-acorus, Carex rostrata, Glyceria fluitans, Sparganium ramosum, Phalaris arundinacea, Comarum palustre, Juncus effusus, Mentha sativa, Œnanthe crocata, Ranunculus Flammula, Polygonum amphibium,

Caltha palustris, Radicula officinalis, Bidens cernua, Hydrocotyle vulgaris, Hypericum elodes, Galium palustre, Spiræa Ulmaria, Peplis Portula, Callitriche stagnalis, Myosotis palustris, Equisetum palustre, etc., and a few common mosses.

[**The lake** at Lochnaw Castle partakes of much the same characteristics as Monreith Lake (p. 140), being also surrounded with wood. Carex pendula grows upon its shores.—J. M'A.]

There are some pools situated upon the Sands of Luce. Thinking, from the nature of the surroundings, that they might afford something of interest, I was disappointed to find they had dried up. Fig. 79 illustrates the site of one of these pools, the appended legend being a sufficient description. These sands extend about 6 miles along the coast, and reach inland about a mile and a half. The dunes, however, are nowhere very large; they begin quite abruptly just above high-water mark, and are immediately covered with Ammophila arundinacea (fig. 80), which binds the otherwise shifting sand. The highest dunes are at some distance from the sea; these also are capped with Ammophila, but are otherwise almost bare of vegetation. Usually, however, the dunes are quite stunted, and there is more vegetation (fig 81), or the ground is more or less flat and moor-like, with a complete plant-covering. The dominant plants are—Ammophila arundinacea, Carex arenaria, Salix repens, Hylocomium triquetum, Rhacomitrium canescens and its variety ericoides, Calluna vulgaris, and Pteris aquilina. These plants frequently form pure associations, or they may be more or less mixed. Occasionally there is a grassy sward, but, being closely cropped by rabbits, the species could not be readily identified. Near the sea a number of the usual halophilous herbs, such as Salsola Kali, Eryngium maritinum, Arenaria peploides, etc., occur scattered over the sand. About Low Torrs, and also at Glenluce, there are extensive salt-marshes with the usual vegetation, amongst which Ruppia rostellata, Scirpus maritimus, and Juncus maritimus (fig. 82) are particularly abundant.

IV.—Area VII.

The examination of the lochs of Fife and Kinross (p. 67) may begin at Lindores Loch in the neighbourhood of Newburgh; and after visiting others in the same district we go to Tents Muir, and thence to Kilconquhar Loch near Elie. From there we travel westwards, following a zigzag route, by way of Clatto Reservoir, Carriston Reservoir, Loch Gelly, Burntisland Reservoir, Loch Fitty, and others, to the lochs situated on the Cleish and Lomond Hills, and thence to Loch Leven. Finally, on an ebb

tide and with a westerly wind, we sail out of Anstruther harbour for the Isle of May.

Lindores Loch is situated 2 miles south-east from Newburgh, amidst a beautifully wooded and agricultural country, where hill and dale follow one another in quick succession. The loch is about $\frac{3}{4}$ mile long and $\frac{1}{4}$ mile broad, its surface being 222 feet above sea level. Its water, which has a maximum depth of 10 feet, is not peaty, but turbid and dead-looking. In many places there is deep, black, fetid mud upon which submersed aquatic plants do not seem to flourish well. In several places, but particularly at the north-west and south-east ends, as well as on the east side, there are large associations of marsh plants. At other places there is a narrow strip of stony or sandy-muddy shore merging into meadow-land. Such shores are usually more or less overgrown with Juncus acutiflorus; this is particularly the case on the west side. Along a considerable portion of the east side runs the public road from Newburgh to Kirkcaldy. This is shut off from the loch by a wall which usually enters the water, and no marsh plants occur there. At other places on the east side there is a stony or sandy shore similar to that of the west side, but usually with less vegetation. In the middle of the loch there is an island formed by a muddy flat, and densely overgrown with Phragmites communis. Many submersed plants have a deposit of lime upon their leaves and stems, and, as is commonly the case with lochs of this nature, filamentous Algæ, particularly Cladophora flavescens, abound. The striking features of the vegetation of this loch are the large quantities of the following plants:—Typha angustifolia, Glyceria aquatica, Scirpus lacustris, Phragmites communis, Phalaris arundinacea, Polygonum amphibium, Nymphæa lutea, Ranunculus circinatus, R. peltatus, and Myriophyllum alterniflorum.

At the south-east end Nymphæa lutea and Polygonum amphibium cover the surface of the water. Nearer the land there are associations of the following:—Glyceria aquatica, Typha angustifolia, Phragmites communis, Iris Pseud-acorus, Heleocharis palustris, Equisetum limosum, and Carex rostrata. Between these associations, and mixed with them, are the following:—Littorella lacustris, Potamogeton natans, Sparganium simplex, Menyanthes trifoliata, Mentha sativa, Alisma Plantago, Caltha palustris, Radicula officinalis, Ranunculus Flammula, Carex disticha, Hydrocotyle vulgaris, Hypnum cuspidatum, etc. Upon the west side, skirting the shore, there are associations of Glyceria aquatica, Scirpus lacustris, and Polygonum amphibium, besides a number of other plants in less abundance. From the middle of the east shore a flat peninsula juts out into the loch. This is considerably overgrown with Typha angustifolia (fig. 85), Heleocharis

palustris, Phalaris arundinacea, Spiræa Ulmaria, and, mixed with them, a variety of other plants. On drier ground, Agrostis vulgaris abounds, whilst the surface of the adjoining water is covered with Nymphæa lutea (fig 85) and Polygonum amphibium. The extensive area of marsh at the north-west end has a very luxuriant vegetation, Typha angustifolia, Glyceria aquatica, Polygonum amphibium, Equisetum limosum, and Carex rostrata being the dominant forms (figs. 83–84).

Besides the above mentioned, the following plants also occur here:—Callitriche autumnalis, Potamogeton filiformis, P. Zizii, P. pusillus, Sparganium ramosum, Comarum palustre, Callitriche stagnalis, Eriophorum polystachion, Epilobium palustre, E. tetragonum, E. hirsutum, Myosotis palustris, Heleocharis acicularis, Juncus bufonius and its var. fasciculatus, Carex flava, C. hirta, Scirpus setaceus, Mentha aquatica, M. arvensis, Ranunculus Flammula, Lycopus europæus, Lythrum Salicaria, Alisma ranunculoides, Hypnum fluitans, H. cuspidatum, and other common marsh mosses.

Black Loch is a small oval pool $\frac{1}{5}$ mile long, situated about a mile south-west of the last-mentioned loch, and surrounded by agricultural land. Excepting for a portion of the south shore, this loch is so entirely surrounded by marsh that the water cannot be approached. Its water is not peaty, but clear and bright, and is entirely encircled by a zone of Castalia speciosa and Nymphæa lutea, the latter being next the shore, which is the reverse of the order in which they usually grow (figs. 86 and 87). At the south side no other plants occur between these and the gravelly-muddy shore, but elsewhere there is a zone of Equisetum limosum between the Nymphæa lutea and the land. Here and there, all around the loch, there are associations of Glyceria aquatica on the shore side of the Equisetum. Upon both the east and west ends of the strip of gravelly shore on the south side, there is a large and pure association of Menyanthes trifoliata (fig. 86). In some places, particularly at the west end, where there is a large bog, the Equisetum limosum is followed by Carex rostrata, and that, in turn, by Juncus effusus on the drier ground. Besides the above, the following plants also occur at this loch:—Littorella lacustris, Myriophyllum alterniflorum, Ranunculus aquatilis, Utricularia vulgaris, Polygonum amphibium, Comarum palustre, Heleocharis palustris, Carex Goodenovii, C. disticha, Iris Pseud-acorus, Alisma Plantago, Caltha palustris, Mentha sativa, M. aquatica, Juncus acutiflorus, Ranunculus Flammula, Cardamine pratensis, Galium palustre, Hydrocotyle vulgaris, and a few common marsh mosses.

Lochmill Loch is beautifully situated amongst the hills, 2 miles south-

west from Newburgh, which it supplies with water. It is about $\frac{1}{3}$ mile long, and half that in width. Low hills of volcanic rock, with grassy or cultivated sides, and occasional plantations of coniferous and deciduous trees, surround it, excepting at the east end, which is more open. Although peat occurs on the higher hills immediately to the south and west, it is doubtful if any appreciable quantity of peaty water gains access to the loch. There is not much marshy ground, although near the effluent at the east end, as well as at other places here and there about the shore, small areas of marsh occur. Excepting for the marshy areas and a rocky part on the south-west, the shores consist of muddy gravel, and merge imperceptibly into the grassy banks. The water is clear, non-peaty, and apparently of a steely-gray colour, probably due to the copious deposit of black mud on the bottom, which arises from the rapid decomposition of a very luxuriant aquatic flora. Many of the submersed plants were heavily coated with a deposit of calcium carbonate. That beautiful species of the Polyzoa, Plumatella repens, was very abundant on sunken twigs, etc., and many of the submersed plants, particularly Littorella lacustris, were overgrown to an extraordinary degree with Diatomaceæ. On the north side there is a large association of Polygonum amphibium (fig. 88), which is frequently mixed with Potamogeton natans, and a belt of the latter extends along the outside of the Polygonum in deeper water. A similar phenomenon also occurs upon the south side, as well as a large and pure association of Potamogeton natans, which in this loch is of typical form, and perfectly distinct from any variety of P. polygonifolius. Potamogeton Zizii and a large form of P. lucens are both very abundant, and cover large areas of the bottom to a depth of 10 feet. Littorella lacustris is abundant upon the shores and in the water to a depth of 3 feet. Heleocharis acicularis frequently forms a sward upon the shore, in which condition it may be mistaken for a fine-leaved grass; it also grows to a depth of 3 feet.

Besides the above mentioned, the following plants are more or less abundant:— Chara aspera, C. hispida and its var. rudis, Callitriche autumnalis, Myriophyllum alterniflorum, M. spicatum, Ranunculus circinatus, R. peltatus, Potamogeton obtusifolius, P. pusillus, Castalia speciosa, Nymphæa lutea, Apium inundatum, Glyceria fluitans, Sparganium natans, Equisetum limosum, Heleocharis palustris, Sparganium simplex, S. ramosum, Carex rostrata, C. flava, Alisma Plantago, Glyceria aquatica, Callitriche stagnalis, Juncus acutiflorus, J. effusus, J. bufonius, Montia fontana, Mentha sativa, Gnaphalium uliginosum, Polygonum Hydropiper, P. Persicaria, Ranunculus Flammula, Caltha palustris, and a few of the ordinary marsh and rock mosses.

Morton Lochs, situated on Tents Muir, and of recent construction, are of considerable interest because of the rapid development of an extensive aquatic flora that has occurred there.

Mr James Morton Christie, the owner of the lochs, formed them in 1906 for fishing purposes by enclosing a natural depression of the land by means of a wide embankment of sand, and by diverting a burn to the site as a feeder. This moor (muir) is an extension of the sand dunes now nearly 3 miles to the east, and is mostly a sandy heath, some of the more favourable spots, however, being under cultivation. In the way indicated, two lochs were formed, having a general depth of from 4 to 8 feet. One of them is about ½ mile long and ¼ mile broad, whilst the other is about half that size; they are connected by a narrow passage through the embankment, which alone separates them. Although there was a thin covering of peat over the sand, still the water is not peaty, because the feeder originates in and passes through a cultivated and non-peaty district.

For the first two years no aquatic Phanerogams were noticeable in the lochs, but there was a great abundance of Algæ, chiefly Rhizoclonium hieroglyphicum, Spirogyræ, and Cladophoræ, which, becoming detached from the sides of the loch, where they chiefly originated, floated about in the water and bade fair to ruin the fishing. These Algæ are now kept down by spraying the sides of the loch with a solution of copper sulphate in the spring.

During the third summer a considerable growth of submersed Phanerogams appeared, but not in sufficient quantity to interfere with the fishing. By the fourth summer, however (*i.e.* in 1909), the plants had increased to such an extent that they would seriously impede the operations of the sportsman were they not subjected to frequent raids by the proprietor of the lochs.

At present there is no development of marsh vegetation at the margin of either loch, such plants being, in fact, practically absent, which is not surprising, in view of the applications of copper sulphate previously mentioned. In both lochs the dead Calluna lying at the bottom is covered with Cladophora, etc. In the smallest loch, which is the southernmost one, Myriophyllum spicatum occurs in extraordinary abundance; somewhat less plentiful are Potamogeton obtusifolius, P. pusillus, and P. crispus; and these four species may be said to fill the loch to the exclusion of other plants. In the largest loch Myriophyllum spicatum is not very abundant, but the three species of Potamogeton just mentioned are very plentiful, whilst P. polygonifolius, P. perfoliatus, and Myriophyllum alterniflorum occur, but are all scarce. Nitella opaca, Chara vulgaris and its var. papillata, C. contraria

and its var. hispidula, C. fragilis and Callitriche hamulata, both with and without floating rosettes, are all frequent, the varieties of the two species of Chara being more abundant than the types. Juncus fluitans, Littorella lacustris, Ranunculus peltatus, R. Baudotii, R. Drouetii, and Hydrocotyle vulgaris occur sparingly, whilst Callitriche stagnalis is plentiful.

As may be imagined from the nature of the embankment, a deal of leakage takes place around these lochs, and a number of little pools have been formed by the accumulation of the water in hollow places, besides which, others have been intentionally made as nurseries for the young trout. Some of these pools are interesting because of the number of plant forms that grow in and around them. Two in particular exhibit a large number of variations in species of Juncus, chiefly of J. acutiflorus, J. supinus, and J. bufonius. Others have curious forms of Ranunculus Flammula, and at one the var. natans of *Persoon* is very abundant (p. 72). In other places Marchantia polymorpha, *L.*, covers considerable tracts of wet ground, producing its reproductive bodies, both asexual and sexual, in extraordinary abundance. On the sides of a drain 2 feet deep that had been cut in the sand only two years previously, Blasia pusilla, *L.*, was growing luxuriantly, as well as numerous commoner Bryophytes. The Blasia also occurs abundantly at some other places which are kept moist by the water escaping from the lochs; and on the sandy-peaty shores of some of the pools exposed in summer by the falling of the water level, Botrydium granulatum, *Grev.*, and Riccia crystallina, *L.*, were very abundant, R. glauca, *L.*, and Aneura pinguis, *Dum.*, were fairly common, whilst Riccia Lescuriana, *Aust.*, and Aneura latifrons, *Lindb.*, were scarce.

The advent of the lochs on the previously dry, sandy moor has wrought considerable changes in the flora of the immediate district even in this short time, and doubtless others will follow. Where the plants came from is an interesting problem, to which a satisfactory answer is not easily found, but in all probability seeds and spores have been brought to the lochs by water-birds migrating eastwards from the lochs of central and western Fife, where most of the plants, excepting the rare Hepaticæ, abound.

Kilconquhar Loch is situated about a mile north of Elie, at an elevation of 49 feet above sea level. It is a very shallow circular loch about ½ mile across, and is so completely surrounded with marsh and reed swamp that the water can only be approached at a few places, consequently there is no definite shore. The village of Kilconquhar is situated on the north side of the loch (fig. 92), and the gardens of the adjacent cottages run down to its margin. The ornamental grounds of Elie House, which are wooded or park-like, adjoin and beautify the south side (figs. 89 and 91). Upon the

east and west sides the loch is surrounded by agricultural land. The bottom of the loch at the north and west sides consists of deep black mud, but at the south and east sides the bottom is less muddy, and in many places is formed of firm sand. From near the margin to some distance out the average depth of water is from 3 to 5 feet, but as the middle is approached the depth increases somewhat, never, however, exceeding 7 feet. The water is non-peaty and clear, but has a stagnant appearance, which may be described as dead in comparison with the sparkling water of a pellucid highland loch. It is probably rich in plant food-salts, and in consequence of such favourable chemical and physical conditions the whole of the bottom of this loch is more or less overgrown with plants. The marginal swamp vegetation (figs. 89 and 90) is chiefly composed of associations of the following species:—Scirpus lacustris, Equisetum limosum and its var. fluviatile, Phragmites communis, Heleocharis palustris, Carex rostrata (in some places a very robust form of this plant occurs with leaves 5 feet long), Hippuris vulgaris, Typha latifolia, Epilobium hirsutum, Menyanthes trifoliata, Sparganium ramosum, and Phalaris arundinacea. At the south-east side of the loch there is a large association of Polygonum amphibium, whose leaves and flowers cover a very considerable area of the water beyond the marsh (fig. 91). Outside this zone of Polygonum a wide space is occupied by associations of Potamogeton pusillus, P. filiformis, P. pectinatus, Zannichellia palustris, var. brachystemon, Myriophyllum spicatum, Callitriche autumnalis, etc. From the outer margin of this space Ranunculus circinatus reaches the surface even from a depth of 7 feet, and continues to the opposite side of the loch, but nearer the village this species thins out somewhat, and there, Ranunculus Baudotii becomes the dominant plant. The white flowers of these two species and the floating leaves of the latter entirely cover the surface of the water over a large area (fig. 92). Although these two species of Ranunculus were growing together very freely in some parts of the loch, no form was found that might be considered a hybrid between them.

Looking across the loch from the village to the south-east corner, the spectacle presented by the surface-flowering species was unique. The expanse of white Ranunculi, followed by the gorgeous pink of the Polygonum, backed by the dark foliage of the deciduous trees, to which a skirting of paler green was afforded by the marginal associations of Phragmites and Typha, and illuminated by the refulgence of a cloudless sky, gave such a combination of vivid colour contrasts, harmoniously pleasing to the eye, as can rarely be seen in this country.

All the plants mentioned as occurring between the associations of Poly-

gonum and Ranunculus are also abundant at other parts of the loch. Zannichellia palustris, var. brachystemon, occurs in extraordinary quantity on the deep black mud at the west side, whilst Myriophyllum spicatum is so plentiful in some places, particularly on the east side, that the surface of the water appears crimson with its flowers. Callitriche autumnalis occurs chiefly in small patches a yard or less across, but in a few places considerable areas of the sandy bottom are covered with it. Lemna trisulca is extremely abundant in shallow water, especially at the north side near the village. Large masses of Cladophora flavescens and Enteromorpha intestinalis were floating about in many places; indeed, I have only seen such a prodigious quantity of the last mentioned at one other place in Scotland, namely, at Duddingston Loch, near Edinburgh. In some places a species of Nostoc was abundant, floating in masses amongst the vegetation of the margin. A single specimen of Hippuris vulgaris was gathered here, having the nodes in the form of a continuous spiral from base to apex. Considering the shallowness of the loch and the other apparently favourable conditions, it seems astonishing that the reeds which grow so luxuriantly at the margin, particularly Scirpus lacustris, Phragmites communis, and Equisetum limosum, have not overgrown it almost completely.

Besides those already mentioned, the following plants were plentiful at this loch:—Chara aspera and its var. capillata, Potamogeton crispus, P. perfoliatus, P. obtusifolius, Lemna minor, Caltha palustris, Sium angustifolium, Comarum palustre, Epilobium tetragonum, Mimulus Langsdorffii, Myosotis palustris, Veronica scutellata, Mentha aquatica, M. sativa, Equisetum arvense, E. palustre, Ranunculus Flammula, Juncus acutiflorus and J. effusus. There are a few common bog mosses, but such are not abundant, as favourable situations are scarce.

Halton Reservoir is a small, irregularly shaped sheet of water situated about 2 miles north of Largo. It has been formed by the widening of the natural gorge of the Halton Burn and by the construction of a dam at the lower end. At the time of my visit the water had fallen about 12 feet below the full water level, leaving upon the exposed mud the remains of a number of aquatic plants. Some of these were growing in terrestrial form upon the mud, *e.g.* Myriophyllum spicatum, Polygonum amphibium, Ranunculus peltatus, Potamogeton natans, Callitriche stagnalis, etc. In some places the mud was thickly covered with dead remains of Chara fragilis, which is extremely abundant there. A large quantity of the same species was still flourishing in the water, which under conditions of normal water level would be at a depth of about 14 feet. Gnaphalium uliginosum is very abundant, and forms a sward upon the sides near the full water level.

Except a few common marsh plants about the affluents at the west and north, nothing else of botanical interest was noticed here. When the water is low, the steep rocky or stony sides give this place the appearance of a flooded quarry, which indeed it is.

Clatto Reservoir is situated about 3 miles south of Springfield, at an elevation of over 500 feet above sea level, in an upland district of which Clatto Hill is the highest point. It is a narrow sheet of water about $\frac{3}{4}$ mile long, made by building a dam across the east end of the valley through which flows the Ceres Burn. The water is clear, and not peaty, and is bordered in many places by a zone of marsh, or a narrow strip of sandy-muddy or stony shore may intervene between the water and the grassy banks. Skirting a portion of both the north and south shores there are plantations of coniferous trees, otherwise the surrounding country is mostly of the agricultural type. At the east end, where the dam is, deep water occurs, and this part bears no plants, but at the shallow west end there is an extensive development of marsh. The vegetation of this marsh consists chiefly of the following species:—Juncus effusus, J. acutiflorus, Deschampsia cæspitosa, Spiræa Ulmaria, Carex Goodenovii, C. rostrata, Heleocharis palustris, Sparganium ramosum, Menyanthes trifoliata, and Comarum palustre. These species occur in definite associations in accordance with the amount of water, whilst the minor plants of this formation are—Cnicus palustris, Stachys palustris, Ranunculus Flammula, R. hederaceus, Caltha palustris, Callitriche stagnalis, Montia fontana, Hydrocotyle vulgaris, Galium palustre, Mentha sativa, M. aquatica, Radicula officinalis, Glyceria fluitans, and Veronica Beccabunga. In deeper water, beyond the associations above mentioned, a large area is occupied by Equisetum limosum, with which are mixed, here and there, small patches of Hippuris vulgaris. These advance into water 3 or 4 feet deep, and succeeding them there are associations of Ranunculus aquatilis, Sparganium natans, Potamogeton natans, P. pusillus, P. obtusifolius, and Myriophyllum spicatum, running out into water 7 or 8 feet deep.

The south shore has a border of marsh along a considerable portion of its length, chiefly composed of Juncus effusus, J. acutiflorus, Cnicus palustris, Deschampsia cæspitosa, Spiræa Ulmaria, Carex Goodenovii, C. rostrata, and Heleocharis palustris, the last being the most abundant, and occupying a zone from 10 to 20 feet wide outside the other species. In deeper water, beyond the zone of Heleocharis palustris, there are patches of the submersed plants mentioned above, Potamogeton natans and Myriophyllum spicatum being the most abundant. Here and there, where the border of marsh is absent, the shore is frequently carpeted with Littorella lacustris, whilst Heleocharis acicularis is scarce.

The north shore resembles that on the south, but vegetation is somewhat less abundant. In some places associations of Equisetum limosum extend into the water beyond the marginal zone of Heleocharis palustris.

At the south-east end, an arm to the reservoir has been formed by constructing a dam across an adjacent valley and excavating a connection. The flora of this arm or bay is similar to that of the main body of water. There is an extensive marsh on its west side, with vegetation similar to that existing at the west end of the main reservoir, excepting that Spiræa Ulmaria is conspicuous by its greater abundance and luxuriance, and that the var. fluviatile of Equisetum limosum is more abundant than the type.

Bryophytes are poorly represented by a few of the common marsh mosses. The Heleocharis palustris growing at this reservoir was much diseased by Claviceps nigricans, *Tul.* (*Sclerotium Eleocharidis*, *Thum.*), some of the inflorescences bearing 3 or 4 ergots $\frac{1}{2}$ inch long. This fungus is one seldom met with in this country.

Carriston Reservoir is a sub-circular sheet of water, $\frac{1}{4}$ mile across, situated 2 miles north-east of Markinch in a rich agricultural district. It was formed by the construction of a long dam across a valley through which flowed a tributary of the River Leven. The water is clear, and not peaty. The dam occupies most of the west side, and there is not much shore on the south, as a bank, which is faced with stonework, frequently enters the water. On the north and east there is a flat, sandy or muddy shore, and the small amount of marsh vegetation about the loch occurs at this place. There is a mixed plantation and a few isolated trees on the north shore (fig. 93), otherwise there is no wood in the immediate vicinity of the water. Heleocharis acicularis forms a dense sward on parts of the sandy-muddy shore, covering it as grass does a meadow, and entering the water to a depth of 3 or 4 feet. I have not seen this plant so abundant anywhere else. On some parts of the exposed shore terrestrial forms of Myriophyllum spicatum were abundant. In one or two places Gnaphalium uliginosum and Juncus bufonius formed a dense sward. Heleocharis palustris is abundant near the winter water level, but the plants are dwarfed, being only 6 or 8 inches high, probably because they are left comparatively dry in summer, owing to the water receding from them. The tiny Ranunculus reptans is very abundant here, even more so than at Loch Leven, where it also occurs, the sandy shore in some places being covered with it (fig. 94). Water-fowl migrating eastwards from Loch Leven, and avoiding the Lomond Hills, would be likely to drop down at Carriston, which is only 10 miles distant, and probably by their agency several plants common at Loch Leven have been introduced to this reservoir.

Besides the above, the following plants also occur here:—Littorella

lacustris, not abundant; Nitella opaca, Callitriche autumnalis, and Myriophyllum spicatum, all very abundant; Potamogeton pusillus, P. perfoliatus, and Polygonum amphibium, not abundant. All the following were comparatively scarce:—Carex rostrata, C. Goodenovii, C. disticha, C. flava, Caltha palustris, Mentha sativa, M. aquatica, Juncus acutiflorus, J. effusus, J. conglomeratus, Phalaris arundinacea, Spiræa Ulmaria, and Ranunculus Flammula. Bryophytes were very scarce.

Kinghorn Loch is a rectangular sheet of water about $\frac{1}{3}$ mile across, situated a mile west of the village of Kinghorn, at an elevation of 204 feet above sea level. The water, which has a maximum depth of 38 feet, is not peaty, but turbid and dead-looking. The west shore, upon which there are a few willow trees, is flat and boggy, and its vegetation merges into that of the adjoining meadow, this being the only side of the loch where there is any development of marsh plants. A public road adjoining the south side is shut off from the loch by a wall and a row of bushes, below which a few marsh plants may be found, but there is practically no shore, except in a dry season when the water has fallen. The east shore is composed of dirty sand or gravel, with stretches of bare volcanic rocks. At the north side either meadow-land or a wall adjoins the water, without the intervention of a shore.

In early summer the water contains a vast quantity of Anabæna Flos-aquæ, var. circinalis, which so discolours the water in some parts, in accordance with the direction of the wind, that it has the appearance of pale green paint. A number of perch, which, I suppose, were killed by the Anabæna, were strewn about the shore. The exact action of this alga upon the fish is not known, but it is usually thought that it clogs the gills, although there may be poisonous properties as well, several species of Myxophyceæ being known to be baneful to horses and cattle that drink water containing them.

The west side of the loch is overgrown with an association of Polygonum amphibium, that extends from the marshy ground outwards until the water is 6 or 7 feet deep, which is an unusual depth for this plant to flourish in. On drier parts of the bog terrestrial forms of Ranunculus peltatus and R. Drouetii cover a considerable tract, mingling with the Polygonum on the one hand, and with the grass of the meadow on the other. I have not seen these plants growing in terrestrial form to such an extent elsewhere. The following marsh plants also occur, but more or less scattered, as there are no large associations; and it will be noticed that while a number of uncommon plants grow at this loch, some of the usual ones are absent:—Equisetum limosum and its var. fluviatile, Sparganium ramosum, Heleo-

charis palustris, Alisma Plantago, Glyceria fluitans, terrestrial form, Veronica Beccabunga, Mentha aquatica, Carex hirta, both tall and dwarf forms; Caltha palustris, Juncus effusus, J. glaucus, J. acutiflorus, Phalaris arundinacea, Apium inundatum, terrestrial form; A. nodiflorum, var. repens, Sium angustifolium, Cardamine pratensis, Ranunculus Flammula, R. sceleratus, Radicula palustris, Myosotis palustris, Equisetum arvense, both tall and dwarf forms, and Hypnum cuspidatum.

The submersed aquatic plants are uninteresting and comparatively scarce. Littorella lacustris is abundant, and in many places forms a sward on the shore when the water has fallen. Potamogeton crispus, as well as the f. serratus, *Huds.*, and Myriophyllum spicatum, are fairly abundant, although they do not appear to be thriving very well. Potamogeton pectinatus and Ranunculus peltatus are scarce. Enteromorpha intestinalis is scarce, whilst Cladophoræ abound. In the paucity of submerged plants this loch agrees with some others in which "water-bloom" occurs in early summer, and, excepting for the occurrence of this phenomenon, the loch would probably support a rich aquatic flora.

This loch was one of the first recorded stations for Potamogeton Zizii in Britain, the spot, indeed, where Mr Boswell (Syme) collected the specimens which he distributed under the name of "P. lucens with floating leaves," subsequently referred to P. Zizii. This plant, however, now appears to be extinct in the loch.

Loch Camilla is a small oval sheet of water about 4 miles east of Cowdenbeath. The water, which is not peaty, is rather turbid, and is surrounded by meadow-land which approaches almost to the margin of the water. The narrow shore on the east, south, and north is stony and bears but few plants, but at the west end there is a considerable development of marsh vegetation. A large association of Equisetum limosum, mixed here and there with patches of Hippuris vulgaris, stands out in the water. Nearer the land there is a large association of Carex rostrata, behind which there is a wide stretch of bog gradually merging into meadow, and this bog is covered with the following plants:—Menyanthes trifoliata, Heleocharis palustris, Radicula officinalis, Caltha palustris, Cardamine pratensis, Mentha sativa, Galium palustre, Myosotis palustris, Ranunculus Flammula, Alisma Plantago, Juncus effusus, J. glaucus, J. acutiflorus, Phalaris arundinacea, Deschampsia cæspitosa, Spiræa Ulmaria, Cnicus palustris, Pedicularis palustris, Comarum palustre, Carex flacca, C. Goodenovii, Angelica sylvestris, Veronica Beccabunga, Ranunculus hederaceus, and a curious semi-terrestrial form of R. peltatus, with purple blotches on the peltate leaves, which suggests a crossing with R. hederaceus. A number of Bryophytes are also

abundant at this marsh, particularly Hypnum cuspidatum, Hylocomium squarrosum, Mnium rostratum, and Marchantia polymorpha. The following plants were abundant in the water:—Chara vulgaris, covering an extensive area of the bottom, and C. contraria. Littorella lacustris, Heleocharis acicularis, Potamogeton crispus, P. filiformis, P. pusillus, Myriophyllum spicatum, Callitriche autumnalis, Ranunculus peltatus, and near the margin a floating form of R. hederaceus. Possibly there are other species, but these are all I could discover without a boat.

Loch Gelly is an oval loch about $\frac{3}{4}$ mile long, situated at an elevation of 351 feet above sea level, 2 miles east of Cowdenbeath, and close to the village of Lochgelly. The loch is surrounded by low hills, which slope gently towards the water, except on the west side, where the country is quite open as far as Cowdenbeath, whilst the effluent passes through a depression of the ground at the east end. The district around is of the agricultural type, with a few acres of rough boggy pasture at the west end of the loch, which was probably a portion of its bottom at a former period. The margins of this shallow loch are so gently inclined, that only in a few places can a boat be brought within 20 feet of the shore because of the shallowness of the water. The sides at the north and east slope gently towards the loch, and are covered with a fine, close grass sward, about which there are a few large deciduous trees. This meadow-land gives place near the water's edge to a narrow shore of dirty sand or gravel, with a few larger stones, but, excepting some patches of Caltha palustris, Ranunculus Flammula, Littorella lacustris, etc., there is little vegetation on these shores. The west shores consist largely of a Phragmites swamp (fig. 95), behind which there is a marsh of varying width, which merges into the area of boggy pasture previously mentioned. Towards the north-west corner, however, about the affluent, the swamp is occupied by a considerable variety of plants an enumeration of which will be given presently. On the drier patches of this portion of the marsh there are bushes of Alnus glutinosa, Salix aurita, etc., and on the better-drained area farther from the loch there is a mixed wood. The south shore has a zone of marsh throughout its length, immediately behind which there is a narrow plantation of conifers, with which are mixed here and there, on the damper spots, alders, poplars, willows, etc. (figs. 95–97).

For several years this loch was used as the common receptacle for the sewage of the populous mining district around. The inflowing burn at the west end was then an evil-smelling open sewer, 6 or 8 feet wide, consequently the water of the loch was extremely foul, and an examination of the flora was by no means a pleasant occupation. The local sanitary authorities,

however, became enlightened regarding the danger of this mode of sewage disposal, and forthwith adopted a more modern method. Meanwhile, certain colliery owners found in the affluent a convenient means of disposing of their mine water as well as the waste from coal-washing machinery, so that now the burn resembles a stream of ink, and the loch is being silted up with a deposit of coal-dust. The influence of such filthy additions is seen over the whole of the loch, particularly at the west end, where the deep, black mud has an insufferable odour. When the loch received the sewage, the water had a turbid, unwholesome appearance, and was everywhere crowded with plankton organisms, besides which all objects about the shores were covered with filamentous Algæ, chiefly Cladophora fracta, whilst there were innumerable floating masses of Enteromorpha intestinalis and Cladophora flavescens. Now the water is black and dead-looking, and the Algæ have considerably diminished, especially the Cladophoræ, whilst everything is covered with black filth. The marginal vegetation previously mentioned is luxuriant, although somewhat restricted in variety, but the submersed plants are scarce, which is not surprising when one considers the vicissitudes through which the loch has passed.

The plants of the marshy area in the north-west corner are chiefly as follows:—Scirpus lacustris, Equisetum limosum, Heleocharis palustris, Carex rostrata, C. Goodenovii, C. paniculata, C. canescens, Menyanthes trifoliata, Epilobium hirsutum, Polygonum amphibium, Hippuris vulgaris, Phalaris arundinacea, Sparganium ramosum, Comarum palustre, Iris Pseud-acorus, Juncus effusus, J. glaucus, J. acutiflorus, Myosotis palustris, Veronica Beccabunga, Alisma Plantago, Ranunculus Flammula, R. sceleratus, Epilobium palustre, Valeriana officinalis, Lysimachia vulgaris, Cardamine pratensis, Caltha palustris, Spiræa Ulmaria, Viola palustris, Hypnum cuspidatum, etc. There are pure groups of Iris Pseud-acorus standing 5 feet high out of shallow water, and the same occurs at Loch Fitty. Some very large clumps of Cardamine pratensis were also found here. Many of these were propagating vegetatively by the production of plantlets from buds at the base of the leaflets. This method of reproduction is probably in this case due to overfeeding, as the soil is very rich at this corner of the loch. I am unable to say whether these plants produce seed as well, because when I saw them the season had passed. A beautiful variegated variety of Phragmites communis was also seen here. In the water in front of this marshy area the following plants occur, and some of them are also found at other parts of the loch:—Scirpus lacustris, Myriophyllum spicatum, Nymphæa lutea, Castalia speciosa, Potamogeton pectinatus, P. filiformis, as well as its variety alpina, *Blytt*, the last being extremely plentiful in the

effluent, and a small submersed form of Alisma Plantago, growing in 18 inches of water, with delicate linear-lanceolate leaves floating on the surface, and linear submerged ones.

The chief plants of the marshy zone along the south shore are—Equisetum limosum, Carex rostrata, C. Goodenovii, Heleocharis palustris, Iris Pseud-acorus, Phalaris arundinacea, Spiræa Ulmaria, Comarum palustre, Caltha palustris, Juncus effusus, etc., all of which grow luxuriantly (figs. 96 and 97).

Burntisland Reservoir is a very irregularly shaped sheet of water about ½ mile long, situated amidst picturesque surroundings 1½ miles north of Aberdour, at an elevation of 290 feet above sea level, and lying between the hills of Dunearn, Balcam, and Cullalo. It was formed by the construction of a short dam at the south-west end, where the maximum depth of 39 feet occurs. Upon the south side, the loose rock and soil have been protected by stonework, which in most places enters the water. Excepting a few lichens and Bryophytes, no vegetation occurs either along this wall or at the dam, but at other parts of the loch, marginal vegetation is generally abundant. The shores, where bare of plants, are either gravelly or muddy, and the water, which is not peaty, has a slightly turbid appearance due to the somewhat impure water of one of the affluents, and to the erosion of the muddy shore by the waves. These matters, however, are about to receive attention from the authorities at Burntisland who own the reservoir, and the proposed alterations will, I fear, eradicate a number of interesting plants from this locality. About the affluent at the east end there is a considerable extent of marsh, which, near the water, is covered with Equisetum limosum and Heleocharis palustris. From this place to about the middle of the loch, where there is a large bay, the flat shore, which is usually exposed in the summer by the falling of the water level, is sandy or muddy, and is covered with vegetation. Littorella lacustris grows out of the water and for some distance up the shore. Then there is a broad zone of Heleocharis palustris, with which a few other species of plants are mixed. Above that a narrow strip of Spiræa Ulmaria grows at the winter water level, and behind the Spiræa there is a luxuriant grass meadow (fig. 98). The Spiræa, as is usually the case with the Rosaceæ, is a gross feeder, and grows uncommonly well along this line, because the storms of winter deposit a supply of rich detrital matter at that place. Similar conditions to those just described for this piece of shore also prevail along the east side of the bay previously mentioned. The wide zone of Heleocharis is cut every summer, and dried for use as bedding for cattle, but chiefly in order to prevent the dead stems being washed into the loch during winter, as the decay of so large a quantity

of vegetable detritus would pollute the water. The margin of the bay, otherwise than upon a portion of the east side, has a very sinuous outline, partly because some old gravel quarries have been connected with it. A considerable portion of this bay is quite shallow, consequently a large area of mud is exposed after a period of drought. Upon this exposed mud I found a few specimens of Riccia crystallina, and Mr James M'Andrew discovered a small spot where it was quite abundant. According to my experience, the Ricciaceæ are very rare about Scottish lochs, but Mr William Evans and Mr James M'Andrew discovered five species in the reservoirs around Edinburgh, when the water level was abnormally low, during the dry summer of 1905.* My friend Miss Helen S. Ogilvie has found Riccia Lescuriana in great abundance just below the normal water level of Loch Long on the Sidlaw Hills; and at pools around the Morton Lochs on Tents Muir, some species of this genus abound on the sandy mud when the water has fallen (p. 152). Possibly such Hepaticæ have been previously overlooked, and a more diligent search might prove them to be of more common occurrence about the lochs than is at present acknowledged. Terrestrial forms of Littorella lacustris are very abundant upon the east shore of this bay. Upon a portion of the shore that is evidently only submerged in winter, there was a large prostrate form of this species, with copious stolons as well as flowers. The submerged subulate leaves were dead, and the flattened aerial leaves were slightly ciliated at the margins. In this bay there was also a curious aquatic form of Bryum pallens growing amongst Littorella, etc. (p. 92). One of the deep pools of the bay was almost filled with Myriophyllum alterniflorum, whilst in another Potamogeton obtusifolius, var. fluitans, was very abundant, and at the same place Myriophyllum alterniflorum and M. spicatum were growing together, which is rather unusual. Fig. 99 affords a view of this loch from the north, looking towards Dunearn Hill.

Besides those enumerated above, the following plants occur here:—Nitella opaca, Chara fragilis, Callitriche autumnalis, Apium inundatum, Potamogeton obtusifolius, P. filiformis, P. pusillus, P. perfoliatus, P. prælongus, P. lucens, P. Zizii, Ranunculus peltatus, Polygonum amphibium, Sparganium simplex, S. ramosum and its var. microcarpum, Carex rostrata, C. Goodenovii, C. aquatilis, C. flacca, C. disticha, Alisma Plantago, Caltha palustris, Radicula officinalis, Myosotis palustris, Comarum palustre, Mentha sativa, M. aquatica, M. arvensis, Juncus bufonius, J. glaucus, J. effusus, J. conglomeratus, J. acutiflorus, Epilobium tetragonum, E. palustre, Galium palustre, Stellaria uliginosa, Phalaris arundinacea, Ranunculus Flammula, R. sceleratus.

* *Trans. and Proc. Bot. Soc. Edin*, 1907, vol. xxiii., part iii., p. 285.

Veronica scutellata, V. Beccabunga, Cnicus palustris, Sagina nodosa, Bartsia Odontites, Stachys palustris, Deschampsia cæspitosa, Achillea Ptarmica, Scutellaria galericulata, Gnaphalium uliginosum, Grimmia apocarpa, var. rivularis, Eurhynchium rusciforme, Bryum argenteum, Hypnum cuspidatum, H. riparium, etc. This a long list of plants for a small loch of comparatively recent formation, and it will be noticed that the greater number of species are those of the marsh zone. A considerable number of water-fowl resort to this reservoir, and they are doubtless responsible for the introduction of so many plants, which, moreover, find the shores a suitable habitat. Scutellaria galericulata, for instance, is not a common plant at the lochs of Fife, but in marshy ground, adjoining the seashore, about 2 miles south-west of Aberdour, it grows in great abundance, and possibly it was transported from there to the reservoir by gulls, etc.

On the top of Balcam Hill, which adjoins this reservoir, there is a small pool about 12 yards long, containing over twenty species of aquatic and marsh plants, two of which, namely Glyceria fluitans and Potamogeton natans, do not occur in the reservoir below.

Otterston Loch is a small shallow sheet of water, 2 miles west of Aberdour. It is closed in by low hills and is entirely surrounded by luxuriant deciduous trees, which also cover a small island in the middle. The water is not peaty, and, although clear, it has a dead, stagnant appearance. The loch is an ornamental sheet of water to Otterston House, which stands upon its north side, whilst the public road from Aberdour adjoins it on the north-east. Except on the west side, where there is an extensive and treacherous bog, the loch is bordered nearly everywhere by low walls or grassy banks (figs. 100–102), so that there is practically no shore, but in several places marsh vegetation overgrows the banks. At the west end the mud at the bottom is deep, black, and fetid; at the east end there is much less mud, and there some parts of the margin are sandy, or a narrow zone of stones may even occur (fig. 102). Ceratophyllum demersum is so abundant that the loch is almost choked with it, and in summer, when the plants are at the surface, the manipulation of a boat over the water is a matter of some difficulty. Doubtless many plants that otherwise would thrive in this loch are excluded by the Ceratophyllum, which appears to be gradually extending, for I have noticed that Ranunculus Baudotii, which was abundant at the east side of the loch in 1903, had in 1908 become almost extinct by the extension of the Ceratophyllum. The latter, however, is not able to hold its own in the marginal zone against the Polygonum amphibium which grows there (figs. 100 and 101), possibly for two reasons. (1) The Ceratophyllum is favoured by water of some depth, to the bottom

of which it sinks in order to hibernate, whilst the Polygonum grows best in comparatively shallow water. (2) Owing to the luxuriant development of the floating leaves of the Polygonum, which obstruct the free access of light to the water below, the Ceratophyllum is not able to thirve in this darkened area, and consequently becomes extinct at the margin of the Polygonum association. Notwithstanding its absence in a few small areas, the quantity of Ceratophyllum in this loch is extraordinary. Equally striking is the flora of the bog at the west end. Considerable portions of it are covered with associations of Menyanthes trifoliata, the individuals of which are so closely compacted that no other plants can thrive amongst them. Ranunculus Lingua is also very abundant and grows in large patches, the vivid yellow of its masses of flowers affording an agreeable tone to the sombre green of the tussocks of Carex paniculata (fig. 103), which dominates the greater portion of the bog.

Besides those already mentioned, the following plants occur here:—Callitriche autumnalis, Potamogeton Zizii, Zannichellia palustris, Ranunculus circinatus, Myriophyllum spicatum, Lemna minor, L. trisulca, Cladophora crispata, Carex rostrata, Heleocharis palustris, Cicuta virosa, Sparganium simplex, S. ramosum, Epilobium hirsutum, Ranunculus Flammula, Caltha palustris, Myosotis palustris, Galium palustre, Spiræa Ulmaria, Mentha aquatica, Juncus effusus, Phalaris arundinacea, Montia fontana, etc. [Scirpus sylvatica, close to the loch.—J. M'A.]. A number of Bryophytes occur about the banks and on fallen timber, particularly at the south and west sides, but as these are chiefly forms that grow in damp woods and similar places they cannot be named as belonging to the loch.

Loch Fitty is situated amidst a mining and agricultural district 3 miles west of Cowdenbeath. It is a mile long by $\frac{1}{3}$ mile wide, is at an elevation of 413 feet above sea level, and has a maximum depth of 17 feet. The water is clear but it has a flat, dead appearance, especially so in autumn when the abundant vegetation, particularly the Characeæ, is decomposing. A fine tow-net used in the middle of the loch at the end of September caught a very pure collection of Asterionella formosa. Many of the submersed plants were heavily incrusted with lime, which proves the presence of that substance in the water. Some plants, *e.g.* Potamogeton Zizii and P. perfoliatus, were so weighted with this substance that many of them were lying on the bottom instead of rising up to the surface. The shore at the north side is stony, gravelly or sandy, and almost destitute of marsh plants. At the south side a portion of the shore is composed of a bank of shale, which has been thrown out from an adjacent mine, and a number of aquatic plants occur in the pools and little bays formed by the irregularities in this

substance. At other places upon this side of the loch the shore is gravelly or sandy, and bears more plants than the opposite side does. The chief affluent enters the loch at the west end, previous to which it has a very sinuous course for about a mile through an alluvial flat consisting of pasture or arable land, doubtless at one time covered by the water of the loch. Near the loch this flat merges gradually into a bog several acres in extent (fig. 104). At the east end, whence flows the effluent, and where also another burn enters the loch, there is a similar but much less extensive bog (fig. 105). The loch is shallow throughout its area, a depth of 10 feet being seldom exceeded. The deepest water occurs opposite the mine on the south shore, where a depth of 17 feet may be sounded; this depth is probably due to the bottom sinking on account of the mining operations below. The photic zone extends to a depth of 12 feet, beyond which black mud occurs and no living vegetation. The stones about the shores are nearly everywhere thickly covered with Cladophoræ, which bear an extraordinary quantity of Diatomaceæ, chiefly of the genera Diatoma, Gomphonema, and Cocconeis. A remarkable feature is the vast amount of Potamogeton pectinatus growing in water from 6 to 12 feet deep, the slender stems carrying the flowers to the surface even from the greatest depth. Here and there at the marshes are pure groups of Iris Pseud-acorus, with leaves 4 or 5 feet high, standing out of the water as little islands. A considerable portion of the bottom is covered with Chara fragilis and its vars. fulcrata and delicatula, C. aspera and its vars. subinermis and capillata, and C. contraria, as well as with a number of other plants. Litorella lacustris and Heleocharis acicularis are scarce. Callitriche autumnalis is very abundant; besides the type there is a form differing slightly in the leaves and fruit. About the marshes aquatic and terrestrial forms of C. stagnalis abound. Potamogeton rufescens, the type as well as a very leafy, barren form, is abundant. P. pusillus, the type as well as a narrower-leaved form, P. filiformis, the type and a much longer-leaved form, and P. pectinatus are all very plentiful; indeed, the three last-mentioned species are so abundant that their fruits, which in the autumn are washed upon the shore, formed in a few places a stratum over the sand an inch deep. Potamogeton perfoliatus, P. Zizii, and P. obtusifolius are all very abundant, while P. prælongus, P. natans, and P. polygonifolius are all scarce. Myriophyllum spicatum is very plentiful, but M. alterniflorum is scarce. Anacharis Alsinastrum occurs sparingly, and the plants are weak. The following also occur, but none of them is abundant:—Nitella opaca, Juncus fluitans, Ranunculus peltatus, Nymphæa lutea, Polygonum amphibium, Sparganium simplex, var. longissimum, the last near the affluent at the west end, and

Fontinalis antipyretica on submersed stones, but chiefly at the north side of the loch. There is a small association of Scirpus lacustris on the south side and a few plants elsewhere, but they are all rather dwarfed, standing only about 3 feet high out of the water. Phragmites communis is represented by a few sparse and dwarfed patches at the marshes. Carex aquatilis is extremely abundant and covers large areas of marsh; when growing in water it is 3 feet high, but in drier places it only attains half that size (fig. 104). The following marsh plants also occur at this loch:—Equisetum limosum, Hippuris vulgaris, Heleocharis palustris, Sparganium ramosum, S. simplex, Iris Pseud-acorus, Alisma Plantago, Epilobium hirsutum, Carex rostrata, C. Goodenovii, Comarum palustre, Menyanthes trifoliata, Myosotis palustris, Phalaris arundinacea, Juncus acutiflorus, J effusus, J. glaucus, J. bufonius and its var. fasciculatus, Galium palustre, Veronica scutellata, Pedicularis palustris, Mentha aquatica, M. sativa, M. arvensis, Caltha palustris, Ranunculus Flammula, Equisetum arvense, E. palustre, Angelica sylvestris, Spiræa Ulmaria, and Deschampsia cæspitosa. Hepatics are scarce, but a number of mosses are common in the marshes, the following being the most abundant:—Climacium dendroides, Hypnum stramineum, H. cuspidatum, H. stellatum, Hylocomium squarrosum, and Aulacomnium palustre.

Town Loch, which is only a few hundreds of yards long, is about 2 miles north of Dunfermline, and close to the mining village of Townhill. At the time of my visit the water had fallen several feet owing to dry weather, and a large expanse of uninviting shore composed of sandy gravel, mud, and coal-dust, was exposed. At the full water level there is a zone of vegetation composed chiefly of plants of the damp meadow type, with which are mixed some of those species usually associated with the shores of a loch. The water is extremely foul, as the loch is used as a receptacle for sewage. It is, in fact, little more than a large and dirty horse-pond. The bottom at the centre is somewhat raised, and I could see from the margin that this part was covered with aquatic plants. There is no boat at this loch, however, and I did not feel inclined for a swim in the filthy water, although, at one end, a number of urchins were disporting themselves with great gusto. These I approached, and by the judicious distribution of a few coppers amongst them heaps of plants from different parts of the loch were quickly brought to shore. There were, however, only two species, viz. Chara fragilis, the type form in a very prolific condition, and a robust form of Potamogeton flabellatus. At the west end there was a large association of Polygonum amphibium, and smaller ones of Equisetum limosum and Carex rostrata. Some parts of the exposed shore were covered with Agrostis vulgaris,

Alopecurus geniculatus, and Polygonum aviculare. The other plants noticed here were—Littorella lacustris, Alisma Plantago, Heleocharis palustris, Juncus effusus, J. acutiflorus, J. bufonius, var. fasciculatus, Mentha arvensis, Equisetum arvense, Gnaphalium uliginosum, and Potentilla anserina.

Between Dunfermline and Saline there are two reservoirs at which I had intended to make detailed observations, but was prevented doing so by unfavourable weather, and I have not been able to visit them again. It is, however, of some interest to note that Potamogeten rufescens grows here in great abundance.

In a small pool at Patricks Walls, which is situated in a hollow surrounded on three sides by steep craigs, I saw with a telescope what was probably Nymphæa intermedia, but could not reach it on account of bog. I am led to record these observations because neither of these plants is common in Fife, but the latter and a variety of the former occur in a loch on the Cleish Hills, a few miles to the north.

Loch Glow is situated in an open, wind-exposed position on the Cleish Hills (fig. 106). These hills are for the most part covered with a grass-like formation of plants, below which in many places there is peat. The loch is the largest of a series of four; it is $\frac{3}{4}$ mile long by $\frac{1}{2}$ mile broad, and is situated at an elevation of 900 feet above sea level. The original loch has been deepened by the construction of a short dam at the east end, and it is now used as a reservoir. The water is clear, but slightly peaty. The north shore is rocky, stony or more rarely sandy, and the south shore is mostly peaty. A few leaves of Littorella lacustris, that had been washed upon the shore, were the only evidence of the existence of submersed aquatic plants in the loch that I could discover without the aid of a boat, which at the time of my visit was out of repair. Marsh plants also are practically absent. It would be reasonable to expect a number of submerged plants in this loch, as Black Loch, which drains into it, has several species. The shores, however, are scarcely suited for the development of a marsh flora.

Black Loch, recently mentioned, is a small sheet of water about $\frac{1}{2}$ mile west of Loch Glow, and is surrounded by hills. The water is somewhat peaty, and there is scarcely any shore save a few stony places here and there, as the grassy moor terminates in a bank at the water's edge. There is a thin association of Phragmites communis stretching along the south shore (fig. 107). Carex rostrata occurs in patches all around the margin, but particularly at the west end, and there are small associations of Equisetum limosum. Nymphæa intermedia occurs abundantly at the west end. Myriophyllum alterniflorum, Potamogeton perfoliatus, and P. prælongus are

fairly common, whilst P. rufescens, var. spathulifolius,* is extremely abundant outside the zone of Nymphæa intermedia in 7 or 8 feet of water. Mr A. Bennett informs me that as regards Scotland he had previously only seen this variety of Potamogeton from Loch Fada, Isle of Colonsay, Argyllshire. Littorella lacustris, Carex Goodenovii, Juncus effusus, J. acutiflorus, and J. lamprocarpus were the other abundant plants, whilst some other common species were less plentiful.

On the 18th of May no new shoots of Phragmites communis had appeared above the water at this loch, which is about 900 feet above sea level; whilst on the previous day at Loch Gelly, which is about 400 feet above sea level and only 8 miles away, the young shoots of the same species were over a foot above the water. At Loch Gelly the plant grows to a height of 8 or even 10 feet, whilst the form at Black Loch attains only half that size. Possibly they are two distinct physiological forms, whose morphological difference is most easily expressed in terms of size. It would be interesting to transpose specimens from one loch to the other in order to discover whether the form would change.

Loch Dow is a small oval sheet of water situated in a hollow of the grassy moor, ½ mile north-east of Loch Glow. The water is slightly peaty, and the stony or rocky shores on the north and east are narrow, with a sparse vegetation, or the moor meets the water without the intervention of a shore. Extending around the south and west sides there is an extensive bog, mostly occupied by Carex rostrata, which advances into the water on the one hand, and merges into the grass formation of the moor on the other (fig. 108). There are associations of Equisetum limosum on the south and west. Hydrocotyle vulgaris is abundant everywhere, and there are also a number of other common species.

Loch Larg is a few hundreds of yards north of the last mentioned, and is very similar to it, excepting that its eastern shore is more stony. There is a flat boggy area along the west side, which is covered near the water with Carex rostrata. Adjoining the moor this bog is overgrown with Calluna vulgaris, Polytrichum commune, P. gracile, Sphagnum cymbifolium, S. intermedium, etc. A slight but sudden rise of the ground causes an abrupt termination to the vegetation just mentioned, and in its place associations of grass-like plants, amongst which Scirpus cæspitosus is dominant, extend towards the moor (fig. 109). The line of demarcation between the Calluna and the grass-like associations is quite sharp, and probably marks the original extent of the loch.

* Fischer, "Die Bayrischen Potamogetonen und Zannichellien," *Ber. Bayr. Bot. Ges.*, xi. (1907), pp. 20–162.

There is no boat on any of the three last-mentioned lochs, so that it is impossible to say anything about their submerged plants except such as could be seen from the shore, or washed there, or gathered by swimming after them. Besides the plants already enumerated the following species were more or less common to these three lochs:—Littorella lacustris, Lobelia Dortmanna, Myriophyllum alterniflorum, Fontinalis antipyretica, Chara fragilis, var. delicatula, Castalia speciosa, Potamogeton polygonifolius, Equisetum limosum, Carex Goodenovii, Ranunculus Flammula, Eriophorum vaginatum, Juncus acutiflorus, J. effusus, Cardamine pratensis, Montia fontana, var. minor, Hydrocotyle vulgaris, Blindia acuta, Sphagnum intermedium, S. cymbifolium, Philonotis fontana, Aulacomnium palustre, Polytrichum commune, P. gracile, Hypnum commutatum, H. revolvens, H. scorpioides, H. cuspidatum, H. fluitans, Webera nutans, Bryum bimum, B. pallens, B. argenteum, Ceratodon purpurens, Rhacomitruim aciculare, Scapania undulata, Pellia epiphylla, and Diplophyllum albicans.

Harperleas Reservoir is situated on the Lomond Hills, at an elevation of 848 feet above sea level. It is about ½ mile long, and is of an irregular shape, with clear but somewhat peaty water. It has been formed by the construction of a long dam at the east end, and there the maximum depth of 41 feet occurs. The south shore is either stony or muddy, and at some places the bank enters the water without the intervention of a shore. At the north and west the shore is flat, and muddy or peaty, and is covered with a luxuriant vegetation, whilst there are very few plants at the south shore, and none along the dam. A zone of Equisetum limosum, behind which are Carex rostrata and Juncus effusus (fig. 110), extends along the greater part of the north side. The Equisetum is intermingled with Littorella lacustris, which also runs up the shore and forms a dense sward in some places. Occasionally considerable areas of exposed mud were covered with Juncus fluitans, which was reverting to the terrestrial type—J. supinus—which it somewhat resembled. Other normally submersed plants were assuming a terrestrial habit and forming a meadow-like sward upon the exposed shore, particularly Ranunculus aquatilis, Heleocharis acicularis, Polygonum amphibium, Potamogeton polygonifolius, and P. heterophyllus. The normal aquatic form of the last mentioned was very abundant at this loch, and those plants left upon the exposed mud were developing new shoots with aerial leaves similar to the coriaceous floating ones but smaller, the shoots with thin submersed leaves having completely withered away. At the north-west end a portion of the shore presented a remarkable appearance through being covered with dead tussocks of Molinia cærulea, which had been drowned during some period when the

water level was abnormally high (fig. 111). Salix repens covers the higher portions of the shore in some places at the west end.

A little to the east of Harperleas is Ballo Reservoir, both being situated on an upland plateau which forms the south flank of the East and West Lomond Hills. These reservoirs are surrounded by moor of the grass or heather type, or by a superior pasture-land which is due to cultivation. Harperleas Reservoir is treeless, but Ballo has a plantation of conifers upon its south-west shore (fig. 112). There are also a few plantations in the neighbourhood of the reservoirs, which pleasingly relieve the sameness of the moor, and add a picturesque charm to this pleasant, although small, stretch of upland country.

Ballo Reservoir has a somewhat pear-shaped outline, with the narrow end towards the south-east, in both of which respects it resembles Loch Leven on a small scale. It is about a mile long by ½ mile wide at the broadest part. In general features it much resembles Harperleas Reservoir, but there is less variety in the species of plants. At the north-west end there is an extensive peaty-muddy flat, covered with an association of Juncus effusus. This flat area extends out into the loch for some distance, and in the dry season is exposed by the falling of the water (fig. 112). It is covered with Littorella lacustris, Heleocharis acicularis, and Juncus fluitans, all of which assume the terrestrial habit when the water has receded. At the same end of the loch, but nearer the north side, Hydrocotyle vulgaris extends over a considerable area and forms a dense sward. Equisetum limosum forms a zone along a portion of the north shore, as at Harperleas Reservoir, behind which there is a strip of boggy ground covered with Carex, etc., and at one place there is an association of Typha latifolia. The shore along the north and east is flat and peaty, and a wide strip of it, exposed by the falling of the water (fig. 113), was more or less covered with Juncus fluitans, which was reverting towards the terrestrial type. Time did not permit me to make use of the boat at either of these reservoirs; but as the water was very low when I was there, probably there were few plants in the water that could not be observed by other means.

Besides those already mentioned, the following species were more or less common to both lochs:—Chara fragilis, var. delicatula, Fontinalis antipyretica, Myriophyllum alterniflorum, Ranunculus aquatilis, Potamogeton natans, P. polygonifolius, Heleocharis palustris, Polygonum amphibium, Callitriche hamulata, C. stagnalis, Comarum palustre, Mentha aquatica, M. sativa, Carex rostrata, C. Goodenovii, C. flava, Juncus supinus, J. acutiflorus, J. lamprocarpus, J. effusus, J. bufonius, Myosotis palustris, Caltha palustris, Peplis Portula, Veronica scutellata, Gnaphalium uliginosum, Ranunculus

Flammula and the little R. reptans, which, however, is rather scarce here; Hydrocotyle vulgaris, Spiræa Ulmaria, Galium palustre, Deschampsia cæspitosa, Hypnum fluitans, H. cuspidatum, and Aulcomnium palustre.

Loch Leven is situated in the lowest part of a somewhat oval strath called the Plain of Kinross, which is bounded by the Cleish Hills, Benarty Hill, the Lomond Hills, and the Ochil Hills (fig. 114). It is somewhat pear-shaped in outline, with the apex lying to the south-east. It is $3\frac{2}{3}$ miles long by $2\frac{2}{3}$ miles wide at the broadest part. The surface of the loch is 350 feet above sea level; and as the land for some distance around is below the 400-feet level, it must at a former period have been very much larger. It was artificially reduced in size in 1845, when its level was lowered $4\frac{1}{2}$ feet. On account of the shallow marginal zone, this slight lowering of the level reduced the area by about 1400 acres. For its size it is an extremely shallow loch, the greater portion of it being less than 15 feet deep. Indeed, along the east shore an area almost 3 miles long by nearly a mile broad is mostly less than 9 feet deep. It has, however, two depressions, each having a depth of about 80 feet, one to the west of St Serf's Island, and the other to the north-east of Scart Island. If the affluent were lowered 22 feet, so as to reduce the level of the loch by that amount, about 3000 acres of land would be reclaimed. There are six islands in the loch. The largest of them, called St Serf's Island, has an area of about 80 acres; it is quite treeless, and is utilised as a rabbit warren. Castle Island is covered with deciduous trees, and has an extent of about 5 acres (fig. 115). The outline of both these islands curiously resembles that of the loch, but their apices lie in the reverse direction. The other islands are quite small. The shores are everywhere flat and usually sandy, particularly on the east side, where the sand is sometimes blown into small dunes. More rarely the shore is composed of stones, or there is no shore because meadow-land comes down to the water's edge. In a few places there is a narrow zone of marsh extending a considerable distance along the shore, as, for example, upon both the east and south sides, opposite St Serf's Island (figs. 116 and 117). In many places there are large quantities of vegetable remains, chiefly those of Chara and Anacharis, lying upon the shore at the winter water level. The flat shores of this loch are in many parts very much exposed to wind, and due to this influence is the fact that some plants which ordinarily grow erect here assume a prostrate habit, such, for example, as Equisetum arvense, E. palustre, Juncus bufonius, J. acutiflorus, J. supinus, Ranunculus Flammula, etc. There are two or three associations of Phragmites communis, as well as of Heleocharis palustris and Equisetum limosum, that enter the water here and there, otherwise there are no plants of semi-aquatic type

in the water of the loch. The water is fairly clear, and is not appreciably peaty. The bottom of the loch from the shore to a depth of about 15 feet consists largely of firm sand, which is, however, frequently dirty and mixed with mud. Where the bottom is of this nature, which is particularly the case along the east side, it is usually carpeted with Chara aspera, or its var. subinermis, and sometimes with C. vulgaris and C. fragilis, to a depth of 14 or 15 feet. The growth of these plants, particularly the first mentioned, at depths of from 4 to 8 feet, is prodigious, but they thin out towards the shallower water on the one hand, and towards the deeper water on the other. Nitella opaca occupies considerable areas, also where the bottom is sandy, and at similar depths to the Chara, but it has a tendency to be most abundant in slightly deeper water than that at which the maximum growth of the Chara occurs. On the few areas where the bottom from near the margin to a depth of 15 feet is of mud, as, for example, in some places at the west side of the loch and in the bay at the east end of St Serf's Island, Anacharis Alsinastrum grows with such extraordinary vigour that in the summer, when the plants are near the surface, it is very difficult to row a boat through them.* At greater depths than about 16 feet no living vegetation of the higher type occurs, and mud covers the bottom nearly everywhere. This mud, which is usually blackish, with a somewhat offensive odour, was in August crowded with worm-like larvæ at many parts of the loch. Among a number of other plants which grow in the water, the most abundant is probably Potamogeton perfoliatus. The boat-keeper at the loch informed me that previous to the extensive development of the Anacharis this Potamogeton was extremely abundant, and that it had been partially exterminated by the former plant. Besides those already mentioned, the following plants occur at this loch:—Littorella lacustris, Callitriche autumnalis, Heleocharis acicularis, Tolypella glomerata, but very scarce; Myriophyllum spicatum, M. alterniflorum, Potamogeton filiformis, P. pusillus, P. obtusifolius, P. heterophyllus, and in pools on the shore the var. terrestris, *Schlecht.*, P. prælongus, P. Zizii, P. lucens,

* It is supposed that this plant was introduced into Loch Leven by an itinerant hawker of gold-fish, who, changing the water in his tanks at the loch, threw out some of the plant. This is quite possible, as Anacharis is commonly used for aerating the water in aquaria, and is sold by dealers for that purpose. In non-peaty water containing a supply of suitable plant food-salts the smallest scrap of this plant bearing a whorl of leaves will grow and increase very rapidly, whether floating or attached to the bottom. That the Anacharis has not become general at other non-peaty lochs of this Area is probably because (1) it propagates vegetatively, as only female plants occur, consequently no seed is produced for dispersal by birds; (2) the form of the plant is such that it is not likely to be carried inadvertently on the legs or bodies of birds; (3) the only effluent of Loch Leven flows directly into the sea without entering any other loch.

Ranunculus aquatilis, Polygonum amphibium, Glyceria fluitans, Lemna minor, Callitriche stagnalis, the two last only in pools on the shore; Carex rostrata, C. aquatilis, C. Goodenovii, C. hirta, on the sandy shores, and growing like C. arenaria on the seashore; Alisma Plantago, A. ranunculoides, Phalaris arundinacea, Veronica Beccabunga, Comarum palustre, Mentha sativa, M. aquatica, M. arvensis, Eriophorum polystachion, Myosotis palustris, Montia fontana. Iris Pseud–acorus, Menyanthes trifoliata, Radicula officinalis, Juncus effusus, J. lamprocarpus, J. acutiflorus, and a dwarf prostrate form of it with scarcely any rhizome, growing on the exposed sandy shores; J. bufonius and its var. fasciculatus, Equisetum palustre, Pedicularis palustris, Mimulus Langsdorffii, Lysimachia nummularia, Galium palustre, Stellaria uliginosa, Cardamine pratensis, Angelica sylvestris, Ranunculus Flammula and its var. pseudo-reptans, as well as intermediate forms; R. reptans, R. hederaceus, Caltha palustris, Spiræa Ulmaria, Hydrocotyle vulgaris, Spergularia rubra, Gnaphalium uliginosum, Sagina nodosa, Sphagnum acutifolium, Philonotis calcarea, P. fontana, Climacium dendrioides, Bryum bimum, Hypnum falcatum, H. commutatum, H. scorpioides, H. cuspidatum and Hylocomium squarrosum. These mosses are most abundant at the marshy places on the south shore. Hepatics are quite scarce.

The Isle of May is situated at the entrance to the Firth of Forth, and is about 5 miles from the coast of Fife. It can be reached most easily by engaging a sailing-boat from either Anstruther or Crail, according to wind and tide. The only inhabitants of the island are the lighthouse keepers and their families; and there is no public communication with the island from the mainland excepting in the summer, when an occasional steamer from Leith may, if the sea is calm, land excursionists there for an hour or so. The island is a little over a mile long by about $\frac{1}{3}$ mile wide, and consists of a mass of volcanic rock. Almost the whole of the eastern side rises from the sea at a gradual inclination (figs. 118 and 119), but the western side is precipitous, and the black dolerite cliffs, which occasionally exhibit magnificent columnar structure, rise perpendicularly from the sea to a height of 150 feet (fig. 122). Thousands of sea-birds of various kinds find a congenial home upon this cliff. The countless little platforms that are formed by the ends of the basalt columns suit them admirably both as resting-places and as sites for nidification. The black cliffs are whitened by the droppings of the birds, and when seen from a distance of 2 or 3 miles, under the soft refulgence of the declining sun of a gentle summer's eve, this side of the island presents a magnificent spectacle.

I was induced to visit this isolated spot in order to investigate a small

loch which is there, thinking it might afford something of interest because of the numerous water-birds that visit the island during their migrations. The loch, which is quite small, is situated in a ravine that divides the island obliquely in the direction S.E. by E. and N.W. by W. From the rocky and precipitous nature of the ravine one might imagine the pool to be a little lochan high on the mountains. The extensive engine-house at the east end and the cement dams at both the east and west ends, however, quickly dispel such a pleasant illusion. The water, which is maintained at a depth of about 7 feet by the dams, is used for the engines that generate the electricity for the lighthouse and the compressed air for the fog-horn. The light from the lighthouse, which is situated at the highest part of the island near its centre, is a revolving one, showing four rays of white light every half-minute. The light is produced from a single pair of very large carbons, and is one of the most powerful around the coast of Britain, although it is said not to penetrate to so great a distance as the light given by oil when the atmosphere is thick. The fog-horn, which is worked by compressed air, sounds four consecutive notes, namely—high, low, high, low. The horn is situated at the south end of the island, the compressed air being conducted to it from the engine-house through a strong iron tube. Upon one occasion I happened to be close to the horn when a fog suddenly descended and it was consequently put into action. The noise was appalling, and reverberated from rock to rock as if it would rend them asunder.

No aquatic Phanerogams or higher Cryptogams exist either in the water or about the shores of the little loch, but the water is coloured yellowish-green by the abundance of minute Myxophyceæ, Bacteria, Infusoria, Entomostraca, and by the waste water from the adjacent engine-house. The water is so discoloured that the bottom can only be seen at a depth of a few inches, and the engineer informed me that the discoloration is maintained throughout the year. It must not be imagined, however, that the cliffs about the loch are bare of vegetation, for, besides grassy slopes and banks, the rocks and crannies are clothed with a variety of plants such as are common to the maritime cliffs of the adjacent mainland.

Being disappointed with the loch, I gave some attention to the terrestrial flora, and the following is a compendium describing what I saw in August:— There is no peat on the island, and plants usually associated with that substance are consequently absent; neither are there any trees, and, beyond two or three small gardens and an enclosure for grass, cultivation is nil. The dominant vegetation, as might be expected, is that of the maritime cliff type, the species of which are frequently dwarfed by the poverty of the soil and by the desiccating action of the wind. In some places a fine close sward

is maintained by rabbits, which abound. There is no sandy shore at any part of the island, because nearly everywhere rock abruptly enters the sea. There is, however, at the north-west end a little beach about 20 yards long, composed entirely of finely broken shells; and at the south-west end, immediately to the east of some bold dolerite stacks that stand out in the water, there is a small stretch of pebbly beach. No vegetation grows upon either of these places, so that the usual maritime sandy or pebbly shore plants are absent from the island. Between the rocks, especially upon the low-lying east side of the island, and even at the highest parts, numerous little pools are formed by the collection of rainwater in the hollows, and a few semi-aquatic plants occur at such places (fig. 121). Near the southern point of the island there is a small damp, open cave, which was probably excavated by the sea at a former period, when the 25-feet raised beaches were formed upon the adjoining mainland. This cave has been excavated in a reddish material, comparatively soft and easily crumbled, which, as Messrs Peach and Horne, of the Geological Survey, inform me, is due to the decomposition of the exfoliating teschenite lying in a fissure of that rock. This place contains a few plants not observed elsewhere upon the island, and it will subsequently be referred to as the cave. Upon the east side of the island the rocks, by their gradual rise from the sea (fig. 118), are admirably adapted for being swept with spray during every stiff easterly breeze, so that over a rather wide zone these rocks are perfectly bare of soil, and the only vegetation that can exist consists of certain maritime lichens. These grow in great abundance and cover almost every rock, imparting a vivid coloration to the otherwise sombre aspect, the grey Lecanora parella, the brown Physcia aquila, the orange Physcia parietina, and the glaucous Ramalina scopulorum being particularly noticeable (fig. 120). A little higher up the incline, where the drenching spray has been insufficient to entirely wash away all soil, and where some mould has been retained in the interstices of the rocks, the vanguard of the phanerogamic vegetation appears. This is usually in the form of dwarf cushion-like tussocks of Armeria maritima, which adds its own peculiar charm to the coloration of the lichen-besprinkled rocks around (fig 119). The Armeria in such situations produces roots of surprising length, which penetrate the fissures of the rocks until an agreeable soil is reached (fig. 123). Algæ are generally scarce, and cannot be said to take any part in the formation of the plant-covering of the island. The same remark applies to the rocks at the margin of the sea, for they are in most places singularly poor in species of marine Algæ, and are often quite destitute of such plants. The larger Fungi also appear to be very scarce. Besides the lichen-covered rocks already mentioned, the

island owes a good deal of its plant-covering to lichens, of which the following species are the most conspicuous:—Ramalina polymorpha, both large and small forms, R. scopulorum, and Alectoria jubata are all abundant, especially on cliffs and the vertical sides of rocks, from which they hang in profusion. Parmelia saxatilis, Cetraria aculeata, and Sphærophoron coralloides cover rocks that are beyond the influence of the sea spray Amphiloma lanuginosum is abundant in shady corners about the rocks and cliffs, and Peltigera polydactyla occurs amongst rough grass, but is not abundant.

As would be inferred from the nature of the environment, Hepaticæ are scarce. Lophocolea bidentata occurs about pools and other damp places, and in the cave. Jungermannia ventricosa grows in damp places below rocks, but is not abundant; Nardia scalaris, growing with Cephalozia Starkii, also Lunularia vulgaris and Conocephalus conicus, carpet the sides of the cave. Mosses are of much more frequent occurrence. Grimmia maritima and Webera nutans are common on rocks; a form of the latter species was also fairly abundant. Respecting it, Mr H. N. Dixon, to whom a specimen was submitted, writes as follows:—"It is a form or variety which I have gathered once or twice, usually in mountainous country, and it comes near the mountain form which I have referred to in the Handbook [of British Mosses], 2nd edition, as like W. commutata." Polytrichum Juniperinum occurs in scattered patches all over the island. Mnium hornum, Amblystegium serpens, and Eurhynchium prælongum are all common in damp places throughout the island.*

The only ferns observed were Asplenium marinum, which grows sparingly about the cliffs and in the cave, and A. Ruta-muraria, which is very scarce. Excluding weeds in the cultivated spots and on ground adjacent to them, the most abundant Phanerogams are as follows:—Ranunculus repens, very common; R. aquatilis, rather dwarf specimens in some of the pools; Cochlearia officinalis, very abundant in places; several variations occur in accordance with the environment, and in the cave there is a very slender and somewhat pellucid form; Cerastium tetrandrum, scarce; C. triviale, very abundant; Stellaria media, common near the houses, on rubbish-heaps, etc.; Silene maritima, very abundant, and in the cave there is a very long-leaved form; Sagina apetala, common; Potentilla anserina and P. tormentilla, both abundant; Sedum anglicum, very abundant about the rocks (fig. 124); Callitriche verna and C. stagnalis, both

* Since the above was written, Mr William Evans has published his observations on the Bryophytes of the Isle of May, the specimens having been collected there at various times from 1885 to 1908. He mentions 18 species of mosses and 7 hepatics, most of which are very scarce.—*Trans. and Proc. Bot. Soc. Edin.*, 1908, vol. xxiii., part iv., pp. 348–351.

abundant in pools; Conium maculatum, sporadic in sheltered places; Apium inundatum, very scarce in a few pools; Heracleum Sphondylium, frequent; Ligusticum scoticum, very abundant and luxuriant on the cliffs, etc.; Galium verum, common; Bellis perennis, scarce, near the houses; Senecio Jacobæa, scarce; Cnicus lanceolatus, plentiful; C. arvensis, very abundant; Leontodon autumnalis, abundant; Glaux maritima, very abundant, being often one of the constituents of the sward; Rumex Acetosa, very abundant; R. crispus, frequent; R. conglomeratus, scarce; Armeria maritima, very abundant all over the island, and forming a close sward where cropped by rabbits or sheep; Plantago media, not abundant; P. maritima, abundant; P. Coronopus, very abundant; Littorella lacustris, abundant in small pools; Urtica dioica, abundant, but generally near the houses; Atriplex patula, various forms abound nearly everywhere, from little fruiting plants 2 inches high in exposed places to specimens a foot high in sheltered spots; sometimes the dwarf forms produce a loose sward; Juncus bufonius and its var. fasciculatus, common about the pools; J. acutiflorus, dwarf forms about the pools, but not abundant; Heleocharis palustris, dwarf forms a foot or less in height fill some of the pools (fig. 121); H. uniglumis, scarce; Carex Goodenovii, dwarf forms about the pools, but not abundant; C. vulpina, abundant about the pools. Agrostis alba, A. vulgaris, Holcus lanatus, and Festuca ovina, var. hispidula, are the dominant grasses, and provide the chief plant-covering to the soil of the island. Festuca ovina, var. glauca, is common about the rocks and drier places. No viviparous forms of F. ovina were observed.

V.—Concluding Remarks.

In the seven Areas included in this and the former paper, about 175 lochs have been visited; these vary in size from what are practically inland seas, such as Loch Ness, to mere ponds, like Lochan Diota. These lochs have to a considerable extent their individual floristic peculiarities, and this fact inhibits the process of condensation of such features into a short summary. The lochs may, of course, be grouped in accordance with their striking physical characteristics, such as elevation above sea level, exposure to wind, nature of shore, depth of water, condition of the bottom, whether rocky, stony, sandy, clayey, muddy, etc., kind of water, whether peaty or non-peaty, rich or poor in plant food-salts, etc.; and these characters very largely depend upon the physiographical features of the surrounding country. When the combination of such factors respecting any loch is known, the plants likely to be found there may be roughly indicated, but this apparent simplicity is frequently modified by other agencies.

The laws which govern the geographical distribution of aquatic plants cannot be fully understood until science has revealed more facts regarding the ecology of the plants than it at present possesses; it is therefore futile to attempt the deduction of general laws, with only an inadequate knowledge of the phenomena to be generalised. During the last great glacial epoch it is certain that all forms of the higher plants were banished from the greater portion of Scotland. Towards the end of that era, as the mantle of ice and snow began to retreat, so would plants encroach again over the country from the region to the south, where its influence had been less severe. What precise causes influenced most this gradual northward march of aquatic and terrestrial plants cannot now be determined, but probably they were such as affect the distribution of plants at the present day. The plants no doubt followed the lines of least resistance and greatest traction, not only in their geographical advance, but also in their adaptations of structure and function to the varying environments. These lines must necessarily be ramified and involved, perhaps to an insoluble degree; yet on them are the secrets of plant geography to be discovered, on the basis of physiological anatomy and plant psychology. By such methods a most interesting inquiry would be—What is the equilibrium that has been attained between the forces of resistance and traction that has caused certain species to arrive at, and remain in, restricted areas? This is a subject bristling with difficult chemical and physical complications, combined with the various influences resulting from the never-ceasing action and reaction, not only between the different members and associations of the flora, but between the flora and fauna as well; and it is at present impossible to set down a complete satisfactory statement of the ecology of any single group of aquatic plants. We must, therefore, for the moment leave the final generalisation of the causes which govern the distribution of plants, and content ourselves with the routine work of taking evidence of that which occurs; not being too eager to surround ourselves with metaphysical hypotheses which seek to explain the observed phenomenon by a noumenon, nor to cloak our ignorance and befool our senses with vague concepts that transcend actual demonstration, and, when analysed, explain to our intelligence nothing whatever. On p. 172 I gave a possible explanation of the introduction of Anacharis Alsinastrum into Loch Leven, and of its restriction to that loch; but I am quite ignorant of the exact reason why it should flourish so exceedingly there, and not in Loch Fitty, where it also occurs, but very sparingly. Again, the restriction of certain plants to particular localities may be accounted for by observing that they are ill adapted for any mode of dispersal to which they are likely to be subjected; it is then difficult to

understand their introduction to their present situation. It is not easily explained why Equisetum limosum, Carex rostrata, Phragmites communis, and others should be so widely distributed about the margins of all kinds of lochs, whereas Cladium Mariscus, an equally dominant species, should be restricted, in the Areas under discussion, to a few places in Wigtownshire. When the sub-science of plant-ecology has taught us the full facts regarding the relationship existing between organism and enviroment, then shall we be able to generalise sets of phenomena regarding the geographical distribution of water plants to some useful purpose.

Whilst I have no desire to enter the contest with those who so boldly wield the cudgels in the arena of the origin of species, yet I may briefly state the impression regarding this subject which the study of the plants of the lochs has left upon me. In the first place, it seems to me that aquatic plants have not always had their origin from terrestrial forms that had been forced into the water by more robust competitors on the land, as is sometimes stated, but, more probably, because certain mutable forms have exhibited a tendency, as some do even now, to take on the aquatic habit, that mode of living being more agreeable to their requirements. Some plants form themselves into dense associations consisting of one species only, which spread over considerable areas, and not only prevent others from growing amongst them, but year by year extend their borders at the expense of neighbouring plants. In the vanguard of such colonies there is doubtless very keen competition for the space, and the weaker or less suitably adapted species will be slowly driven before the stronger. This, however, is unlikely to go on continuously, because the stronger species will sooner or later meet with physical or chemical barriers which it is ill adapted to overcome, but to which the weaker species may be better adapted. Quite commonly, it is not that competition for available space is so great, but that the local conditions favour the dominant growth of a few individual species. One frequently finds normal terrestrial or marsh species taking on the aquatic habit: instance Ranunculus Flammula, Juncus supinus, J. acutiflorus, Peplis Portula, etc., but always of their own free will, so to speak, *i.e.* by the exercise of the subtle power of adaptability which is more or less the common possession of all plants; never have I observed the case of a plant being driven into the water by a stronger competitor.

From another aspect of this interesting subject, it appears to me that other causes for variation, with the consequent production of new forms, lie in the fact that although the conditions for plant life are so often remote from the ideal, yet the plastic power possessed by plants, enabling

them to adapt themselves to the various combinations of edaphic and climatic conditions, is so great that there are comparatively few spots, where existence is possible, in which some plant or other is not able to thrive and carry on its metabolic activities. Now in order to maintain a proper tone of health a plant has of necessity to respond in suitable ways to all the varying external impressions. A plant is therefore in a constant and continual state of change, owing to the never-ceasing mechanical, physical, and chemical changes of its unstable environment. The plastic nature of many plants enables them to modify their organs in reciprocation to any fairly constant set of environmental conditions, and it is in this endeavour to accommodate themselves for the maintenance of healthy existence in places that are either inhospitable or too luxurious, that certain deviations, either fixed or transient, from the usual forms of more normal environments are to be accounted for, and such variations occur in almost every loch. That some of such variants may doubtless be concerned in the origin of new species and varieties is the impression that I have received, but I hasten to add that other causes also contribute towards that process.

The rapid increase of aquatic and marsh plants in reservoirs that are used for the public water supply is occasionally a matter of anxiety and expense to the owners. Enormous sums of money are frequently paid by public bodies for advice respecting the construction of reservoirs to persons wholly unacquainted with the local geological features, as well as with the flora and fauna of the district. Whilst it is very unwise to construct a reservoir over a geological fault and expect it to hold water (and this has been done), it is equally vain to make a shallow reservoir in the line of the constant migration of water-fowl (*i.e.* between their resorts), and expect it to maintain a freedom from water plants. By consulting the table on pp. 97–99 it will be seen that the greatest depth at which aquatic plants will flourish in Scottish waters is about 40 feet. It is very unlikely, however, that the species capable of growing at such a depth will ever become a nuisance in a reservoir. But at a depth of 20 feet it will be found that, in suitable water, many species capable of giving trouble will flourish. Upon consideration of these facts, it seems advisable, as a prevention against the development of water plants, to construct reservoirs with sides so steep that a minimum depth of from 20 to 25 feet will be maintained within a few yards of the margin. Moreover, the sides, unless of natural rock, should be faced with stonework, which will further impede the growth of plants, as well as prevent discoloration of the water by wave-erosion.

The record of the field work that I have done on behalf of the Scottish Lake Survey is now completed, and I trust that the reader will receive from this contribution a share of the pleasure that its preparation has afforded me. It has always been my endeavour to describe as plainly as possible what I have learned

> " From the great lakes of the Northland,
> From the mountains, moors, and fen-lands."

To that end I hope the accompanying illustrations will immediately convey more perfect ideas of the subjects they represent than could be obtained from tedious verbal descriptions.

The lochs mentioned in this paper may be found in Bartholomew's half-inch maps of Scotland, numbers 1, 2, 4, 8, and 13.

An account of the physical and biological features, together with bathymetrical maps on a large scale, of most of the lochs enumerated in the two parts of this contribution, will be found in a series of volumes on the Fresh-water Lochs of Scotland, now being prepared by Sir John Murray. In the same publication the reader will also find an epitome of the work that I have done, together with some remarks and illustrations not previously published.

In conclusion, I wish to acknowledge with gratitude my indebtedness to Sir John Murray and Mr Laurence Pullar for the help and kindly encouragement which they have at all times given me.

University College,
Dundee.

(*Issued separately January* 12, 1910.)

"——The greatest thing a human soul ever does in this world is to see something and tell what it saw in a plain way."—RUSKIN.

Fig. 1.—View of the south end of Loch Doon from Portmark, looking towards Merrick, which is the most distant mountain in the centre of the picture; that in the distance on the left is Mullwharchar. The barren stony shore merging into moor is clearly shown.

Fig. 2.—View of the south end of Loch Doon from near Craigmulloch (opposite Portmark), showing the rocky shore, Castle Island, etc. The highest distant mountains are Mullwharchar on the right and Meikle Craigtarson on the left; between them runs the glen leading to Loch Dee.

Fig. 3.—View from the north end of Loch Doon looking south-east, showing bare stony shore and evidences of glaciation on the land. Only a portion of the loch can be seen from this point, owing to its curvature. It runs parallel to the distant hills, of which the one on the right centre of the picture is Coran of Portmark.

Fig. 4.—The top of Craiglee, looking towards Curleywee, the distant mountain in centre of picture, showing bare, rocky mountain-top, with numerous perched rocks. The pool on the left has a few dwarf specimens of Juncus, Carex, Eriophorum, etc.

FIG. 5.—Loch Enoch and Merrick, view from the north-east shore looking west. The outline is very irregular, and the rocky shore supports but few Phanerogams.

FIG. 6.—Loch Enoch and Mullwharchar, view from the south-west shore looking north-east. The largest island has upon it a little loch. The distant hill on the right is Carlin's Cairn, upon the opposite side of the glen.

Fig. 7.—View from the Nick of the Dungeon looking south, showing Lochs Neldricken and Valley. The distant hills are Lamachan and Larg. The portion of Loch Neldricken shown in fig. 8 is the arm to the right of the picture, partially hidden by the hill in the foreground.

Fig. 8.—Loch Neldricken, view from the north-west corner, looking east towards Craignaw, showing in the foreground a very regularly shaped "murder-hole."

Fig. 9.—Loch Neldricken from the south shore looking north, showing extensive shore of white sand; at the head of the glen in the centre of the picture is seen the Nick of the Dungeon, which joins Dungeon Hill with Craignaw.

Fig. 10.—Loch Valley, looking west from a shoulder of Craignaw, showing rocky shore and white sandy bays, almost bare of vegetation. Buchan Hill is seen at the end of the loch, below which flows the Gairland Burn that drains Loch Valley into Loch Trool.

Fig. 11.—Round Loch of Glenhead and Long Loch of Glenhead, from a shoulder of Craiglee. The marginal flora of the lochs is very sparse, whilst that of the adjacent moor is principally of grass or heather associations. The distant mountain on the right centre is Benyellary.

Fig. 12.—Loch Dee, with Craiglee rising from its north-west shore. On the right is seen a portion of the Kells Range, between which and Craiglee runs the glen leading to Loch Doon (10 miles). The White Laggan Burn, meandering through the foreground, enters Loch Dee at its south corner.

Fig. 13.—Loch Trool, from the hillside above Glenhead, looking west. Half-way down the loch, wooded promontories project into the water from either side, leaving but a narrow passage; this is seen from the other side in figs. 14 and 15.

Fig. 14.—Loch Trool from the west end, looking east. Associations of Carex rostrata, etc. occur about the margin. The greater portion of the loch is hidden from view by the promontories mentioned above.

FIG. 15.—Loch Trool. View from the Earl of Galloway's boat-house, looking across the loch. The narrow channel formed by the two wooded promontories (see also figs. 13 and 14) is indicated at the left centre of picture. The narrow zone of rocky shore is very bare of littoral plants.

FIG. 16.—Loch Grennoch (by Cairnsmore of Fleet). View from the north-west end, looking south. The sandy bays are not well shown as the scale is so small. This wind-exposed loch has scarcely any littoral vegetation.

Fig. 17.—Loch Skerrow. View from the east side, looking south-west, showing rocky shore, island-rocks capped with Calluna, Vaccinium, etc.; there are larger islands with dwarf trees at the other side of the loch.

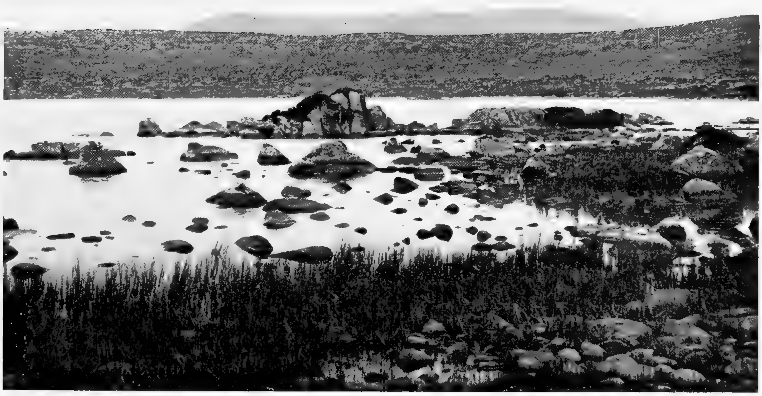

Fig. 18.—Loch Skerrow. View from the east side, looking north-west, showing rocky shore, which in the foreground is overgrown with Juncus lamprocarpus, Ranunculus Flammula, etc.

FIG. 19.—Loch Skerrow. A small bay on the west side, overgrown with Carex filiformis.

FIG. 20.—View looking north-west up the River Deė, at its junction with the Airie Burn, whose embouchure is on the left. A zone of Carex rostrata borders the river on either side; outside this zone, in the foreground, is Castalia speciosa, mixed with scattered Equisetum limosum and Scirpus lacustris.

Fig. 21.—Another view of the vegetation bordering the river, from a position close to that of fig. 20, showing the zone of Castalia speciosa, amongst which are a few stems of Equisetum limosum and Scirpus lacustris; beyond this zone, Carex rostrata appears as in the preceding picture.

Fig. 22.—A view looking into the zone of Carex rostrata which borders the river, as shown in the two preceding figures.

FIG. 23.—Loch Stroan from the south-east corner, looking north-west, showing Scirpus lacustris bordering the stony shore, River Dee in the distance, etc.

FIG. 24.—Loch Stroan. East shore, showing its rocky nature and a bank of vegetable remains, to which the man is pointing, washed into its present position by winter floods. The moor is clothed with associations of grass, bracken, and heather, but a deal of bare rock appears; stragglers from these associations occur upon the shore.

FIG. 25.—Loch Stroan. Scirpus fluitans floating on the surface of the water.

FIG. 26.—Loch Stroan. Hypericum elodes, not yet in flower, growing in a peaty pool.

Fig. 27.—Loch Stroan. Carum verticillatum growing amongst other herbage in marshy ground. A characteristic marsh plant in the Galloway district.

Fig. 28.—Woodhall Loch from the north-west end, looking towards Laurieston. The sides of the hill on the right are covered with mixed wood, and an association of Scirpus lacustris occurs in the foreground.

Fig. 29.—Woodhall Loch, from south-east to north-west, showing Scirpus lacustris bordering the loch. The wooded hill on the left is a continuation of that seen in fig. 28.

Fig. 30.—Galium palustre, growing amongst Carex and Juncus. This scrambling marsh plant is particularly abundant at some of the lochs of Galloway.

Fig. 31.—Loch Ken. View near New Galloway from the west shore, looking north, showing a large association of Nymphæa lutea extending along the loch for about a quarter of a mile outside a zone of Scirpus lacustris. The water over the Nymphæa area is from 2 to 7 feet deep.

Fig. 32.—Loch Ken. View from the west shore, looking south-east, with Burned Island and Green Island in the distance, showing a zone of Scirpus lacustris skirting the shore.

Fig. 33.—Loch Ken. View on the west side, looking south-east, showing Scirpus lacustris growing upon the dry shore 3 or 4 feet high, the water having receded somewhat through drought.

Fig. 34.—Loch Ken. View from the west shore 2 miles from the head of the loch, looking south-east, showing Scirpus lacustris growing out of the water, and the bush association of the littoral, of which Myrica Gale, here about 5 feet high, is dominant; the taller bushes are Alnus glutinosa. Cnicus palustris, Œnanthe crocata, ferns and other herbaceous plants are mixed with the bush.

Fig. 35.—Loch Ken. View from the west side about $1\frac{1}{2}$ miles north of the viaduct, looking east across the loch, showing gradual change from terrestrial to semi-aquatic vegetation. In the foreground there are various grasses, with Juncus effusus, then a wide zone of Eriophorum polystachion, beyond which are associations of Carex rostrata, Menyanthes trifoliata, Heleocharis palustris, etc., whilst out in the water a belt of Scirpus lacustris extends along the loch.

Fig. 36.—Loch Ken. View along the west shore about a mile north of the viaduct, looking towards the islands. The shore is here stony and almost bare of vegetation.

Fig. 37.—Loch Ken. View along the east side near the viaduct, showing the wooded shore and Ranunculus heterophyllus flowering upon the surface of the water, and extending over a considerable area ; a few specimens of Nymphæa intermedia are seen on the left.

Fig. 38.—A closer view of a portion of the association of Ranunculus heterophyllus shown in fig. 37.

Fig. 39.—View on the west shore of Loch Ken, showing a dwarf prostrate form of Ranunculus Flammula (larger than var. pseudo-reptans) overgrowing the stony beach.

Fig. 40.—A nearer view of a portion of the beach shown in fig. 39, showing the plants more distinctly. The dull green colour of the plants and the grey stones are not sufficiently contrasted in the photograph.

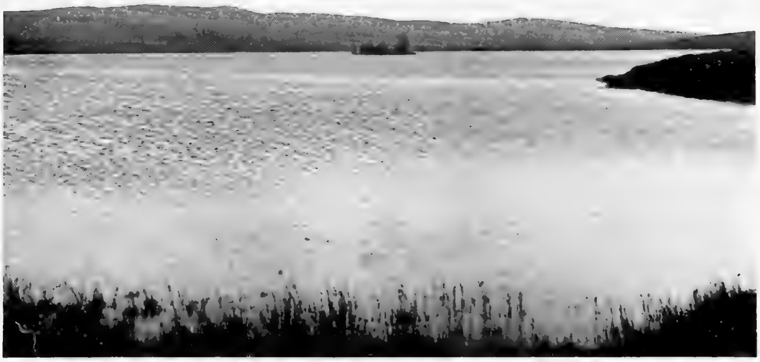

Fig. 41.—Lochinvar, showing the island on which there is a ruined castle, moorland vegetation approaching to the water's edge, so that there is no shore, and the treeless scenery around.

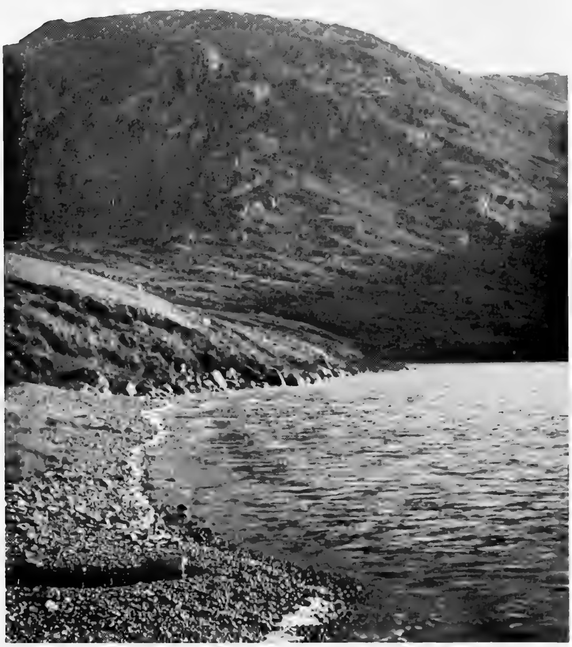

Fig. 42.—Loch Dungeon. View from the east side looking south, showing stony shore in the foreground, beyond which the littoral is very rocky, and the north flank of Meikle Millyea rising precipitously from the south shore.

Fig. 43.—Loch Dungeon from the north-west side, looking east, showing the burn from the North Garry top entering the loch. The flat foreground on either side of the burn is an alluvial cone, and consists of detrital gravel from a large moraine through which the burn has cut its way. Previous to this reduction there must have been another loch of considerable extent, because the moraine formed a dam across the glen through which the burn flows.

Fig. 44.—Loch Dungeon from the north-east corner, looking south towards Meikle Millyea, showing the large association of Phragmites communis at this corner of the loch.

Fig. 45.—Loch Minnoch from the north-east, looking towards Loch Dungeon, showing the affluent and its fan of detrital matter covered with vegetation, as described in the text.

Fig. 46.—Loch Corsock. General view from the south-west, looking north-east, showing coniferous wood about the margin of the loch, and an extensive marsh in the foreground.

Fig. 47.—Loch Corsock. View from the north-west corner, looking east, showing the shore, exposed through drought, covered in the foreground with Polygonum amphibium.

Fig. 48.—Loch Roan. View from the south-west, looking north-east, showing the plantation which surrounds the loch, excepting on the south side. The arm of the loch in the foreground is overgrown with Carex rostrata.

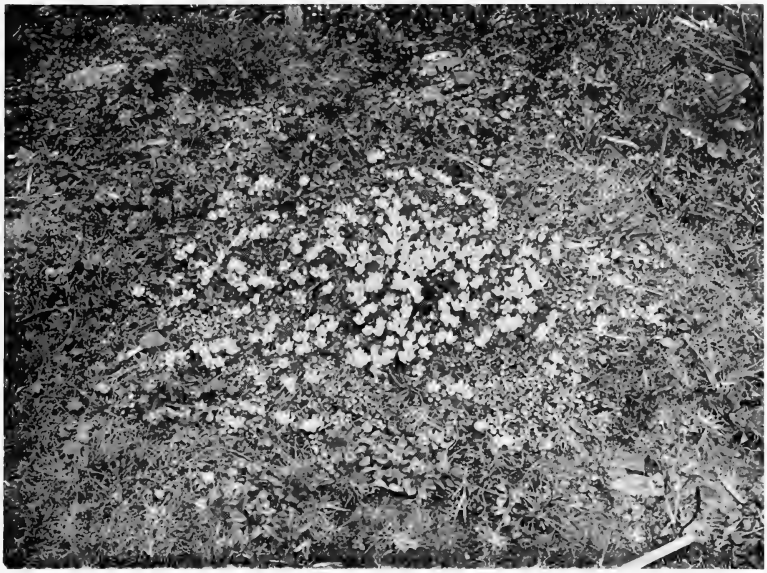

Fig. 49.—Loch Roan. A patch of Anagallis tenella on the west shore.

Fig. 50.—Loch Ernerogo from the north end, looking south-south-east, showing the swampy margin. Beyond the beds of Carex rostrata the shallower portions of the loch are overgrown with Castalia speciosa and Nymphæa lutea.

Fig. 51.—Loch Dornell. View from the south-east side, looking north, showing the stony shore overgrown with Juncus articulatus and Ranunculus Flammula, and the mixed wood upon the north side of the loch.

Fig. 52.—Loch Glentoo. View from the south-west, looking north-east, showing half the loch overgrown with Phragmites communis, Scirpus lacustris, Carex rostrata, etc.

Fig. 53.—Carlingwark Loch. View from the east shore, looking north-west towards Castle-Douglas.

Fig. 54.—Carlingwark Loch, showing Glyceria aquatica upon the west shore, two of the islands with trees, and a strip of deciduous wood upon the opposite shore.

Fig. 55.—Carlingwark Loch. View at the south end, looking east along the margin, which is here a wide marsh, showing luxuriant vegetation in the following order from the foreground:—Carex rostrata, Ranunculus Lingua, Cicuta virosa, Equisetum limosum, a narrow arm of water with Nymphæa lutea, then a large area of Carex rostrata with a small association of Phragmites to the left of it, and Typha latifolia near the trees in the distance.

Fig. 56.—Carlingwark Loch. Another view at the south end, looking east, showing Carex rostrata in the foreground, with a group of Cicuta virosa at the margin of the water, beyond which are Equisetum limosum and Phragmites communis, with Nymphæa lutea in the water and deciduous trees in the distance.

Fig. 57.—Lochrutton Loch. View from the south-east shore, looking north, showing an island with dwarf trees in the middle of the loch, and the village of Lochfoot at the north end. The open and wind-exposed situation is manifest.

Fig. 58.—Lochaber Loch. View from the north-west, looking south-east, showing Myrica Gale in the foreground, beyond which, in the water, are colonies of Carex rostrata, Phragmites communis, Equisetum limosum, etc. The loch is surrounded by forest to the water's edge, excepting on the north-west.

Fig. 59.—Loch Ronald. View from the south-west shore, looking north-east, showing flat stony shore without vegetation, and the plantation of conifers upon the east side of the loch.

Fig. 60.—Castle Loch, and Mochrum Loch beyond it. View from Challochglass Moor at the north-west of Castle Loch, looking south-east. The island with dwarf trees has upon it the remains of a castle. The other islands are merely bare rocks, and are occupied by cormorants that breed there.

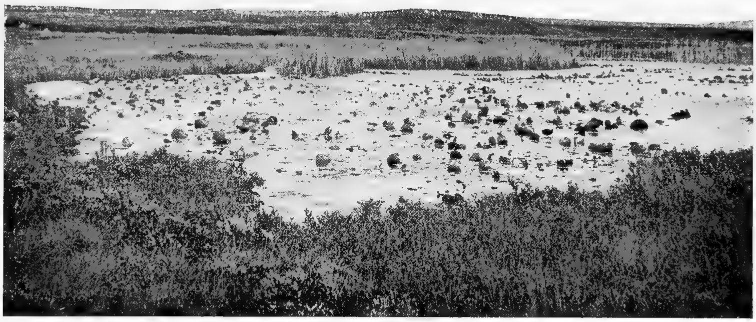

Fig. 61.—A tarn on the moor near Castle Loch, bordered by various plants, such as Phragmites communis, Carex rostrata, Schœnus nigricans, Hypericum elodes, Juncus effusus, Myrica Gale, Calluna vulgaris, etc. Its surface is covered with Castalia speciosa, whose leaves are being raised more than usual by a strong wind.

Fig. 62.—A small loch on Anabaglish Moss, with associations of plants in the following order from left to right:—(1) Potamogeton natans; (2) Castalia speciosa; (3) Cladium Mariscus; (4) Schœnus nigricans, Eriophorum polystachion, etc., the last group merging into moorland types.

Fig. 63.—White Loch on Anabaglish Moss, showing a portion of the zone of Cladium Mariscus, which entirely surrounds the pool. In the water in front of that species a belt of Castalia speciosa encircles the loch. Behind the Cladium, Carex filiformis, Schœnus nigricans, etc. merge into the types of the moor.

Fig. 64.—Peat Loch on Anabaglish Moss. A near view of a portion of the association of Cladium Mariscus which encircles the loch.

Fig. 65.—Barhapple Loch. View from the north shore, looking south-west, showing the dense association of Phragmites communis which borders the north side; also the island with dwarf trees in the middle of the loch.

Fig. 66.—Barhapple Loch. View on the south-east side, looking north-west, showing large tussocks of Juncus effusus extending over the gravelly shore; also the island seen in the preceding illustration.

Fig. 67.—Loch Dernaglar. View from the affluent at the south-west, looking north-east, showing Castalia speciosa on the water, dense colonies of Carex rostrata, and behind them a few stems of Phragmites communis, Juncus effusus, etc.

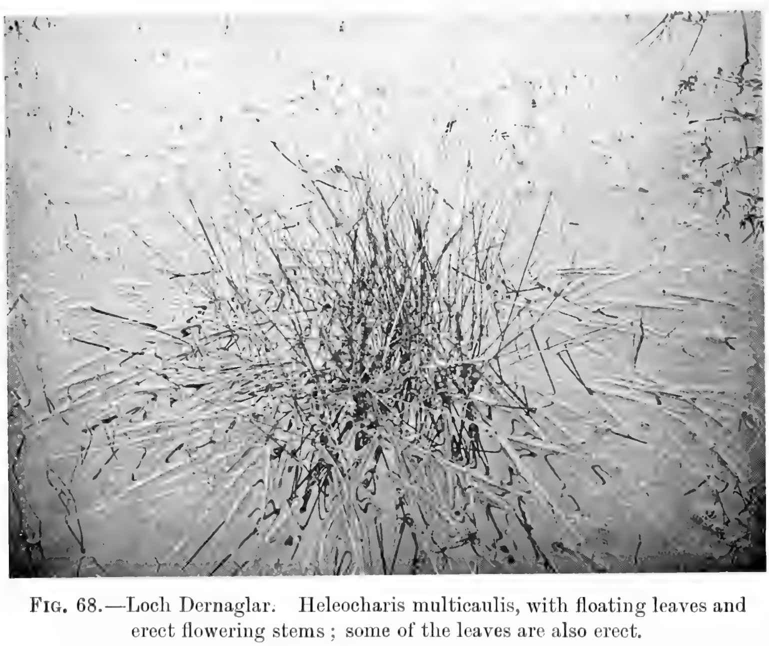

Fig. 68.—Loch Dernaglar. Heleocharis multicaulis, with floating leaves and erect flowering stems; some of the leaves are also erect.

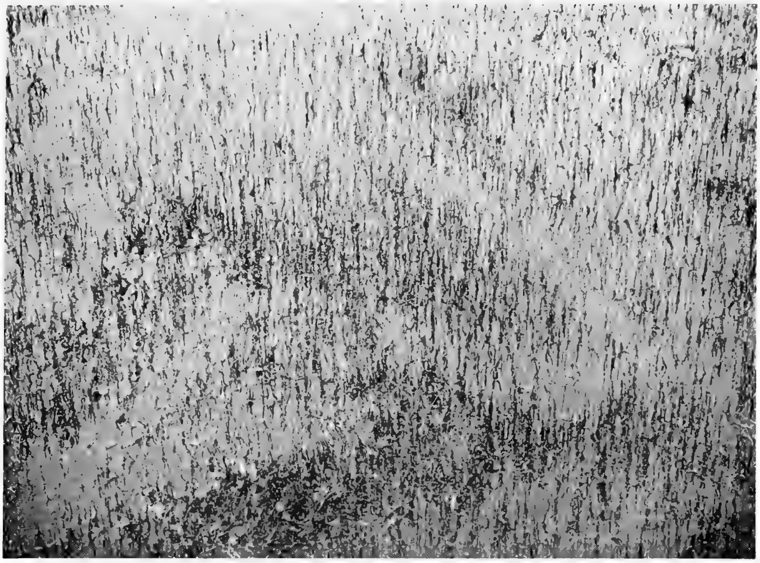

Fig. 69.—Loch Dernaglar. Pilularia globulifera overgrowing a shallow portion of the shore.

Fig. 70.—Barlockhart Loch. View at the east end, looking south-west, showing the associations of Phragmites, Equisetum, Castalia, etc., as described in the text.

Fig. 71.—White Loch, Castle-Kennedy. View from the large island on the west side, looking north towards Lochinch Castle. Phalaris arundinacea, in the foreground, forms a marginal zone about the island.

Fig. 72.—Black Loch, Castle-Kennedy. View from the south-east end, looking north-west, showing marsh vegetation in the foreground, island with luxuriant trees, and the wooded east shore of the loch. Lochinch Castle and the terraced grounds appear on the left of the island.

FIG. 73.—Black Loch, Castle-Kennedy. Typha latifolia at the margin of the loch, with its inflorescences: also with those of the previous year (the pale fluffy ones) that have not yet shed the whole of their seed.

FIG. 74.—Black Loch, Castle-Kennedy. Typha latifolia bordering the loch, behind which Lythrum Salicaria abounds.

Fig. 75.—Cults Loch. View at the east end of the loch, looking north-west along the shore, showing the narrow zone of marsh, occupied chiefly by Juncus effusus, that intervenes between the water and the meadow. The terrace indicates that the loch has once been of greater extent.

Fig. 76.—A small loch adjoining the railway, a mile west of Castle-Kennedy station. It is almost entirely overgrown by various species of plants common to the district.

Fig. 77.—Marshy ground of great extent, bordering a small loch near the ruins of Culhorn House. This marsh was probably once a portion of the loch, and the exuberance of the vegetation, which may be judged by the cattle standing amongst it, is reciprocal to the richness of the soil.

Fig. 78.—Bidens cernua, growing with other herbage in the marsh shown in fig. 77 ; illustrating the luxuriant vegetation of this rich area.

Fig. 79.—Site of a pool on the sands of Luce, dry because of drought. The bottom is now covered with a sward composed of Hydrocotyle vulgaris, Peplis Portula, and Carex arenaria. On the border of the pool great tussocks of Juncus effusus are in close proximity to Ammophila arundinacea, which overgrows the sand hill in the rear.

Fig. 80.—View of the sands of Luce bordering the sea, looking along the sandy shore towards Glenluce. The dunes are everywhere capped with Ammophila arundinacea; and at their base, quantities of dead Algæ are partially engulfed in the sand.

Fig. 81.—Sands of Luce, showing low dunes capped with Ammophila. Upon the lower slopes a kind of sward has been formed, chiefly by Carex arenaria, Salix repens, Hylocomium triquetrum, Rhacomitrium canescens and its var. ericoides.

Fig. 82.—Juncus maritimus in a brackish marsh at the east end of the sands of Luce, near Low Torrs.

FIG. 83.—Lindores Loch. View from the north-west shore, looking north across the loch. The foreground is covered with an association of Glyceria aquatica, amongst which are groups of Typha angustifolia. In the water there are four pure associations of Menyanthes trifoliata. On the island the vegetation is chiefly composed of Glyceria aquatica, Typha angustifolia, and a few alders. Upon the other side of the loch, to the left, the marshy ground is covered with similar plants.

FIG. 84.—Lindores Loch. View from the north-east, looking west along the end of the loch, showing a large association of Polygonum amphibium on the water, which is bordered on the land side by Glyceria aquatica that has been eaten down by cattle. At the other side of the loch a large association of Typha angustifolia extends completely across the picture.

Fig. 85.—Lindores Loch. Typha angustifolia bordering a portion of the east shore, with Nymphæa lutea on the water in front of it, and deciduous trees upon the shore behind.

Fig. 86.—Black Loch, North Fife. View from the south shore, looking north-east, meadow-land in the foreground, then a narrow strip of gravelly-muddy shore, close to which is Nymphæa lutea and a broader zone of Castalia speciosa (fig. 87). To the right of the latter there is an association of Menyanthes trifoliata; farther away, and to the right of the Castalia and Nymphæa, there is an association of Equisetum limosum, and behind it Glyceria aquatica. Grain fields cover the distant hills.

Fig. 87.—Black Loch, North Fife. A portion of the zone of Castalia speciosa and Nymphæa lutea which encircles the loch. The Nymphæa is in the foreground next to the shore, with its leaves flat on the surface of the water, and its flowers raised above the surface. The Castalia is beyond, and has its leaves partially above the surface, and its flowers on the surface of the water (compare figs. 21, 31, and 61).

Fig. 88.—Lochmill Loch, North Fife. View from near the effluent at the east end, looking west, showing in the foreground marsh vegetation, which is largely composed of Glyceria fluitans, and beyond this a wide zone of Polygonum amphibium mixed with Potamgeton natans, which extends along the north shore. Hills, with plantations, enclose the loch, excepting at the east end.

Fig. 89.—Kilconquhar Loch. View of the east shore, showing the dense marginal zone of vegetation, which is composed of an association of Scirpus lacustris in the foreground and Phragmites communis beyond it. The distant trees are in the grounds of Elie House.

Fig. 90.—Kilconquhar Loch. View on the north side, showing the dense marginal vegetation. Hippuris vulgaris in the foreground, behind which an association of Phragmites communis shuts off the view of the loch. Over the top of the latter species appears the cultivated land to the east of the loch.

Fig. 91.—Kilconquhar Loch. View from a boat at the south-east corner of the loch, looking towards Elie House, showing a portion of the large association of Polygonun amphibium, with its floating leaves and flowers above the surface of the water.

Fig. 92.—Kilconquhar Loch. View from a boat near the middle of the loch, looking towards the village. In the foreground the surface of the water is covered with the flowers of Ranunculus circinatus, but nearer the village the same species is mixed with R. Baudotii.

Fig. 93.—Carriston Reservoir. View from the island at the east side, looking west towards the Lomond Hills, of which the East Lomond appears on the right. The extensive sandy shore in the foreground has been exposed through the falling of the water level, and a few aquatic plants which grew there are now dead.

Fig. 94.—Carriston Reservoir. Ranunculus reptans, on the sandy shore at the north side.

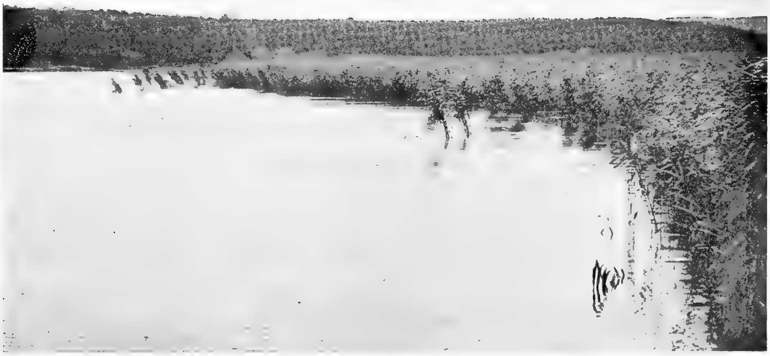

Fig. 95.—Loch Gelly. View of the west shore, looking south along the margin of the large association of Phragmites communis, showing here and there isolated groups that extend into the loch beyond the main body. In the distance is seen the coniferous wood that extends along the south shore.

Fig. 96.—Loch Gelly. View of the south shore looking east, showing the zone of marsh that extends along this side of the loch, and behind it a strip of coniferous wood. Carex rostrata occupies the foreground, and the small island is overgrown with Spiræa, Carex, etc.

Fig. 97.—Loch Gelly. View of a portion of the south shore from the water In the centre is shown a large schistose rock of glacial origin, and the only feature of this kind about the loch. On the left Carex rostrata, on the right Iris Pseud-acorus, and the plantation in the rear as seen in figs. 95 and 96.

Fig. 98.—Burntisland Reservoir. View along the north-east shore, looking west towards Cullalo Hill. Near the water the shore is covered with Littorella lacustris, then there is a broad zone of Heleocharis palustris, followed by a narrow strip of Spiræa Ulmaria, which is succeeded by meadow-land.

Fig. 99.—Burntisland Reservoir. View from the north-west side, looking east towards Dunearn Hill. In the foreground are Spiræa Ulmaria, Juncus acutiflorus, and Heleocharis palustris; Pinus sylvestris and Ulex europæus occupy the mound on the left.

Fig. 100.—Otterston Loch. View along the north shore, looking west, showing the public road, which is separated from the loch by a grassy bank that dips into the water, the luxuriance of the surrounding trees, and the surface of the water covered with Polygonum amphibium.

Fig. 101.—Otterston Loch. View from a similar position to that of fig. 100, but looking in the opposite direction, showing a portion of a large association of Polygonum amphibium on the water in the foreground, and the bank below the trees covered with marsh plants, such as Carex rostrata, C. paniculata, Phalaris arundinacea, etc.

Fig. 102.—Otterston Loch. View along the east shore, looking north, showing Iris Pseud-acorus, etc. at the margin, which is stony in some places. The ruin in the centre of the picture is of historical interest.

FIG. 103.—Otterston Loch. Tussocks of Carex paniculata growing in the deep bog at the west end of the loch, where they extend over a large area.

FIG. 104.—Loch Fitty. View from the south shore, looking towards the extensive marsh at the west end, which is seen in the distance. The plant in the foreground next the water is Carex aquatilis.

Fig. 105.—Loch Fitty. View of the marshy area at the east end of the loch, from the north shore.

Fig. 106.—Loch Glow. View from the south shore, looking north. The foreground is covered with grass and grass-like moor plants, below which there is deep peat, and the shore at this side is peaty. At the opposite side a thin layer of peat has been washed away, exposing either glacial gravel or volcanic rock, and the shore there is formed of these materials. The margin of this loch is almost devoid of either marsh or aquatic vegetation.

Fig. 107.—Black Loch, Cleish Hills. View from the east end, looking west, showing Phragmites communis skirting the south shore, and the grassy moor extending to the margin of the water. The effluent, which flows into Loch Glow, is seen in the foreground.

Fig. 108.—Loch Dow, Cleish Hills. View of the west side from the south end, looking north-west, showing Carex rostrata bordering the west side of the loch. The darker patch standing out of the water on the right is an association of Equisetum limosum.

Fig. 109.—Loch Larg, Cleish Hills. View from the south-east, looking north-west, showing the boggy area on the west side as described in the text. The Ochil Hills appear in the distance.

Fig 110.—Harperleas Reservoir. View from the north-west end, looking east along the north shore, with East Lomond Hill in the distance. Juncus effusus and Carex rostrata occur in the foreground. Nearer the water, a zone of Equisetum limosum, which extends along this side of the loch, has more or less died down owing to the receding of the water.

Mr George West.   [Plate 55.—*Referred to on pages* 168 *and* 169.

Fig. 111.—Harperleas Reservoir. View on the north-west shore, showing tussocks of Molinia cærulea that have been drowned. Those in the foreground, which is next the water, are quite dead, having suffered a longer submersion than those in the background, which are still living, whilst the intermediate tussocks exhibit a little vitality.

Fig. 112.—Ballo Reservoir. General view from the hillside at the north-west, looking south-east. In the immediate foreground is shown a portion of the rough hill pasture which predominates over the Lomond Hills. A superior pasturage is being raised upon the enclosed ground at the other side of the wall, whilst nearer the loch there are excellent meadows. The area of flat ground lying at the west end of the loch, between the two plantations, is covered with water in winter.

Fig. 113.—Ballo Reservoir. View from the south-east shore, looking towards West Lomond Hill. The flat peaty shore, with pools here and there, has been exposed through drought, and is now more or less covered with Juncus supinus, var. fluitans, growing as a terrestrial plant, and reverting towards the type.

Fig. 114.—Loch Leven. View from the hillside above Cleish Church (4 miles from the loch). The Lomond Hills appear in the distance; the top of East Lomond Hill is seen just above St Serf's Island on the right of the picture. Most of the other islands are also shown. The low-lying land in the middle distance, forming the Plain of Kinross, is intensely cultivated.

Fig. 115.—Loch Leven. View from the west shore, looking east, showing the great extent of barren, flat, sandy-stony shore, and the islands Roy's Folly and Castle Island. Behind the latter, White Craigs Hill appears in the distance.

Fig. 116.—Loch Leven.—View on the south shore opposite St Serf's Island, looking towards Kinross, showing the marshy shore. The undergrowth is chiefly Juncus effusus and Carex rostrata. The bushes are Salix aurita, Alnus glutinosa, and Betula glutinosa.

FIG. 117.—Loch Leven. View of the east shore opposite St Serf's Island, looking north-west. The scattered marsh plants are chiefly Equisetum limosum, Heleocharis palustris, and Carex rostrata. Dry parts of the shore, as in the foreground, are clothed with grasses. Then follows a strip of Alnus and Salix, behind which there is a plantation of Pinus sylvestris.

FIG. 118.—Isle of May. View from the north-east end of the island, looking south-east along the eastern shore, showing the gradual inclination of the land from the sea. The whole of this area is swept by sea-spray during every stiff easterly breeze, and the vegetation consists chiefly of the lichens enumerated under fig. 120.

Fig. 119.—Isle of May. View from near the same place as fig. 118, but looking in a westerly direction towards the higher portion of the island. The zone of rock that produces nothing but lichens lies between this spot and high-water mark. In the foreground of the present view is seen the vanguard of the phanerogamic vegetation, dwarf tussocks of Armeria maritima having obtained a hold in the interstices of the lichen-covered rocks.

Fig. 120.—Isle of May. Nearer view of the lichens mentioned under fig. 118. They are as follows:—Lecanora parella, Physcia parietina, P. aquila, and Ramalina scopulorum.

Fig. 121.—Isle of May. One of the pools on an exposed part of the island near the lighthouse. A dwarf form of Heleocharis palustris, with which H. uniglumis is mixed, overgrows the entire area of the pool.

Fig. 122.—Isle of May. Precipitous cliff on the west of the island, consisting of black columnar basalt. The cliff is inhabited by great numbers of sea-birds, whose nests are placed chiefly upon the exposed ends of the columns, the black rock being whitened by the droppings from the birds. It is nearly low water, and the light zone at the base of the cliff is caused by the abundance of Balanidæ, etc.

Mr George West. [Plate 61.—*Referred to on pages* 173 *to* 177.

Fig. 123.—Isle of May. A cushion-like tussock of Armeria maritima. The front of the rock on which it is growing was removed in order to exhibit the great length of root which had penetrated a fissure of the rock.

Fig. 124.—Isle of May. Sedum anglicum growing in a crevice of lichen-covered rocks. This plant is abundant on the island.

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Schäfer, E. A.—On the Existence within the Liver Cells of Channels which can be directly injected from the Blood-vessels. Proc. Roy. Soc. Edin., vol. , 1902, pp. .

Cells, Liver,—Intra-cellular Canaliculi in.
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Liver,—Injection within Cells of.
E. A. Schäfer. Proc. Roy. Soc. Edin., vol. , 1902, pp. .

PROCEEDINGS

OF THE

ROYAL SOCIETY OF EDINBURGH.

SESSION 1909–10.

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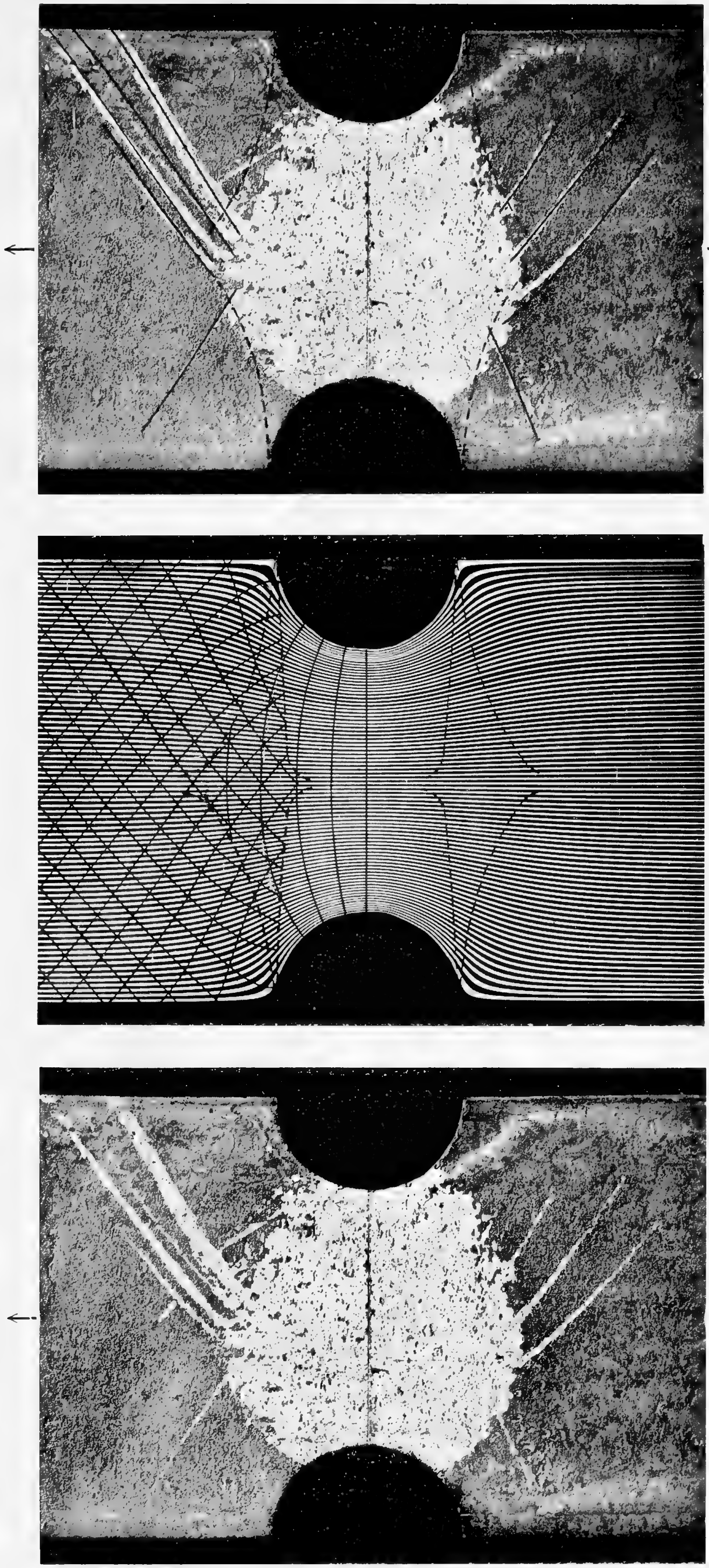

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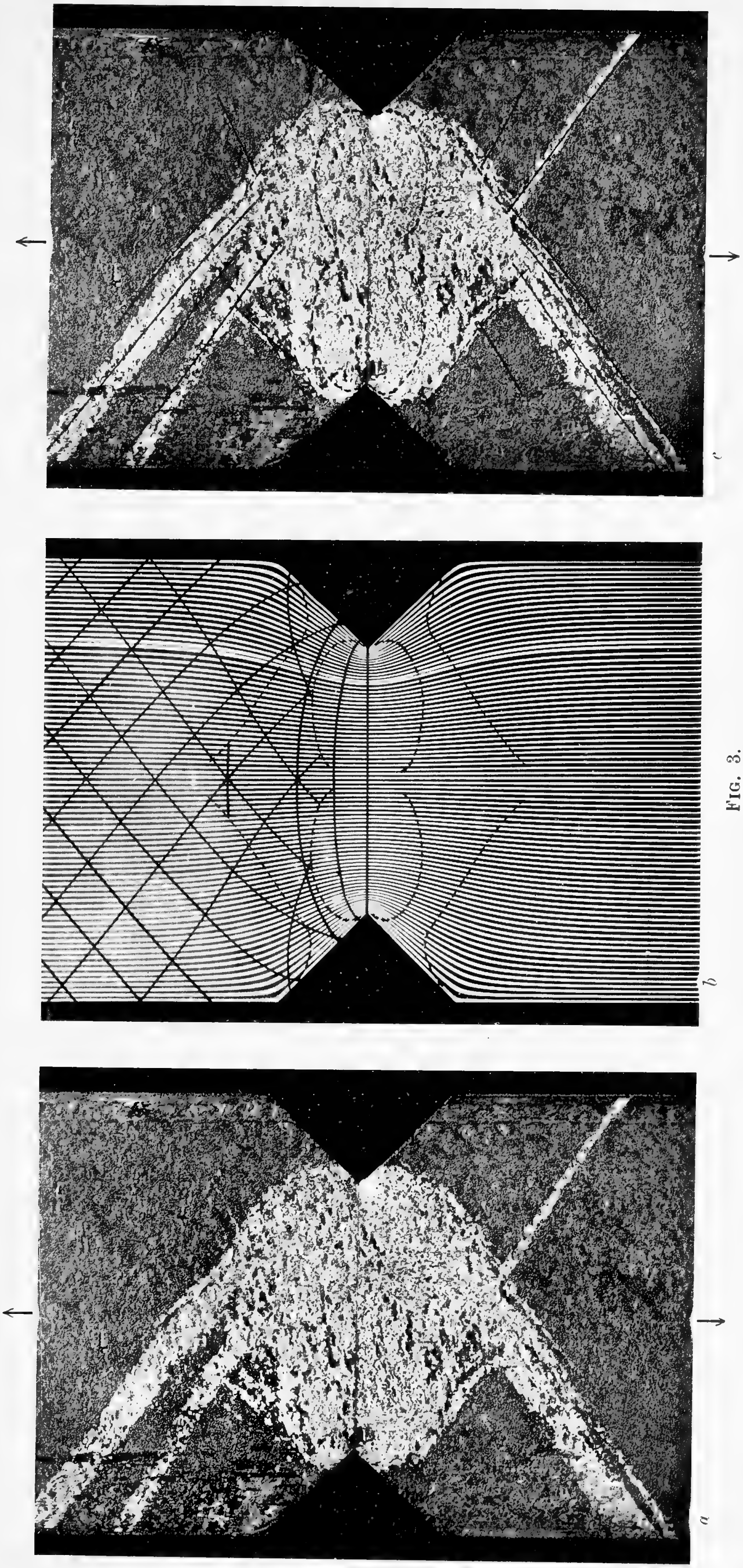
Fig. 2.



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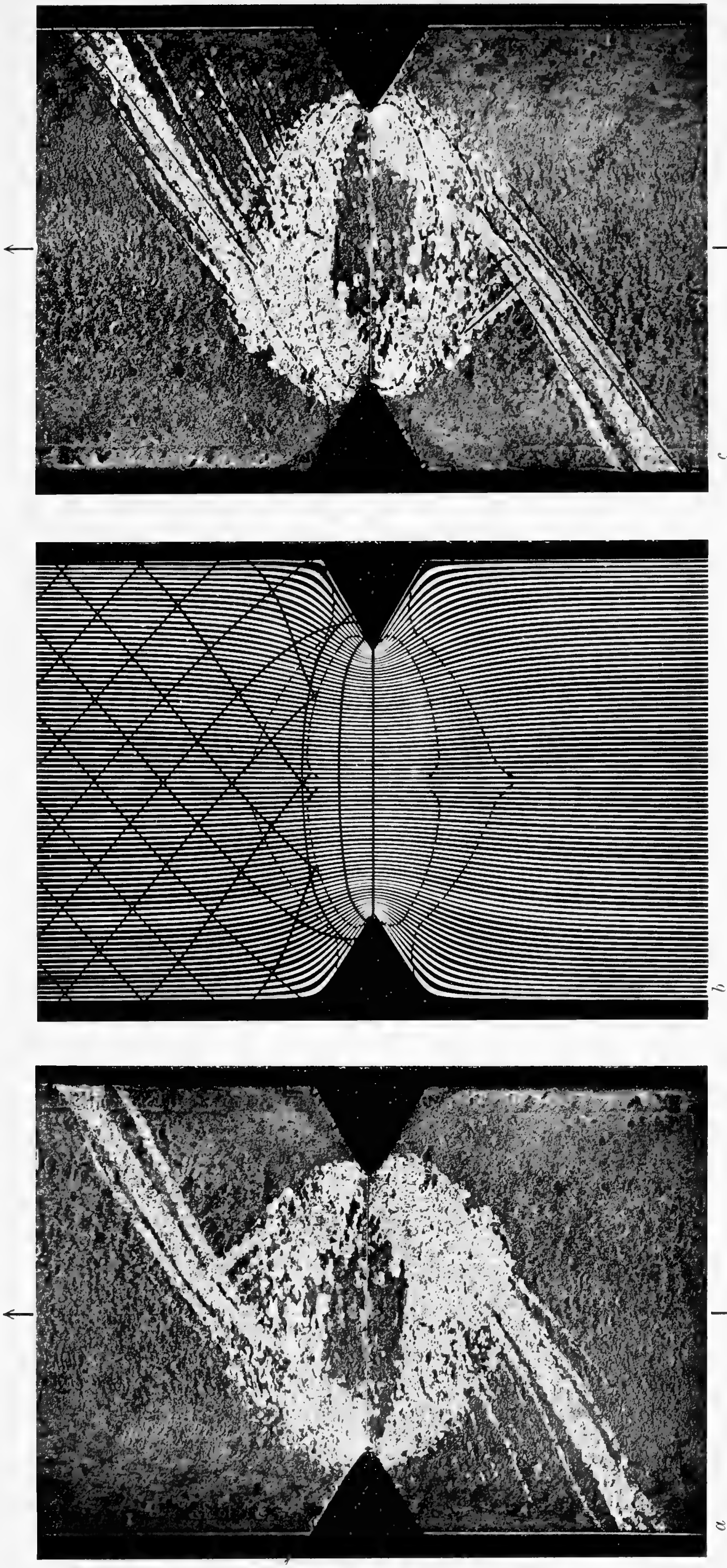

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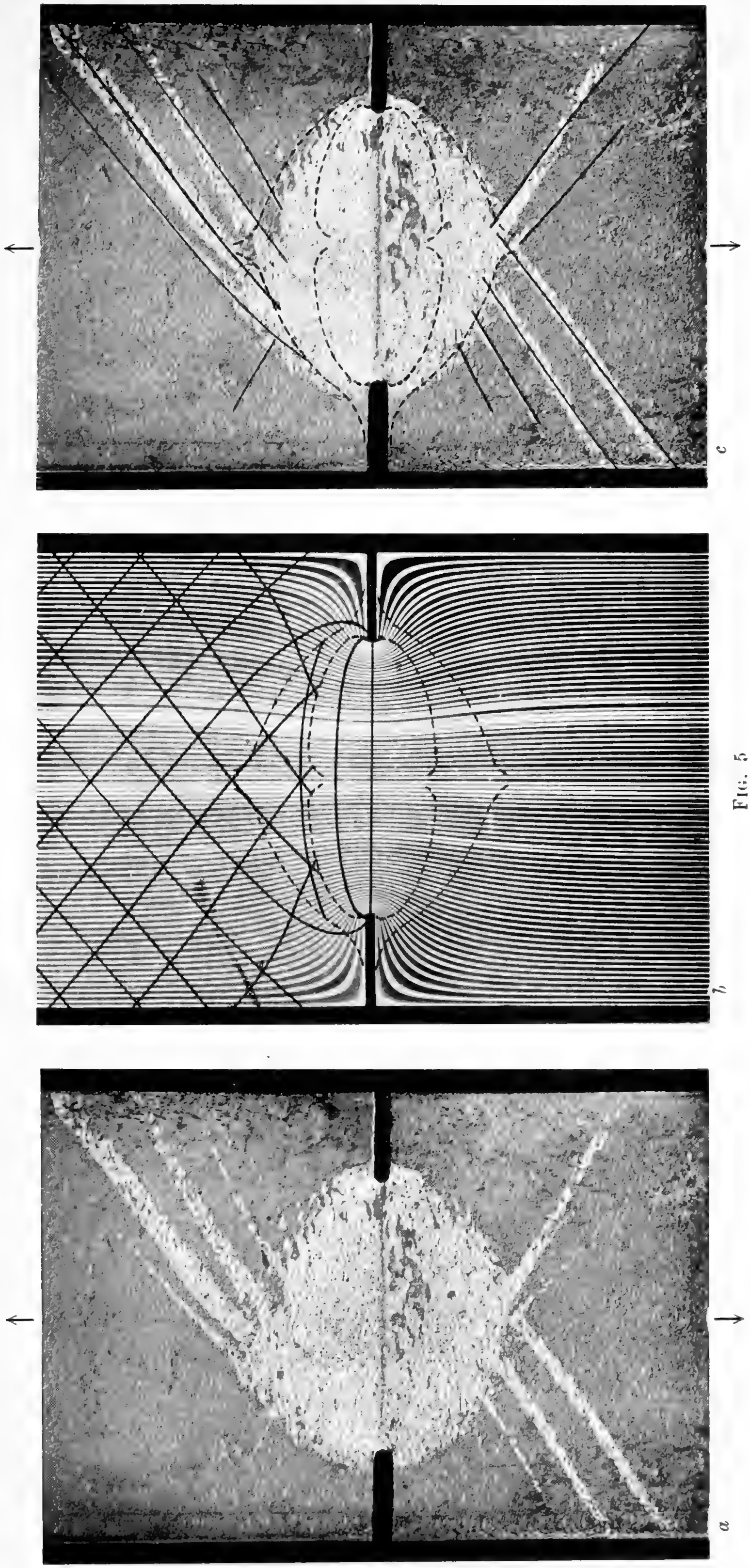

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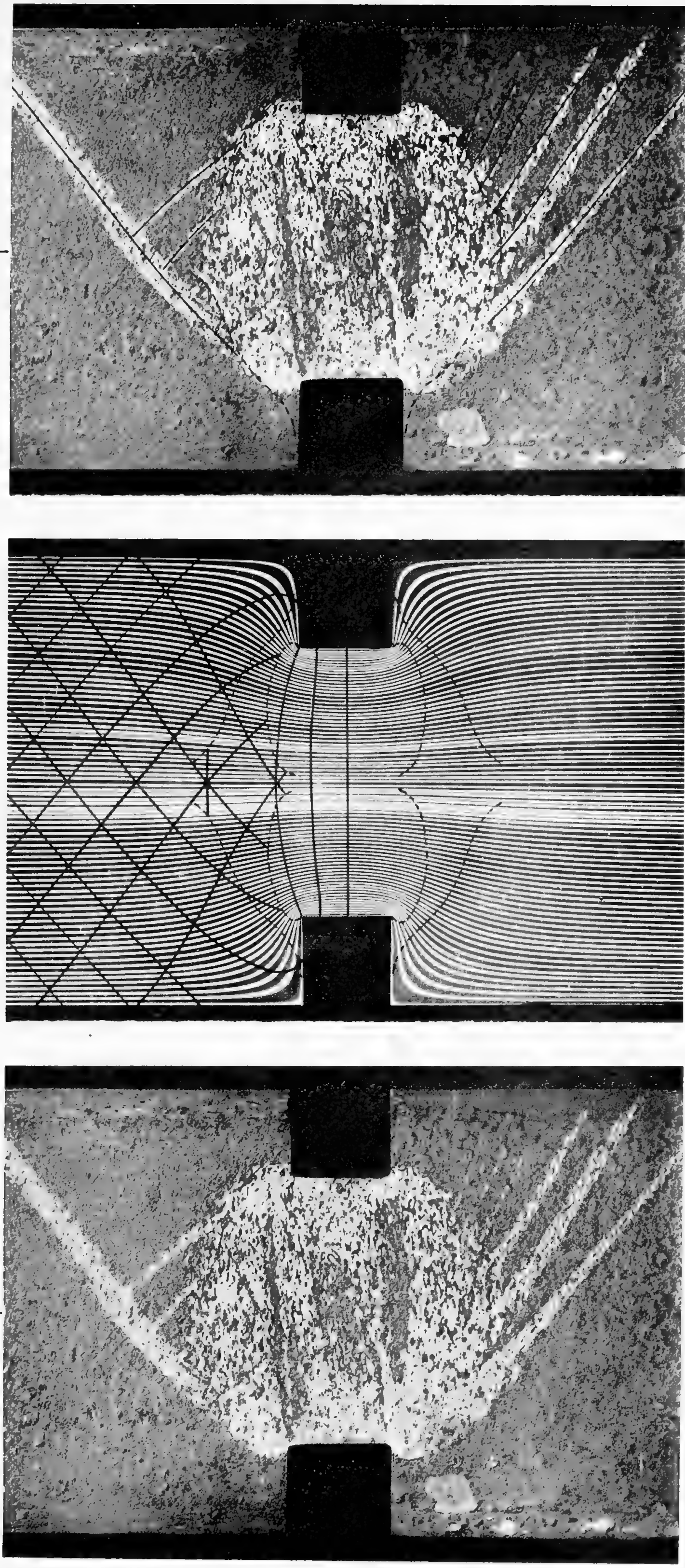

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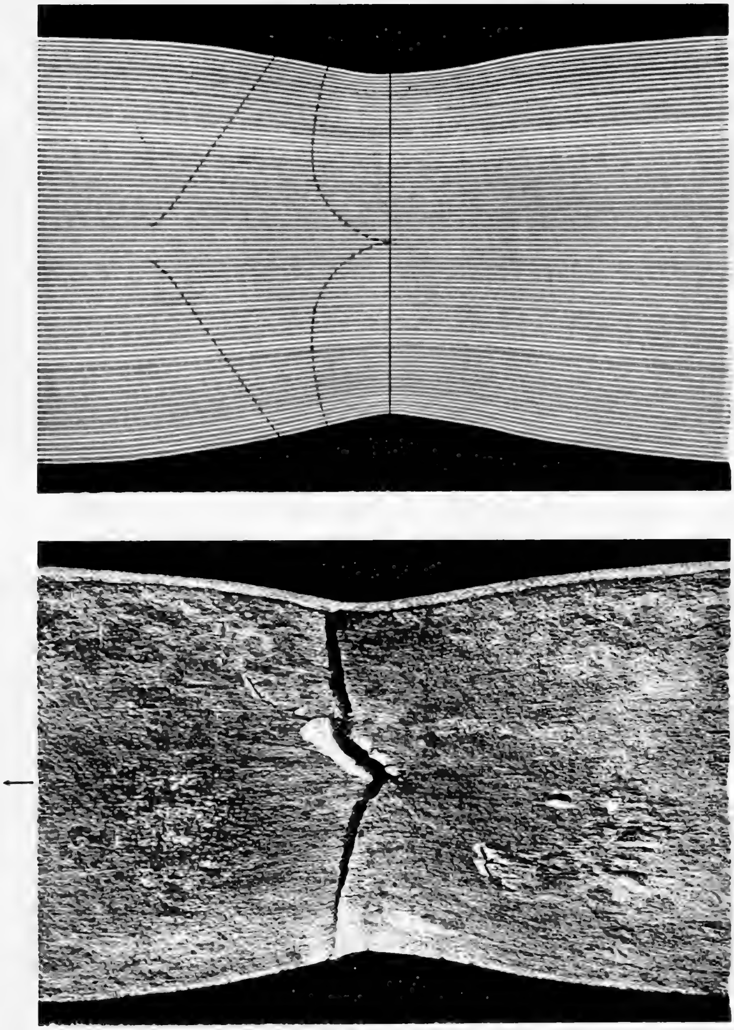

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Fig. 7.

VII.—The Composition and Character of Oceanic Red Clay. By W. A. Caspari, B.Sc., Ph.D., F.I.C., Assistant at the *Challenger* Office. *Communicated by* Sir John Murray, K.C.B., F.R.S.

(MS. received November 25, 1909. Read July 12, 1909.)

The chemistry of this not the least interesting of deep-sea deposits, though it has received attention from several investigators in the past, still presents some uncertainties as regards both the analytical composition of the material and its chemical nature. It has therefore been suggested by Sir John Murray, to whom directly or indirectly all our knowledge of Red Clay is due, that the subject might with advantage be reopened from the chemical standpoint. The results of this revision, which was carried out in the *Challenger* Laboratory at Edinburgh, are set forth and discussed in the ensuing pages.

Red Clay deposits were first met with by the naturalists of H.M.S. *Challenger* in 1873, some hundred miles west of the Morocco coast, and were so called from their resemblance, in consistency, to terrestrial clay. Regarded at first as a biological precipitate,* Red Clay was shown by Murray † to be produced mainly by the decomposition in the sea itself of the volcanic minerals, especially pumices, with which the deepest parts of the ocean are strewn. The general distribution of Red Clay, as mapped in the *Challenger Reports*,‡ was but little modified by the more recent soundings of the *Valdivia*,§ *Gauss*,‖ and *Albatross*.¶ Usually Red Clay borders upon and gradually merges into Globigerina Ooze deposits; there are consequently intermediate types of deposit which cannot be classified, except on an arbitrary system, as one or the other. Typical Red Clays should not contain above 20 per cent. of calcium carbonate, and those from 3000 fathoms or deeper contain very little, or none at all.

All over the ocean Red Clay may be expected wherever the depth exceeds 2000 fathoms. The total area occupied by it is about 51 million square miles; in the Pacific it is the deposit *par excellence*, whereas in the Atlantic and Indian Oceans it occurs rather as patches in the Globigerina Ooze.

* Wyville Thomson, *Proc. Roy. Soc. Edin.*, xxiii. p. 47, 1874.

† *Proc. Roy. Soc. Edin.*, ix. p. 247, 1877.

‡ In which the contemporary or shortly subsequent soundings of several other vessels are also made use of.

§ Murray and Philippi, *Wiss. Erg. Deutsch. Tiefsee-Exp.*, x. p. 4, Jena, 1908.

‖ Philippi, *Verh. XV Deutsch. Geographentages*, Danzig, 1905, pp. 28–33.

¶ Murray and Lee, *Mem. Mus. Comp. Zoöl.*, xxxviii., 1909.

Here and there Red Clay borders upon and merges into deposits which are not of a calcareous nature. In the middle of the Pacific there is an approximately equatorial belt of Red Clay mixed with biologically precipitated silica, to which deposit the specific name "Radiolarian Ooze" is applied. Along the Arctic and Antarctic circles Red Clay borders upon another kind of biological siliceous deposit, viz. Diatom Ooze, whilst around the Pacific coast-lines and off the Newfoundland bank direct transitions from terrigenous deposits (Blue Muds) to Red Clay have been observed.

As it lies at the bottom of the ocean, Red Clay is macroscopically far from homogeneous. Objects such as manganese nodules, otoliths, sharks' teeth, lumps of pumice and of palagonitic tuff, etc., are disseminated in it. Among microscopic admixtures mineral fragments are never absent, whilst the following may or may not be present:—calcareous and siliceous products of organised life, phillipsite crystals (formed *in situ* and limited to the South Pacific and Indian Oceans), and cosmic particles. All these have an interest of their own which is outside the scope of the present considerations, and in point of quantity they play no part whatever in comparison with the matrix itself. It is this latter to which the term "Red Clay" is here applied.

Now all Red Clays are composed of two chemically distinct ingredients, viz. hydrated amorphous silicates of an argillaceous character and finely divided anhydrous silicates, partly vitreous and partly crystalline, which represent what is left of the mother-substances of the former. The first-named component is that which imparts to the Red Clay its characteristic consistency, and it does not differ in general physical properties from the essential constituent of terrestrial clay. That clay-substance is of a colloidal nature had been recognised when or before oceanic Red Clay was discovered,* and has been amply confirmed in the course of the great advance of our knowledge of colloids made during the last decade or two.† A corollary from this fact which was formerly overlooked is that a hard and fast chemical composition is not to be attributed to clay-substance. It so happens that kaolin, which used to be regarded (erroneously, because it often consists largely of fine crystalline matter) as ideally pure clay, answers approximately to $Al_2O_3 . 2SiO_2 . 2H_2O$, and a compound of this formula was at first assumed to be the essential ingredient of Red Clay. This view, however, can no longer be considered unassailable. As a matter of experimental fact there is far more silica in Red Clay than corresponds to the "ideal clay" formula.

* *Challenger Reports*, "Deep-Sea Deposits," 1891, pp. 190, 340.

† For an account of the colloidal characteristics of clay see Rohland, Abegg's *Handb. d. anorg. Chem.*, iii. 1, pp. 90–102, 1906.

The hydrous and anhydrous constituents of Red Clay cannot be separated by mechanical means, except in so far as undecomposed silicates are present in coarse grains; but a tolerably accurate chemical separation may be effected by decomposing and bringing into solution the hydrous portion, leaving the anhydrous portion intact. Preliminary treatment of the material on these lines renders it possible to furnish dual analyses of Red Clay, each of which consists of two complete silicate-analyses.

Of the twenty-seven analyses of Red Clay hitherto published, twenty-one *Challenger* analyses by Brazier * and one by Harrison and Jukes-Browne † are of the dual type, whilst in four *Challenger* analyses by Hornung, Klement, and Renard ‡ the material was dealt with as a single silicate, as also in a recent composite analysis, published by Clarke, § of twenty-one samples selected by Sir John Murray. It need scarcely be pointed out that the true nature of Red Clay is not brought out unless some attempt be made to separate it into its primary and secondary constituents. Unfortunately the numerous and laborious analyses by Brazier have certain imperfections which limit their utility. On the other hand, the analysis of an average mixture of Red Clays published by Clarke, even though it takes the form of a single silicate-analysis, is invaluable, not only because it was carried out with more care than its predecessors, but because it gives information as to the several rare and minor elements present in Red Clay: no less than twenty-two distinct constituents are enumerated. It is noteworthy that qualitatively the same minor oxides were detected by Clarke as find a place in Gibson's || exhaustive analysis of South Pacific manganese nodules—viz. TiO_2, Cr_2O_3, NiO, CoO, SrO, BrO, V_2O_5, P_2O_5, MoO_3, CuO, PbO, ZnO. Clarke, in addition, reports As_2O_5, and Gibson Tl_2O. Quantitatively the percentages present in the nodules are almost throughout higher than in Red Clay, and markedly so in the case of NiO, CoO, and CuO. That is, manganese nodules have the property of concentrating these oxides out of the solid and aqueous surroundings. It should be mentioned that Gibson's nodules were from one single locality, whilst Clarke's Red Clay was a mixture from several places where nodules are scarce or absent.

In the present re-examination of the chemical composition of Red Clay the principle of separating each sample into its two integral parts was observed; the alkalies, omitted in the *Challenger* analyses, were determined throughout, but rare and minor elements, with the exception of

* *Challenger Reports*, "Deep-Sea Deposits," pp. 425–433.

† *Quart. Jour. Geol. Soc.*, li. p. 315, 1895.

‡ *Challenger Reports*, "Deep-Sea Deposits," pp. 434, 435.

§ *Proc. Roy. Soc. Edin.*, xxvii. p. 170, 1907.

|| *Challenger Reports*, "Deep-Sea Deposits," p. 424.

barium, were left out of account. Furthermore, since a great wealth of material was generously placed at disposal by Sir John Murray out of his unique collection of deep-sea deposits, the opportunity was taken of choosing a highly representative series of Red Clays from all parts of the ocean. For the specimens numbered 12 and 13 I am indebted to the kindness of Dr Emil Philippi.

The subjoined list of Red Clays analysed gives topographical, bathymetrical, and other particulars, together with the associated species of coarse minerals, as determined by the original investigators of the deposits. The proportion of these minerals separable by elutriation is in all cases insignificant and well under 1 per cent. Grains of manganese peroxide were usually reported as present in the coarse washings.

		Depth (fathoms).	Locality.	Minerals.
		North Atlantic.		
1.	*Challenger* Station 9 (trawl).	3150	23° 23′ N.; 35° 11′ W., about 800 miles N.W. of Cape Verde Islands.	Felspar, magnetite, biotite, augite, pumice, quartz.
2.	*Challenger* Station 13 (trawl): residue from a Glob. Ooze of 74 per cent. $CaCO_3$.	1900	21° 38′ N.; 44° 39′ W., about 1200 miles W.N.W. of Cape Verde Islands.	Lapilli, sanidine, augite, magnetite, palagonite, volcanic glass.
		South Atlantic.		
3.	*Seine* Station 33.	2815	17° 16′ S.; 23° 10′ W., about 1000 miles due E. of Porto Seguro (Brazil).	Glassy particles.
		North Pacific.		
4.	*Challenger* Station 256 (trawl).	2950	30° 22′ N.; 154° 33′ W., about 600 miles due N. of Sandwich Islands.	Felspar, volcanic glass, biotite, hornblende, magnetite, palagonite.
5.	*Albatross* Station 2 (trawl).	2368	28° 23′ N.; 126° 57′ W., about 450 miles due W. of Guadalupe Island.	Felspar (orthoclase and plagioclase), volcanic glass, augite, palagonite.
6.	*Challenger* Station 244 (trawl).	2900	35° 22′ N.; 169° 53′ E., about 1800 miles due E. of Yokohama.	Felspar, sanidine, pumice, magnetite, cosmic spherules.
7.	*Challenger* Station 227.	2475	17° 29′ N.; 141° 21′ E., about 300 miles W. of the Ladrones.	Pumice, volcanic glass, plagioclase, felspar, augite, hornblende, magnetite.

		Depth (fathoms).	Locality.	Minerals.
		South Pacific.		
8.	*Challenger* Station 288.	2600	40° 3′ S.; 132° 58′ W., about midway between Chili and New Zealand.	Phillipsite, felspar, volcanic glass.
9.	*Albatross* Station 4701: contains 11 per cent. of $CaCO_3$.	2265	19° 11′ S.; 102° 24′ W., about 1900 miles due W. of Pisagua.	Decomposed basic mineral (greenish flakes), plagioclase, augite, phillipsite.
10.	*Challenger* Station 165A: contains 19 per cent. of $CaCO_3$.	2600	36° 41′ S.; 158° 29′ E., about midway between New South Wales and New Zealand.	Quartz, felspar, hornblende, mica, volcanic glass, magnetite.
11.	*Challenger* Station 171A.	2900	25° 5′ S.; 172° 56′ W., about 1000 miles N.N.E. of New Zealand.	Plagioclase, magnetite, hornblende, quartz, pumice, red glassy particles, basaltic fragments.
		Indian Ocean.		
12.	*Valdivia* Station 176.	2933	24° 0′ S.; 95° 8′ E., about 1100 miles due W. of Cape Cuvier (West Australia).	Pumice, volcanic glass, felspar; very angular.
13.	*Gauss* Station 96.	2700	26° 0′ S.; 54° 0′ E.; about 400 miles S.E. from Madagascar.	

Experimental.—All samples were prepared for analysis by being deprived of calcium carbonate and, as far as possible, of coarse minerals and sea-salts. Calcium carbonate was removed, if present, by means of dilute acetic acid, coarse minerals by simple elutriation, and sea-salts by repeated decantation with distilled water. The last trace of sea-salt cannot be extracted in this way, because the wash-water ultimately becomes so poor in electrolytes that the clay refuses to settle within practicable time; however, the purified material, when boiled out with dilute nitric acid, gives only a faint opalescence with silver nitrate and no reaction whatever with barium chloride, so that it was deemed unnecessary to take residual sea-salt into account in the analytical statements.

The separation into anhydrous and hydrous silicates (the "insoluble" and "soluble" of the older analyses) was effected by the method originally due to Forchhammer,* which is now universal in ceramic analysis:† the hydrous

* *Pogg. Ann.*, xxxv. p. 331, 1835.

† For a full account see Berdel, *Sprechsaal*, 1902, pp. 881, 919, 959.

silicates are decomposed by means of hot concentrated sulphuric acid, and the silica set free from them is brought into solution with hot dilute caustic soda solution. A circumstance which renders the method peculiarly suitable to Red Clays is the scarcity or absence in them of micaceous minerals, which are rather easily attacked by hot sulphuric acid.

It cannot be pretended that a theoretically perfect separation is afforded by this means, since the extremely fine pumiceous and other mineral particles present in Red Clay are not likely to resist the attack of hot acid absolutely. Nevertheless, the procedure described below may be regarded as reasonably satisfactory, considering the variability of the material under examination. Preliminary experiments with No. 4 showed that treatment with hydrochloric instead of sulphuric acid does not completely remove the hydrous silicates; *e.g.* in the case of No. 4 the residues amounted to about 40 per cent. instead of 30 per cent., and gave ignition losses of about 3 per cent. Further, it was always found that the proportion of anhydrous residue obtained from a given sample by the sulphuric acid method is constant within 1 or 2 per cent. of the residue itself. Unduly prolonged treatment with the hot acid was found to be neither desirable nor necessary in the case of Red Clays.

One gram of pulverulent material is mixed with 5 c.c. of water in a tall 150-c.c. beaker; 5 c.c. of concentrated sulphuric acid are added, and the mixture is evaporated down until fumes of SO_3 begin to come off freely (time, $\frac{3}{4}$–1 hour). The cooled magma is digested on the water-bath with 100 c.c. of dilute ($\frac{1}{1}$ n.) hydrochloric acid during one hour with frequent stirring, and the beaker is allowed to stand in a slightly inclined position for at least six hours. After this the clear supernatant liquid can be decanted off to within a few cubic centimetres. The solid settlings are washed into a hot mixture of 35 c.c. of 10 per cent. NaOH solution and 25 c.c. of water contained in a larger beaker, and the whole is kept on the boil during five minutes. Settling then proceeds rapidly, and is complete within an hour. The clear alkaline liquid is poured off; the residue is treated with 10 c.c. of concentrated hydrochloric acid, filtered upon a Gooch crucible, washed, dried, and weighed.

The final treatment with hydrochloric acid is essential to a correct separation. An excessive proportion of sesquioxides, and especially of magnesia in the "insoluble" portion shown by some of the older analyses, may perhaps be due to neglect of this precaution.

The united acid and alkaline extracts thus obtained contained the "soluble" or hydrous silicates. They were evaporated down and analysed, allowance being made for silica and alumina imported by the caustic soda.

The "insoluble" residues, which (excepting No. 7, where black specks of ferruginous mineral were plainly visible) consisted of fine white powders sometimes rendered bluish by organic matter, were separately analysed.

The analyses were carried out with as much refinement as was considered adequate for so variable a material. Double precipitations were not made except in the determination of sesquioxides. Manganese was precipitated by the bromine method, and a correction (not to be neglected) for the manganese escaping into the Ca and Mg precipitates was made by dissolving these latter, after weighing, in dilute nitric and sulphuric acids respectively, oxidising with sodium bismuthate, and titrating the permanganate formed. Silica was invariably corrected by evaporation with hydrofluoric and sulphuric acids. Titanic and phosphoric acids were not determined, and may be supposed to lie hidden in the item Al_2O_3. Barium eventuated principally in the residue as sulphate, but was apt also to go partially into the "soluble" portion; in both portions it was brought down together with silica and duly allowed for; the BaO reported below, however, though classed in the "soluble" category, refers to total baryta determined on a separate batch of material. Alkalies in the anhydrous residue were determined by the Berzelius method; for "soluble" alkalies a separate gram of material was treated with sulphuric followed by hydrochloric acid, the filtrate was evaporated to expulsion of free sulphuric acid, and the subsequent procedure was as usual. All suspicious or unexpected results were checked by at least one redetermination.

The analytical figures are displayed in Table A, arranged in what seems to be the most rational manner. The item "residue" represents the total percentage of anhydrous silicates, and its composition is stated independently at the foot of each main analysis. In all cases the main analyses are referred to material dried in an air-bath at 110°. It is true that the degree of hydration of a clayey mineral depends not only on the temperature, but also on the tension of water-vapour in the surrounding atmosphere, so that in theory the material ought to be neither air-dried nor oven-dried, but brought into equilibrium (time required, several months) with air of a definite moistness at a definite temperature. However, drying at 110° was found to give sufficient constancy for present purposes.

It is seen that the proportion of anhydrous silicates in Red Clays fluctuates somewhat: it lies mostly in the neighbourhood of 30 per cent., but is exceptionally low in South Atlantic (No. 3) and South Pacific (Nos. 8 and 9) specimens and exceptionally high in Nos. 7 and 11. A glance at the detailed analyses of these residues shows that they consist in general of highly acid alkali-aluminium silicates with vanishing

TABLE A.

No.	1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.	13.	14.
Residue	27·35	28·74	20·15	31·58	31·16	34·24	42·37	14·49	18·07	25·51	40·34	23·20	31·12	...
Ignition loss	6·94	9·14	8·02	5·56	5·97	7·08	5·08	9·52	8·76	8·41	5·51	8·23	5·21	10·75
SiO_2	33·20	31·12	36·61	31·38	31·18	32·71	27·61	39·80	31·22	34·74	28·10	35·79	33·32	47·72
Al_2O_3	19·07	16·67	19·60	17·25	16·21	14·43	11·60	17·45	15·06	21·32	12·19	15·80	15·42	20·80
Fe_2O_3	8·75	8·85	9·88	8·18	8·64	6·28	7·43	9·47	14·33	4·76	7·63	8·88	8·77	12·91
MnO_2	0·32	0·39	1·24	1·34	0·62	0·65	0·60	1·24	5·00	1·08	0·62	3·28	0·63	0·55
CaO	0·51	1·02	0·71	0·43	0·62	0·68	1·10	1·15	2·82	0·96	1·33	0·79	1·13	0·89
BaO	0·05	trace	0·11	0·11	0·45	0·22	0·18	0·80	1·16	0·21	0·21	0·15	0·05	trace
MgO	2·31	2·28	2·52	2·70	3·16	2·60	2·73	2·49	2·06	1·86	2·88	2·65	3·25	3·37
K_2O	1·98	1·71	1·82	1·98	2·11	1·48	1·12	3·07	1·85	1·18	1·44	1·42	0·86	2·75
Na_2O	0·32	0·39	0·32	0·30	0·47	0·48	0·28	1·18	0·68	0·13	0·47	0·43	0·39	0·61
	100·80	100·31	100·98	100·81	100·59	100·85	100·10	100·66	101·01	100·16	100·72	100·62	100·15	100·35
Analysis of Residue.														
SiO_2	84·85	81·51	77·68	80·75	78·60	77·82	66·65	75·52	73·90	85·40	71·38	79·02	64·30	...
Al_2O_3	9·62	11·55	13·15	11·84	13·49	14·04	17·02	14·01	17·28	9·86	14·86	12·66	17·16	...
Fe_2O_3	0·15	0·70	0·42	0·20	0·21	0·20	4·56	0·26	0·40	0·35	2·85	0·69	3·38	...
CaO	1·14	1·13	2·07	1·10	1·27	1·06	4·65	1·36	2·59	0·53	3·57	1·03	6·90	...
MgO	0·33	0·53	0·22	0·36	0·36	0·08	1·57	0·10	0·25	0·16	1·03	0·39	2·44	...
K_2O	2·96	3·02	4·35	2·74	2·86	4·46	3·21	4·42	2·88	2·08	3·65	4·36	3·38	...
Na_2O	1·73	2·27	3·09	2·94	3·15	2·47	3·41	4·44	3·42	1·29	3·05	2·38	3·21	...
	100·78	100·71	100·98	99·93	99·94	100·13	101·07	100·11	100·72	99·67	100·39	100·53	100·77	...

contents of iron, calcium, and magnesium. Their composition thus approximates to that of white liparitic pumice, and in so far confirms the accepted view that Red Clays are produced by the degradation of such pumice. But it is clear that the original substance or mixture of substances from which Red Clay originates must have been rather differently composed, since we find in the hydrous portion considerable percentages of iron. Now much, if not all, of this constituent must have been derived either from more highly basic (basaltic) pumices or from ferruginous minerals accompanying or enclosed in acid pumice. Iron having in most cases disappeared from the residues, it may be concluded that basic volcanic glasses and pumices, and such individuals as augite, hornblende, biotite, and magnetite, are less resistant to submarine decomposition than liparitic pumice. This view is supported by the peculiarities of No. 7 (between Australia and Japan) and No. 11 (South Seas), where the residues are not only abnormally high in amount, but abnormally basic and ferruginous in composition; here the Red Clay would appear to be of more recent formation than in the other examples. The residue of No. 13 (off Madagascar) presents similarly abnormal features, but it is possible that there may be terrigenous admixtures in this instance. Again, the two Red Clays (Nos. 8 and 9) from the open South Pacific, where pumices of a basic habit are known to predominate, illustrate the effects of sufficiently prolonged action of sea-water: the proportion of residue is very small—less than 20 per cent.—but its composition is much the same as that of Red Clay residues from the North Pacific, where the volcanic silicates supplied are predominantly acid.

Quartz, supposed to be wind-blown, had been detected microscopically in the Red Clays Nos. 1 and 10; its presence is reflected by the residue-analyses, which show unusually high silica contents.

In general, chemically separated residues, once the coarse minerals have been removed by washing, yield little additional information to the microscope. The particles are for the most part under 0·005 mm. in diameter, and too small to react with polarised light or to show vesicles; sometimes, however, a few scattered units are large enough to be recognised as felspar, or, by their areolar structure, as pumice. Contours are invariably very much rounded. The undecomposed minerals isolated by chemical treatment (compared with which the mechanically separable coarse minerals are insignificant in amount) cannot, differences in density notwithstanding, be separated from the secondary constituents by elutriation, and it appears probable that they exist as cores within envelopes of argillaceous secondary matter.

As regards the total analyses, Nos. 1 and 2 claim a moment's attention. No. 1 is a typical Red Clay, quite free from calcium carbonate, in a deep basin of the North Atlantic; No. 2 was originally a Globigerina Ooze from a much higher level not many hundred miles distant; the analysis refers to the siliceous matter remaining (about one-fourth of the whole) after the calcareous bulk had been removed by means of dilute acetic acid. It will be observed from the marked similarity between the two analyses that this siliceous matter is itself a characteristic Red Clay. Additional proof, if any such were needed, is thus afforded that the formation of Red Clay is a process independent of the biological activity which brings the Globigerina Ooze deposits into being. The percentage of lime in such an ooze gives the relative rates at which, at the locality concerned, calcareous shells and Red Clay (from floating pumice) reach the bottom.

The elements which show the greatest absolute variation in these analyses are manganese and barium. Both are most abundant in the South Pacific and least in the North Atlantic. Manganese appears always to be present as peroxide: even the feebly manganiferous No. 1 evolves chlorine when heated with hydrochloric acid. In the more manganiferous Red Clays opaque specks of black oxide (mean diameter not exceeding 0·005 mm.) can be observed under the microscope, and it seems probable that manganese exists in Red Clays in this independent form, though part of it may perhaps be an ingredient of the colloidal aggregates constituting the argillaceous portion. Certain it is that no other element is so readily detached from the Red Clay mass, either on the floor of the ocean (*e.g.* by organic matter in decay) or in the laboratory (*e.g.* by dilute sulphurous acid).

The experimental data at hand leave it undecided whether any important proportion of the barium present belongs properly to the anhydrous residue.* Some of it, at any rate, is adsorbed by the hydrous silicates in the same way as magnesium and alkalies, since it can be partially extracted by ammonium chloride or nitrate solutions. The abundance of barium in South Pacific deposits is suggestive with regard to the proneness of this element to co-precipitate radioactive elements. Possibly the high activity found by Joly † in one of the Red Clays examined by him may thus be accounted for. Joly's figures for activity in Red Clays, together with the barium-contents now determined, are as follows:—

	Radium.	BaO.
Challenger Station 5 (North Atlantic) . .	$15 \cdot 4 \times 10^{-12}$	0·02
,, ,, 276 (South Pacific) . .	52·6 ,,	0·50

* The volcanic glass from *Challenger* Station 302 (analysis, *Challenger Reports*, "Deep-Sea Deposits," p. 307) was tested and found to contain not a trace of barium.

† *Radioactivity and Geology*, London, 1909, p. 50.

Outside of the South Pacific the contents of iron in Red Clay tend to run fairly uniform—surprisingly so, since the supply of iron to the floor of the ocean would be expected to be not less variable than that of manganese. Of the specimens here dealt with the only ones exceptional as to iron are No. 9 (South Pacific), where it is very high, and No. 10 (off Australia), where it is very low.

The origin of the iron in Red Clays of the Southern Hemisphere, where the mother-substances are mainly basic and highly ferruginous, is evident; but the 8 per cent. or so of Fe_2O_3 found in specimens from the northern oceans (Nos. 1, 2, 4, 5) are not so easily accounted for. The mother-substance in these regions is mainly white, acid (liparitic) pumice, which usually contains less than 1 per cent. of iron oxide. It seems probable, therefore, that this iron has been imported from the waters of the ocean and is ultimately of terrigenous origin. Much iron is undoubtedly carried into the sea by rivers in a soluble form, as carbonate or humate. It may well be that some of this finds its way into pelagic waters and deposits, though it is not known with certainty that any dissolved iron exists in sea-water, and we have no evidence to explain by what processes it is carried into the deposits.

It may be remarked that the figures now found for iron are in general rather lower than those of the older analyses, and agree fairly well with the average value stated by Clarke.

In No. 2 the ignition loss is known to be too high, as there was far more extraneous organic (carbonaceous) matter present that in ordinary Red Clay. No. 8 is a highly phillipsitic deposit, and the exceptional content of alkalies is thus accounted for. No. 9 also contains phillipsite, but it scarcely betrays itself in the analysis.

The hydrous or argillaceous portion of Red Clay, which not only preponderates in amount but imparts to the deposit its characteristic plastic properties, is also interesting as representing the typical degradation-product of igneous minerals under submarine weathering. It is well, therefore, to compare the percentage composition of the several secondary portions by themselves. To this end Table B has been drawn up. The comparatively accidental items MnO_2 and BaO have been omitted, and so much has in each case been subtracted from the ignition loss as corresponds to the conversion of $MnO_2 . \frac{1}{2}H_2O$ into Mn_3O_4.

[Table.

Table B.

No.	1.	2.	3.	4.	5.	6.	7.
Ignition loss	9·4	12·7	9·9	7·8	8·5	10·4	8·8
SiO_2	45·5	43·9	46·2	46·5	46·9	49·8	48·6
Al_2O_3	26·1	23·4	24·7	25·6	23·6	22·0	20·4
Fe_2O_3	12·0	12·4	12·4	12·1	12·6	9·6	13·1
CaO	0·7	1·4	0·9	0·6	0·9	1·1	1·9
MgO	3·1	3·2	3·2	4·0	4·6	4·0	4·8
K_2O	2·7	2·4	2·2	2·9	3·1	2·3	1·9
Na_2O	0·4	0·5	0·4	0·4	0·6	0·7	0·4

No.	8.	9.	10.	11.	12.	13.	14.
Ignition loss	11·0	10·0	11·2	9·1	10·3	7·5	10·6
SiO_2	47·4	41·3	47·4	47·3	48·9	48·8	47·9
Al_2O_3	20·8	19·9	29·1	20·5	21·6	22·6	20·9
Fe_2O_3	11·3	19·0	6·5	12·8	12·1	12·8	12·9
CaO	1·5	3·7	1·3	2·2	1·0	1·7	0·9
MgO	2·9	2·7	2·6	4·8	3·6	4·7	3·4
K_2O	3·6	2·4	1·6	2·4	1·9	1·3	2·7
Na_2O	1·4	0·9	0·2	0·8	0·5	0·5	0·6

The above figures, it will be perceived, present a considerable degree of uniformity and there are no very serious divergences from the mean of Nos. 1 to 13, which runs as follows:—

Loss	9·8
SiO_2	46·8
Al_2O_3	23·1
Fe_2O_3	12·2
CaO	1·4
MgO	3·7
K_2O	2·4
Na_2O	0·5
	99·9

Chemically, then, this substance is composed in the main of silica, alumina, ferric oxide, and water. On calculating molecular ratios from the above average we find

$$SiO_2 : Al_2O_3 : Fe_2O_3 = 1 : 0·287 : 0·096,$$

whence

$$Al_2O_3 : SiO_2 = 1 : 3·5,$$

or

$$(AlFe)_2O_3 : SiO_2 = 1 : 2·6\,;$$

that is, the molecules are in no simple proportion. This is only to be expected in view of the mixed nature and origin of the clay-substance.

But the irregular ratios are not to be accounted for by a mixture of definite chemical individuals, unless the presence of very finely divided kaolinite, for which there is no direct evidence, be assumed; rather is it probable that definite chemical compounds, in the strict sense, are altogether absent. It will be observed that the argillaceous portion of Red Clay is considerably more acid than terrestrial clays, for which, if the analytical data available are to be trusted, the ratio

$$Al_2O_3 : SiO_2 = 1 : 2$$

is understood to hold good.

Under the microscope the characteristic portion of Red Clay is seen to consist of transparent isotropic rounded particles, mean diameter about 0·001 mm., of which the majority are colourless, whilst others, doubtless owing to the presence of iron, are more or less deeply brownish-yellow. With the exception of crystalline and vitreous fragments not coated with decomposed matter, and of opaque manganese specks, all the particles are readily stained by a dye such as methylene blue; that is, they are of colloidal habit. On extraction with dilute acid and alkali alternately (the specimen experimented upon was No. 4) the successive extracts contain silica and sesquioxides in gradually varying non-stoichiometrical proportions, like the soils and clays studied by Van Bemmelen,* and the amounts extracted vary greatly with the concentration, temperature, etc., of the solvent. Chemically well-defined silicates, therefore, appear to be absent.

Hydrochloric acid alone, even at boiling temperature, is unable to dissolve the whole of the clay-substance. In this respect one Red Clay is apt to differ from another: that from the South Pacific, for instance, yields more readily to acid attack than that from the North Atlantic.†

As to the iron in Red Clay, there is no warrant for supposing it to be present in a distinct form, as limonite. If the ferric hydroxide existed independently it would tend, under the influence of salt-water and deep-sea pressure, to become anhydrous and crystalline,‡ and would be found as crystals of hematite, like those occurring, *e.g.*, in carnallite. As it is, the iron behaves altogether as if it formed part of the argillaceous agglutinate. At the same time, it is undoubtedly more loosely bound than alumina, since it is more easily extracted by acids; and, as the microscope shows, it appears to be not quite regularly distributed in the clay particles.

Turning now to the molecular constitution of submarine clay, it seems difficult to hold other views than those arrived at by Van Bemmelen § with

* *Zeitschr. Anorg. Chem.*, xlii. 265–324, 1904.

† Caspari, *Mem. Mus. Comp. Zoöl.*, xxxviii. p. 169, 1909.

‡ Wittstein, *Vierteljahrsschr. f. Pharm.*, i. p. 275; Spring, *Neues Jahrb.*, 1899, p. 47.

§ *Loc. cit.*

respect to the amorphous products of subaerial weathering. In so far as they are amorphous and colloidal, these hydrous silicates are to be regarded not as definite chemical compounds or as mixtures of such, but as agglutinates of colloidal silica, alumina, etc., in inconstant proportions. Just as clay itself attaches water of hydration not in a series of stoichiometrical proportions (like, for instance, the hydrates of $MgSO_4$ or Na_2CO_3) but in continuously variable proportions depending on such conditions as temperature, pressure, and medium, so the chief constituents of clay form an agglutinate in proportions similarly governed by active masses and other extraneous factors. What the affinity is which binds the constituents together we do not know, but it is certainly not exclusively chemical.

The irregular ratios which obtain when alumina combines with silica to form an amorphous hydrated solid are not only illustrated in nature, but are also well brought out by synthetic experiments such as those of Lemberg* and of Stremme.† The latter investigator, who analysed not only artificial alumino-silicic precipitates, but also minerals of the allophane, halloysite, and montmorillonite groups, comes to the conclusion that all these bodies are merely mechanical mixtures of colloidal hydrated alumina and silica. The same view with respect to clay has also been expressed by Rohland.‡ There are two considerations, however, which may be urged against the suggestion that clays are nothing more than mixtures. In the first place, there is the resistance offered by clays to acid attack; if they were mixtures, extraction of all the argillaceous alumina by dilute acid ought to be a simple matter, whereas in reality no less drastic a reagent than hot concentrated sulphuric acid is required for the complete decomposition of clay. In the second place, it cannot be overlooked that a certain hazy constancy of the ratio between silica and alumina prevails in both submarine and continental clays, whereby a tendency on the part of these constituents to combine rather than exist independently side by side is indicated. Thus acid minerals (*e.g.* orthoclase, in which Al_2O_3 ; $SiO_2 = 1 : 6$) weather to clays which have for the most part ratios ranging from 1 : 2 to 1 : 3; now, if the latter were mixtures, it is hard to understand why the same agencies which have removed so much silica should not also remove the remainder and leave bauxite instead of clay. Degradation-products intermediate between clay and bauxite do indeed occur, but they are comparatively uncommon minerals and are therefore doubtless produced by exceptional means.

* *Zeitschr. deutsch. geol. Ges.*, xxviii. p. 519, 1896.

† *Centralbl. f. Min.*, 1908, pp. 622, 661.

‡ *Loc. cit.*

Whilst, therefore, it is not at all improbable that colloidal hydrated alumina and silica are present as such in the argillaceous portion of Red Clay, one may incline to the belief that the dominant molecular species in the argillaceous portion of Red Clay is an "absorption-compound" (the expression is Van Bemmelen's) $SiO_2 . mAl_2O_3 . nFe_2O_3 . pH_2O$, where on the average $m = 0{\cdot}20$, $n = 0{\cdot}10$, and $p = 0{\cdot}69$.

The ultimate physical structure of the simpler gelatinous and flocculent inorganic colloids is now known * to consist of a reticulated or honeycombed framework of no great elasticity, filled with colloidal liquid matter. No doubt the particles of clay-substance are similarly constituted. The extremely variable behaviour of a given colloid according to its mode of preparation, previous history, etc., may be set down to variations in the shape, dimensions, and elasticity of this framework. Similar considerations may well apply to Red Clay. Thus Red Clays produced mainly from basic glasses (Southern Hemisphere) might be expected to differ physically from such as are derived mainly from acid pumice (Northern Hemisphere); and indeed the No. 9 type, as brought up from the bottom, is much more gelatinous and unctuous than the No. 1 or No. 4 type, and is relatively more easily decomposed by dilute acid. Differences in ultimate physical structure may also account for the rather wide differences in hydration, expressed as ignition loss, which appear from Table B.

It has been established, as was mentioned above, that Red Clays originate in the main from the degradation of acid and basic volcanic glasses. The chemical processes involved cannot differ in essence from those associated with the subaerial weathering of silicates. A peculiarity of vitreous silicates is that their decomposition begins with an absorption of water without loss of coherence on the part of the substance. This admits of direct proof in the case of basic glasses (South Pacific), which pass first into a hydrated hyaline mineral, palagonite. Acid pumices doubtless behave similarly, though the existence of an intermediate hyaline stage has not been observed in deep-sea deposits, probably because acid glass, having more material (silica) to lose in its passage towards clay, cannot long remain in a coherent hydrated form. Laboratory experiments by Lemberg † and by Barus ‡ have shown that glasses in general have a marked tendency to take up water, and that silicates in vitreous form are consequently more easily decomposed than the same silicates as crystals. It

* Bütschli, *Untersuchungen über Strukturen*, Leipzig, 1898; Hardy, *Journ. of Physiol.*, xxiv. p. 158, 1899.

† *Zeitschr. deutsch. geol. Ges.*, xxxix. p. 594, 1887; xl. p. 637, 1888.

‡ *Amer. Journ. Sci.*, vi. p. 270, 1898; *Phil. Mag.*, xlvii. p. 461, 1899.

agrees with these observations that in Red Clays anhydrous crystalline silicates often occur in a state of great freshness, whereas pumice fragments are invariably much corroded and basic glass is generally (except when preserved in manganese nodules) wholly palagonitised.

An interesting feature of deep-sea weathering is that it takes place under conditions which admit of finality. In the Red Clay areas we have a temperature of 1°–3°, a pressure of 400–600 atm., and a uniform medium (sea-water) which have scarcely changed for millions of years. As the result, we find a degradation-product of much the same composition all over the globe, and we note that it is a more acid silicate than the corresponding continental material. This acidity cannot be due to non-elimination of silica, since the mother-substances themselves are far more acid. Clearly silica can escape into the hydrosphere just as well as alkalies and alkaline earths.* On the whole, there seems to be something approximating to genuine equilibrium between the argillaceous portion of Red Clay and sea-water, modified only by the inability of the two sesquioxides to transfer themselves freely between deposit and water. If there were no iron, then, since silica can pass in and out of solution, a constant ratio of Al_2O_3 to SiO_2 would be striven for in equilibrium; but, as things are, so much iron must remain in the deposit as happens to be on the spot. Hence the variations illustrated by Nos. 9 and 10.

In subaerial weathering, *per contra*, the conditions are of the widest conceivable variety, and are such that a state of chemical equilibrium is rarely, if ever, reached. Thus we find that in temperate climates products having roughly one molecule of alumina to two of silica are the rule, whereas in the tropics the tendency is toward the highly basic laterites, culminating in almost non-siliceous bauxites.

On land, circumstances occasionally arise in which constituents of the colloidal clay-agglutinate can combine to true chemical individuals. Such are the crystalline minerals kaolinite, $Al_2Si_2O_7 . 2H_2O$, hydrargillite, $Al_2O_3 . 3H_2O$, and perhaps nontronite, $Fe_2Si_2O_7 . 2H_2O$, not to mention micas, chlorites, staurolites, felspars, etc., of secondary origin. No such formations are known in deep-sea deposits; at any rate, no crystalline matter of secondary growth, derived from colloidal materials, can be detected microscopically. On the other hand, there are conditions in parts of the South Pacific which favour the deposition of well-crystallised zeolites (sodio-potassic phillipsite) in the midst of Red Clay deposits, a phenomenon which has no parallel in continental clays. In the reaction by which these crystals were formed

* Silica cannot, however, accumulate in solution, but is returned to the bottom by biological agencies as diatom or radiolarian ooze.

colloidal solid matter played no direct part; they separated out of aqueous solution in the interstices between particles or lumps of Red Clay, and it may be surmised that in these regions diffusion of aqueous solutions from the deposit into the sea is, or once was, exceptionally slow.

One of the oldest observations in the chemistry of colloids is that many inorganic colloidal solids may be brought into colloidal solution by minute quantities of acid or alkali ("peptisation"). This can also be accomplished, to a limited extent, with Red Clay. When shaken up with caustic soda solution of not more than $\frac{1}{100}$ n. strength, Red Clay yields a suspension which settles with great difficulty. After several months at rest the upper liquid is a clear yellow solution, transparent but slightly opalescent, of clay. This solution passes unchanged through ordinary filters, but is deprived of its dissolved matter when forced through a layer of clay; on making it faintly acid or adding salts or excess of caustic, the dissolved matter is precipitated in flakes. The precipitate thus obtained from a colloidal solution of No. 4 was analysed and figures in Tables A and B as No. 14; this material was prepared from a second brew, and it is noteworthy that the product of the first extraction (not fully analysed) contained as much as $3\frac{1}{2}$ per cent. of MnO_2, which constituent apparently goes independently and preferentially into alkaline solution. There is a close resemblance in composition between No. 14 and the hydrous portion of Red Clays generally, as shown by Table B; that is to say, no ultimate constituent (barring manganese) of the clay is selectively dissolved or "peptised." We have here strong evidence that iron is not an admixture but an integral constituent of the clay-substance, seeing that ferric hydroxide by itself, which is a positive colloid, would require a trace of acid to go into solution and would be precipitated by a trace of alkali.

When the colloidal nature of Red Clay is realised, the invariable presence of calcium, magnesium, and alkalies causes no surprise. Colloids of the clay type are able to adsorb much crystalloid matter from solutions with which they are in equilibrium. This retention of highly soluble matter may be ascribed to capillary effects at the enormous surfaces presented by the fine grains of clay and their internal framework, but the possibility that chemical affinities are also exerted is not to be disregarded. In Red Clay the four elements named are withdrawn, in approximately constant proportions, out of the surrounding sea-water. They are adsorbed not as salts but as hydroxides or perhaps as ions, since there is not nearly enough Cl or SO_4 present to balance them.

In general, the amount of a given element adsorbed varies with the

physical peculiarities of the adsorbent, but also, and especially, with the specific adsorbability of the element itself. On comparing the percentage in Red Clay with the concentration in sea-water, the order of adsorbability is found to run K, Ca, Mg, Na, the latter element being a very bad fourth; this sequence exactly reverses that of abundance in sea-water. Evidence that these elements are held by adsorption is furnished by the fact that they can be extruded to some extent by neutral salts of a more adsorbable base, such as ammonia. By means of dilute acid, which partially breaks down the clay aggregate, still more adsorbed matter is set free. The subjoined figures show (*a*) the percentage of certain constituents, in hydrous form, in No. 4; (*b*) the amounts extracted by boiling for a few minutes with seminormal hydrochloric acid; and (*c*) the amounts extracted in the same way by 3 per cent. ammonium sulphate:—

	a.	*b.*	*c.*
Al_2O_3	14·43	4·18	...
Fe_2O_3	6·28	3·59	...
CaO	0·65	0·60	0·36
MgO	2·60	1·15	0·41
K_2O	1·48	0·66	0·33

The tendency of soils and clays to adsorb potash and magnesia has long been known, and has been investigated quantitatively by Van Bemmelen. In the deep sea a very striking case of potash-adsorption in palagonite was directly demonstrated by Murray and Renard.*

Continental clays which have been in contact with sea-water or other saline medium commonly retain small percentages of calcium and magnesium, which sometimes puzzle analysts owing to the absence of sufficient carbonic acid to account for their presence as carbonate.† The opportunity may here be taken of remarking that Brazier's habit of stating magnesia as $MgCO_3$ in all deposits,‡ which has led more than one naturalist into speculations on dolomitisation, cannot be regarded as justifiable, at any rate for the less calcareous deposits.

In all probability retention of potash by bottom-deposits (by no means necessarily as glauconite) has something to do with the secular impoverishment of the ocean in potash, relatively to soda. It is well to remember, however, that if Red Clay is a deposit formed *in situ*, the adsorbed potash

* *Challenger Reports*, "Deep-Sea Deposits," p. 307.

† *Cf.* Vesterberg and Mauzelius, *Bull. Geol. Inst. Upsala*, v., No. 9, p. 125, 1900.

‡ *Challenger Reports*, "Deep-Sea Deposits," *passim*.

cannot be wholly debited to the superjacent ocean, since much of it must have been derived from the volcanic mother-substances of the Red Clay itself. A not less serious item in the inorganic economy of the ocean would seem to be the adsorption by bottom-deposits of magnesia, the bulk of which must certainly represent a net withdrawal from the salts dissolved in the ocean.

(*Issued separately February* 1, 1910.)

VIII.—The Short Muscles of the Hand of the Agile Gibbon (Hylobates agilis), with Comments on the Morphological Position and Function of the Short Muscles of the Hand of Man.* By Duncan C. L. Fitzwilliams, M.D., Ch.M., F.R.C.S., Surgeon-in-Charge of Out-Patients, St Mary's Hospital, London. *Communicated by* Sir Wm. Turner. (With Two Plates.)

(MS. received November 22, 1909. Read December 6, 1909.)

The kindness of Professor Cunningham has enabled me to carry out the dissection of a gibbon in his possession. The dissection was performed in 1905, in the Anatomical Department of the University of Edinburgh. The following paper is a description of the short muscles of the hand of the animal, to which is added a consideration of the primitive position and function of the short muscles of the human hand.

In dealing with the nerve supply of the various muscles, it is necessary to explain that the median nerve, high up in the forearm, gave off a strong branch of communication to the ulnar nerve. The fibres thus supplied to the ulnar were traced down to the muscles in which they ended. This explains the phrase constantly met with in which the nerve supply is described as coming from the median by way of the deep division of the ulnar nerve. In this animal the nerves freely communicated with one another in the forearm. The median, even, gave off a large branch which passed backwards with the posterior interosseous artery, joined the posterior interosseous nerve, and ended in the pronator quadratus. This is one of the very few reported instances in which the anterior and posterior divisions of the nerves forming the brachial plexus have been found to communicate with one another. Hepburn† noted that the pronator quadratus muscle was supplied by the posterior interosseous nerve in a gibbon, but did not state the true origin of the fibres.

Short Muscles of the Thumb.

Abductor Pollicis.

Origin.—From the sesamoid bone (prepollex) and the ligaments which bind that bone to the scaphoid and trapezium, from the outer side of the

* This paper is an abstract of part of my thesis entitled "An Essay on the Anatomy of the Gibbon (*Hylobates Agilis*), with Notes on Comparative Anatomy," presented for the degree of M.D. in the University of Edinburgh, 1905.

† *Journ. Anat. and Physiol.*, 1893.

anterior annular ligament, and from the common head of origin of the opponens and the superficial head of the flexor brevis pollicis.

Insertion.—Into the outer side of the base of the first phalanx of the thumb.

Nerve Supply.—From the outer division of the median.

Structure.—The muscle has a belly distinct from the other muscles of the thumb. The tendon is separate all the way to its insertion, and is the outer and most posterior of the tendons inserted into the first phalanx.

Opponens Pollicis.

This muscle is partly blended with the superficial head of the flexor brevis pollicis.

Origin.—From the sesamoid bone or prepollex, scaphoid, trapezium, and outer part of the anterior annular ligament.

Insertion.—Into the outer side of the first metacarpal. It is also prolonged down with the flexor brevis pollicis, from which it is at this point quite inseparable, to the outer side of the first phalanx.

Nerve Supply.—From the outer division of the median.

Structure.—The muscle is composed of short bundles of fibres running outwards and downwards.

Flexor Brevis Pollicis.

This muscle is composed of two heads, of which the superficial is partly fused with the last-mentioned muscle.

Origin.—The superficial head.—By fleshy fibres from the scaphoid, trapezium and sesamoid bones, and by tendon (not shown in the plate) from the front of the anterior annular ligament. Deep head.—From the part of the trapezium which projects between the 1st and 2nd metacarpals, from the front of the base and the inner side of the 1st metacarpal bone.

Insertion.—Into the sesamoid bone on the front of the 1st metacarpo-phalangeal joint; the fibres are also prolonged downwards, to be inserted on the outer side of the shaft of the proximal phalanx.

Structure.—The portion of the muscle which arises from bone is fleshy. The part of the superficial head which arises from the anterior annular ligament does so by thin tendons. These tendons end in small bellies, which join the rest of the muscle. The muscle is fleshy at its insertion.

Short Muscles of the Little Finger.

These are three small and relatively ill-developed muscles, which hardly form any prominence on the palm of the hand.

Abductor Minimi Digiti.

Origin.—A slip from the pisiform and the inner part of the anterior annular ligament. A deeper head from the inner part of the unciform, situated behind a similar head of the flexor brevis muscle.

Insertion.—Into the inner side of the base of the proximal phalanx of the little finger.

Nerve Supply.—From the deep division of the ulnar nerve which passes between the two heads of origin.

Structure.—The belly formed by the two heads is very short, the tendon proportionately long. The latter seems much too strong for the feeble muscle.

Flexor Brevis Minimi Digiti.

Origin.—There are, as in the last muscle, two heads of origin; one from the front and inner side of the pisiform, another (lying in front of a similar slip to the last muscle) from the hook of the unciform and the inner part of the anterior annular ligament.

Insertion.—By short tendinous fibres into the sesamoid bone on the front of the 5th metacarpo-phalangeal joint, and by fleshy fibres into the front of the base of the proximal phalanx of the little finger. On following the fleshy fibres down the finger they are found to give place to a tendon which passes downwards to find attachment to the inner margin of the 1st and 2nd phalanges and to the pulp of the finger.

Nerve Supply.—By two twigs from the deep division of the ulnar nerve which passes between its two heads of origin.

Structure.—The pisiform head is muscular, the other is tendinous. The remainder of the muscle is fleshy till it approaches its insertion, of which the structure has been fully described.

Opponens Minimi Digiti.

Origin.—From the hook of the unciform process, the anterior annular ligament, and the anterior carpal ligaments.

Insertion.—Into the inner side of the front of the 5th metacarpal bone in its whole length.

Nerve Supply.—By a twig which is given off from the deep division of the ulnar before that nerve pierces the muscle. The twig accompanies its parent trunk till it reaches the deeper plane of the palm.

Structure.—The muscular fibres are largely intermixed with fibrous tissue. The muscle is blended partly with the small palmar interosseous

muscle of this digit; the inner or fourth contrahens is also largely blended with its fibres.

We now come to the description of muscles lying deeper in the palm of the hand. In describing them it is well to keep before one's mind the three layers of muscles of the typical manus—the contrahentes, the palmar, and the dorsal interossei muscles; or, according to the better anatomical nomenclature of Professor Cunningham, the adductores, the flexores breves, and the abductores.

There are in the hand of the gibbon certain muscles which do not at first sight fall into any of these three layers. These muscles I have termed musculi accessorii interossei.

The morphological position of these as well as of the other muscles will be dealt with after a description of each has been given.

First Layer: Contrahentes or Adductores.

All four of these muscles are present, the largest being that of the thumb, which shows a segmentation into an adductor transversus and an adductor obliquus pollicis. They all take origin from the central portion of the palm in the neighbourhood of the 3rd metacarpal bone, and from a well-marked tendinous fascia which occupies the hollow of the hand and is best marked over the 3rd and 4th metacarpals.

Contrahens 1.

This muscle is segmented into two parts, an adductor transversus and an adductor obliquus pollicis.

ADDUCTOR TRANSVERSUS POLLICIS.

Origin.—From the front of the anterior annular ligament, and from the front of the bases of the 2nd and 3rd metacarpals.

Insertion.—Into the inner side of the distal half of the 1st metacarpal bone, slightly in front of the insertion of the obliquus.

Structure.—The muscle is fleshy throughout its entire extent. It is fan-shaped, being widest at its base.

ADDUCTOR OBLIQUUS POLLICIS.

Origin.—From the front of the proximal half of the 3rd metacarpal, and from the layer of fascia over the metacarpals.

Insertion.—Into the inner side of the distal half of the 1st metacarpal rather behind the attachment of the last muscle, and into the inner side of the proximal phalanx of the 1st digit.

Nerve Supply.—From the deep division of the ulnar nerve—the fibres, however, come from the median by a communication in the forearm. The nerve supply to both muscles is the same.

Structure.—The origin is the thinnest and widest part of the muscle which becomes thicker and narrower as it passes outwards. The muscle is twisted on itself so that the fibres which have the highest origin have the lowest insertion, and *vice versa.*

Contrahens 2.

Origin.—From the front of 3rd metacarpal close to its base, and from the fascia over the metacarpals.

Insertion.—Into the inner side of the base of the proximal phalanx of the index.

Contrahens 3.

Origin.—From the front of the bases of the 3rd and 4th metacarpals, proximal three-quarters of the shaft of the latter bone, and fascia over them.

Insertion.—Into the outer side of the base of the proximal phalanx of the 4th digit, and into the dorsal extensor expansion.

Contrahens 4.

Origin.—Chiefly from the front of the base of the 4th and slightly from the front of the base of the 3rd metacarpals; a small slip from the inner lumbrical and the fascia over the metacarpals.

Insertion.—Into the outer side of the base of the proximal phalanx of the 5th digit, into the dorsal extensor expansion in common with the inner lumbrical, and into the sesamoid bone on the front of the 5th metacarpo-phalangeal joint.

Nerve Supply.—This has already been mentioned in referring to the 1st. In the case of the others the nerve fibres come from the median by way of the deep division of the ulnar, but I am not quite certain of this in the case of the 4th.

Structure.—As regards size, the 1st is the largest, next comes the 3rd, then the 4th, and the smallest is the 2nd. The inner three have small tendons, which in the case of the 3rd and 4th are inserted partly into the dorsal extensor expansion just below the lumbricals.

Second Layer: Palmar Interosseous or Flexores Breves.

Of this group only two are found; they belong to the index and little fingers, corresponding to the 1st and 3rd in man.

1st Palmar Interosseous.

Origin.—From the front and inner aspect of the 2nd metacarpal, and very slightly from the base of the 3rd.

Insertion.—Into the inner side of the base of the proximal phalanx of the index, and into the dorsal extensor expansion on that bone.

Nerve Supply.—Fibres from the median by way of the deep division of the ulnar nerve.

Structure.—The muscle is strong and well developed. The fibres are arranged in a penniform manner, and end in a small tendon. This muscle lies in front of the second dorsal interosseous and behind the adductors of the thumb and index (contrahentes 1 and 2).

The *2nd Palmar Interosseous Muscle* is absent, and its function is taken by the strong contrahens of this digit.

3rd Palmar Interosseous.

Orgin.—From a small part of the outer and front surface of the 5th metacarpal, behind the contrahens of this digit.

Insertion.—Into the outer side of the base of the proximal phalanx of the 5th digit, and partly into the ligaments of the joint.

Nerve Supply.—Ulnar fibres from the deep division of the ulnar nerve.

Structure.—This muscle was very rudimentary; it had a small belly and a thin, weak tendon.

The 3rd palmar interosseous being completely hidden by the contrahens of this digit, appears to be absent till the latter muscle is pulled aside.

Third Layer: Musculi Interossei Accessorii.

This is a group of four muscles which pass from the palm to the fingers (Plate I., M.A. 1, 2, 3, 4). They all lie posterior to the deep branch of the ulnar nerve, and therefore belong to the two groups of muscles which lie deeper than that nerve, namely the palmar and dorsal interossei muscles. Their exact morphological position will be settled later when the general morphology of the muscles is considered.

Musculus Accessorius Interosseus 1.

Origin.—From the front of the lower half of the 2nd metacarpal bone, between the palmar and dorsal interossei muscles of this digit, and closely associated with them.

Insertion.—Into the outer surface of the middle phalanx of the index,

the base and also the outer margin of the shaft. In addition, some tendinous processes pass to the pulp of the finger.

Nerve Supply.—Fibres from the median by way of the deep division of the ulnar.

Structure.—This is the largest muscle of the group, measuring 9 cm. in length. It is slightly thicker than is depicted in Plate I. The shape of the muscle is that of a cylinder tapered towards both extremities. The origin is fleshy; the insertion is by means of tendon which appears opposite the first interphalangeal joint. The muscle lies, to begin with, on the front of the metacarpal bone, and then gradually passes towards the outer side of the finger. An interosseous muscle lies on each side of its origin. The tendon of the first lumbrical passes between the muscle and the outer side of the first phalanx.

Musculus Interosseus 2.

This is partly segmented into two bellies placed the one in front of the other.

Origin.—The posterior part arises from the outer side of the head and shaft of the 3rd metacarpal, in front of the dorsal interosseous, through which it may have some slight attachment to the proximal half of the 2nd metacarpal. The anterior part arises from the front of the head and the lower half of the 3rd metacarpal on its outer aspect.

Insertion.—The two bellies are inserted almost together. The posterior half is inserted partly into the outer side of the proximal phalanx of the middle digit, but mostly into the dorsal extensor expansion. The anterior half is inserted partly with the tendon of the posterior half, and partly by fleshy fibres, into the dorsal expansion below that tendon.

Nerve Supply.—Fibres from the median nerve by way of the deep division of the ulnar. The nerve appears from under cover of the contrahentes, and before entering the muscle gives off a twig which runs down the front of the metacarpal bone to the proximal phalanx, where it probably forms a communication with the digital branch of the median, which supplies the skin in this region.

Structure.—As already stated, the muscle is in two parts. The anterior half is the smaller, measuring only 4 cm., while the posterior measures fully 6 cm. in length. Each half is pointed at both ends, arises fleshily from the bone, and is for the most part inserted by tendon. The two bellies lie between the adductors of the pollex and index in front, and the 2nd dorsal interosseous muscle behind. The 1st palmar interosseous lies to the outer side.

Musculus Accessorius Interosseus 3.

This muscle is situated on the inner aspect of the metacarpal and proximal phalanx of the 3rd digit. Like the last muscle, it is partly segmented into two parts, one of which, however, is very minute.

Origin. — The larger, anterior half arises from the inner and front aspects of the lower half of the 3rd metacarpal, and slightly from the layer of fascia which gives origin to the contrahentes. The posterior part arises from the lower part of the same bone behind the anterior portion.

Insertion.—The anterior portion is inserted into the inner edge of the dorsal expansion of this digit near the first interphalangeal joint. The posterior part finds attachment to the inner side of the proximal phalanx near its base.

Nerve Supply.—Fibres from the median by way of the deep division of the ulnar.

Structure.—The anterior half is nearly 6 cm. long, and quite hides the posterior part, which only measures about $2\frac{1}{2}$ cm.

The muscle at its origin lies on a slightly posterior plane to the adductor of the 4th digit (contrahens 3). Behind it lies the 3rd dorsal interosseous muscle.

Musculus Accessorius Interosseus 4.

This muscle is situated on the inner side of the 4th metacarpal bone and first phalanx of the 4th digit.

Origin.—From the front and inner surfaces of the 4th metacarpal in its lower three-quarters.

Insertion.—Into the inner margin of the first phalanx and into the inner side of the dorsal expansion of the 4th digit.

Nerve Supply.—The same as the muscle just described.

Structure.—This muscle resembles the immediately preceding in shape and size, but is unsegmented. Behind the muscle lies the 4th dorsal interosseous muscle, while to the inner side and rather in front is the adductor of the little finger (contrahens 4).

Fourth Layer: Dorsal Interossei or Abductores.

(See Plate II. fig. 6.)

These muscles resemble the corresponding muscles in the hand of man. They all abduct the digits on which they act from a line which passes through the centre of the middle digit.

1st Dorsal Interosseus (Abductor Indicis).

Origin.—(1) From the base of the 1st metacarpal on its inner side; (2) from the whole length of the outer side of the metacarpal of the index, but rather more on its anterior than on its posterior surface.

Insertion.—Into the outer side of the proximal phalanx, and slightly into the dorsal extensor expansion.

Structure.—The muscle is penniform, fleshy at its origin but tendinous towards its insertion. It is separated below from the first palmar interosseous by the first musculus accessorius interosseus.

2nd, 3rd, and 4th Dorsal Interossei.

These correspond closely to the same muscles in man, and need no detailed description.

Origin.—From the sides of the two metacarpals between which they lie, additional fibres being received from the metacarpal bone of the digit on which they act.

Insertion.—Nos. 2 and 3 are inserted into the outer and inner sides respectively, near the base of the proximal phalanx of the middle digit, and into corresponding margins of the dorsal extensor expansion. No. 4 is inserted into the inner side of the base of the proximal phalanx of the 4th digit, and into the edge of the dorsal extensor expansion.

Nerve Supply.—From the deep division of the ulnar; the fibres in the case of the outer three coming from the median in the forearm, while the fourth is supplied by fibres which travel all the way in the ulnar nerve.

Structure.—The muscles are all bipenniform, and end in small tendons. They are situated dorsal to all other muscles, but can easily be seen from the front.

Before discussing peculiarities of muscular development, it is always well to consider the particular functions which the muscles are called upon to perform. Function has a profound influence on the outward form of all organs; modification of shape and position is frequently an adaptation to special requirements. In no organ perhaps is this better exemplified than in muscle, which readily and rapidly responds to any persistent demand of nature. The function of the hand of the gibbon has therefore to be called to mind if the muscular arrangement is to be understood.

The upper limb of the animal is of extraordinary length. The humerus alone is longer by 5 cm. than the head and trunk, measured from the vertex to the ischial callosities. When it is borne in mind that the

European brachio-radial index is only 74 while that of the gibbon is 116, the great elongation of the animal's forearm will readily be appreciated. The hand and fingers participate in this elongation. The hand measures, from the crease in front of the wrist to the tip of the middle finger, $15\frac{1}{2}$ cm., while at its widest part it does not exceed $3\frac{1}{4}$ cm. in breadth. The agile gibbon is largely arboreal in its habits, and the hands are used as hooks by which to suspend its body from the branches. The animal progresses by swinging its body to and fro by its long arms until sufficient impetus is gained to project it through space to the next branch. Distances of 40 feet are cleared in this manner with apparently little effort. It is this peculiar facility of flying through space that gains for this animal the distinctive title of Agile. The fingers lie parallel to one another, and are kept half bent into the palm. They show little tendency to oppose the thumb. The use to which this hook-like extremity is put calls for great strengthening of the muscles which produce flexion of the fingers. Flexion is needed not only at the metacarpo-phalangeal joints, but also at the proximal interphalangeal articulations, so as to complete the hook-like attitude. Increased flexion at the distal interphalangeal joints is uncalled for.

It will have been noticed that nearly all the above muscles exhibit a tendency to wander down the phalanges. This is a mechanical gain, for the muscles act to better advantage when inserted well down the shaft of the long phalanx than when only attached to the base of the bone. Some muscles have even passed down as far as the middle of the 2nd phalanx. Many find partial attachment to the dorsal extensor expansion. The dorsal extensor expansion is rather a misnomer in this case, for it has a double function to perform. The long extensor tendons of the forearm acting through the middle of the dorsal expansion produce extension in the ordinary way; but the margins of the expansion extend so far round the sides of the fingers that the lateral portions, into which the short muscles are inserted, produce, when pulled upon, flexion not only of the metacarpo-phalangeal joints, but also of the proximal interphalangeal articulations. Indeed, as far as the short muscles are concerned the chief function of the dorsal expansion seems to be the production of flexion at these two joints. Moreover, the flexor brevis minimi digiti and musculus accessorius interosseus 1 actually pass far enough down to gain attachment to the shaft of the middle phalanx. As the fibrous pulp of the front of the finger serves as a partial attachment for these same two muscles, the action is well transferred from the dorsal to the palmar aspect of the digit. The lumbrical muscles, though not included in this description, are inserted as far down

as the first interphalangeal joints, and, like the other muscles attached to the lateral parts of the dorsal expansion, produce flexion of the joints.

We must now pass to the discussion of the true morphological position of the muscles, and to do this with any hope of success we must first call to mind the muscular arrangement of the typical mammalian manus.

To Professor Cunningham* belongs the credit of pointing out the three primitive layers of muscles in the typical mammalian manus. These primitive layers are:—

(1) A palmar layer of adductores.

(2) An intermediate layer of flexores breves.

(3) A dorsal layer of abductores.

Between the first and second layers runs the deep division of the ulnar nerve. These three layers correspond to the contrahentes, the palmar, and the dorsal interossei muscles of Halford

The first or palmar layer consists of a group of four adductor muscles, one being supplied to each digit, with the exception of the third. The third digit has no need of an adductor, for the others are adducted towards a line which runs down its centre. The two abductors supplied to the third digit serve to bring it back to the middle line. In man this group disappears, with the exception of the adductor of the thumb, which becomes greatly developed with the growth of importance and opposability of this digit. (According to Quain the flexor brevis minimi digiti is a representative of this group of muscles, but strong reasons are adduced later to show that this muscle really belongs to the intermediate or flexor group.) In order to cope with the increased amount of work that opposability involves, the adductor to the thumb becomes segmented into two, the adductor obliquus and the adductor transversus pollicis.

In the gibbon all the muscles of this layer are found present. The adductor to the thumb is the largest, and its partial segmentation has already been noted.

It is convenient next to consider the third or dorsal layer. This layer in the typical manus consists of six muscles, of which two are attached to the third and one to each of the other digits. They all abduct from a line which runs down the centre of the third digit. These muscles are fully represented both in the hand of man and in that of the gibbon by the abductor pollicis, the abductor minimi digiti, and the four dorsal interossei muscles. The unequal appearance given to them in Plate II. fig. 2 is intended to represent the greater attachment they possess to the digit on which they act. Ruge has shown that these muscles are primarily developed on the

* *Challenger Reports.*

palmar aspect of the hand. In sections through the fœtal hand the metacarpal bones are found pressed tightly together with the muscles lying on their anterior surfaces. It is only as development advances that the bones separate and the muscles become pressed back into the position which they occupy in the adult.

Having found that the first and the third layers are completely represented in the hand of the gibbon, we can now proceed with more confidence to relegate the remaining muscles to their true morphological position and function.

The second or intermediate layer is typically represented by a double-headed or paired muscle to each digit. All these ten muscles are flexors. In the hand of man there exist six of these ten muscles. They are the two heads of the flexor brevis pollicis, the three palmar interossei, and the flexor brevis minimi digiti muscles (see over page). The remaining four have completely disappeared (Plate II. figs. 1, 3, and 4).

In the hand of man, flexion is so well performed by the long flexors of the forearm, which have taken on increased development, that these short and relatively weak muscles are superfluous. In the 3rd digit they are quite gone. In the 2nd, 4th, and 5th digits one head becomes the palmar interosseous muscle. By becoming palmar interossei, both their position and function are altered. Whether the change of position and function was the cause or the effect of the disappearance of the adductor (contrahentes) muscles to these digits it is quite impossible to say. But certain it is that the migration and development of the one are proportionate to the degeneration in the other. This is well demonstrated in the hand of the gibbon, where the size of the muscles of the one layer bears an inverse ratio to the size of the muscles in the other. In this animal only two palmar interossei are present (Plate II. fig. 2, P.I.1 and P.I.3), but there are in addition the muscles we have termed musculi accessorii interossei. We will therefore consider the digits *seriatim*, to decide the position of each muscle.

The 1st digit has both heads represented by the two heads of the flexor brevis pollicis.

In the 2nd digit there is a strong palmar interosseous muscle to produce adduction, and the contrahens or true adductor is only rudimentary. This palmar interosseous is the inner of the two heads of the flexor brevis muscle of this digit. Musculus accessorius interosseus 1, which lies to the outer side of the first palmar interosseous, must therefore represent the outer head of the flexor brevis muscle.

The 3rd, being the middle digit, has no palmar interosseous muscle, and the abducting dorsal interossei serve to bring the finger back to the middle

line. In the typical hand, it will be remembered, there is no adductor supplied to this digit. The musculi accessorii interossei 2 and 3 are therefore the two heads of the flexor brevis unaltered in function.

In the 4th digit the outer head of the flexor brevis has disappeared. It has acquired neither the position nor the function of a palmar interosseous muscle, as there is a well-developed contrahens producing adduction. The outer head of the flexor brevis muscle to the 4th digit is the only one of this series of muscles which is absent from the hand of this ape. The inner head is present as the musculus accessorius interosseus 4.

The 5th digit has both heads of the typical muscle. The outer head is represented by a poorly developed palmar interosseous muscle, a fairly strong contrahens carrying out the function of adduction. The inner head is present as the flexor brevis minimi digiti.

This analysis shows clearly the relations borne by the palmar interossei to the adductor or contrahentes muscles, and in a satisfactory manner establishes the true position of the musculi accessorii interossei. These muscles are no new development in this animal. They are merely deviations from the original type, the modification being the response to the special requirements of function. The habits of the animal demanding, as they do, a strengthening of the short flexors, have caused Nos. 2 and 3 of these muscles to become partially segmented into two bellies; and in the same way increase in the function of the thumb has caused the segmentation of contrahens 1 into adductors transversus and obliquus pollicis. The same demand on the part of nature has caused the insertions of the muscles to migrate downwards to a point where they can act to better mechanical advantage.

In mentioning the representation of the second or intermediate layer of muscles in the hand of man and this ape, it will be noticed that the flexor brevis minimi digiti is counted as belonging to this layer, although hitherto it has been the custom to regard this muscle as belonging to the first or adductor layer. (See Plate II. fig. 4.)

The reason this muscle is usually grouped with the adductor layer is the position it holds with regard to the deep division of the ulnar nerve. The deep division of the ulnar nerve lies, in the typical manus, between the adductor and the flexor brevis layers of muscles. In the human hand the flexor brevis minimi digiti lies superficial to this nerve, and has, in consequence, been adjudicated to the former group of muscles. The change of function from an adductor to a flexor cannot be urged strongly against such a view, as we have already seen that many of the original muscles of the flexor layer adopt the functions of adductors when they become

palmar interosseous muscles. But I bring forward the arguments mentioned below in favour of the view that, although changing its position, the muscle retains its primary function of flexion, and really belongs to the layer of short flexors.

It has, I submit, migrated across the nerve the better to act as a flexor of the digit to which it belongs.

In the first place, the muscle so obviously corresponds in function and arrangement to the flexor brevis pollicis, that it is natural to conclude that they are both derived from the same layer.

Secondly, the flexor brevis minimi digiti cannot possibly be considered to represent the inner contrahens muscle in the hand of the gibbon, as that muscle is itself present in a well-developed condition (Plate II. fig. 2).

Thirdly, the relationship of the muscle to the deep division of the ulnar nerve is not an infallible guide to the morphological position of the muscle. In this ape the muscle is pierced by the nerve which passes between its two heads of origin, and the bulk of the muscle is on a deeper level than the nerve. The muscle is here seen in process of migration across the nerve, and the fascial expansion of the anterior annular ligament over the vessels and nerve forms the bridge over which the muscular origin travels. The abductor minimi digiti, a muscle of the third layer, is in an exactly analogous position in the hand of the gibbon. The call for flexion in the hand of this animal causes the abductor minimi digiti to migrate at its insertion so as to become a flexor of the little finger, and, the better to act as such, it has migrated also at its origin so as to be placed, like the flexor brevis minimi digiti, partially superficial to the deep division of the ulnar nerve. No one could, however, class the abductor minimi digiti as belonging to any but the third or abducting layer.

Another good instance of a muscle travelling for functional reasons across a nerve is to be found in the relationship the supinator radii brevis muscle bears to the posterior interosseous nerve. Professor Hepburn * drew attention to the varying positions of these two structures in the series of the anthropoid apes and man. He writes: "The posterior interosseous nerve of the chimpanzee deserves special attention because it affords some explanation of the position of this nerve in the substance of the supinator brevis muscle. As the nerve passes from the anterior to the posterior aspect of the forearm, it is never hidden altogether from view, being merely covered by a very thin aponeurotic fascia on the surface of the supinator brevis, and it can be readily understood how an increase in the size of the muscle and in the amount of its fibres taking origin from this investing

* *Journ. Anat. and Physiol.*, 1893.

fascia, would cause a submergence of the nerve, and produce the characteristic appearance of the nerve piercing the muscle."

I believe that the increased need for flexion has caused both the origins of the abductor and the flexor brevis minimi digiti partially to cross the nerve in the hand of the gibbon, and the widening of the sphere of action and opposability have caused the latter muscle to cross the nerve completely in the hand of man.

This view leaves the adductors of the thumb the only representatives in man of the first or adductor layer of muscles.

No mention has up till now been made of the opponens pollicis and the opponens minimi digiti muscles. The former is plainly a segmentation from the outer head of the flexor brevis pollicis, from which indeed it is scarcely differentiated in the gibbon. The opponens minimi digiti is poorly developed, but suffices to clothe the front of the 5th metacarpal bone. It is closely associated with the flexor brevis minimi digiti at its origin, and with the third palmar interosseous at its insertion. According to the view adopted by *Quain's Anatomy*,* these two muscles belong originally to different layers, and the opponens is represented as being composed of portions from each layer. I have above endeavoured to prove that these two muscles in question are derived originally from the same layer. Ruge has shown by sections through the developing foot that the opponens is a segmentation from off the short flexor. In the ape the migration of the short flexor across the nerve has carried the opponens along with it. If the position of the nerve is to be looked upon as an infallible guide to the morphological position of the muscle, the opponens, flexor brevis, and abductor minimi digiti muscles must all be regarded as the representatives of the already present contrahens 4. The position of the opponens is therefore an additional reason for rejecting such a view. The view has, however, been adopted by the editors of *Quain's Anatomy*: in dealing with the opponens in the hand of man they represent the muscle as being segmented partly from the adductor and partly from the short flexor layer.

The diagram is stated to be "based on Cunningham," though I have not found in the writings of that author sufficient to warrant the idea that this is the position he attributes to this muscle. I believe the opponens and flexor brevis minimi digiti belong solely to the second or flexor brevis layer.

* *Quain's Anatomy*, vol. ii. pt. ii. p. 276.

EXPLANATION OF PLATES.

Plate I.

The short muscles of the hand of the gibbon, slightly over the natural size. The different layers are shown as follows:—1st, unshaded; 2nd, moderately shaded; 3rd, darkly shaded.

Ad.P. Adductors transversus and obliquus pollicis, or contrahens 1.
F.P. Tendon of the flexor profundus digitorum to the thumb.
F.B.P. Flexor brevis pollicis, both heads.
A.P. Abductor pollicis.
Op.P. Opponens pollicis.
A.An.Lig. Anterior annular ligament cut.
P. Pisiform bone.
A.M.D. Abductor minimi digiti.
F.B.M.D. Flexor brevis minimi digiti.
Op.M.D. Opponens minimi digiti.
P. 1 and P. 3. The first and third palmar interossei; the latter muscle is only just seen, but its outline is dotted.
C. 2, 3, 4. The second, third, and fourth contrahentes; the first is Ad.P.
D. 1, 2, 3, 4. The dorsal interossei according to their numbers.
M.A. 1, 2, 3, 4. The musculi interossei accessorii according to their numbers.

Plate II.

Fig. 1. The typical manus, after Cunningham.
1, 2, 3, 4, 5. The metacarpal bones.
D. 1, 2, 3, 4, 5, 6. The dorsal layer (abductores).
M. 1, 2, 3, 4, 5. The intermediate layer (flexores breves).
P. 1, 2, 3, 4. The palmar layer (adductores).
D.U. The deep division of the ulnar nerve.
S.U. The superficial division of the ulnar nerve.
Fig. 2. The hand of the gibbon.
Fig. 3. The hand of man (as I believe).
Fig. 4. The hand of man as depicted in *Quain's Anatomy*; the diagram has in this case been reversed. The figures are the same for each.
1, 2, 3, 4, 5. The metacarpals.
D.I. 1, 2, 3, 4. The dorsal interossei.
P.I. 1, 2, 3, 4. The palmar interossei.
M.A. 1, 2, 3, 4. The musculi interossei accessorii.
C. 2, 3, 4. The contrahentes.
A.P. The abductor pollicis.
Ad.P. The adductor pollicis.
F.B.P. The flexor brevis pollicis.
Op.P. The opponens pollicis.

A.M.D. The abductor minimi digiti.

F.B.M.D. The flexor brevis minimi digiti.

Op.M.D. The opponens minimi digiti.

D.U. The deep division of the ulnar nerve.

S.U. Superficial division of the ulnar nerve.

The muscles marked in a dotted manner are not represented. 1st layer, unshaded; 2nd layer, shaded vertically; 3rd layer, shaded transversely.

Fig. 5. Showing the origins of the abductor and flexor brevis minimi digiti and the relations borne by their two heads to the deep division of the ulnar nerve.

P. Pisiform bone.

Un. Hook of unciform.

F.B.M.D. Flexor brevis minimi digiti.

Ab.M.D. Abductor minimi digiti.

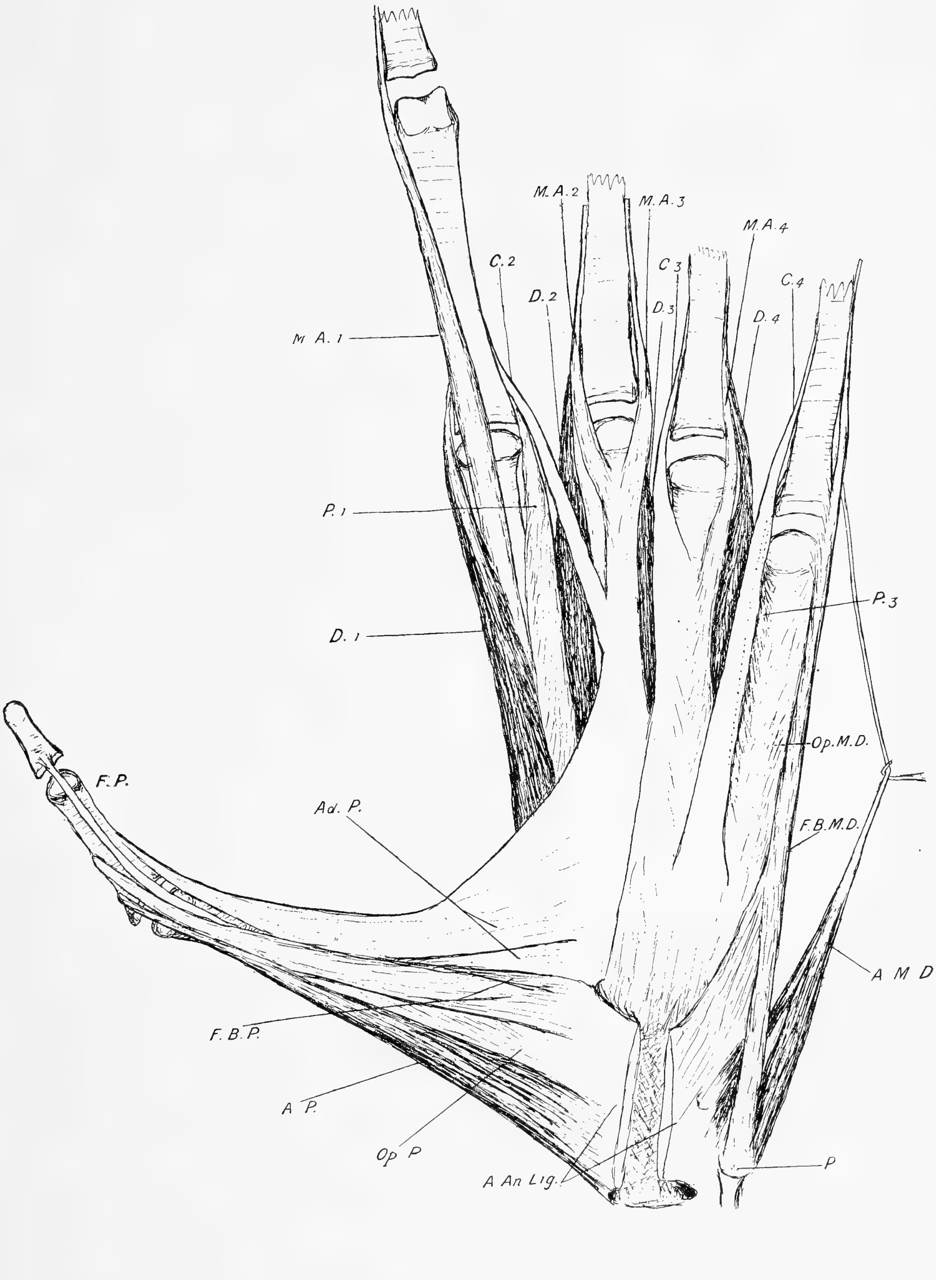
Op.M.D. Opponens minimi digiti.

Ul.A. & N. Ulnar artery and nerve.

A.An.Lig. Anterior annular ligament.

Fig. 6. The dorsal interossei muscles.

(*Issued separately February* 2, 1910.)


M.A.2
M.A.3
M.A.4
C.2
C 3
C.4
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D.4
M A. 1
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P. 3
D. 1
Op.M.D.
F.P.
Ad. P.
F.B.M.D.
A M D
F.B.P.
A P.
Op P
A An Lig.
P

Fig. 1.

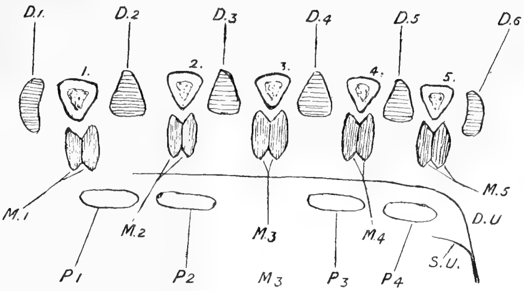


Fig. 2.

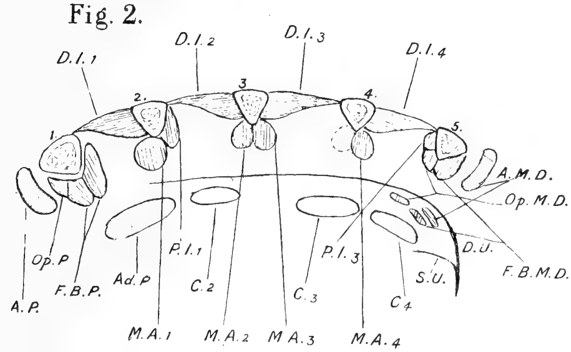


Fig. 3.

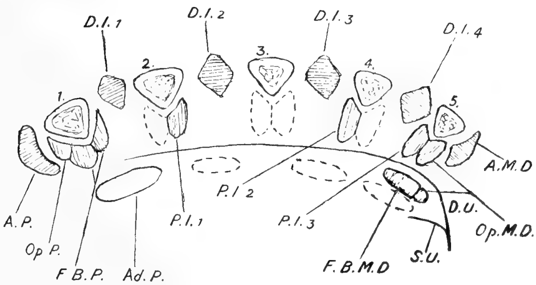


Fig. 4.

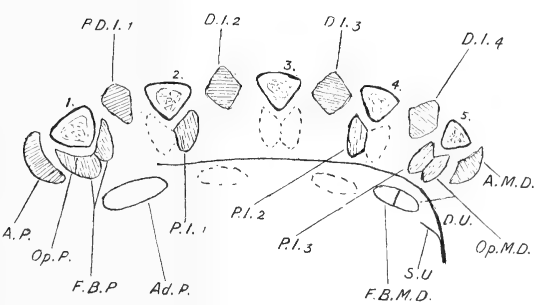


Fig. 5.

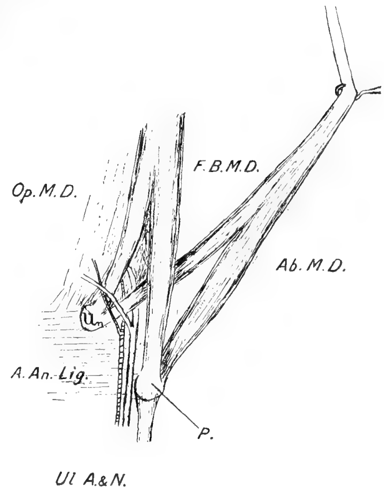
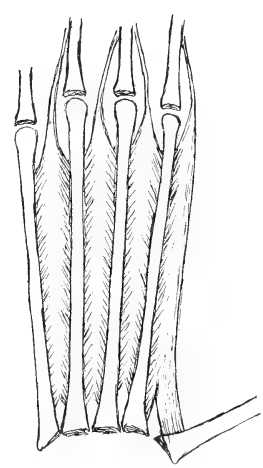


Fig. 6



DORSAL INTEROSSEI MUSCLES.

IX.—Observations on some Spark-Gap Phenomena. By John M'Whan, M.A., George A. Clark Scholar of the University of Glasgow. *Communicated* by Professor A. Gray, F.R.S.

(MS. received November 26, 1909. Read January 10, 1910.)

In a recent note to the Royal Society of Edinburgh,* Dr Dawson Turner remarks on a peculiarity observed by him in the behaviour of an induction-coil spark-gap when a piece of mica, glass, sulphur, ebonite, etc., is placed between the terminals, near or against the + one. The surprising variety of the phenomena exhibited on the interposition of a solid dielectric plate between the two poles of such a coil (or between the poles of an influence machine) may make, in view of Dr Turner's note, the following observations of some interest. The observations in question were made during the course of an inquiry into the causes of the Lullin effect (according to which a thin dielectric plate interposed between two sparking terminals *not* opposite to one another is, as a general rule, perforated at the negative pole), and the apparatus employed consisted essentially of the details shown in fig. i., viz. of a motor-driven influence machine whose terminals were connected through a battery of Leyden jars in cascade to a specially designed spark-gap which allowed of any three-dimensional motion of the electrodes being accurately measured, and of any desired type or form of electrode (spherical, conical, disc type, etc.) being inserted at will. A second spark-gap, fitted with standard spherical electrodes and a micrometer screw motion to widen or narrow the gap and measure it accurately, was connected in parallel with the first. By adjusting this second just to spark synchronously with the first, and no more, the potential difference of the first could easily be deduced, no matter in what sort of medium it was immersed.† This method was found preferable to the use of an electrometer, very high potentials such as 100,000 volts being determined rapidly and with sufficient accuracy. In using such high potentials, even heavily insulated wires *leak*, *i.e.* give off, at weak points in the insulation, strong brush discharges into the surrounding air. These discharges are facilitated by abrupt bends in the wire, or by the proximity (within a few feet) of foreign bodies. The wires were therefore kept as straight or as gently curved as possible, were embedded in paraffine wax, and the main leads in addition enclosed in long

* *Proc. Roy. Soc. Edin.*, 1908–9, vol. xxix. p. 414.

† Landolt, Börnstein, and Meyerhoffer, *Physikalische-Chemische Tabellen*, p. 778.

glass tubes. Brass terminals and so on were heavily coated with sealing-wax; the battery of Leyden jars stood on a thick glass base-board, which in turn stood on ebonite pegs.

The main object of the investigation, as already mentioned, was to give an explanation of the Lullin experiment. It will at once be obvious that such would have a special interest for makers of large induction coils, who often experience strange and unaccountable trouble with their insulations, perforations occurring between the coils.* It would, however, be out of place here to enter on the results of these experiments: we desire only to

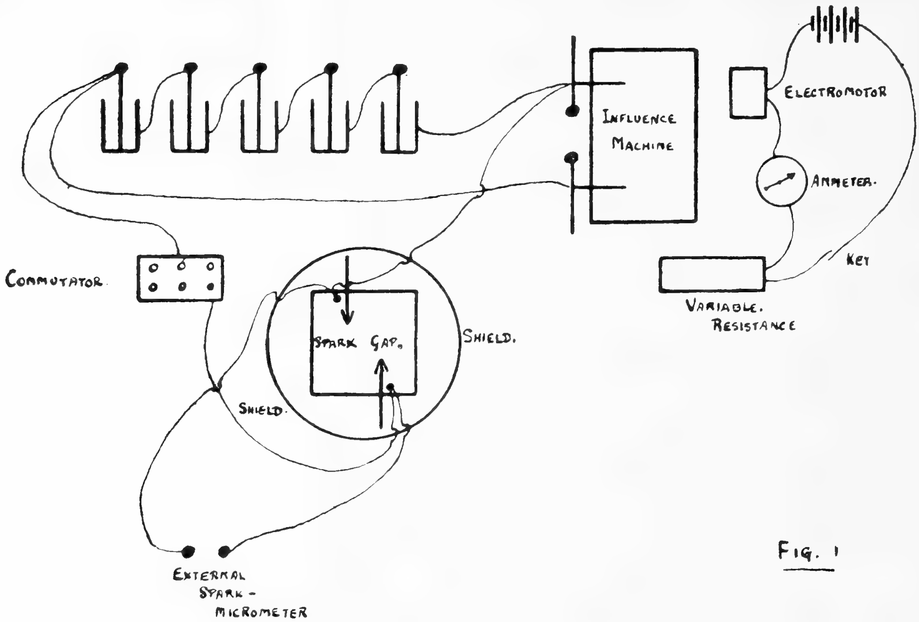


Fig. 1

mention a few phenomena, met with as the experiments proceeded, which bear on that described by Dr Turner.

I. Dr Turner says: "The introduction of a dielectric between the poles will greatly facilitate the sparking, provided the dielectric be placed near or against the positive pole." The statement is somewhat loose, ignoring, as it does, the questions of dielectric thickness and superficial area (if both be great enough, sparking will certainly not be facilitated) and the question of whether the spark passes by perforation of the dielectric or round about its edge. It is certainly the case, however, that within certain limits for the dimensions of the dielectric, sparking may often be induced by placing it "near or against the positive pole"; and the opposite effect may be obtained

* Kiessling and Walter, *Annalen d. Physik* (4), Bd. xi., 1903, p. 586.

by setting it at or near the negative pole. [This experiment, however, and most of the others to follow, will time and again *seem* to fail, owing to non-fulfilment of some one of the many limiting conditions. Thus the potential difference may not be near enough to the spark potential for free air, the dielectric may be too thick or too thin, or too broad, etc.] Even at the negative pole, however, it may be observed that if a dielectric plate be suddenly dropped vertically between the poles, a violent discharge at once takes place, and, curiously enough, more easily as a rule when the plate is so thin as to be easily ruptured by the discharge. In other words, the possibility of perforation seems to act as an extra inducement for the spark to pass, and it will not pass so freely if its path must lie round the edge of the plate. The whole phenomenon, and particularly the violence of the discharge, is most strongly marked when, at the instant of the plate's fall, the terminals of the coil are giving off the characteristic brush discharge.* Again, if the dielectric (a sheet of cardboard, a thin plate of wax, glass, mica, etc.) be so set up that it is at right angles to the lines of the electrodes and can be moved freely in the direction from one electrode to the other, some interesting results may be noted. Thus, if each electrode is spherical, *i.e.* has, for example, a brass ball screwed on to its sparking end, motion of the card in the direction from the + electrode to the − electrode results in a great facilitation of the spark, while motion in the opposite direction distinctly acts as a deterrent. Thus, even in the case of a fairly wide gap (6–8 inches, say) with the dielectric set vertical in the middle a quick, sharp motion of quarter or half an inch towards the − pole will often induce a perforating discharge even though the potential difference is not high enough to produce a brush discharge in free air. And so a little paddle-wheel, whose paddles are made of the dielectric in use, may be set up so that the paddles when rotating pass, at the lowest point, between the electrodes from + to − and provoke a steady stream of sparks. The converse of this experiment—the "quenching" of an already sparking gap by rotation in the opposite sense—is more difficult to perform, but usually can be got to succeed with a little patience. The same results are to be observed when each electrode is fitted with a conical cap, or point. But if now one be spherical and one conical, an important difference comes in, for now the direction of motion for provocation of a spark is found to be *always* towards the conical electrode, no matter whether it be + or −. The dielectric itself, as will appear in § II., is in this case under a continuous and very sensible repulsive force tending to drive it towards the spherical electrode again quite independently of the polarity of the electrodes.

* *Cf.* Faraday, *Experimental Researches in Electricity*, series xii.

There are, of course, limiting conditions to the above experiments. Thus, if the angle of the conical electrode be increased indefinitely until the cone degenerates into a plane surface, the preference of the spark for that electrode would naturally be expected to diminish; and so it is found in actual fact.

It may also be noted that when a discharge followed by perforation of the dielectric has once been induced by motion of the dielectric, the difficulty of provoking a second spark discharge becomes still greater than formerly: on examining the apparatus in the dark it is usually easy to discern the faint lines of an almost silent discharge threading the hole from electrode to electrode.

II. *Phenomena of Repulsion.*—Doubrava has pointed out * that in general any dielectric plate set between discharging electrodes experiences a definite force tending to displace it in the direction of one of the electrodes. He declares, however, that "the experiment only succeeds when the electrodes are sufficiently separated to give a crackling brush discharge." This is not the case. Repulsion effects on all kinds of plates are observable even when the electrodes are sparking freely (*through* the plate, or round its edge) or discharging by the so-called "silent" discharge. [See § IV. below for some notes on the form of these discharges.] In observing the effect in the present instance, dielectric plates (16 cm. × 16 cm. is a convenient size) were suspended midway between the electrodes by a fine silk bifilar suspension from insulators on the ceiling. The suspension was about 15 feet long, a length sufficient to permit of even very small forces producing sensible deflections, and the plate itself was screened off from draughts (fig. i.). In the case of metal plates, which were also found to exhibit repulsion phenomena, one thread can conveniently be replaced by a fine copper filament leading to an electroscope screened from electrical action by fine wire gauze. The behaviour of the leaves of the electroscope then indicates the condition of the plate as regards charge. Dielectric plates were removed by tongs when it was desired to investigate their charge. No difference in the results was ever experienced on earthing one of the electrodes.

In general it was found that when the electrodes were of similar shape and size, discharge between the electrodes resulted in a strong repulsion of the plate in the direction from the + to the −. [If the potential was not high enough to permit of discharge of any nature, then the plate could easily be induced to perform regular vibrations from pole to pole, after the

* *Wiedemann Annalen*, Bd. viii., p. 476, "Ueber die Bewegung von Platten zwischen den Elektroden einer Holtz'schen Maschine."

manner of the "electrical chimes" experiment, by alternate charge and discharge, attraction and repulsion. This pendulum motion could be shown in its most regular form by staying down the plate lightly with another silk bifilar.] Preconceived theories of this repulsion, such as that of a possible hydro-electric excitation of the surface of the plate by the discharge,* are upset when we, once more, employ electrodes of different shapes. Thus if the sphere and cone be again used, it does not then matter which is + and which − : the repulsion is always violent, and in the direction from the conical electrode to the spherical one, regardless of polarity. If the electrodes are not opposite one another, then, on reaching the sphere, the plate is rotated through a considerable angle (fig. ii.). If the plate is not allowed to touch either electrode, but is tested immediately

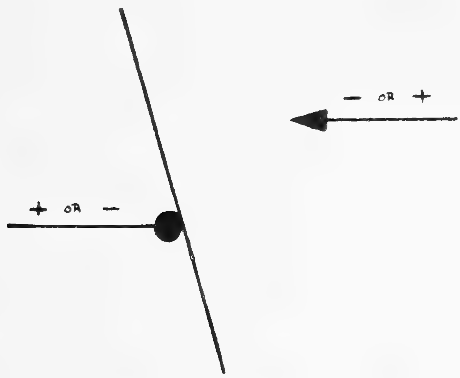


Fig. ii.

after the discharge, it shows always the sign of the conical electrode. When the electrodes are similar, it shows always the sign of the repelling electrode, *i.e.* +. Another curious phenomenon occurs in the case of similar electrodes, *e.g.* spheres. Let the plate be repelled against the − electrode, and remain there during discharge. If the electrodes are not opposite one another, it will again show rotation, but this time of the opposite sense to that in fig. ii. That is to say, it behaves as if attracted simultaneously by both electrodes, but with the − electrode in predominance (see fig. iii.). This effect could not be obtained with metal plates: it may hence be inferred that different parts of the dielectric plate are in this case in different electrical conditions as regards charge.

III. If the plate be light enough, and the electrodes placed so near the edge of the plate that the path of least resistance for the discharge is

* Doubrava also disposed of the hydro-electric theory by using as the plate a thin sheet of some dielectric or conductor, enclosed in an envelope of material of the opposite hydro-electric sign, and showing that the effect remained unaltered (*Wiedemann Annalen*, *loc. cit.*).

round the edge of the plate instead of through it, a considerable horizontal displacement of the plate in the direction of its own length may take place, as shown in fig. iv. Observed in the dark, the phenomenon is a very beautiful one, the rays of the discharge appearing like luminous elastic strings attempting, in a contractile effort to straighten out, to pull the

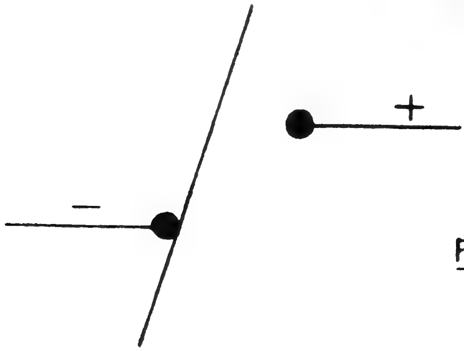


Fig. iii

plate aside—in fact, like luminous Faraday lines of force. The electrification of the plate is slight, and practically indeterminate as regards sign.

IV. *Characteristics of the Brush Discharge.*—When a dielectric is placed between two electrodes excited to a potential sufficient to produce

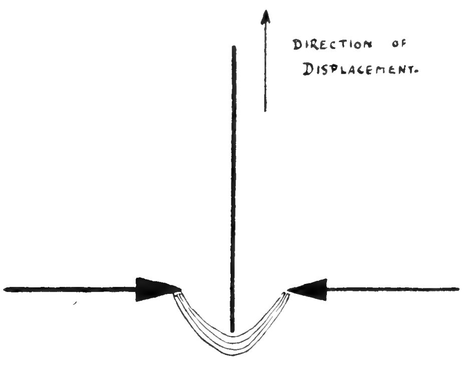


Fig. iv.

the brush discharge, examination in the dark reveals some interesting details. The discharge is seen to be made up, as in ordinary cases, of a bundle of luminous threads, each pursuing a quite independent path; but at one part of their course these threads converge, and pass almost wholly through one point. This point lies *on* the dielectric (usually on the + side), is brilliantly illuminated, and is found to be in every case *the site of*

the perforation consequent on the exciting of the electrodes to spark potential. Thus if the electrodes are circular brass discs pressed flat against a sheet of cardboard, one on either side, the appearance of the brush is as shown in fig. v.: it lies entirely on the + side of the card (thus in this case lending colour to von Waltenhofen's hydro-electric theory *), and spreads out from opposite one single point on the – to the whole opposing side of the + electrode. After it has been passing for some time, the card is found to have a well-marked black band alongside the + pole. If now a hole be made in the card (*e.g.* a pin-hole) at any point between the electrodes, but not midway between them, the brush will pass through this hole, and will therefore lie partly on one side, partly on the other side of the card. With all the specimens of card examined it was found that, irrespective of polarity, the longer part of the brush

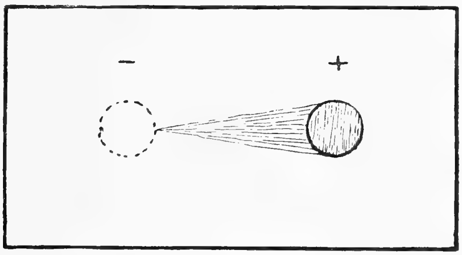


Fig. V.

assumed a violet-white tint, while the shorter part was of a bright light-greenish colour.

Although, however, the point of concentration of the rays usually determines the point of perforation, the spark does not, except in simple cases, follow the path of the brush discharge. Fig. vi. illustrates a typical case of this, ABC being the path of the brush, ADC the approximate spark-path. The point of concentration, A, is common to both paths. In this case a hole made in the card between the electrodes would have no appreciable effect on the path of the brush.

Some remarkable "dark-space" phenomena can also be easily observed or photographed. Fig. vii. gives an example: the observer is looking from above on to the spark-gap below. The + side of the card for the whole distance between the electrodes is strongly illuminated: from the – (conical) electrode proceeds little more than a point of luminous discharge—between

* *Pogg. Ann.*, Bd. cxxviii. p. 589.

that and the card all is dark. The rays from the + (spherical) electrode have the peculiarity of lying entirely in one horizontal plane: they, too, after bridging about one-half the gap between card and electrode, vanish abruptly, the end of the discharge being perfectly straight and regular. If

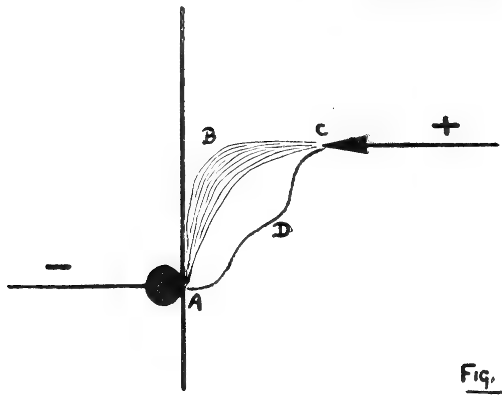


Fig. vi.

now the card be moved slowly towards the anode, a change takes place in the character of the brush. Fig. viii. is a side view showing that change. At a certain stage in the motion, the horizontal brush suddenly splits into two brushes, each uniplanar; and as the card continues to approach, these

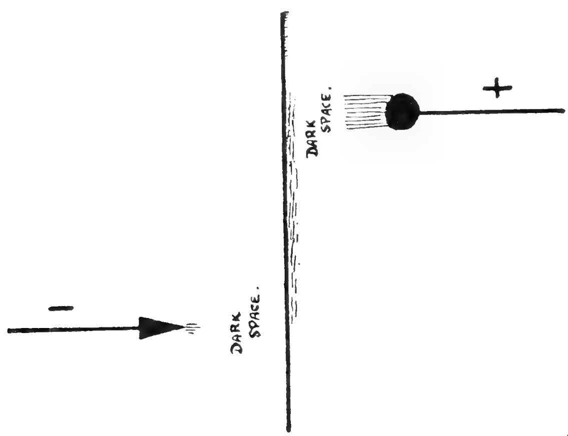


Fig. vii.

continue to diverge, each making at any time the same angle as the other with the plane of the electrodes.

Many varieties of these experiments can be designed by making holes at various points in the plates between or outside the electrodes, or by bending the plate itself into various shapes. An instance of this last is

shown in fig. ix., and fig. x. gives an example of a hole made just beyond the + (spherical) electrode, in which case it will be noted that two distinct paths are available.

V. An attempt was made to observe the influence of low pressures in

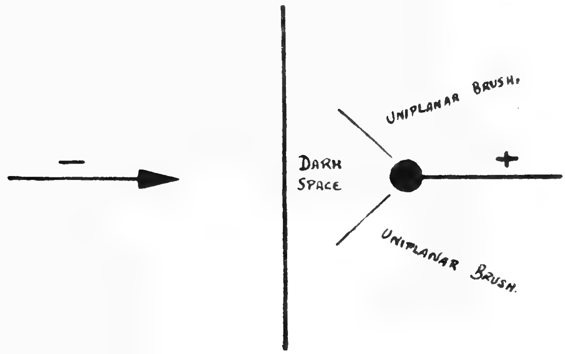


Fig. viii.

air and other gases on these phenomena, and on the Lullin effect. A strong glass bell-jar was accordingly made to contain the spark-gap and a contrivance for supporting or suspending the dielectric. The dimensions of the jar were approximately:—height, 40 cm.; average thickness, 1 cm.;

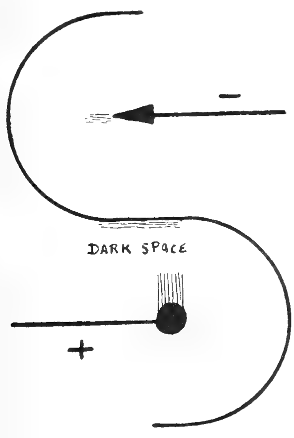


Fig ix.

mean inside diameter, 25 cm. It was made in two parts for convenience of access to the interior, and was fitted with stop-cocks for exhaustion and for admission of other gases. In the first experiment the air was exhausted to a pressure of about 4 mm. Hg. and a spark passed. Instantly the lower half of the jar split, with a loud report, from top to bottom—one great open

rent, through which at any point a thin card could be passed—but fortunately the damage was confined to that. Lehmann has observed the same phenomenon when working on discharges through gases in large vessels: * in his case, however, working with an "egg" 60 cm. long and 45 cm. broad, whose two parts had been separately tested beforehand to make sure of their strength, "after it had stood untouched for about five hours it was suddenly shattered, with a loud report, into countless little pieces. These were driven as far as 15 metres off, some of them going through the window-panes. All breakables in the neighbourhood were damaged, and some splinters of glass even forced themselves so violently into the wooden doors of cupboards and other wooden fittings that they stuck there."

The danger of breakage is, however, not entirely confined to *large* vessels.

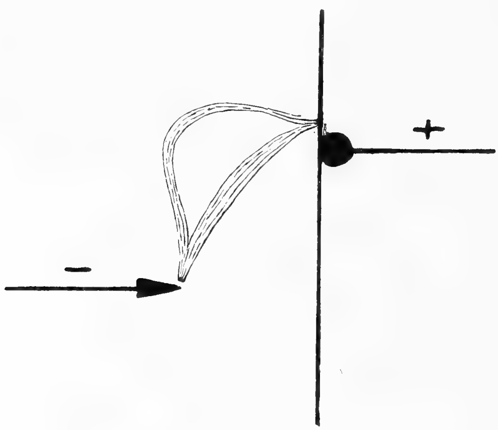


Fig. X.

After the bell-jar had broken, it was decided to abandon the idea of getting the special spark-gap bodily into the vessel. A cylindrical glass tube, 14 cm. in length and 5 cm. inside diameter, was taken instead. The ends were open, but could be closed by two plane slabs of glass, bored to allow stout pieces of copper wire (which were to do duty as the electrodes) to enter, and the whole was rendered air-tight with sealing-wax. This stood successfully the first and second discharges, but on the third the lower end of the tube was completely shattered.

In such a series of experiments as described above, the vagaries common to most electrostatic experiments performed in the free air of a laboratory are of course prominent, especially during adverse atmospheric conditions; and results on occasion display such consistent inconsistency as to baffle

* O. Lehmann, *Annalen d. Physik* (4), Bd. vii., 1902, p. 1.

serious consideration. For this reason only a few phenomena, substantiated beyond reasonable doubt, have been quoted. These should be sufficient, nevertheless, to draw attention to the complete absence of a theory regarding the presence of dielectrics in a spark-gap, which will deal satisfactorily with all cases. The further surprising divergences met with when the dielectric has imposed upon it a drop or drops, of various shapes and sizes, of wax, stearine, etc. (see the various papers quoted—von Waltenhofen, *Wiedemann Annalen*, Bd. viii. p. 460, etc.), make the problem still more difficult. Kiessling and Walter (*loc. cit.*), in considering the bright "threads" of the brush discharge, especially when these are crowded together into one bright line by a stearine canal, as lines of ionic concentration, seem to have made the most important recent advance. It is obvious, however, that even that theory cannot be made to fit all cases—some of the experiments given here are cases in point.

These observations are suggested by some researches which I have been carrying out in the Physical Institute of the University of Göttingen, and which are being prepared for publication elsewhere. My best thanks are due to Professor Riecke for suggesting the line of research, and for the interest he has taken in the progress of the work.

(*Issued separately February* 2, 1910.)

X.—The Structure of the Reproductive Organs in the Free-Martin, with a Theory of the Significance of the Abnormality.

By **D. Berry Hart**, M.D., etc., Edinburgh.* (With Two Plates.)

(MS. received November 15, 1909. Read November 22, 1909.)

The apparent prodigality with which Nature provides for the reproduction of plants and animals has a marked exception in the case of sterile organisms, often produced on a large scale, in which every function except that of propagation may be carried on in a quite perfect manner. The cases of the sterilisation of an organism are best seen in insects, but I wish at present to consider its occurrence high up in the animal kingdom in the well-known case of the Free-Martin, an apparent sterile cow born co-twin with a potent bull.

The nature of the free-martin is considered one of the unsolved problems of anomalous sex. John Hunter, who first drew attention to it, speaks of it as an unnatural hermaphrodite. "Hermaphrodites may be divided," he says, "into two kinds, the natural and the unnatural. The natural hermaphrodite belongs to the inferior and more simple genera of animals, of which there is a greater number than of the more perfect: just as animals become more complicated, have more parts, and each part is confined to its particular use, a separation of the two necessary families for generation has taken place" (*op. cit.*, p. 46). Sir James Simpson, in his well-known paper, after enumerating the paradoxes in the knowledge at that time of the free-martin, says in conclusion: "The whole series of circumstances, when considered in conjunction with each other, seems to form, in relation to the origin of malformations, one of the strangest and most inexplicable facts to be met with in the study of anormal development" (*op. cit.*, p. 835).

Spiegelberg looked on the free-martin as a transverse hermaphrodite, and Geddes and Thomson say: "No theory has yet explained the facts of this case" (*op. cit.*, p. 41).

The explanation I wish to bring forward is based on the views expressed in a previous paper on "Mendelian Action on Differentiated Sex."

I must first describe the free-martin, next consider the facts known as to it, and then give my view as to its anomalous sex-condition.

* From the Laboratory of the Royal College of Physicians, Edinburgh.

The free-martin is a sterile twin, usually co-twin with a potent bull.* It has, in this case, the lower part of its genital tract to the naked eye like that of a cow, the upper part defective, and is usually considered as a cow sterile from incomplete development of its upper vaginal and uterine tract.

John Hunter described three specimens:—

(1) *Mr Arbuthnot's Free-Martin* (*op. cit.*, p. 52).—This animal was seven years old; went with the cows and bull; never showed any desire for either. The external parts were rather smaller than in the cow; the vagina was contracted above the urethral opening, becoming continuous with a small canal; uterus with two horns, two ovaria, and two testicles: vasa deferentia to the testicles; the left one did not come near the testicle, the right one only came near to it but did not terminate in the epididymis. They were both pervious, and opened into the vagina near the urethra. Vesiculæ seminales were present, smaller than in the bull; "the ducts opened along with the vasa deferentia." "This was more deserving of the name hermaphrodite than the following, for it had the mixture of all the parts, although all were imperfect."

(2) *Mr Wright's Free-Martin*, fig. 1, five years old (*op. cit.*, p. 53).—This animal was more like the ox or spayed heifer than the bull or cow. The vagina had a blind end near the urethral orifice; a two-horned uterus; testicles, and not ovaria; this based on their size nearly equal those of a bull. Vesiculæ seminales opening into vagina, but nothing like vasa deferentia; a clitoris was present. "This animal cannot be said to have been a mixture of all the parts of both sexes."

(3) *Mr Well's Free-Martin* (*op. cit.*, p. 54).—Three to four years when killed, more like a heifer; no desire for male. Beginning of vagina as in cow, but obliterated beyond the urethra, although a solid part was continued. Two horns and two ovaria; vas deferens in interrupted portions. Between vagina and bladder the vesiculæ seminales, and between them the termination of the vasa deferentia. "This could not be called an exact mixture of all the parts of both sexes, for here was no appearance of testicles (*op. cit.*, p. 54). He thus considered Arbuthnot's case to have testes and ovaries; Well's case to have ovaries, and Wright's case testes.

Sir James Simpson, in the paper already quoted on "The Alleged Infecundity of Females born co-twin with Males," states as his conclusions that the human female co-twin with a male is as likely to be fertile and have as many children as the normal woman not a twin (*op. cit.*, p. 835).

* The reason for this limitation will be seen afterwards.

The third paper I have now to consider is one by Professor Otto Spiegelberg, the late distinguished obstetrician of Breslau, who examined the parts carefully by the naked eye and microscopically in a free-martin full-time calf, and threw great light on the nature of the free-martin by showing that it was *an imperfect male calf and not an imperfect female calf.* Spiegelberg had, of course, the advantage over Hunter and Simpson of the more advanced knowledge of his time in microscopical technique and embryology. Rueff also noted in two cases that the sexual glands were testes (1851), and Gurlt, in 1832, states the same.

Spiegelberg's cases were two in number, and are briefly as follows:—

Case 1.—This was one of twin calves killed a few days after birth. Externally, both calves seemed normal. The male had the testes in the scrotum, the internal genitals were quite normal. In the apparent female there were no traces of uterus, tubes, and ovaries (*nichts von Uterus, Tuben und Ovarien*); instead there was a special complex of organs, which, beginning a little distance from the kidneys in a peritoneal fold, ran down between the rectum and bladder (*vide* Pl. I.). There the external genitals are opened from the front and turned to the side. Vulva and clitoris normal; the former passes into a narrow canal 1½ inch long and 1 inch broad, with smooth walls, on whose anterior wall the urethra opens, and is accordingly the vestibule. No trace of the openings of Gartner's canals was evident. At the apex of the canal is an opening the size of a linseed, with no bridge of hymen, through which one can carry a sound up and back into a cavity scarcely an inch long. The wall of this cavity is ½ inch thick, with connective tissue and transverse muscular fibres. Laterally from this rudiment of the vagina there run up two hollow processes, blind above, about 15 lines long; below, they unite with the wall of the cavity, and have the knob of the sound *c* between them; above, they end free in the connective tissue; both have a narrow lumen, blind at both ends. Between them lie two cords close to one another, *e* springing from the upper wall of the vagina; the left, 16 lines long, loses itself in the peritoneal fold; the right runs tortuously up as a fine thread, ultimately to a structure *g*. Both cords form a relatively wide canal, from which a few drops of white, slimy fluid can be squeezed. The canals are closed at both ends; *f* is solid. At the upper end of this arrangement there lie on each side two structures, at first glance the sexual glands. The vulvar canal is thus vestibule or female urogenital canal: the cavity *b* is rudimentary uterus; *d d* are rudimentary vesiculæ seminales; *e* and *f* vasa deferentia; the double organs *g* and *h* not ovaries and testes, but testes and rudimentary

tubes, with a canalicular structure. The testes are thus seen at *g*, and at *h* rudimentary portions of the Wolffian body.

Spiegelberg considers this to be a case of transverse hermaphroditism in a bull calf, the upper part being male and the lower female in type. He summarises the literature carefully, and concludes that "if the twins are both female or of a different sex, their sexual organs are, as a rule, well developed; if both male, then frequently one is hermaphrodite. ("Sind die Zwillinge weiblich oder verschiedenen Geschlechts, so sind ihre Geschlechtsorgane in der Regel wohlgebildet; sind sie beide männlich, so ist sehr haufig das eine derselben ein Hermaphrodit" (*op. cit.*, S. 130).)*

Spiegelberg quotes Numan's article in the *Journal Vétérinaire et Agricole de Belgique par Brognier, etc.*, année 1844, as a valuable contribution. This it undoubtedly is, as I have found on studying it.†

The summary given by Spiegelberg of Numan's conclusions is as follows:—

(1) If a cow has twins of different sex the female calf has almost invariably incompletely developed organs and is sterile.

(2) This fact based on observations has exceptions, and cannot be considered as indicating an exclusive law.

(4) The developmental error of the genitals does not happen exclusively to the female of similar twin couples, but occasionally to the male, and then the female is well developed: such cases are, however, rare.

(5) Multiple births are frequent in cattle, and, so far as the female is concerned, are to be considered the most certain and constant cause of sterility; the more so as the condition of the sexual organs

* *Case* 2.—In this case Spiegelberg found the twins of different sex and normal. They evidently came from separate zygotes.

† Numan's monograph was entitled "Verhandeling over de onvruchtbaren rundern bekend onder den naam van Kweewen," etc., met 23 platen; Utrecht, N. van der Monde, 1843. This monograph is not accessible in this country, so far as I have been able to ascertain. It is translated in the *Journal Vétérinaire et Agricole de Belgique* for 1844, and a copy of this was obtained by the Royal College of Physicians, Edinburgh. All the references to Numan's plates are given, but unfortunately not the plates themselves. Numan figures nine specimens, of which six are the same as Hunter's, but three show the sexual glands in the free-martin to be evident testes with the Müllerian element much less represented. A rudimentary preputial sheath is present in the latter, so that the animal is very like a bull. Numan terms the six like John Hunter's specimens "Heifer Free-martins," and the three remaining as "Steer Free-martins." He considers only the six as heifers, but all the nine are sterile bulls. He figures the vesiculæ seminales in all, but considers them in the six heifers as diverticula of Gartner's canal. In the points as to the sexual glands and vesiculæ seminales he is in error, but his whole monograph is a very able and valuable one.

which causes the anomaly does not as yet seem to have been observed in simple born calves. One finds, however, sometimes in such cases incomplete organs in a male.

The conclusion stated under (4) is the most novel, as it points to a free-martin with a sound female twin. It is of great interest theoretically in regard to Mendelism in this connection. The necessary modification of (1) will be seen presently.

As Hunter's specimens are still in the Museum of the Royal College of Surgeons, London, it was therefore a matter of great interest that the sexual glands should be examined microscopically. This has just been done under the direction of Dr Arthur Keith, the Curator, and by permission of the Council; and I am greatly indebted to Dr Keith for permission to examine the slides and make microphotographs. I was able also to see the naked-eye specimens themselves during a visit I paid to the museum.

These specimens, although now about 140 years old, are in perfect preservation, and it is still more remarkable that the microscopical sections, cut in celloidin and stained with logwood and eosin, show even the finer details in a recognisable manner.

The special fact that emerges is that all the sexual glands are testes in Hunter's cases, that adjacent structures are epididymis, and that in none of the sexual glands are ova present. The characteristic testicular tissue is in the form of tubuli seminiferi, and in only one are spermatozoa present. They thus show the condition usually found in undescended testes in man and in the testis normally before descent is complete. I add a report of Hunter's specimens, and the structures are illustrated in the figures given.

The examination of the slides prepared from John Hunter's free-martin specimens gave the following results:—

Slide 678, testes with tubuli seminiferi; 682, epididymis; 686, epididymis and testes; 688, epididymis; 689, testes and hydatid testis (Müllerian); 690, testes; 691, testes and hydatid testis; 692, testes (Arbuthnot); 693, testes (Wright); 694, testes, scanty elements. In 691 the Müllerian hydatid is well shown (*vide* figs. 3 and 4, Pl. II.).

It seems to me, therefore, fully established that the free-martin, when the co-twin is a potent male, is a sterile male, and not a sterile female, *i.e.* they are identical male twins except in their genital tract and secondary sexual characters.

The following table shows the main cases (thirty in number) I have been able to collect.

Author.	Conditions of Organs.	Remarks.
John Hunter (1786).	(1) *Arbuthnot's Free-Martin.* Vagina, upper part narrow ; testes ; two cornua ; vasa defer., vesiculæ seminales.	
	(2) *Wright's Case.* Vagina with blind end ; two horns ; testes really, external parts like cow ; clitoris.	"More the appearance of the ox or spayed heifer" (*op. cit.*, p. 53).
	(3) *Well's Case.* Teats and udder small ; vagina as in (1) ; vagina and uterus not pervious ; testes ; vasa deferentia ; segments of vesiculæ seminales.	More like a heifer ; no microscopic examination made of the tissues.
Numan (1843).	Describes nine cases : six like Hunter's and three somewhat different.	
Spiegelberg (1861).	External genitals and urinogenital sinus as in cow ; rudimentary vagina and uterus ; parts of seminales and vasa ; parts of W. bodies ; testes rudimentary (microscopical examination).	Potent bull twin and free-martin calves slaughtered one day after birth ; microscopical examination of structures.
Rueff.	Three cases : two had testes ; no vesiculæ seminales ; external genitals female ; vagina (?) blind in all three. *Rep. d. Thierheilk. von Hering*, 12 Jahrg. 1851, p. 103.	
Hering.	Two cases : data only in one. No genitals apart from vagina. ("Ausser der Scheide nichts von Genitalien vorhanden gewesen und von Grunde jener, blos eine dünne Falte des Bauchfells nach beiden Seiten hin abgegangen sein.")	
Anonymous.	In Fuchs' *Thierarztl Zeitung*, Jahrg. iii., note given of five female calves from twins with defective uterus. In another, note is made by Fuchs of a preparation at the Carlsruher Vet. School, of case with both male and female genitals.	
Scarpa.	One year old animal with external female genitals and internal male.	Presence of twin not stated in Spiegelberg's summary.
Gurlt.	Conditions like Spiegelberg's ; union between testis epididymis and vas more evident.	
Allnatt.	Vagina long cul-de-sac with rudimentary uterus ; solid vesiculæ seminales ; vas ended in prostate.	Allnatt states that he was informed of cases where sexual desire was not absent in the free-martin. (There is no reason why it should be absent as a rule, as the testes are usually represented.)

Author.	Conditions of Organs.	Remarks.
SIMPSON, J. Y. (1844).	Mentions dissection of two adult and one calf free-martin, all like Hunter's cases. Quotes also a case of Allen Thomson's, *Simpson's Obst. Mem.*, i. p. 823.	Simpson does not consider the question of the female calves co-twin with males and fertile as being derived from separate zygotes. This mistake was due to the still current error of holding that sex is not determined, as it really is, on fertilisation, but by subsequent changes in the zygote.
	In *Todd's Cyclopædia*, he quotes a free-martin where he found testicles.	A free-martin is derived along with its potent twin from one zygote.

I have now to explain how the free-martin arises; and in considering this, the cardinal fact must be kept in mind that the potent and sterile twins arise from one zygote, not from two, and that the genitals of each have to be provided from a single zygote, which normally might give rise to one perfect male. Simpson's mistake in his paper was his not recognising that a male and female twin, perfect in development, arise from separate zygotes.

I must now consider—

(1) The General Anatomy of the Genital Tract in the Female Calf.
(2) The Action of Mendelism in producing the Free-Martin condition.
(3) Consideration of the View that the Free-Martin is a transverse Hermaphrodite.
(4) Summary and Literature.

(1) *The General Anatomy of the Genital Tract in the Female Calf.*

This consists of ovaries, Fallopian tubes, cornua, vagina (Müllerian and urinogenital sinus), urethral opening, lateral folds of skin comparable with the human labia majora and minora, and of the glans phalli. The vagina is urinogenital sinus in its lower half and Müllerian in its upper. The urinogenital sinus portion is the lower end of the anterior division of the pars penultima of the primitive gut or entodermal cloaca of other investigators. The lower half of the effective vagina is thus urinogenital sinus, and this must be carefully noted. Developmentally, the sexual glands and urogenital tracts in boves arise as in allied mammals, *i.e.* we have Wolffian bodies, etc., and on these the ovaries or testes develop; subsequent changes in the Wolffian bodies and ducts give us in the female

a persistent Wolffian duct, the canal of Gartner, instead of its uppermost and lowest segments as in most mammals; while the ducts of Müller give the vagina tubes and cornua. The urinogenital sinus, urethra, and bladder arising from the anterior coronal division of the pars penultima of the primitive gut make up the rest of the tract. The ova and spermatozoa arise by reduction from the primitive germ-cells, and these arise not from the germ epithelium but from the primitive germ-cell mass, itself a part of the unreduced zygote in its earliest stage. This view of the origin of the gametes may be termed the zygotic origin of the gametes, or Owen-Weismann law.

In the normal female bovine tract we have not only the potent elements (ovary, cornua, and vagina), but also the epoöphoron with more than the ductus epididymis, viz. the whole Wolffian duct. We have thus in the female the equivalents of the epididymis and vas deferens of the male. These are not functionally active, and histologically are degenerated, but are quite recognisable in the broad ligament.

In the normal bovine male we have not only the potent male organs, but also the hydatid testis (Müllerian duct) and the prostatic utricle, the equivalent of the hymen usually, and a varying amount of degenerated female genital structures.

These potent and non-potent elements of the developed tract mean causal unit-characters in the zygote, of unequal value, usually coupled in the human male and autonomous, Dominant and Recessive in their nature, and it is by their separation or segregation into the soma of the potent male and sterile male (free-martin) that we get the explanation of this anomaly.

(2) *The Action of Mendelism in producing the Free-Martin.*

When a male zygote twins we may get—

(1) Identical male twins.

(2) One perfect male and one sterile male, the free-martin.

In (1) there is an equivalent division of genital and somatic determinants, and identical twins is the result.

How do a perfect male and imperfect male arise?

In the free-martin (2) there are present, as Spiegelberg shows, urinogenital sinus, rudimentary vesiculæ seminales, very rudimentary vagina and knob of uterus; testes and Wolffian bodies, both imperfectly developed.

Now, as already said, we have to account for the genital organs in the potent bull and free-martin from one zygote—a male zygote. Had this male zygote developed a single bull, it would have had potent male organs

(testes and phallus) and non-potent female ones, viz. Müllerian hydatid attached to the testis, and prostatic utricle, the analogue of the hymen, with a varying amount of the rest of the female Müllerian tract. When the twinning of a male zygote in black cattle takes place, this potent and non-potent complex of the genital organs may be divided so that the potent part goes to the potent bull-calf, the non-potent to the free-martin. This being the general scheme of the division, we must now account as exactly as possible for the various parts in the genital tract of the free-martin in Spiegelberg's case, and these may be grouped as follows for explanation:—(*a*) Testes; (*b*) Urinogenital sinus and external genitals; (*c*) Rudimentary vagina; (*d*) Vesiculæ seminales, vasa deferentia, and Wolffian bodies.

(*a*) *Testes.*—The male zygote gave, of course, sperm-germ cells in the sperm-epithelium of its Wolffian ridges, and the future testes became divided between the twins. This division took place most probably after the primitive sperm-cell mass had formed. The testes are undescended.

(*b*) *Urinogenital sinus and external genitals.*—The former is probably derived from the somatic division of the twins. The labia and glans are non-potent elements.

(*c*) *Rudimentary vagina and uterus.*—These are undoubtedly derived from the Müllerian hydatid and prostatic utricle (Pl. II. fig. 4).

(*d*) *Vesiculæ seminales, vasa deferentia, and Wolffian bodies.* The first two arise from the Wolffian ducts which are present and derived from the non-potent elements; and the Wolffian bodies are given in somatic division.

Thus the free-martin with a potent bull twin is the result of a division of a male zygote, so that the somatic determinants are equally divided, the genital determinants unequally divided, the potent going to the one twin, the potent bull, the non-potent genital determinants to the free-martin. The potent organs are dominant, the non-potent recessive, and the Mendelian scheme may be figured as follows (diag. 1):—

DIAG. 1. Male sex gamete X Female non-sex gamete P

Male zygote which if it twins may give F¹

D — R F²

Bull with equivalent soma and potent genital organs. | Bull with equivalent soma and non-potent genital organs (female type).

In three of Numan's cases the sterile twin had testes and a rudimentary preputial sheath. This is a rare free-martin where the Müllerian elements remain in the potent bull, but the preputial element of the external genitals

is thrown into this variety of free-martin. There are thus two varieties of bull free-martin, viz. John Hunter's free-martin (so-called heifer free-martin) and Numan's free-martin (steer free-martin).

Before I knew of Numan's paper, I saw that there should be also in Mendel's scheme a free-martin with a potent female.* I could admit this as a possibility not yet demonstrated, but Numan's cases show its existence clinically, and indicate the value of the Mendelian scheme and the Owen-Weismann law in studying anomalous sex. This very rare occurrence is shown in the Mendelian scheme (diag. 2). We rarely get the following:—

Diag. 2. Female sex gamete X Male non-sex gamete P

Female zygote which if it twins may give F[1]

D — R F[2]

Female with equivalent soma and potent genital organs. | Female with non-potent genital organs (male type).

One point not yet determined so far is as follows:—

While the vaginal and uterine rudiments in the free-martin are undoubtedly derived from the normal non-potent elements of the normal male segregated as a recessive element, as in F^2 of Mendel's crossing pea-experiments, I am in doubt yet as to whether urinogenital sinus and the external genitals are derived from these or from a division of the soma.

(3) *Consideration of the View that the Free-Martin is a Hermaphrodite.*

I may remark here that the non-potent elements in the human genital tract vary in amount. Normally in the male we have only the prostatic utricle and hydatid testis (Müllerian), but occasionally we have the non-potent organs more extensive, producing a uterus and vagina in a male, as in Shepherd's case. These are described as tubular or transverse hermaphrodites. They are not hermaphrodites at all, as no organism is hermaphrodite if the sexual glands are similar. Such anomalies are due to an excessive amount of the non-potent elements; and as these are of the opposite sex type, they give rise to the idea of hermaphroditism. Free-martins thus are not hermaphrodite, and the term pseudo-hermaphrodite means nothing.

(4) *Summary and Literature.*

General Conclusions.—Nature can thus, in a very simple and effective way, sterilise an organism. The zygote, male or female, is, for this purpose,

* As yet there is no actual anatomical demonstration, but Numan gives one of these.

unequally divided by twinning (or otherwise in single cases), so that only recessive or non-potent genital determinants are allotted to the one twin, and in this way sterility is absolutely secured.

On one point we have as yet no information, viz. as to whether or not the potent bull co-twin with the free-martin has its prostatic utricle normal. Theoretically it seems to me this may be defective or wanting, but here actual dissection is necessary.

The free-martin is, according to Mendelian phraseology, a pure or extracted recessive *quâ* its genital determinants, and the potent twin a pure or extracted dominant, both of F^2 in the Mendelian scheme. Occasionally, but very rarely, as in three of Numan's cases, the recessive element is less complete.

The potent bull alone can have offspring, and some of its males must breed true to the dominant genital determinants, in certain cases at any rate, as Mendel's scheme demands and theory indicates. This will introduce a variation, *i.e.* we may get a bull not possessing a hydatid testis, and thus varying from the normal bull, and we get here one factor in the mechanism of variation; but I defer the consideration of this.*

According to J. A. H. Murray, the origin of the term "free-martin" is unknown. In Holland they are termed *Kween*, and in Brabant, *Bouquetin*; in France, *Taur*; in ancient Rome, *Taura* was the term (Simpson and Spiegelberg). According to Simpson and others, the Romans did not seem to have been aware of the association of the "Taura" with twinning.

I have to thank Professor J. Arthur Thomson for valuable references, and Mr Henderson for one undoubted specimen.

LITERATURE.

Allnatt, *Lond. Med. Gaz.*, vol. xviii., 1836, p. 528.

Geddes and Thomson, *The Evolution of Sex*, 2nd edit., London, Walter Scott, 1901.

Gurlt, *Lehrbuch der path. anat. Haussäugethiere*, 1833, Tab. xxii. figs. 2–5.

Hart, D. Berry, "Mendelian Action on Differentiated Sex," Abstract, *Proc. Roy. Soc. of Edin.*, July 1909; *Edin. Obstr. Trans.*, 1908–9, *in extenso.*

Hering, *Repert. der Thierheilk.*, 12 Jahrgang, 1851, p. 107.

* Observations are still needed on such specimens. We need to know the exact condition of the organs in the potent animal, and this could best be done if such twin calves or specimens obtained in the cornua of slaughtered animals were examined as to details. Free-martins are not uncommon, but in Hunter's days they were obtained from the farm and a history given; now, in the large sales where animals are bought, the history can seldom be obtained.


h
h
g
g
f
e
e
d
d
c
b
a

Fig. 1.

Fig. 2.

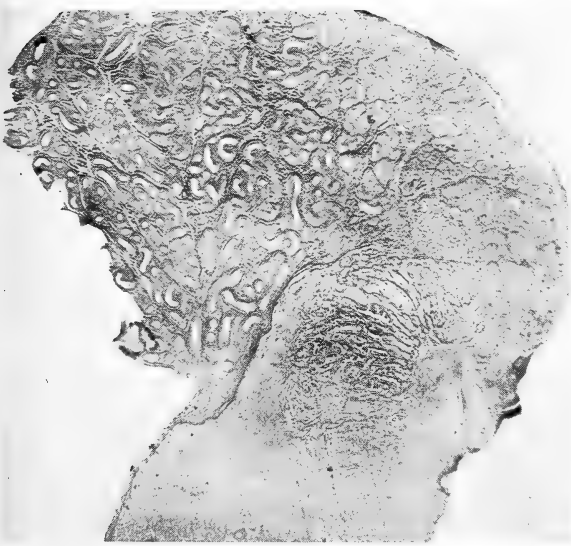


Fig. 3.

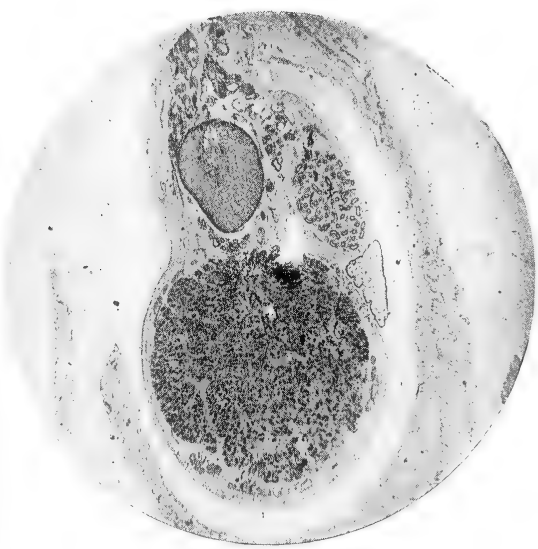


Fig. 4.

Hunter, John, *Account of the Free-Martin; Observations on certain parts of the animal economy:* London, sold at No. 13 Castle Street, Leicester Sq., 1786 (*vide* also Atlas attached to Palmer's edition of Hunter's Works).

Numan, Dr A., "Mémoire sur les vaches stériles," etc., *Journal vétérinaire et agricole de Belgique,* 1844. This is a translation of Numan's Dutch paper, but unfortunately the illustrations are not given.

Rueff, *Repertor. d. Thierheilkunde von Hering,* 12 Jahrg., 1851, p. 103.

Sanson, A., "Sur la Stérilité des Génisses jumelles de taureaux," *Bul. Soc. centr. de médecine vétérinaire,* 4e Série, Tome iii. (1880), p. 132. Tome iv. (1881), p. 103. (This I have not seen.)

Scarpa, *Opere varie del Cav. A. Scarpa, etc.,* Vannoni, Firenze, 1845, i. p. 175.

Simpson, Sir J. Y., *On the alleged Infecundity of Females born co-twin with Males.* The Works of Sir J. Y. Simpson, vol. i., edited by J. Watt Black, M.D.; Edinburgh, A. & C. Black. See also Article on "Hermaphroditism," *Todd's Cyclopædia of Anat. and Phys.,* vol. ii., 1836–39.

Spiegelberg, "Ueber die Verkümmerung der Genitalien bei (angeblich) verschieden geschlechtlichen Zwillingskälbern," *Ztsch. für rationelle Medicin,* Henle and Pfeufer, Drt. Reihe, Bd. xi.

EXPLANATION OF PLATES.

Plate I.

Genital organs in full-time free-martin calf (Spiegelberg) ($\frac{1}{1}$).

a, urinogenital sinus laid open from the front and turned back: a sound is passed into the Müllerian non-potent element with the knob ending at *c*, the uterus; *d d*, vesiculæ seminales; *e e f*, vasa deferentia; *g g*, imperfect testes; *h h*, Wolffian bodies. In John Hunter's free-martin cases, Owen suggested that in what Hunter figured as double and differing sexual glands, one might be Wolffian body (*vide* Palmer's edition of Hunter, with Owen's notes).

Plate II.

Fig. 1. One of Hunter's free-martin cases. It has the head and horns of the ox or spayed animal, but the fore and hind quarters of a bull. Rudimentary nipples are present.

Fig. 2. Sexual gland in Hunter's free-martin; tubuli seminiferi, slide 678.

Fig. 3. Sexual gland in Hunter's free-martin; epididymis in upper left angle, with Müllerian epithelium and glands below (hydatid testis, slide 691).

Fig. 4. Testis in scrotum of newly-born human fœtus. Testis is below; above, on right side, is seen the hydatid testis unattached and then epididymis attached. To left of epididymis lies accessory suprarenal body.

(*Issued separately February* 11, 1910.)

XI.—On Waves in a Dispersive Medium resulting from a Limited Initial Disturbance. By George Green, M.A., B.Sc., Assistant to the Professor of Natural Philosophy in the University of Glasgow. *Communicated by* Professor A. Gray, F.R.S.

(MS. received November 11, 1909. Read December 6, 1909.)

§ 1. In a former paper "On Group-Velocity and on the Propagation of Waves in a Dispersive Medium" (*Proc. R.S.E.*, xxix. pp. 445–470, 1909), it was shown that group-velocity, or the principle of "stationary phase," provides us with a satisfactory explanation of the *modus operandi* of dispersion; and the principle was applied to obtain an expression for the effect of a single impulse confined to the neighbourhood of a point of the medium. The present paper is intended to fulfil a promise given in § 29 of that paper, to show that by means of this principle we can arrive at the general features of the wave-system in a dispersive medium resulting from any limited initial disturbance.

It is proposed to examine the effect of the same initial disturbance in several dispersive media in which the group-velocity is positive, that is, always in the same direction as the individual waves travel. Using the notation of the paper referred to above, we take V as the velocity of an infinite train of waves of wave-length λ, and period $2\pi/kV$, where $k=2\pi/\lambda$, and where $V=f(k)$, since the medium is such that the wave-velocity varies with the wave-length. For convenience the investigation is restricted to media for which the wave-velocity is given by $V=Ak^n$. The group-velocity U, corresponding to k, is then given by equation (16) of the former paper, namely, $U=(1+n)Ak^n$; so that we have the further restriction, $n>-1$, to make this always of the same sign as the wave-velocity.

§ 2. The wave-system resulting from a given initial displacement has already been investigated for several different forms of initial displacement in the particular cases where $n=-\frac{1}{2}$, which corresponds to deep-water waves, and where $n=1$, which corresponds to flexural waves in an elastic rod. Professor Schuster, in his paper on "The Propagation of Waves through Dispersive Media,"* has also dealt with the cases included in the formula $V=a+bk^{-1}$, where a and b are constants. The form of initial displacement

$$\xi=\frac{1}{a^2+x^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1),$$

* Boltzmann, *Festschrift*, 1904.

where a is a constant, has been chosen by Professor Schuster in the above paper, and also by Professor Burnside * for deep-water waves; so that it is convenient, for the sake of comparison with their results, to begin by considering this form of initial displacement in the general case where $n > -1$. Keeping, however, in the meantime to any initial form given by $\xi = f(x)$, we proceed first to show that in all cases included in the formula $V = a + bk^{-1}$ there is no resolution of the original disturbance by the medium depending on the wave-length of the constituent Fourier trains, as is understood by dispersion.

§ 3. The problem to be solved is:—to find the displacement, ξ, at any point x, at time t, having been given the initial displacement of the medium, $\xi = f(x)$, with the additional initial condition $\frac{d\xi}{dt} = 0$: and the solution, obtained by the regular Fourier synthesis, is

$$\xi = \frac{1}{\pi} \int_0^\infty dk \int_{-\infty}^{+\infty} dx_1 f(x_1) \cos k(x - x_1) \cos kVt \quad . \quad . \quad . \quad (2).$$

This fulfils the initial conditions, provided

$$f(x) = \frac{1}{\pi} \int_0^\infty dk \int_{-\infty}^{+\infty} dx_1 f(x_1) \cos k(x - x_1) \quad . \quad . \quad . \quad . \quad . \quad (3),$$

which we know to be the case by Fourier's theorem. If, following Professor Schuster, we now take $V = a + bk^{-1}$, and define $\psi(x)$ by the equation

$$\psi(x) = \frac{1}{\pi} \int_0^\infty dk \int_{-\infty}^{+\infty} dx_1 f(x_1) \sin k(x - x_1) \quad . \quad . \quad . \quad . \quad . \quad (4),$$

we easily obtain, by expressing the product of the cosines in (2) as the sum of two cosine terms and two sine terms,

$$\xi = \tfrac{1}{2}\{f(x + at) + f(x - at)\} \cos bt - \tfrac{1}{2}\{\psi(x + at) - \psi(x - at)\} \sin bt \quad . \quad (5).$$

This equation shows that the displacement at point x at time t may be regarded as due to two initial disturbances each of which moves along without change of form at the constant velocity a; half the total energy of each disturbance going in the positive direction, and half in the negative. If a and b are both finite, the original form of the disturbance reappears at regular intervals, $2\pi/b$, displaced in each from its former position by a distance $2\pi a/b$. When a is zero the above equation reduces to

$$\xi = f(x) \cos bt \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6),$$

* *Proc. Lon. Math. Soc.*, t. xx., 1888.

which indicates that each particle of the medium performs a simple harmonic motion, of period $2\pi/b$, and of amplitude equal to the initial displacement of the particle; in exactly the same way as a row of Reynold's disconnected pendulums would swing after being held at rest in a displaced position and then let go. The form of equations (5) and (6) shows that there is no resolution of the initial disturbances by the medium according to wave-length whereby effects due to the separate constituent trains are observable at distinct parts of the medium.

§ 4. Taking now the particular initial conditions

$$\xi = \frac{1}{a^2+x^2}\,; \quad \frac{d\xi}{dt} = 0 \quad . \quad . \quad . \quad [(1) \text{ repeated}],$$

where a is a constant, in the general case where $\mathrm{V} = \mathrm{A}k^n$, we shall consider first the solution corresponding to equation (2) above:

$$\xi = \frac{1}{\pi}\int_0^\infty dk \int_{-\infty}^{+\infty} dx_1 \frac{1}{a^2+x_1^2} \cos k(x-x_1) \cos \mathrm{A}k^{n+1}t \quad . \quad . \quad . \quad (7).$$

The integration with respect to x_1 can be performed by means of the well-known integrals

$$\int_{-\infty}^{+\infty} \frac{dx_1 \cos kx_1}{a^2+x_1^2} = \frac{\pi\epsilon^{-ak}}{a}\,; \quad \int_{-\infty}^{+\infty} \frac{dx_1 \sin kx_1}{a^2+x_1^2} = 0\,;$$

the result being given by the equation

$$\xi = \frac{1}{a}\int_0^\infty dk\epsilon^{-ak} \cos kx \cos \mathrm{A}k^{n+1}t \quad . \quad . \quad . \quad . \quad (8).$$

By expanding $\cos \mathrm{A}k^{n+1}t$, and integrating the series term by term, we obtain finally the following value for ξ:

$$\left.\begin{aligned} \xi &= \sum_{r=0}^{r=\infty} \frac{(-1)^r}{a} \frac{(\mathrm{A}t)^{2r}}{2r\,!} \cdot \frac{\Gamma\{2r(n+1)+1\}}{(a^2+x^2)^{nr+r+\frac{1}{2}}} \cos\left[\{2r(n+1)+1\}\chi\right] \\ \text{where} \quad \chi &= \tan^{-1}\frac{x}{a} \end{aligned}\right\} \quad . \quad (9).$$

This expression can be used to calculate the displacement at any point x, with moderate labour so long as t is small compared with x. As t increases, however, the series becomes sluggishly convergent, and even ultimately divergent * when $n > 0$, so that it is necessary to adopt another method of evaluating the right-hand side of (7) when t is of the same order of magnitude as, or greater than, x.

* See Lord Rayleigh, *Phil. Mag.*, July 1909.

§ 5. Going back to equation (2), we assume that it is permissible to reverse the order of the integrations, in which case the equation may be written in either of the forms:

$$\left.\begin{aligned}\xi &= \frac{1}{\pi}\int_{-\infty}^{+\infty} dx_1 f(x_1)\int_0^{\infty} dk \cos k(x-x_1)\cos k\mathrm{V}t \\ &= \frac{1}{2\pi}\int_{-\infty}^{+\infty} dx_1 f(x_1)\int_0^{\infty} dk\{\cos k(x-x_1-\mathrm{V}t)+\cos k(x-x_1+\mathrm{V}t)\}\end{aligned}\right\} \quad . \quad (10).$$

In either of these forms the first integration gives the effect at point x, at time t, of a single initial disturbance confined to the neighbourhood of a point x_1, and the second integration then gives the sum of the effects of all the initial disturbances at all points of the medium, which together comprise the total initial disturbance represented by $f(x)$. From the second of equations (10), we can write down the effect of a single initial impulse confined to the neighbourhood of the origin in the form:

$$\xi = \frac{1}{2\pi}\int_0^{\infty} dk\{\cos k(x-\mathrm{V}t)+\cos k(x+\mathrm{V}t)\} \quad . \quad . \quad . \quad (11),$$

of which the right-hand member is the integral evaluated by Lord Kelvin in his paper of 1887,* and used in my former paper referred to in § 1 above. From equation (11) we see that half the total number of constituent wave-trains move in the positive direction and half in the negative; and it is clear from the argument adopted in my former paper that the displacement at any point whose x is positive is due to positively moving trains alone, and the displacement at any point whose x is negative is due entirely to negatively moving trains. Equation (1) of my former paper should in fact be replaced by equation (11) above, as it represents a single impulse at the origin, with $\frac{d\xi}{dt}=0$ when $t=0$, only under the restriction $\mathrm{V}=f(k)=-f(-k)$. The principle underlying the evaluation given by Lord Kelvin applies to a single impulse with or without this restriction, giving, as it does, the value of ξ in

$$\xi = \frac{1}{2\pi}\int_0^{\infty} dk \cos k(x \mp \mathrm{V}t) \quad . \quad . \quad . \quad . \quad . \quad . \quad (12),$$

or in the corresponding equation with sin instead of cos, where the negative sign is taken for the positively moving trains (x positive) and the positive sign is taken for the negatively moving trains (x negative).

* *Phil. Mag.*, vol. xxiii., March 1887.

§ 6. The evaluation of (12) given by Lord Kelvin and repeated in § 14 of my former paper is

$$\xi = \frac{\cos\left[k_0\{x - tf(k_0)\} \pm \frac{\pi}{4}\right]}{2^{\frac{1}{2}}\pi^{\frac{1}{2}}t^{\frac{1}{2}}[\mp\{2f'(k_0) + k_0f''(k_0)\}]^{\frac{1}{2}}} \quad . \quad . \quad . \quad . \quad (13),$$

where k_0 determines the particular group of wave-trains which predominate at point x, at time t, t being great, though probably not very great. The ambiguous sign of the expression in the denominator is to be chosen so as to make the expression positive, and in the numerator the opposite sign is required. The relation between k_0 and x at time t is given by the equation

$$x = \{f(k_0) + k_0f'(k_0)\}t = \mathrm{U}t \quad . \quad . \quad . \quad . \quad (14),$$

where U is the group-velocity for waves of wave-length $2\pi/k_0$. Putting now $f(k_0) = \mathrm{A}k_0^{\,n}$, and eliminating k_0 from (13) by means of (14), we obtain finally, as the value of the integral in (12),

$$\left.\begin{aligned} \xi &= \mathrm{P}\frac{x^{\frac{1}{2n}-\frac{1}{2}}}{t^{\frac{1}{2n}}}\cos\left[\mathrm{B}\frac{x^{\frac{n+1}{n}}}{t^{\frac{1}{n}}} \pm \frac{\pi}{4}\right] \\ \text{where} \qquad \mathrm{P} &= \left[2\pi n\{(n+1)\mathrm{A}\}^{\frac{1}{n}}\right]^{-\frac{1}{2}}; \ \mathrm{B} = \frac{n}{(1+n)^{\frac{n+1}{n}}\mathrm{A}^{\frac{1}{n}}} \end{aligned}\right\} \quad . \quad . \quad . \quad (15).$$

The condition to which this evaluation is subject is given in § 14 of my former paper:—that the process of dispersion described in § 12 is exceedingly far advanced, and t therefore very great. The further condition given in Lord Kelvin's paper of 1887, that the denominator of (13) must be very great, is also stated. But this can be proved to be inconsistent with the well-known condition to which (13) is subject in the case of deep-water waves (equation (14) of Lord Kelvin's paper), namely, that $\frac{gt^2}{4x}$ is very great.

When this expression for the effect of a single initial impulse is inserted in equation (10), the displacement ξ at any point x, at time t, due to the initial disturbance $f(x)$ can be written in the form

$$\xi = \frac{\mathrm{P}}{t^{\frac{1}{2n}}}\int_{-\infty}^{+\infty} dx_1 f(x_1)(x - x_1)^{\frac{1}{2n}-\frac{1}{2}}\cos\left[\mathrm{B}\frac{(x-x_1)^{\frac{n+1}{n}}}{t^{\frac{1}{n}}} \pm \frac{\pi}{4}\right] \quad . \quad . \quad (16),$$

the positive sign being taken when n lies between 0 and -1.

§ 7. If the whole disturbance, or if the most important part of the disturbance, $f(x)$, be confined to a small part of the medium on each side of the origin, we can simplify further the expression (16) by introducing the condition that x is so great that we may consider only the first power of (x_1/x) for the greatest value of x_1 that need be included in the above integration in obtaining a first approximation to the value of ξ. The simplified expression of (16), which is true approximately when x and t are both very great, is obtained by expanding $(x-x_1)^{\frac{1}{2n}-\frac{1}{2}}$ and $(x-x_1)^{\frac{n+1}{n}}$ so far as the first power of x_1, and then transforming the cosine term by the addition theorem. It is:

$$\left.\begin{aligned}\xi = \mathrm{P}\frac{x^{\frac{1}{2n}-\frac{1}{2}}}{t^{\frac{1}{2n}}}\Bigg[&\cos\theta\int_{-\infty}^{+\infty} dx_1 f(x_1)\left\{1-\left(\frac{1}{2n}-\frac{1}{2}\right)\frac{x_1}{x}\right\}\cos\left\{\mathrm{B}\frac{n+1}{n}\left(\frac{x}{t}\right)^{\frac{1}{n}}x_1\right\}\\ &+\sin\theta\int_{-\infty}^{+\infty} dx_1 f(x_1)\left\{1-\left(\frac{1}{2n}-\frac{1}{2}\right)\frac{x_1}{x}\right\}\sin\left\{\mathrm{B}\frac{n+1}{n}\left(\frac{x}{t}\right)^{\frac{1}{n}}x_1\right\}\Bigg]\\ \text{with}\qquad \theta = &\left\{\mathrm{B}\frac{x^{\frac{n+1}{n}}}{t^{\frac{1}{n}}}\pm\frac{\pi}{4}\right\}\end{aligned}\right\} \quad (17).$$

The lower sign is to be taken in θ when n is positive.

§ 8. We can now find a second solution of our problem, called for at the end of § 4. Introducing the form of initial displacement of equation (1), (17) becomes

$$\begin{aligned}\xi = \mathrm{P}\frac{x^{\frac{1}{2n}-\frac{1}{2}}}{t^{\frac{1}{2n}}}\Bigg[&\cos\theta\int_{-\infty}^{+\infty} dx_1\frac{\left\{1-\left(\frac{1}{2n}-\frac{1}{2}\right)\frac{x_1}{x}\right\}}{a^2+x_1^2}\cos\left\{\mathrm{B}\frac{n+1}{n}\left(\frac{x}{t}\right)^{\frac{1}{n}}x_1\right\}\\ &+\sin\theta\int_{-\infty}^{+\infty} dx_1\frac{\left\{1-\left(\frac{1}{2n}-\frac{1}{2}\right)\frac{x_1}{x}\right\}}{a^2+x_1^2}\sin\left\{\mathrm{B}\frac{n+1}{n}\left(\frac{x}{t}\right)^{\frac{1}{n}}x_1\right\}\Bigg]\end{aligned} \quad . \quad (18).$$

The integrals in this expression belong to the same class as those dealt with in § 4, and can all be evaluated, the result obtained when this is done being

$$\xi = \mathrm{P}\frac{\pi x^{\frac{1}{2n}-\frac{1}{2}}}{a t^{\frac{1}{2n}}}\epsilon^{-\frac{n+1}{n}\left(\frac{x}{t}\right)^{\frac{1}{n}}\mathrm{B}a}\left\{\cos\theta-\left(\frac{1}{2n}-\frac{1}{2}\right)\frac{a}{x}\sin\theta\right\} \quad . \quad . \quad (19).$$

The second term on the right-hand side of (19), being of the order $(1/x)$ of the first term, is negligible compared with it by the condition stated in § 7. The displacement at point x due to our initial disturbance of equation (1)

during all the time when it is comparable in magnitude with its maximum value is therefore given with sufficient accuracy by the equation

$$\xi = \mathrm{P}\frac{\pi}{\alpha}\,\frac{x^{\frac{1}{2n}-\frac{1}{2}}}{t^{\frac{1}{2n}}}\,\epsilon^{-\frac{n+1}{n}\left(\frac{x}{t}\right)^{\frac{1}{n}}\mathrm{B}\alpha}\cos\left\{\mathrm{B}\frac{x^{\frac{n+1}{n}}}{t^{\frac{1}{n}}} \pm \frac{\pi}{4}\right\} \quad . \quad . \quad . \quad (20).$$

§ 9. In the case of waves in deep water, we have $\mathrm{V} = g^{\frac{1}{2}}k^{-\frac{1}{2}}$, or $\mathrm{A} = g^{\frac{1}{2}}$ and $n = -\frac{1}{2}$; these give $\mathrm{P} = \frac{g^{\frac{1}{2}}}{2\pi^{\frac{1}{2}}}$ and $\mathrm{B} = -\frac{g}{4}$, and the equation to the water surface becomes

$$\xi = \frac{\pi^{\frac{1}{2}}}{\alpha x}\left(\frac{gt^2}{4x}\right)^{\frac{1}{2}}\epsilon^{-\frac{gt^2\alpha}{4x^2}}\cos\left\{\frac{gt^2}{4x} - \frac{\pi}{4}\right\} \quad . \quad . \quad . \quad . \quad (21),*$$

which is in agreement with the result given by Professor Burnside in his paper "On Deep-water Waves resulting from a Limited Original Disturbance," where it is obtained by an entirely different method.

For flexural waves in a thin elastic rod, we have $\mathrm{V} = \kappa bk$, or $\mathrm{A} = \kappa b$ and $n = 1$; $\mathrm{P} = (4\pi\kappa b)^{-\frac{1}{2}}$, and $\mathrm{B} = (4\kappa b)^{-1}$. The shape of the rod is then given by

$$\xi = \frac{\pi^{\frac{1}{2}}}{2(\kappa bt)^{\frac{1}{2}}\alpha}\epsilon^{-\frac{x\alpha}{2\kappa bt}}\cos\left\{\frac{x^2}{4\kappa bt} - \frac{\pi}{4}\right\} \quad . \quad . \quad . \quad . \quad (22).$$

For capillary waves we have $\mathrm{V} = \mathrm{T}^{\frac{1}{2}}k^{\frac{1}{2}}$, or $\mathrm{A} = \mathrm{T}^{\frac{1}{2}}$ and $n = \frac{1}{2}$; $\mathrm{P} = (\frac{9}{4}\mathrm{T}\pi)^{-\frac{1}{2}}$, and $\mathrm{B} = (\frac{27}{4}\mathrm{T})^{-1}$. The water surface is given by

$$\xi = \frac{2\pi^{\frac{1}{2}}}{3\mathrm{T}^{\frac{1}{2}}}\,\frac{x^{\frac{1}{2}}}{t}\,\epsilon^{-\frac{4x^2\alpha}{9\mathrm{T}t^2}}\cos\left\{\frac{4x^3}{27\mathrm{T}t^2} - \frac{\pi}{4}\right\} \quad . \quad . \quad . \quad . \quad (23).$$

§ 10. We can now find by means of equation (20) the time at which the amplitude of the disturbance at any chosen point x has its maximum value. The amplitude is given by the right-hand member of (20), with the cosine term omitted:

$$a = \mathrm{P}\frac{\pi}{\alpha}\,\frac{x^{\frac{1}{2n}-\frac{1}{2}}}{t^{\frac{1}{2n}}}\,\epsilon^{-\frac{n+1}{n}\left(\frac{x}{t}\right)^{\frac{1}{n}}\mathrm{B}\alpha} \quad . \quad . \quad . \quad . \quad . \quad . \quad (24).$$

By differentiating this with respect to t, we find that the time of maximum disturbance at x is given by the equation

$$\left(\frac{x}{t}\right)^{\frac{1}{n}} = \frac{n}{2(n+1)\mathrm{B}\alpha} \quad . \quad . \quad . \quad . \quad . \quad . \quad (25),$$

and the amount of this greatest amplitude is then given by

$$a = \frac{\pi^{\frac{1}{2}}}{2\alpha^{\frac{1}{2}}}\,\frac{1}{(nx)^{\frac{1}{2}}}\epsilon^{-\frac{1}{2}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (26),$$

* After this paper had been completed I found this result given in a very comprehensive paper by Dr T. H. Havelock, "The Propagation of Groups of Waves in Dispersive Media," in *Proc. Roy. Soc.*, lxxxi., 1908, obtained by a similar application of Lord Kelvin's 1887 result to that given in §§ 6, 7 above. The view of group-velocity given in my paper, *Proc. R.S.E.*, 1909, is there fully discussed and illustrated.

where for n we take its absolute value. The greatest amplitude of the disturbance at any point therefore varies inversely as the square root of its distance from the centre of the initial disturbance, no matter what value we give to n.

To find the wave-length of the disturbance at x, when the amplitude there has its maximum value, we put

$$\frac{B}{t^{\frac{1}{n}}}\left\{\left(x+\frac{\lambda}{2}\right)^{\frac{n+1}{n}}-\left(x-\frac{\lambda}{2}\right)^{\frac{n+1}{n}}\right\}=2\pi \quad . \quad . \quad . \quad . \quad (27),$$

since the phase varies by 2π in passing along one complete wave-length. From this we obtain, as in § 16 of my former paper,

$$\left.\begin{aligned}\lambda&=\frac{2n\pi}{(n+1)B}\left(\frac{t}{x}\right)^{\frac{1}{n}}\\ \text{or by (23),}\qquad \lambda&=4\pi a\end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad (28),$$

which shows that the wave-length at any point when the displacement there is greatest depends only on the form of the initial disturbance, and is the same in all dispersive media for the particular disturbance we are considering.

§ 11. If, on the other hand, we regard t as fixed, and proceed to find the place at which the disturbance has its maximum amplitude at the time chosen, we differentiate with respect to x, and then by proceeding exactly as before obtain

$$\lambda=\frac{4\pi a}{1-n} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (29),$$

as the value of the wave-length at the place of maximum displacement, for all times. The place of maximum displacement at a given time is determined by

$$\left(\frac{x}{t}\right)^{\frac{1}{n}}=\frac{n(1-n)}{2(1+n)Ba} \quad . \quad . \quad . \quad . \quad . \quad . \quad (30),$$

and the amount of the maximum displacement at the time and place so determined is given by

$$a'=\frac{\pi^{\frac{1}{2}}}{2a^{\frac{3}{2}}}\left(\frac{1-n}{nx}\right)^{\frac{1}{2}}\epsilon^{-\frac{1}{2}(1-n)} \quad . \quad . \quad . \quad . \quad . \quad . \quad (31),$$

where, as before, the absolute values of n and $(1-n)$ are to be taken.

§ 12. Remembering that in these equations the presence of a indicates in what manner the wave-length, amplitude, etc., of the displacement curve at the point of the medium observed are determined by the form of the initial disturbance, and the presence of n indicates in what manner they depend on the dispersive nature of the medium, we can state the results con-

tained in equations (25)–(31), for our initial form of disturbance $1/(a^2+x^2)$, as follows. Equation (28) shows that the wave-length associated with the maximum disturbance at any point x of the medium depends only on the form of the initial disturbance, and is independent of the dispersive nature of the medium or of the part of the medium considered. The greatest amplitude of disturbance, however, at any point of a particular medium falls off according to the inverse square root of the distance of the point from the centre of the initial disturbance, as we see by equation (26).

It was shown in my former paper that the position of any given wave-length in a wave-system changes uniformly at the group-velocity corresponding to that wave-length. Hence, or by equation (25) above, we see that in any one medium the time which elapses before the disturbance at any point reaches its maximum increases uniformly according to the distance of the point observed from the place of the original disturbance. If we observe the displacements at points corresponding to a fixed value of x in several dispersive media, the times of the maximum displacements vary from one medium to another according to equation (25); and, by (28), the wave-length, and therefore the velocity of this maximum in each medium depends only on the form of the initial disturbance. Its amplitude in each medium varies as $x^{-\frac{1}{2}}$. If we observe all the media at a fixed time, the places of maximum displacement, and the amounts, vary from one medium to another according to equation (30).

As to the beginning of wave-disturbance, we see by § 12 of my former paper that there is practically instantaneous propagation of disturbance from the middle to each point of the medium, by wave-trains whose group-velocities are very great.

§ 13. The association of a certain wave-length with the maximum disturbance at any point in any medium, and also of a distinct wave-length with the maximum disturbance at any time for each medium, is in accordance with the general views of wave-propagation expressed in § 21 of my former paper, and illustrates the fact that the amplitude associated with any chosen wave-length at any time depends on the manner in which the energy is distributed initially among the effective Fourier trains. This is made clearer by an examination of the law of falling off of the amplitude of disturbance corresponding to a definite wave-length, by equation (20). The position in the wave-system of the wave-length λ, at time t, is given by equation (28), and from this equation combined with equation (20) we find that the amplitude associated with wave-length λ is given by

$$a = \mathrm{P}\frac{\pi}{\alpha}\left\{\frac{2\pi n}{(n+1)\mathrm{B}\lambda}\right\}^{\frac{1}{2}} \cdot \frac{1}{x^{\frac{1}{2}}}\epsilon^{-\frac{2\pi}{\lambda}\alpha} = \frac{\pi}{\alpha} \cdot \frac{1}{(n\lambda x)^{\frac{1}{2}}}\epsilon^{-\frac{2\pi}{\lambda}\alpha} \quad . \quad . \quad (32),$$

which shows that the amplitudes fall off according to $x^{-\frac{1}{2}}$, or according to $(Ut)^{-\frac{1}{2}}$, where U is the group-velocity corresponding to the wave-length considered. From (32) we can obtain the relation between energy and wave-length at any fixed place of observation.

§ 14. We can now trace the general course of the disturbance in any

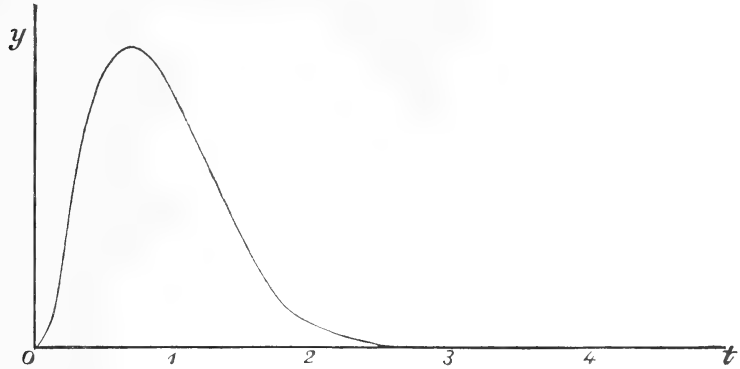


FIG. 1.

medium at a chosen place of observation, x; and we can also obtain an idea of the state of the disturbance throughout the medium at any particular time, t. It is convenient for this purpose to distinguish between media for which n is positive and those for which n is negative. In the case of n positive, the

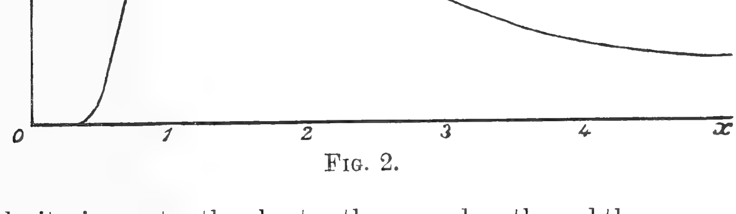


FIG. 2.

wave-velocity is greater the shorter the wave-length, and the group-velocity for any chosen wave-length exceeds the corresponding wave-velocity; in the case of n negative, the wave-velocity is greater the greater the wave-length, and the group-velocity is less than the corresponding wave-velocity. In both cases the general course of the disturbance at a very distant point x is evident from the following statements. When t is very small, we see

by equation (9) that the amplitude of the disturbance is of the order $1/x^2$. When t becomes comparable with x, equation (20) shows that the amplitude rises to a maximum of the order $1/x^{\frac{1}{2}}$, and begins to diminish. When t becomes great compared with x, the amplitude becomes vanishingly small.

§ 15. But for n negative, the rate of rise to the maximum value is much quicker than the rate of subsidence from it, for in this case the successive waves of the disturbance at the point observed are of smaller and smaller wave-length, and take relatively longer to pass, owing to their smaller wave-velocity. When we observe the whole disturbance at any particular time, however, we find that the greater part of the medium sensibly disturbed lies beyond the point where the maximum displacement occurs—that is, towards the side where the greater wave-lengths predominate. The general form of the amplitude curve at any point x, throughout all time, is shown in fig. 1; and the general form of the amplitude curve throughout the medium at any time, t, is shown in fig. 2. These curves are reproduced from Professor Burnside's paper, referred to in § 2 above, where they were drawn to represent the particular case of deep-water waves ($n = -\frac{1}{2}$), but it is clear from the equations given below that we can obtain from them, by suitable modification of scales used, an indication of the general features of the wave-disturbance in any medium for which n is negative. The equations to these curves are:

$$\xi = c'z\epsilon^{-d'z} \text{ for fig. 1, where } z = t^{\frac{1}{n}} \quad . \quad . \quad . \quad . \quad (33),$$

$$\xi = \frac{c}{z^{\frac{1+n}{2}}}\epsilon^{-\frac{d}{z}} \text{ for fig. 2, where } z = x^{\frac{1}{n}} \quad . \quad . \quad . \quad . \quad (34),$$

c, d, c', d' being constants, and the absolute value of n being taken throughout.

When n is positive, the amplitude curve at any point x, throughout all time, is given by

$$\xi = \frac{c_1}{z^{\frac{1}{2}}}\epsilon^{-\frac{d_1}{z}}, \text{ where } z = t^{\frac{1}{n}} \quad . \quad . \quad . \quad . \quad . \quad (35),$$

and the amplitude curve throughout the medium at any fixed time, t, is given by

$$\xi = c'_1 z^{\frac{1-n}{2}}\epsilon^{-d'_1 z}, \text{ where } z = x^{\frac{1}{n}} \quad . \quad . \quad . \quad . \quad . \quad (36).$$

These equations, being respectively of the same types as (34) and (33) above, show that for n positive and less than unity fig. 1 may be taken as representing the general features of the amplitude curve throughout the medium at any fixed time, and fig. 2 may be taken as representing the general features of the amplitude curve at any fixed place x, throughout all

time. Remembering that, for n positive, the successive waves of the disturbance are of increasing wave-lengths, it is clear from the curves that in this case again the most important part of the disturbance at any time is on one side of the point where the maximum amplitude occurs, being associated in all cases with the greater wave-lengths.

§ 16. In conclusion, the chief results which have been established are: (1) that the wave-length associated with the greatest disturbance at any point depends only on the form of the initial disturbance; (2) that the wave-length associated with the maximum disturbance at any time is constant for any one medium; (3) that the amplitude corresponding to any wave-length falls off according to $x^{-\frac{1}{2}}$, and the energy per wave-length therefore according to x^{-1}; (4) that the front of a wave-system, for $n<0$, continually grows in importance relatively to the rear, while for $n>0$ the front is always of less importance than the rear.

These results are proved only for the particular initial disturbance chosen, but it seems certain from the nature of the case that they are quite general, being applicable to any initial form of disturbance. This has, in fact, been verified for several initial forms, including the case of a finite constant displacement over a given length of the medium, and also the case of a regular sinusoidal displacement over a finite length of the medium.

(*Issued separately February* 18, 1910.)

XII.—On an Electrically Controlled Thermostat and other Apparatus for the Accurate Determination of the Electrolytic Conductivity of Highly Conducting Solutions. By John Gibson, Ph.D., and G. E. Gibson, B.Sc.

(MS. received November 19, 1909. Read June 22, 1908.)

Kohlrausch and others have recently published investigations which show that the electrolytic conductivity of dilute solutions of inorganic salts may be determined with a maximum error of two or three in ten thousand.

Hitherto such accuracy has not been attained with highly conducting concentrated solutions. With such solutions different difficulties are encountered from those met with in dilute solutions. Temperature variations originating outside the cell, the heating effect of the current within the cell, and polarisation are sources of error which are particularly troublesome in the case of highly conducting solutions.

By means of the electrically controlled thermostat shown in fig. 1 the temperature of the bath can be kept constant to about one thousandth of a degree centigrade.* The temperature variations are observed through a reading telescope on a Beckmann thermometer graduated to one-hundredth of a degree centigrade. When the thermostat is working properly the thermometer suspended in the bath remains perfectly steady for hours at a time. On one occasion no visible movement of the mercury was observed during four days. The whole apparatus has been in nearly constant use during the last four years.

The cylindrical copper tank is painted white inside, and on the outside is covered with thick felt. It has a capacity of about 100 litres.

The stirrer, shown in fig. 1, is of the screw propeller type, and has a diameter about one-third of the diameter of the tank. It is driven by an electromotor, and rotated so that the water circulates upwards at the centre and downwards at the sides. The lower end of the vertical brass shaft *s* is rounded off so as to run in a foot-bearing of gun-metal fixed to the bottom of the tank. The upper end runs in a bearing in a horizontal cross-bar, not shown in fig. 1, and is reduced from $\frac{1}{2}$ inch diameter to $\frac{1}{4}$ inch so as to form a

* It is interesting to note that so long ago as 1865 Kohlrausch described an apparatus for controlling temperature electrically. At that time the use of the electric current for technical purposes was quite undeveloped, and the appliances now so familiar were unknown. *Poggendorf Annalen*, Bd. cxxv. p. 126.

collar which prevents upward displacement. The adjustable driving pulley, not shown in fig. 1, is fixed a little lower down. If the full degree of constancy is to be attained, rapid stirring, low capacity for heat of the heating apparatus, and high sensibility of the thermo-regulator are essential. For most purposes a constancy of about one-hundredth of a degree centigrade is amply sufficient, and the rate of stirring may be correspondingly reduced.

The heating apparatus consists of electric glow-lamps with elongated glass necks placed in the water of the bath as shown in fig. 1. These

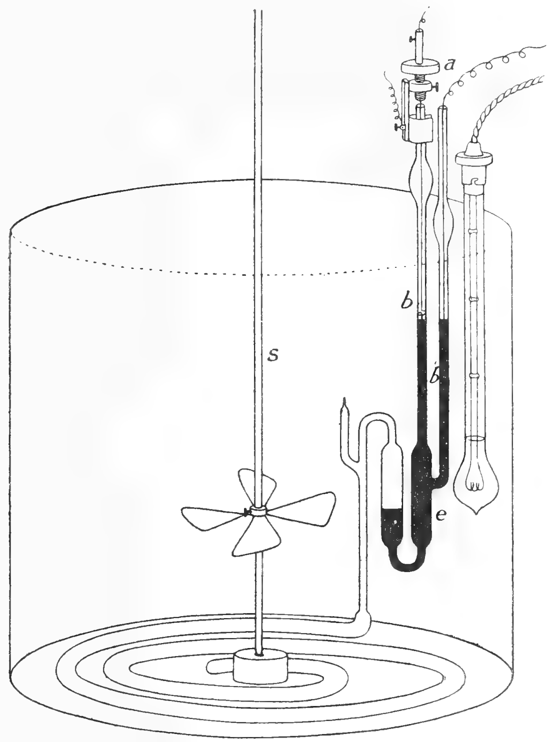


Fig. 1.

lamps were made by the Ediswan Company, according to Professor Hugh Marshall's specifications.

Glow-lamps with water-tight metal covers to protect the insulation of the wires were also tried, but they were not satisfactory. The air inside the cover and the glass of the glow-lamp acted as a store of heat, which continued to raise the temperature of the bath after the lamp had been extinguished. This caused a "lag" of several hundredths of a degree.

The thermo-regulator finally adopted consisted of a long spiral glass tube filled with toluol, and terminating in the U-shaped reservoir *e* (figs. 1 and 2) containing mercury.

The spiral shape was found most suitable, as a large surface is essential. The layer of toluol nearest the glass is the only part appreciably affected by the changes in temperature, and a large volume of toluol is therefore unnecessary provided the surface exposed to the water of the bath is sufficiently large.

The tubes *b* and *b′* have an internal diameter of about 2·5 mm., and are both left open at the top. Through *b′* a platinum wire passes, which is long enough to be constantly in contact with the mercury in the reservoir *e*. The tube *b* bears a movable platinum wire and its adjusting screw.

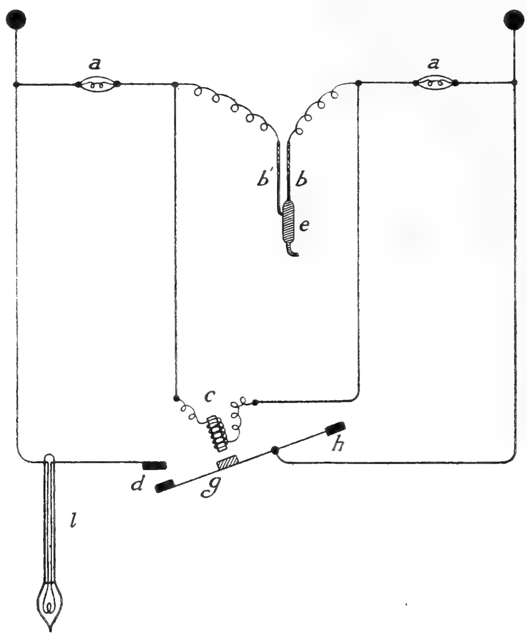


Fig. 2.

When the temperature of the bath rises, the expansion of the toluol raises the mercury in the tubes *b* and *b′* (fig. 1), makes contact in the tube *b*, and extinguishes the heating-lamps *l* (fig. 2).

The first rough adjustment of temperature is made by adding or removing mercury until the meniscus is about 6 inches from the top of the tubes *b* and *b′*. The final adjustment is made by raising or lowering the platinum wire in *b* by means of the screw adjustment *a* (fig. 1). The lower end of the adjustable platinum wire is twisted like the end of a cork-screw so that contact is made in the centre of the mercury surface. When contact is made at the side the delicacy of the adjustment is impaired owing to capillary action.

The source of electricity used was the Edinburgh Corporation lighting supply at 230 volts.

A diagram of the connections is shown in fig. 2.

The current for the relay and thermo-regulator is reduced by means of the two 8-candle-power glow-lamps *a*, *a*, which also prevent all risk from an accidental earthing of the wires. One of the chief difficulties encountered was the deterioration of the mercury contact due to sparking. This was overcome by arranging the mercury contact in parallel with the electromagnets of the relay which controls the heating circuit. When contact is broken the current goes through the electromagnets, and the potential difference across the contact in *b* (fig. 1) is comparatively small. Sparking is thereby greatly reduced.

When the thermostat is in continuous use it is necessary to clean the mercury contact about once a fortnight. To do this a small quantity of

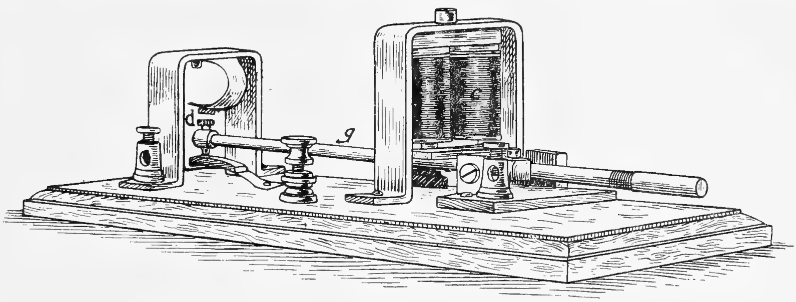


Fig. 3.

mercury is removed from the tube *b* by means of a long, narrow mercury pipette. The sides of the capillary are then rinsed once or twice with clear toluol. The mercury contact is finally covered with a little toluol. The whole operation requires less than five minutes. The contact in *b* is then re-adjusted.

The relay is shown in figs. 2 and 3.

The two electromagnets (figs. 2 and 3, *c*) are wound with 2620 turns of copper wire, 36 standard wire gauge (0·155 mm. diam.). The resistance of the electromagnets in series is 95·9 ohms. The diameter of the core is $\frac{3}{8}$ inch. When contact is made in *b* (fig. 2) by the thermo-regulator, practically no current goes through the electromagnets, and the rod *g* (figs. 2 and 3) falls by its own weight, breaking contact at *d* and extinguishing the heating-lamps. The shorter arm of the rod bears a nut which acts as a counterpoise, and can be screwed out or in until an adjustment is arrived at which gives the greatest sensibility compatible with certainty of action. This arrangement is much less liable to get out of order than any spring

adjustment. Owing to the sparking at d it was found necessary to arm the terminals with pieces of platinum. These pieces were riveted on, not soldered. They are 2–3 mm. thick, and their flat polished surfaces are about 2 mm. square.

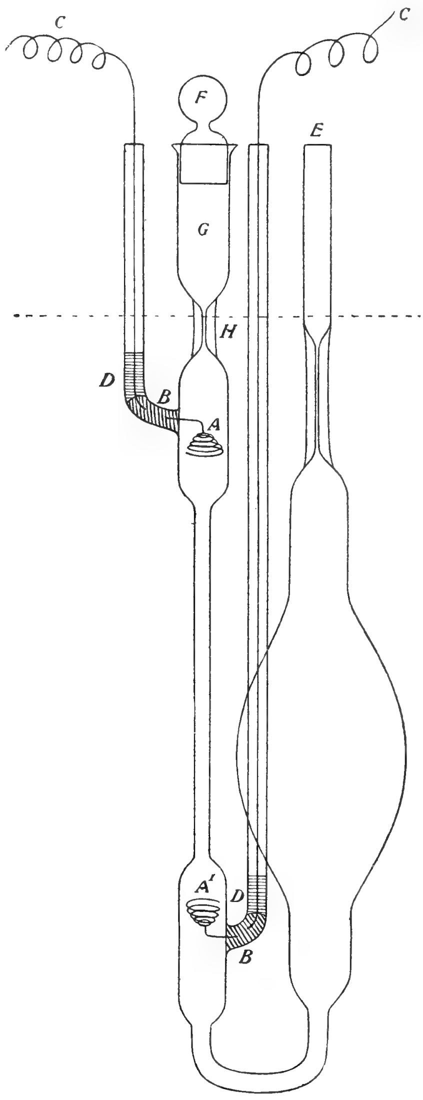


Fig. 4.

By means of the cell shown in fig. 4, polarisation and the heating effect of the current are avoided.

The following points were found to be important:—(1) The cross-section of the liquid between the electrodes must not be too small, or heating cannot

be avoided with a current which is strong enough to give a distinct minimum in the telephone. (2) The "capacity" of the cell must be high in order to prevent polarisation. (3) The cross-section must not be too large, or the cell becomes unwieldy.

A tube 6 cm. in length and 1·5 mm. in diameter was found to be suitable for concentrated sulphuric acid.

Sulphuric acid was chosen as, on the whole, the most suitable electrolyte with which to test the degree of accuracy attainable with highly conducting solutions. Hydrochloric acid has a higher conductivity than sulphuric acid, but at high concentration its volatility causes difficulties which affect the maintenance and determination of the concentration rather than the determination of the conductivity.

In fig. 4, A, A are electrodes of thick platinum wire wound in the form of a hollow cone. This form of electrode gives a uniform distribution of the lines of flow over the whole surface.

The ends of the electrodes are fused through the glass, and project into the mercury-cups B, B. The mercury and the platinum connecting wires (C, C) are kept in position by means of the layers of solid paraffin (D, D) so that the cell can be inverted without loss of mercury. The free ends of the wires dip into mercury-cups, which are connected by means of stout copper wires to the terminals of the Kohlrausch bridge. The resistance of the connections is determined by filling the cell with mercury and measuring the resistance.

The method of procedure with this cell is as follows:—

The cell is first cleaned with chromic acid mixture and washed with water. It is then dried in a current of air and weighed. The stopper F is then temporarily removed, a quantity of solution of known concentration introduced at G, the stopper replaced, and the cell weighed again.

The stopper is again temporarily removed. A piece of rubber tubing, provided with a clip and guarded by an air-purifying tube, is now attached to the cell at E. By blowing air through this tube the solution can be stirred, and different portions of it brought between the electrodes. The level of the solution is fixed above A by replacing the stopper. The cell is finally immersed in the thermostat to the level of the dotted line, and the conductivity determined. The cell is re-weighed again to show that there has been no increase in weight during the determination.

The solution adhering to the sides of the wider tube G above the capillary H is then washed into the cell with pure distilled water. The inside of the tube G is carefully dried with filter-paper, and the cell with solution is weighed again. The liquid in the cell is stirred and mixed as

described above, and the conductivity is again determined. The cell with solution is weighed again, and more water is added, and so, on till a series of readings of conductivity at different concentrations has been obtained. Weighings of the cell when empty, or the consecutive weighings with a given solution, never varied by more than one milligram, which is of the order of $\frac{1}{10,000}$ of the weight of the solution.

The bridge used is of the large roller type described in Kohlrausch and Holborn's *Leitvermogen der Electrolyte,* p. 42, and was made by Hartman and Braun. Calibrated by the method of Strouhal and Barus, no appreciable errors were found.

Extension resistances of 4·5 times the resistance of the bridge wire were connected to each end of the bridge. The resistances were found to be concordant among themselves.

The source of current is a small induction coil actuated by two storage cells in series. A Fischer interruptor independently actuated by two other cells, and having a very high and steady note, is used to make and break the primary circuit.

The strength of current going through the cell is regulated by a shunt put across the terminals of the secondary coil.

The shunt should be varied at each determination to prove the absence of heating in the cell. When a reduction in the shunt produces no change in the reading, the heating effect may be considered to be eliminated.

The introduction of a small condenser at some part of the circuit, as described by Kohlrausch and Maltby, was generally found to improve the minimum.* For the conductivity determinations recorded below, the sharpness of the minimum was not considered satisfactory unless a movement of the bridge wire through $\frac{4}{100,000}$ of its length to either side produced a distinctly perceptible increase of sound in the telephone. It was generally sharper.

The sulphuric acid used had been re-distilled, and was specially pure.

Twenty grammes of the acid evaporated in a platinum crucible gave no weighable residue.

By Nessler's test the acid was found to contain less than five parts per million of ammonia. It was free from nitrous, nitric, and sulphurous acids.

A stock solution was prepared by mixing equal parts of this acid and pure water. This solution was preserved in a stoppered glass bottle which had contained concentrated sulphuric acid for several years. The stopper was provided with a cover, which was sealed with paraffin to prevent any

* *Wissenschaftliche Abhandlungen der Physikalisch-Technischen Reichs Anstalt,* Band iii. (1900), p. 165.

absorption from the air. The concentration of this stock solution was determined by precipitation with barium chloride. All precautions were taken in the determinations, and a platinum Gooch crucible was used for the filtrations.

About 0·5 grammes of the solution were rapidly pipetted with a special pipette into a small weighed glass bottle containing about 10 c.c. of water. The increase in weight of the bottle gave the weight of solution taken. In this way any increase in weight during the weighing was avoided.

The volume of the filtrate and washings was measured, and a correction applied for the weight of $BaSO_4$ dissolved. The solubility was taken as 1 mg. of $BaSO_4$ in 400 c.c. of filtrate.

	Determination I.	*Determination II.*
Weight of solution taken (in air) .	0·5306 grms.	0·5331 grm.
Weight of $BaSO_4$ obtained (in air)	0·7674 grms.	0·7710 grm.
Volume of filtrate . . .	400 c.c.	390 c.c.

The molecular weight of $BaSO_4$ was taken as 233·46, and of H_2SO_4 as 98·076.

The weights used were of platinum, and were carefully calibrated. To allow for the buoyancy of the air the uncorrected percentages were multiplied by 0·9991. The percentages of H_2SO_4, as calculated from these data, are:—

$$(1)\ 60{\cdot}78_5\,; \quad (2)\ 60{\cdot}78_3.$$

The above details are recorded in order that the true percentages of H_2SO_4 may be calculated should the figures assumed above for the molecular weight and solubility of barium sulphate prove to be incorrect.

The capacity of the cell was determined with a normal solution of Merck's purest potassium chloride, which by previous trials had been found to give the same conductivity as specially pure chloride prepared from potassium perchlorate with all precautions. The conductivity for this concentration of potassium chloride is given by Kohlrausch and Maltby as 0·09828.

The molecular weight of barium sulphate is only known to four significant figures. The concentrations can therefore only be vouched for to 1 in 2000 at most.

For the higher concentrations the conductivity varies rapidly with the concentration, and the accuracy of the conductivity determination is greater than is required. Near the maximum, however, the conductivity varies slowly with concentration, and the full accuracy is required. The limit of error of the conductivity determination is less than 3 in 10,000, as can be seen from Table I.

The determinations recorded in Tables I. and II. were all made at 18°·00 centigrade. From first to last the temperature of the bath never varied by as much as $\frac{1}{500}$° C.

TABLE I.

I.	II.	III.		IV.	V.	VI.	VII.	VIII.
Date.	Hour.	Capacity in parallel with		Shunt on Secondary of Coil.	Resistance.	Reading of Bridge	Percent. Concentration of H_2SO_4.	Specific Conductivity (mean).
		(A) Cell.	(B) Resistance.					
4/3/07	11.20 a.m.	18·0	0	30	500	5·0918	60·78	3662·0
”	12.5 p.m	16·5	0	20	500	5·0918	”	
5/3/07	9.30 a.m.	16·5	0	20	500	5·0918	”	
”	4.0 p.m.	18·5	0	20	500	5·0923	”	3662·8
”	4.40 p.m.	18·5	0	10	500	5·0923	”	
18/3/07	6.5 p.m.	18·0	0	30	500	5·0922	”	3662·5
”	6.10 p.m.	18·0	0	20	500	5·0920	”	
5/3/07	10.45 a.m.	22·5	0	30	500	5·3470	58·45	4056·6
”	10.50 a.m.	22·5	0	50	500	5·3470	”	
”	11.5 a.m.	22·5	0	50	500	5 3468	”	
”	11.10 a.m	22·5	0	20	500	5·3470	”	
6/3/07	10.45 a.m.	0	0	30	400	5·1767	54·45	4735·7
”	10.50 a.m.	0	0	20	400	5·1767	”	
”	10.55 a.m.	0	0	20	400	5·1766	”	
”	11.0 a.m.	0	0	20	400	5·1765	”	
”	11.40 a.m.	6	0	20	400	5·4703	50·86	5329·0
”	11 50 a.m.	6	0	20	400	5·4702	”	
”	11.55 a.m.	6	0	30	400	5·4702	”	
”	4.45 p.m.	0	0	20	300	5·0577	46·43	6022·1
”	4.50 p.m.	0	0	20	300	5·0578	”	
”	5.0 p.m.	0	0	30	300	5·0577	”	
”	5.5 p.m.	0	0	20	300	5·0577	”	
14/3/07	12.15 p.m.	0	0	20	300	5·0576	”	
”	1.20 p.m.	10	0	20	300	5·4530	37·86	7057·4
”	1.25 p.m.	10	0	40	300	5·4531	”	
”	1.30 p.m.	10	0	40	300	5·4530	”	
16/3/07	11.0 a.m.	12	0	40	300	5·4531	”	
”	12.20 p.m.	0	12	40	200	4·5452	33·50	7355·6
”	12.45 p.m.	0	12	20	200	4·5453	”	
”	12.50 p.m.	0	12	20	200	4·5453	”	
18/3/07	10.25 a.m.	0	13	40	200	4·5667	30·91	7418·4
”	10.30 a.m.	0	13	20	200	4·5667	”	
”	10.35 a.m.	0	13	20	200	4·5665	”	
”	11.20 a.m.	0	12	20	200	4·5267	26·44	7300·1
”	11.30 a.m.	0	12	30	200	4·5269	”	

The columns are:—

I. and II. The date and hour of the conductivity determinations.

III. The capacity necessary for a sharp minimum—

(*a*) When in parallel with the cell.

(*b*) When in parallel with the resistances.

The figures refer to square centimetres of tinfoil, and are only relative.

IV. The shunt in ohms, across the terminals of the secondary coil. A rise in the shunt means an increase in the current through the cell. This column along with Column VI. shows the elimination of heating effect.

V. The resistance with which the resistance of the cell was compared. The resistances used have the certificate of the Reichsamt in Berlin.

VI. The actual readings of the bridge. The middle part of the bridge wire was used throughout, so that 1 in the fourth place of decimals corresponds to 1 in 25,000 of the resistance to be measured.

VII. The concentration of the solution used in each case. The concentration of the stock solution was determined gravimetrically with every precaution. The percentages of H_2SO_4 found were :—

$$(1)\ 60{\cdot}78_5\,; \quad (2)\ 60{\cdot}78_3.$$

Those that follow in the column were calculated from the weight of water added to the stock solution. As a check, the concentration of the last and most dilute solution was determined by precipitation with barium chloride as well as by dilution. The percentages of H_2SO_4 found were :—

(1) by dilution, 26·44 ; (2) gravimetrically 26·44.

VIII. The specific conductivity in $\overset{-1}{\text{ohm}}\ \overset{-1}{\text{cm}}$ multiplied by 10^4. These were calculated from the means of the corresponding bridge readings.

TABLE II.

I.	II.	III.		IV.	V.	VI.
Date.	Hour.	Capacity in parallel with (A) Cell.	Capacity in parallel with (B) Resistances.	Shunt on Secondary of Coil.	Resistance.	Reading of Bridge.
1/3/07	4.30 p.m.	0	15	30	1700	4·8635
"	4.40 p.m.	0	15	20	1700	4·8635
2/3/07	11.45 a.m.	0	16·5	20	1700	4·8636
"	11.50 a.m.	0	16·5	30	1700	4·8636
5/3/07	12.0 p.m.	0	5·5	50	1700	4·8635
"	12.15 p.m.	0	5·5	30	1700	4·8636
18/3/07	3.15 p.m.	0	7	20	1700	4·8637
"	3.20 p.m.	0	7	50	1700	4·8638

In this table are given the determinations of the capacity of the cell. What has been said above for columns I. to VI. of Table I. applies also to

this table. The capacity was calculated from the mean of all these readings. It was found to be 176·45.

The figures in Tables I. and II. demonstrate the accuracy of which the method is capable.

We desire, in conclusion, to acknowledge gratefully that the greater part of the cost of the apparatus used for this investigation was met by a Research Grant from the Carnegie Trust.

Heriot-Watt College,
Edinburgh.

(*Issued separately February* 18, 1910.)

XIII.—The Theory of Orthogonants in the Historical Order of Development up to 1860. By Thomas Muir, LL.D.

(MS. received October 25, 1909. Read November 22, 1909.)

NOTWITHSTANDING the generalisations made by Jacobi and Cauchy, the special case with which the whole theory originated continued from time to time to attract attention. In 1843 William Thomson, afterwards known as Lord Kelvin, published under the signature "T." in the *Cambridge Math. Journ.*, iii. pp. 247–248, a short note in which he proved the detached theorem that if $l_1, m_1, n_1, l_2, \ldots$ be nine quantities such that

$$\begin{aligned} l_1^2+m_1^2+n_1^2 &= 1, & l_1l_2+m_1m_2+n_1n_2 &= 0, \\ l_2^2+m_2^2+n_2^2 &= 1, & l_2l_3+m_2m_3+n_2n_3 &= 0, \\ l_3^2+m_3^2+n_3^2 &= 1, & l_3l_1+m_3m_1+n_3n_1 &= 0, \end{aligned}$$

then it follows that

$$\begin{aligned} l_1^2+l_2^2+l_3^2 &= 1, & l_1m_1+l_2m_2+l_3m_3 &= 0, \\ m_1^2+m_2^2+m_3^2 &= 1, & m_1n_1+m_2n_2+m_3n_3 &= 0, \\ n_1^2+n_2^2+n_3^2 &= 1, & n_1l_1+n_2l_2+n_3l_3 &= 0. \end{aligned}$$

This led to a short paper by A. Göpel in the *Archiv d. Math. u. Phys.*, iv. (1843) pp. 244–246. The subject was again taken up in 1848 by L. Schläfli in the *Mitteilungen d. naturf. Ges. in Bern*, Nos. 112, 113, pp. 27–33,* and in 1850 by V.-A. Lebesgue in the *Nouv. Annales de Math.*, ix. pp. 46–51. Details of these papers need not be given. We may take the opportunity to note, however, that after the appearance of Cayley's paper on matrices in 1857 the known general theorem embracing that just mentioned might have been briefly formulated by saying that—*If* $MM'=1$, *where* M *is any square matrix and* M′ *its conjugate, then also* $M'M=1$.

KUMMER, E. E. (1843).

[Bemerkungen über die cubische Gleichung, durch welche die Haupt-Axen der Flächen zweiten Grades bestimmt werden. *Crelle's Journ.*, xxvi. pp. 268–272.]

To prove the reality of all the roots of the equation mentioned in the title of his paper—a problem first solved by Lagrange in 1773—Kummer

* Published also in *Archiv d. Math. u. Phys.*, xiii. pp. 276–281.

sought to show that the expression for the product of their squared differences was inherently positive. This he succeeded in doing by transforming the said expression into a sum of squares, the result being reached by proceeding from particular to general, and by a combined process of guess and test. The equation being

$$\begin{vmatrix} a-x & h & g \\ h & b-x & f \\ g & f & c-x \end{vmatrix} = 0,$$

or, say,

$$x^3 - Px^2 + Qx - R = 0,$$

the expression referred to is*

$$P^2Q^2 - 4P^3R + 18PQR - 4Q^3 - 27R^2;$$

and Kummer's equivalent for it is

$$\begin{aligned} &15\overset{\circ}{\sum}[gh(b-c)+f(g^2-h^2)]^2 \\ &+\overset{\circ}{\sum}[2(b-c)(c-a)h+(2c-a-b)fg+(2h^2-f^2-g^2)h]^2 \\ &+\quad [(b-c)(c-a)(a-b)+(b-c)f^2+(c-a)g^2+(a-b)h^2]^2, \end{aligned}$$

where $\overset{\circ}{\Sigma}$ indicates the summing of the expressions obtained by performing simultaneously the cyclical substitutions

$$\begin{pmatrix} a & b & c \\ b & c & a \end{pmatrix}, \quad \begin{pmatrix} f & g & h \\ g & h & f \end{pmatrix}.$$

JACOBI, C. G. J. (1844, March).

[Sulla condizione di ugualianza di due radici dell' equazione cubica, dalla quale dipendono gli assi principali di una superficie del second' ordine. *Giornale Arcadico,* xcix. pp. 3–11: or *Crelle's Journ.*, xxx. pp. 46–50: or *Gesammelte Werke,* i. pp. 271–276.]

By using A, B, . . . for $bc-f^2$, $ca-g^2$, . . . Jacobi first puts Kummer's sum of squares in a neater form, namely,

$$\begin{aligned} 15\overset{\circ}{\sum}(gH-hG)^2 &+ \overset{\circ}{\sum}(bF-fB+cF-fC-2aF+2fA)^2 \\ &+(bC-cB+cA-aC+aB-bA)^2, \end{aligned}$$

and he then gives a lengthy but thorough verification of its accuracy.

From the fundamental identity

$$\begin{aligned} &ax^2+by^2+cz^2+2fyz+2gzx+2hxy \\ &= L(\alpha_1x+\beta_1y+\gamma_1z)^2+M(\alpha_2x+\beta_2y+\gamma_2z)^2+N(\alpha_3x+\beta_3y+\gamma_3z)^2, \end{aligned}$$

* *I.e.* $-\frac{1}{3}$ of what afterwards came to be called the *discriminant* of $x^3-Px^2y+Qxy^2-Ry^3$.

where L, M, N are, as in his paper of 1827, the roots whose reality is to be established, he obtains at once

$$\begin{aligned} a &= L\alpha_1^2 + M\alpha_2^2 + N\alpha_3^2, & f &= L\beta_1\gamma_1 + M\beta_2\gamma_2 + N\beta_3\gamma_3, \\ b &= L\beta_1^2 + M\beta_2^2 + N\beta_3^2, & g &= L\gamma_1\alpha_1 + M\gamma_2\alpha_2 + N\gamma_3\alpha_3, \\ c &= L\gamma_1^2 + M\gamma_2^2 + N\gamma_3^2, & h &= L\alpha_1\beta_1 + M\alpha_2\beta_2 + N\alpha_3\beta_3, \end{aligned}$$

and thence derives

$$\begin{aligned} A &= MN\alpha_1^2 + NL\alpha_2^2 + LM\alpha_3^2, & F &= MN\beta_1\gamma_1 + NL\beta_2\gamma_2 + LM\beta_3\gamma_3, \\ B &= MN\beta_1^2 + NL\beta_2^2 + LM\beta_3^2, & G &= MN\gamma_1\alpha_1 + NL\gamma_2\alpha_2 + LM\gamma_3\alpha_3, \\ C &= MN\gamma_1^2 + NL\gamma_2^2 + LM\gamma_3^2, & H &= MN\alpha_1\beta_1 + NL\alpha_2\beta_2 + LM\alpha_3\beta_3. \end{aligned}$$

From these it can be shown with more or less trouble * that

$$\begin{aligned} gH - hG &= \Pi \cdot \alpha_1\alpha_2\alpha_3, \\ bF - fB + cF - fC - 2aF + 2fA &= \Pi \cdot (\alpha_1\beta_2\beta_3 + \alpha_2\beta_3\beta_1 + \alpha_3\beta_1\beta_2 - \alpha_1\gamma_2\gamma_3 - \alpha_2\gamma_3\gamma_1 - \alpha_3\gamma_1\gamma_2), \\ bC - cB + cA - aC + aB - bA &= \Pi \cdot (\alpha_1\beta_2\gamma_3 + \alpha_2\beta_3\gamma_1 + \alpha_3\beta_1\gamma_2 + \alpha_1\beta_3\gamma_2 + \alpha_2\beta_1\gamma_3 + \alpha_3\beta_2\gamma_1), \end{aligned}$$

where Π stands for $(L-M)(M-N)(N-L)$. Kummer's sum of squares is thus made to take the form

$$\begin{aligned} \Pi^2 \cdot \Big[15\overset{\circ}{\sum}(\alpha_1\alpha_2\alpha_3)^2 + \overset{\circ}{\sum}\{\alpha_1(\beta_2\beta_3 - \gamma_2\gamma_3) + \alpha_2(\beta_3\beta_1 - \gamma_3\gamma_1) + \alpha_3(\beta_1\beta_2 - \gamma_1\gamma_2)\}^2 \\ + \Big\{\alpha_1(\beta_2\gamma_3 + \beta_3\gamma_2) + \alpha_2(\beta_3\gamma_1 + \beta_1\gamma_3) + \alpha_3(\beta_1\gamma_2 + \beta_2\gamma_1)\Big\}^2 \Big], \end{aligned}$$

where we use $\overset{\circ}{\Sigma}$ to indicate the sum of a set of terms produced by the cyclical substitution $\alpha \rightarrow \beta$, $\beta \rightarrow \gamma$, $\gamma \rightarrow \alpha$. After this the cofactor of Π^2 is shown with seeming ease to be 1, and the desired result is reached.

A knowledge of the relationships existing between the elements of the orthogonant $|\alpha_1\beta_2\gamma_3|$ is, of course, a constant requirement throughout the demonstration; and to two of these relationships special attention is drawn by Jacobi himself. The first is

$$\begin{aligned} 2\{\alpha_1^2\alpha_2^2\alpha_3^2 + \beta_1^2\beta_2^2\beta_3^2 + \gamma_1^2\gamma_2^2\gamma_3^2\} \\ = \alpha_1\beta_2\gamma_3 \cdot \alpha_2\beta_3\gamma_1 + \alpha_2\beta_3\gamma_1 \cdot \alpha_3\beta_1\gamma_2 + \alpha_3\beta_1\gamma_2 \cdot \alpha_1\beta_2\gamma_3 \\ + \alpha_1\beta_3\gamma_2 \cdot \alpha_2\beta_1\gamma_3 + \alpha_2\beta_1\gamma_3 \cdot \alpha_3\beta_2\gamma_1 + \alpha_3\beta_2\gamma_1 \cdot \alpha_1\beta_3\gamma_2, \end{aligned}$$

* The modern reader would do well to use Binet's theorem regarding the determinant which is viewable as the product of two rectangular arrays. Thus

$$\begin{aligned} A = \begin{vmatrix} b & f \\ f & c \end{vmatrix} &= \begin{Vmatrix} L\beta_1 & M\beta_2 & N\beta_3 \\ L\gamma_1 & M\gamma_2 & N\gamma_3 \end{Vmatrix} \cdot \begin{Vmatrix} \beta_1 & \beta_2 & \beta_3 \\ \gamma_1 & \gamma_2 & \gamma_3 \end{Vmatrix}, \\ &= LM\,|\beta_1\gamma_2|^2 + MN\,|\beta_2\gamma_3|^2 + NL\,|\beta_1\gamma_3|^2 = LM\alpha_3^2 + MN\alpha_1^2 + NL\alpha_2^2 : \end{aligned}$$

$$\begin{vmatrix} g & h \\ G & H \end{vmatrix} = \begin{Vmatrix} L & M & N \\ MN & NL & LM \end{Vmatrix} \cdot \begin{Vmatrix} \gamma_1\alpha_1 & \gamma_2\alpha_2 & \gamma_3\alpha_3 \\ \alpha_1\beta_1 & \alpha_2\beta_2 & \alpha_3\beta_3 \end{Vmatrix} = N(L^2 - M^2) \cdot \alpha_1\alpha_2\,|\gamma_1\beta_2| + \ldots .$$

and so forth.

and the second is

$$\alpha_1^2\alpha_2^2\alpha_3^2 + \beta_1^2\beta_2^2\beta_3^2 + \gamma_1^2\gamma_2^2\gamma_3^2 = \alpha_1^2\beta_1^2\gamma_1^2 + \alpha_2^2\beta_2^2\gamma_2^2 + \alpha_3^2\beta_3^2\gamma_3^2,$$

the latter's existence being due to the fact that the right-hand member of the former is not altered by the interchange

$$\begin{pmatrix} \alpha_2 & \alpha_3 & \beta_3 \\ \beta_1 & \gamma_1 & \gamma_2 \end{pmatrix}.$$

BORCHARDT, C. W. (1845, January).

[Neue Eigenschaft der Gleichung, mit deren Hülfe man die seculären Störungen der Planeten bestimmt. *Crelle's Journ.*, xxx. pp. 38–45: or, in an extended form, *Journ. (de Liouville) de Math.*, xii. pp. 50–67: or *Werke*, pp. 3–13.]

Borchardt's "new property" is the naturally desirable generalisation of Kummer's identity. In his mode of designating the equation* he does not follow Kummer and Jacobi, but goes back to Cauchy (1829), the implied reference being to Laplace's *Mécanique Céleste*, partie i., livre ii., § 56 (1799).

The set of equations, from which by elimination there is obtained the equation referred to in the title, being

$$\left.\begin{array}{rcl} gx_1 & = & a_{11}x_1 + a_{12}x_2 + \ . \ . \ . + a_{1n}x_n \\ gx_2 & = & a_{21}x_1 + a_{22}x_2 + \ . \ . \ . + a_{2n}x_n \\ . & . & . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \\ gx_n & = & a_{n1}x_1 + a_{n2}x_2 + \ . \ . \ . + a_{nn}x_n \end{array}\right\},$$

where $a_{ik} = a_{ki}$, Borchardt multiplies the two sides of each equation of the set by g, and then on the right-hand side substitutes for $gx_1, gx_2, \ldots, gx_n$ their equivalents as given. There thus results the new set

$$\left.\begin{array}{rcl} g^2x_1 & = & a_{11}^{(2)}x_1 + a_{12}^{(2)}x_2 + \ . \ . \ . + a_{1n}^{(2)}x_n \\ g^2x_2 & = & a_{21}^{(2)}x_1 + a_{22}^{(2)}x_2 + \ . \ . \ . + a_{2n}^{(2)}x_n \\ . & . & . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \\ g^2x_n & = & a_{n1}^{(2)}x_1 + a_{n2}^{(2)}x_2 + \ . \ . \ . + a_{nn}^{(2)}x_n \end{array}\right\},$$

where $a_{ik}^{(2)} = a_{ki}^{(2)} = \sum_{s=1}^{s=n} a_{is}a_{sk}$. Repeating this operation he finds generally that

* The most appropriate designation would seem to be "Lagrange's determinantal equation," because of this mathematician's early (1773) and successful investigation of the cubic. See his *Œuvres complètes*, iii. pp. 600–603.

$$\left.\begin{array}{lcl} g^m x_1 &=& a_{11}^{(m)}x_1 + a_{12}^{(m)}x_2 + \ . \ . \ . + a_{1n}^{(m)}x_n \\ g^m x_2 &=& a_{21}^{(m)}x_1 + a_{22}^{(m)}x_2 + \ . \ . \ . + a_{2n}^{(m)}x_n \\ . \ . \ . & & . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \\ g^m x_n &=& a_{n1}^{(m)}x_1 + a_{n2}^{(m)}x_2 + \ . \ . \ . + a_{nn}^{(m)}x_n \end{array}\right\} ,$$

where

$$a_{ik}^{(m)} = a_{ki}^{(m)} = \sum_{s_1=1}^{s_1=n} \sum_{s_2=1}^{s_2=n} \cdots \sum_{s_{m-1}=1}^{s_{m-1}=n} a_{is_1}a_{s_1s_2}a_{s_2s_3} \ldots a_{s_{m-1}k} .$$

In the next place, $g_1, g_2, \ldots . g_n$ being the roots of the resultant of the initial set of equations, it is readily seen, from the expression for the said resultant when arranged according to descending powers of g, that

$$g_1 + g_2 + \ . \ . \ . + g_n = a_{11} + a_{22} + \ . \ . \ . + a_{nn} .$$

Similarly, by considering the resultant of the second set of equations we learn that

$$g_1^2 + g_2^2 + \ . \ . \ . + g_n^2 = a_{11}^{(2)} + a_{22}^{(2)} + \ . \ . \ . + a_{nn}^{(2)} ,$$

and generally that

$$g_1^m + g_2^m + \ . \ . \ . + g_n^m = a_{11}^{(m)} + a_{22}^{(m)} + \ . \ . \ . + a_{nn}^{(m)} .$$

Consequently, if we use s_m to stand for the sum of the m^{th} powers of the g's we have

$$s_m = \sum_{i=1}^{i=n} a_{ii}^{(m)} .$$

In the third place the difference-product of the g's being

$$\sum (\pm g_1^0 g_2^1 g_3^2 \ . \ . \ . \ g_n^{n-1}),$$

Borchardt has only to use the multiplication-theorem of determinants to obtain as an equivalent for the product of the squared differences the determinant of the system

$$\begin{array}{ccccc} s_0 & s_1 & s_2 & \ldots & s_{n-1} \\ s_1 & s_2 & s_3 & \ldots & s_n \\ s_2 & s_3 & s_4 & \ldots & s_{n+1} \\ . & . & . & \ldots & . \\ s_{n-1} & s_n & s_{n+1} & \ldots & s_{2n-2} . \end{array}$$

It is this last determinant, therefore, which he has to aim at expressing as a sum of squares.

The process devised by him for doing so is very interesting. Returning to the original set of equations and the sets derived therefrom, he takes the μ^{th} set and multiplies both sides of each equation by g^ν and then on the right-hand side substitutes for $g^\nu x_1, \ g^\nu x_2, \ . \ . \ . \ , \ g^\nu x_n$ their equivalents as

obtainable from the ν^{th} set. A comparison of the results with the equations the $(\mu+\nu)^{\text{th}}$ set, he says, gives the noteworthy result

$$a_{ik}^{(\mu+\nu)} = \sum_{s=1}^{s=n} a_{si}^{(\mu)} a_{sk}^{(\nu)},$$

or

$$a_{ik}^{(m)} = \sum_{s=1}^{s=n} a_{si}^{(r)} a_{sk}^{(m-r)}.$$

This includes, of course, the recurrent law of formation

$$a_{ik}^{(m)} = \sum_{s=1}^{s=n} a_{si} a_{sk}^{(m-1)}$$

if we remember that by implication $a_{si}^{(1)}$ must be viewed to be the same as a_{si}. The variety of expressions for $a_{ik}^{(m)}$ which the identity gives makes possible a like variety for

$$a_{11}^{(m)} + a_{22}^{(m)} + \ . \ . \ . \ + a_{nn}^{(m)},$$

that is, for s_m. We may, in fact, put as an equivalent for s_m any one of the $m-1$ expressions got from

$$\sum_{i=1}^{i=n} \sum_{s=1}^{s=n} a_{si}^{(r)} a_{si}^{(m-r)}$$

by taking $r=1, 2, \ . \ . \ . \ , m-1$. We may even obtain an m^{th} equivalent by making $r=0$ if we agree to consider $a_{i}^{()}=0$ or 1 according as s is different from i or the same as i; in other words, if we agree to place before the original set of equations the set

$$\left.\begin{array}{lcl} g^0 x_1 & = & 1x_1 + 0x_2 + \ . \ . \ . \ + 0x_n \\ g^0 x_2 & = & 0x_1 + 1x_2 + \ . \ . \ . \ + 0x_n \\ . \ . & . & . \ . \ . \ . \ . \ . \ . \ . \\ g^0 x_n & = & 0x_1 + 0 \cdot x_2 + \ . \ . \ . \ + 1x_n \end{array}\right\}$$

the truth of which is incontestable. As a consequence the array of s's above given may be replaced by

$$\begin{array}{lllll} \sum\sum a_{ik}^{(0)} a_{ik}^{(0)} & \sum\sum a_{ik}^{(0)} a_{ik}^{(1)} & \sum\sum a_{ik}^{(0)} a_{ik}^{(2)} & \ . \ . \ . & \sum\sum a_{ik}^{(0)} a_{ik}^{(n-1)} \\ \sum\sum a_{ik}^{(1)} a_{ik}^{(0)} & \sum\sum a_{ik}^{(1)} a_{ik}^{(1)} & \sum\sum a_{ik}^{(1)} a_{ik}^{(2)} & \ . \ . \ . & \sum\sum a_{ik}^{(1)} a_{ik}^{(n-1)} \\ \sum\sum a_{ik}^{(2)} a_{ik}^{(0)} & \sum\sum a_{ik}^{(2)} a_{ik}^{(1)} & \sum\sum a_{ik}^{(2)} a_{ik}^{(2)} & \ . \ . \ . & \sum\sum a_{ik}^{(2)} a_{ik}^{(n-1)} \\ . \ . \ . \ . & . \ . \ . \ . & . \ . \ . \ . & . \ . \ . & . \ . \ . \ . \\ \sum\sum a_{ik}^{(n-1)} a_{ik}^{(0)} & \sum\sum a_{ik}^{(n-1)} a_{ik}^{(1)} & \sum\sum a_{ik}^{(n-1)} a_{ik}^{(2)} & \ . \ . \ . & \sum\sum a_{ik}^{(n-1)} a_{ik}^{(n-1)}, \end{array}$$

where s_2, for example, is represented in the 1st, 2nd, 3rd rows by

$$\sum\sum a_{ik}^{(0)} a_{ik}^{(2)}, \quad \sum\sum a_{ik}^{(1)} a_{ik}^{(1)}, \quad \sum\sum a_{ik}^{(2)} a_{ik}^{(0)}$$

respectively, and where by reason of the range of the two Σ's each element is the sum of n^2 binary products. Any said element may thus be represented as the product of two rows of n^2 elements each, and a little examination shows that only n rows of the latter kind are necessary for the representation of all. In other words, the array of s's can be represented by the product obtained by multiplying the array

$$\begin{array}{cccccccccccc} a_{11}^{(0)} & a_{12}^{(0)} & \ldots & a_{1n}^{(0)} & a_{21}^{(1)} & a_{22}^{(0)} & \ldots & a_{2n}^{(0)} & a_{31}^{(0)} & \ldots\ldots & a_{nn}^{(0)} \\ a_{11}^{(1)} & a_{12}^{(1)} & \ldots & a_{1n}^{(1)} & a_{21}^{(1)} & a_{22}^{(1)} & \ldots & a_{2n}^{(1)} & a_{31}^{(1)} & \ldots\ldots & a_{nn}^{(1)} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ a_{11}^{(n-1)} & a_{12}^{(n-1)} & \ldots & a_{1n}^{(n-1)} & a_{21}^{(n-1)} & a_{22}^{(n-1)} & \ldots & a_{2n}^{(n-1)} & a_{31}^{(n-1)} & \ldots\ldots & a_{nn}^{(n-1)} \end{array}$$

by itself, and therefore is, by Binet's theorem, expressible as a sum of squares.

By way of illustration, Borchardt takes the case where $n=3$. The product of the squared differences of the roots is then, in later notation,

$$\left\| \begin{array}{ccccccccc} 1 & \cdot & \cdot & \cdot & 1 & \cdot & \cdot & \cdot & 1 \\ a & h & g & h & b & f & g & f & c \\ r_1r_1 & r_1r_2 & r_1r_3 & r_2r_1 & r_2r_2 & r_2r_3 & r_3r_1 & r_3r_2 & r_3r_3 \end{array} \right\|^2$$

where $r_\alpha r_{\beta\rfloor}$ means the product of the α^{th} and β^{th} rows of

$$\left| \begin{array}{ccc} a & h & g \\ h & b & f \\ g & f & c \end{array} \right| .$$

By performing on the 3-by-9 array the operation

$$\text{row}_3 - (a+b+c)\,\text{row}_2 + (ab+bc+ca-f^2-g^2-h^2)\,\text{row}_1$$

there is obtained

$$\left\| \begin{array}{ccccccccc} 1 & \cdot & \cdot & \cdot & 1 & \cdot & \cdot & \cdot & 1 \\ a & h & g & h & b & f & g & f & c \\ \mathrm{A} & \mathrm{H} & \mathrm{G} & \mathrm{H} & \mathrm{B} & \mathrm{F} & \mathrm{G} & \mathrm{F} & \mathrm{C} \end{array} \right\|^2$$

which is readily shown to be equal to Kummer's sum of squares.

It is a little curious that Borchardt nowhere draws attention to the fact that the determinant of the coefficients in the right-hand members of his m^{th} set of equations is the m^{th} power of the determinant of the corresponding coefficients of the original set.

JACOBI, C. G. J. (1845, August).

[Ueber ein leichtes Verfahren die in der Theorie der Säcular-störungen vorkommenden Gleichungen numerisch aufzulösen. *Crelle's Journ.*, xxx. pp. 51–94: or *Gesammelte Werke*, i. pp. 227–270: or *Nouv. Annales de Math.*, x. pp. 258–265.]

This long memoir being intended for astronomical mathematicians and computers, there is little of it that concerns us except two of the introductory sections (§§ 2, 3, pp. 52–56); and even these need not detain us, as they are in effect but a well-constructed abstract of Cauchy's paper of 1829, the starting-point being the set of $n+1$ equations

$$\left.\begin{array}{rrcrl} (a_{11}-\theta)x_1 + & a_{12}x_2 + & \dots & + a_{1n}x_n & = 0 \\ a_{21}x_1 + & (a_{22}-\theta)x_2 + & \dots & + a_{2n}x_n & = 0 \\ \dots & \dots & \dots & \dots & \dots \\ a_{n1}x_1 + & a_{n2}x_2 + & \dots & + (a_{nn}-\theta)x_n & = 0 \\ x_1^2 + & x_2^2 + & \dots & + \quad x_n^2 & = 1 \end{array}\right\}$$

considered without any regard to the mode in which they may have originated.

CAYLEY, A. (1846).

[Sur quelques propriétés des déterminants gauches. *Crelle's Journ.*, xxxii. pp. 119–123: or *Collected Math. Papers*, i. pp. 332–336.]

There is clear evidence that Rodrigues' paper of 1840 made a strong impression upon Cayley. In a paper published in 1843 * he introduces his subject by speaking of Rodrigues as having "given some very elegant formulæ for determining the position of two sets of rectangular axes with respect to each other, employing rational functions of three quantities only"; and he proceeds at once to demonstrate these formulæ as a necessary preliminary to the essential part of his paper. In another paper published in 1845,† the first part of which deals with a quaternion identity, he makes the important observation that a set of nine coefficients which occur in the identity is precisely the same as the set of nine given in Rodrigues' transformation; and he adds, "It would be an interesting question to account *à*

* Cayley, A., "On the motion of rotation of a solid body." *Cambridge Math. Journ.*, iii. pp. 224–232: or *Collected Math. Papers*, i. pp. 28–35.

† Cayley, A., "On certain results relating to quaternions." *Philos. Magazine*, xxvi. pp. 141–145: or *Collected Math. Papers*, i. pp. 123–126.

priori for the appearance of these coefficients here." We are thus not wholly unprepared for a communication from Cayley himself on the subject of the construction of a linear substitution for the transformation of $x_1^2+x_2^2+\ \ldots$ into $\xi_1^2+\xi_2^2+\ \ldots$. The following is his procedure, four variables being used in place of his n.

With unity and any six quantities whatever there is first formed the square array

$$\begin{array}{cccc} 1 & l_{12} & l_{13} & l_{14} \\ -l_{12} & 1 & l_{23} & l_{24} \\ -l_{13} & -l_{23} & 1 & l_{34} \\ -l_{14} & -l_{24} & -l_{34} & 1, \end{array} \qquad \text{or say} \qquad \begin{array}{cccc} l_{11} & l_{12} & l_{13} & l_{14} \\ l_{21} & l_{22} & l_{23} & l_{24} \\ l_{31} & l_{32} & l_{33} & l_{34} \\ l_{41} & l_{42} & l_{43} & l_{44} \end{array}$$

where $l_{rr}=1$ and $l_{rs}=-l_{sr}$. Then taking a new set of four variables $\theta_1, \theta_2, \theta_3, \theta_4$, and using for their coefficients the quantities in the square array, firstly as disposed in rows, and secondly as disposed in columns, he puts

$$\left.\begin{array}{rcl} l_{11}\theta_1+l_{12}\theta_2+l_{13}\theta_3+l_{14}\theta_4 &=& x_1 \\ l_{21}\theta_1+l_{22}\theta_2+l_{23}\theta_3+l_{24}\theta_4 &=& x_2 \\ l_{31}\theta_1+l_{32}\theta_2+l_{33}\theta_3+l_{34}\theta_4 &=& x_3 \\ l_{41}\theta_1+l_{42}\theta_2+l_{43}\theta_3+l_{44}\theta_4 &=& x_4 \end{array}\right\}$$

and

$$\left.\begin{array}{rcl} l_{11}\theta_1+l_{21}\theta_2+l_{31}\theta_3+l_{41}\theta_4 &=& \xi_1 \\ l_{12}\theta_1+l_{22}\theta_2+l_{32}\theta_3+l_{42}\theta_4 &=& \xi_2 \\ l_{13}\theta_1+l_{23}\theta_3+l_{33}\theta_3+l_{43}\theta_4 &=& \xi_3 \\ l_{14}\theta_1+l_{24}\theta_4+l_{34}\theta_3+l_{44}\theta_4 &=& \xi_4 \end{array}\right\},$$

thereby ensuring that $x_1{}^2+x_2{}^2+\ldots=\xi_1{}^2+\xi_2{}^2+\ldots$ Solving the two sets of equations separately for each of the θ's and equating the results he next obtains

$$\left.\begin{array}{rcl} L_{11}x_1+L_{21}x_2+L_{31}x_3+L_{41}x_4 &=& L_{11}\xi_1+L_{12}\xi_2+L_{13}\xi_3+L_{14}\xi_4 \\ L_{12}x_1+L_{22}x_2+L_{32}x_3+L_{42}x_4 &=& L_{21}\xi_1+L_{22}\xi_2+L_{23}\xi_3+L_{24}\xi_4 \\ L_{13}x_1+L_{23}x_2+L_{33}x_3+L_{43}x_4 &=& L_{31}\xi_1+L_{32}\xi_2+L_{33}\xi_3+L_{34}\xi_4 \\ L_{14}x_1+L_{24}x_2+L_{34}x_3+L_{44}x_4 &=& L_{41}\xi_1+L_{42}\xi_2+L_{43}\xi_3+L_{44}\xi_4 \end{array}\right\},$$

where L_{rs} is used for the cofactor of l_{rs} in the determinant (Δ say) of the initial array. It only then remains to obtain from this the x's in terms of the ξ's, or the ξ's in terms of the x's. This Cayley does by using as multipliers, in the former case the elements of any row of the original array, and in the latter case the elements of any column. Thus, multiplying by $l_{11}, l_{12}, l_{13}, l_{14}$ respectively and adding he obtains

$$\Delta x_1 = (2l_{11}L_{11}-\Delta)\xi_1+2l_{11}L_{12}\xi_2+2l_{11}L_{13}\xi_3+2l_{11}L_{14}\xi_4,$$

the full substitution being

$$\left.\begin{array}{lll}x_1 &=& \left(\frac{2L_{11}}{\Delta}-1\right)\xi_1 + \frac{2L_{12}}{\Delta}\xi_2 + \frac{2L_{13}}{\Delta}\xi_3 + \frac{2L_{14}}{\Delta}\xi_4 \\ x_2 &=& \frac{2L_{21}}{\Delta}\xi_1 + \left(\frac{2L_{22}}{\Delta}-1\right)\xi_2 + \frac{2L_{23}}{\Delta}\xi_3 + \frac{2L_{24}}{\Delta}\xi_4 \\ x_3 &=& \frac{2L_{31}}{\Delta}\xi_1 + \frac{2L_{32}}{\Delta}\xi_2 + \left(\frac{2L_{33}}{\Delta}-1\right)\xi_3 + \frac{2L_{34}}{\Delta}\xi_4 \\ x_4 &=& \frac{2L_{41}}{\Delta}\xi_1 + \frac{2L_{42}}{\Delta}\xi_2 + \frac{2L_{43}}{\Delta}\xi_3 + \left(\frac{2L_{44}}{\Delta}-1\right)\xi_4 \end{array}\right\} .$$

We may add, that had the relation of the reverse substitution to this not been already known it would have been evident from the set of equations which here produce both. The result reached is that *the* n^2 *coefficients* $a_{11}, \ldots, a_{nn}$ *for the transformation of rectangular co-ordinates can be expressed rationally in terms of* $\frac{1}{2}n(n-1)$ *arbitrary quantities* l_{rs} *satisfying the conditions* $l_{rs} = -l_{sr}$, $l_{rr} = 1$ *by forming the determinant* $|l_{11}l_{22} \ldots l_{nn}|$, *or* Δ *say, and thereafter the adjugate determinant* $|L_{11}L_{22} \ldots L_{nn}|$, *and taking*

$$a_{rs} = \frac{2L_{rs}}{\Delta}, \quad a_{rr} = \frac{2L_{rr}}{\Delta} - 1 .$$

By way of illustration Cayley works out the cases where $n=3$ and where $n=4$. For $n=3$ he begins with three quantities

$$\begin{array}{cc} \nu & -\mu \\ & \lambda \end{array}$$

and obtains the substitution-coefficients

$$\begin{array}{ccc} \frac{1+\lambda^2-\mu^2-\nu^2}{1+\lambda^2+\mu^2+\nu^2} & \frac{2(\lambda\mu+\nu)}{1+\lambda^2+\mu^2+\nu^2} & \frac{2(\nu\lambda-\mu)}{1+\lambda^2+\mu^2+\nu^2} \\ \frac{2(\lambda\mu-\nu)}{1+\lambda^2+\mu^2+\nu^2} & \frac{1+\mu^2-\nu^2-\lambda^2}{1+\lambda^2+\mu^2+\nu^2} & \frac{2(\mu\nu+\lambda)}{1+\lambda^2+\mu^2+\nu^2} \\ \frac{2(\nu\lambda+\mu)}{1+\lambda^2+\mu^2+\nu^2} & \frac{2(\mu\nu-\lambda)}{1+\lambda^2+\mu^2+\nu^2} & \frac{1+\nu^2-\lambda^2-\mu^2}{1+\lambda^2+\mu^2+\nu^2} , \end{array}$$

remarking, in passing, on Rodrigues' introduction of them (but on this point see Euler's memoir of 1770) and on their connection with the theory of quaternions. For $n=4$ he begins with the six arbitrary quantities

$$\begin{array}{ccc} a & b & c \\ & -h & g \\ & & -f \end{array}$$

and obtains for the substitution-coefficients the following quantities all divided by Δ

$$\begin{array}{cccc}
\Delta - 2(a^2+b^2+c^2+\theta^2) & 2(f\theta+a+bh-cg) & 2(g\theta+b+cf-ah) & 2(h\theta+c+ag-bf) \\
2(-f\theta-a+bh-cg) & \Delta - 2(g^2+h^2+a^2+\theta^2) & 2(-c\theta-h+fg-ab) & 2(b\theta+g+hf-ca) \\
2(-g\theta-b+cf-ah) & 2(c\theta+h+fg-ab) & \Delta - 2(h^2+f^2+b^2+\theta^2) & 2(-a\theta-f+gh-bc) \\
2(-h\theta-c+ag-bf) & 2(-b\theta-g+hf-ca) & 2(a\theta+f+gh-bc) & \Delta - 2(f^2+g^2+c^2+\theta^2)
\end{array}$$

where $\theta = af+bg+ch$ and $\Delta = 1+a^2+b^2+c^2+f^2+g^2+h^2+\theta^2$.

Before leaving Cayley's very interesting paper it should be noted that the essential part of it is contained in the first few lines, where in effect he says that *if we put*

$$\left.\begin{array}{rcl}
\theta_1+\lambda\theta_2+\mu\theta_3+\ldots &=& x_1 \\
-\lambda\theta_1+\ \theta_2+\nu\theta_3+\ldots &=& x_2 \\
-\mu\theta_1-\nu\theta_2+\ \theta_3+\ldots &=& x_3 \\
\cdots\cdots\cdots & &
\end{array}\right\} \textit{and} \left\{\begin{array}{rcl}
\theta_1-\lambda\theta_2-\mu\theta_3-\ldots &=& \xi_1 \\
\lambda\theta_1+\ \theta_2-\nu\theta_3-\ldots &=& \xi_2 \\
\mu\theta_1+\nu\theta_2+\ \theta_3-\ldots &=& \xi_3 \\
\cdots\cdots\cdots & &
\end{array}\right.$$

then $x_1, x_2, x_3, \ldots$ *and* $\xi_1, \xi_2, \xi_3, \ldots$ *are orthogonally related, the coefficients of the linear substitutions connecting them being rational functions of* $\lambda, \mu, \nu, \ldots$ The rest of the paper is taken up with the finding of these coefficients, that is to say, with the elimination of $\theta_1, \theta_2, \theta_3, \ldots$ and the expression of each of the remaining variables as a linear function of all the variables of the set to which this variable does not belong.

HERMITE, CH. (1849, January).

[Sur une question relative à la théorie des nombres. *Journ.* (*de Liouville*) *de Math.*, xiv. pp. 21–30.]

The problem here solved has only a distant connection with our subject. What is given is a set of mutually prime integers forming the first column of a determinant, and the requirement is to find all the other elements so that the square of the determinant may be 1.

SPOTTISWOODE, W. (1851).

[ELEMENTARY THEOREMS RELATING TO DETERMINANTS. viii+63 pp., London.]

Following Cayley, Spottiswoode places the construction of an orthogonal substitution at the opening of his section (§ 9) on skew determinants. The mode of treatment differs from Cayley's in being verificatory rather than investigative. Starting with the two sets of equations

$$\left.\begin{array}{rcl}
l_{11}\theta_1+l_{12}\theta_2+l_{13}\theta_3+l_{14}\theta_4 &=& x_1 \\
\cdots\cdots\cdots & &
\end{array}\right\}$$

and

$$\left.\begin{array}{l} l_{11}\theta_1 + l_{21}\theta_2 + l_{31}\theta_3 + l_{41}\theta_4 \; = \; \xi_1 \\ \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \end{array}\right\}$$

he does not seek to ascertain therefrom the linear substitutions connecting the x's with the ξ's, but bringing forward the coefficients of these substitutions as found by Cayley, namely,

$$\left.\begin{array}{cccc} \frac{2L_{11}}{\Delta} - 1 & \frac{2L_{12}}{\Delta} & \frac{2L_{13}}{\Delta} & \frac{2L_{14}}{\Delta} \\ \cdot \quad \cdot \quad \cdot & \cdot \quad \cdot \quad \cdot & \cdot \quad \cdot \quad \cdot & \cdot \quad \cdot \quad \cdot \end{array}\right\}$$

he affirms that by using as multipliers, along with the first set of equations, the elements of the various columns of this array in succession we shall have

$$\left.\begin{array}{l} \left(\frac{2L_{11}}{\Delta} - 1\right)x_1 + \frac{2L_{21}}{\Delta}x_2 + \frac{2L_{31}}{\Delta}x_2 + \frac{2L_{41}}{\Delta}x_3 \; = \; \xi_1 \\ \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \end{array}\right\},$$

and that by using along with the second set of equations the elements of the various *rows* of the array in succession we shall have

$$\left.\begin{array}{l} \left(\frac{2L_{11}}{\Delta} - 1\right)\xi_1 + \frac{2L_{12}}{\Delta}\xi_2 + \frac{2L_{13}}{\Delta}\xi_3 + \frac{2L_{14}}{\Delta}\xi_4 \; = \; x_1 \\ \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \end{array}\right\}.$$

At the close of a preceding section (§ 6) he devotes three pages (pp. 35–37) to an investigation of the conditions under which Lagrange's determinantal equation shall have all its roots positive. The result is not so interesting in connection with our present subject as a theorem made use of in the process of attaining it, namely:—*If we have given the set of equations*

$$\left.\begin{array}{l} a_{11}x_1 + a_{12}x_2 + \; . \; . \; . \; + a_{1n}x_n \; = \; \theta x_1 \\ a_{21}x_1 + a_{22}x_2 + \; . \; . \; . \; + a_{2n}x_n \; = \; \theta x_2 \\ \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \\ a_{n1}x_1 + a_{n2}x_2 + \; . \; . \; . \; + a_{nn}x_n \; = \; \theta x_n \end{array}\right\}$$

where $a_{rs} = a_{sr}$, *and if we put* Δ *for* $| a_{11}a_{22} \; . \; . \; . \; a_{nn} |$, *then*

$$\left.\begin{array}{l} A_{11}x_1 + A_{21}x_2 + \; . \; . \; . \; + A_{n1}x_n \; = \; \frac{\Delta}{\theta}x_1 \\ A_{12}x_1 + A_{22}x_2 + \; . \; . \; . \; + A_{n2}x_n \; = \; \frac{\Delta}{\theta}x_2 \\ \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \\ A_{1n}x_1 + A_{2n}x_2 + \; . \; . \; . \; + A_{nn}x_n \; = \; \frac{\Delta}{\theta}x_n \end{array}\right\}.$$

No proof of this is given, but one is readily got by using the elements of

the r^{th} column of the adjugate of Δ as multipliers in connection with the given set of equations and performing addition, when there results the r^{th} equation of the required set.*

HESSE, O. (1851, April).

[Ueber die Eigenschaften der linearen Substitutionen, durch welche eine homogene ganze Function zweiten Grades, welche nur die Quadrate von vier Variabeln enthält, in eine Function von derselben Form transformirt wird. *Crelle's Journ.*, xlv. pp. 93–101: or *Werke*, pp. 307–317.]

Starting with the supposition that the substitution

$$y_k = a_{k1}x_1 + a_{k2}x_2 + \ldots + a_{kn}x_n \Big\}_{k=1}^{k=n}$$

makes

$$b_1y_1^2 + b_2y_2^2 + \ldots + b_ny_n^2 = a_1x_1^2 + a_2x_2^2 + \ldots + a_nx_n^2 ,$$

Hesse obtains by differentiation with respect to x_1, x_2, x_3, x_4 the reverse substitution

$$a_kx_k = a_{1k}b_1y_1 + a_{2k}b_2y_2 + \ldots + a_{nk}b_ny_n \Big\}_{k=1}^{k=n} ;$$

and having thus found that the latter substitution will make

$$a_1x_1^2 + a_2x_2^2 + \ldots + a_nx_n^2 = b_1y_1^2 + b_2y_2^2 + \ldots + b_ny_n^2$$

he is able by putting η_k for $a_k\, x_k$ and ξ_k for b_ky_k to say that the substitution

$$\eta_k = a_{1k}\xi_1 + a_{2k}\xi_2 + \ldots + a_{nk}\xi_k \Big\}_{k=1}^{k=n}$$

* It should be noted that the theorem holds when Δ is any determinant whatever. Further, there is implied in it another of at least equal importance, namely:—*If* Δ *stand for* $|a_{11}a_{22} \ldots a_{nn}|$, *the equation whose roots are* Δ *times the reciprocals of the roots of the equation*

$$\begin{vmatrix} a_{11}-\theta & a_{12} & \ldots & a_{1n} \\ a_{21} & a_{22}-\theta & \ldots & a_{2n} \\ \cdot & \cdot & \cdot & \cdot \\ a_{n1} & a_{n2} & \ldots & a_{nn}-\theta \end{vmatrix} = 0$$

is

$$\begin{vmatrix} A_{11}-\Theta & A_{12} & \ldots & A_{1n} \\ A_{21} & A_{22}-\Theta & \ldots & A_{2n} \\ \cdot & \cdot & \cdot & \cdot \\ A_{n1} & A_{n2} & \ldots & A_{nn}-\Theta \end{vmatrix} = 0.$$

An independent proof of this is readily obtained by substituting Δ/Θ for θ in the original equation, expanding the determinant in a series arranged according to descending powers of Δ/Θ, using Θ^n/Δ as a multiplier, substituting $A_{11}, A_{12} \ldots$ for their equivalents, and returning to the determinant form.

will make

$$\frac{1}{a_1}\eta_1^2 + \frac{1}{a_2}\eta_2^2 + \ldots + \frac{1}{a_n}\eta_n^2 = \frac{1}{b_1}\xi_1^2 + \frac{1}{b_2}\xi_2^2 + \ldots + \frac{1}{b_n}\xi_n^2 .$$

The result is the theorem that—*If the substitution*

$$y_k = a_{k1}x_1 + a_{k2}x_2 + \ldots + a_{kn}x_n \Big\}_{k=1}^{k=n}$$

changes

$$b_1y_1^2 + b_2y_2^2 + \ldots + b_ny_n^2 \quad into \quad a_1x_1^2 + a_2x_2^2 + \ldots + a_nx_n^2$$

the conjugate substitution will change

$$\frac{1}{a_1}y_1^2 + \frac{1}{a_2}y_2^2 + \ldots + \frac{1}{a_n}y_n^2 \quad into \quad \frac{1}{b_1}x_1^2 + \frac{1}{b_2}x_2^2 + \ldots + \frac{1}{b_n}x_n^2 .$$

The rest of the paper is occupied with theorems which hold only in the case of four variables.

SYLVESTER, J. J. (1852, July).

[A demonstration of the theorem that every homogeneous quadratic polynomial is reducible by real orthogonal substitutions to the form of a sum of positive and negative squares. *Philos. Magazine*, (4) iv. pp. 138–142: or *Collected Math. Papers*, i. pp. 378–381.]

The terms "orthogonal transformation" and "orthogonal substitution" date from the year 1852, the former appearing in a paper of Sylvester's published in the February part of the *Cambridge and Dub. Math. Journ.* (see vol. vii. p. 57), and the latter in the title of the paper now reached. In the former paper, too, the word "unimodular," as applied to a transformation, is first used (see p. 52), the meaning being that the modulus—that is to say, the determinant of the coefficients of transformation—is then unity.

As has been already noted * when dealing with axisymmetric determinants, this opens with the proposition that when $a_{rs} = a_{sr}$,

$$\begin{vmatrix} a_{11}+x & a_{12} & \ldots & a_{1n} \\ a_{21} & a_{22}+x & \ldots & a_{2n} \\ \cdot & \cdot & \cdot & \cdot \\ a_{n1} & a_{n2} & \ldots & a_{nn}+x \end{vmatrix} \cdot \begin{vmatrix} a_{11}-x & a_{12} & \ldots & a_{1n} \\ a_{21} & a_{22}-x & \ldots & a_{2n} \\ \cdot & \cdot & \cdot & \cdot \\ a_{n1} & a_{n2} & \ldots & a_{nn} \end{vmatrix}$$

$$= \begin{vmatrix} q_{11}-x^2 & q_{12} & \ldots & q_{1n} \\ q_{21} & q_{22}-x^2 & \ldots & q_{2n} \\ \cdot & \cdot & \cdot & \cdot \\ q_{n1} & q_{n2} & \ldots & q_{nn}-x^2 \end{vmatrix}$$

* On verifying this, see also the account of the related paper published in the *Nouv. Annales de Math.* for November 1852.

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E. A. Schäfer. Proc. Roy. Soc. Edin., vol. , 1902, pp. .

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where

$$q_{rs} = (a_{r1}\, a_{r2} \;.\;.\;.\; a_{rn} \between a_{1s}\, a_{2s} \;.\;.\;.\; a_{ns}),$$

and where therefore

$$|\, q_{11}\, q_{22} \;.\;.\;.\; q_{nn} \,| \;=\; |\, a_{11}\, a_{22} \;.\;.\;.\; a_{nn} \,|^2 .$$

It is then pointed out that the last determinant multiplied by $(-1)^n$ is expressible in the form

$$(x^2)^n - Q_1(x^2)^{n-1} + Q_2(x^2)^{n-2} - \;.\;.\;.\;.\;.\; ;$$

that $Q_1, Q_2, \ldots$ can be shown to be sums of squares; that consequently the values of x^2 in the equation

$$(x^2)^n - Q_1(x^2)^{n-1} + Q_2(x^2)^{n-2} - \;.\;.\;. \;=\; 0$$

are all positive; and therefore, finally, that the values of x in the equation

$$\begin{vmatrix} a_{11}-x & a_{12} & \ldots & a_{1n} \\ a_{21} & a_{22}-x & \ldots & a_{2n} \\ \cdot & \cdot & \cdot & \cdot \\ a_{n1} & a_{n2} & \ldots & a_{nn}-x \end{vmatrix} = 0$$

are all real.*

The remainder of the paper deals with the "Law of Inertia for Quadratic Forms," this law being "that by whatever linear substitutions, orthogonal or otherwise, a given polynomial is reduced to the form $\Sigma A_1\xi_1^2$, the number of positive and negative coefficients is invariable."

Lamé, G. (1852).

[Leçons sur la Théorie Mathématique de l'Elasticité des Corps Solides. xvi+336 pp., Paris.]

While discussing (§§ 18–22) the axes of the ellipsoid of elasticity Lamé gives in substance the theorem that *if* $|\alpha_1\, \beta_2\, \gamma_3|$ *be an orthogonant, and the ordinary multiplication-theorem produce the identity*

$$\begin{vmatrix} \alpha_1 & \beta_1 & \gamma_1 \\ \alpha_2 & \beta_2 & \gamma_2 \\ \alpha_3 & \beta_3 & \gamma_3 \end{vmatrix} \cdot \begin{vmatrix} a & f & e \\ f & b & d \\ e & d & c \end{vmatrix} \cdot \begin{vmatrix} \alpha_1 & \alpha_2 & \alpha_3 \\ \beta_1 & \beta_2 & \beta_3 \\ \gamma_1 & \gamma_2 & \gamma_3 \end{vmatrix} = \begin{vmatrix} P_1 & Q_3 & Q_2 \\ Q_3 & P_2 & Q_1 \\ Q_2 & Q_1 & P_3 \end{vmatrix},$$

then

$$P_1 + P_2 + P_3 \;=\; a + b + c,$$

$$\begin{vmatrix} P_1 & Q_3 \\ Q_3 & P_2 \end{vmatrix} + \begin{vmatrix} P_2 & Q_1 \\ Q_1 & P_3 \end{vmatrix} + \begin{vmatrix} P_1 & Q_2 \\ Q_2 & P_3 \end{vmatrix} = \begin{vmatrix} a & f \\ f & b \end{vmatrix} + \begin{vmatrix} b & d \\ d & c \end{vmatrix} + \begin{vmatrix} a & e \\ e & c \end{vmatrix},$$

* This proof, for the case where $n=3$, is given free of determinants by Grunert in the *Archiv d. Math. u. Phys.*, xxix. (1857), pp. 442–446.

and of course

$$\begin{vmatrix} P_1 & Q_3 & Q_2 \\ Q_3 & P_2 & Q_1 \\ Q_2 & Q_1 & P_3 \end{vmatrix} = \begin{vmatrix} a & f & e \\ f & b & d \\ e & d & c \end{vmatrix}.$$

No determinant notation, however, is used, nor are determinants spoken of.

Hermite, Ch. (1853, May).

[Sur la théorie des formes quadratiques ternaires indéfinies. *Crelle's Journ.*, xlvii. pp. 307–312: or *Œuvres*, i. pp. 193–199.]

[Remarques sur un mémoire de M. Cayley relatif aux déterminants gauches. *Cambridge and Dub. Math. Journ.*, ix. pp. 63–67: or *Œuvres*, i. pp. 290–295.]

In his paper of 1846 Cayley, as we have seen, gave a general solution of the problem of the transformation of $x_1^2+x_2^2+\ \ldots$ into $\xi_1^2+\xi_2^2+\ \ldots$ by means of a linear substitution. Hermite now faces a more general problem, namely, "la transformation en elle-même d'une forme quadratique *quelconque*," a problem which in itself is rather outside our subject, but which, by reason of the important modification made in the initial step of the solution, deserves attention.

The quadric being $f(x_1, x_2, \ldots)$, the problem is to find the most general linear substitution which will transform

$$f(x_1, x_2, \ldots) \quad \text{into} \quad f(\xi_1, \xi_2, \ldots);$$

and Hermite having before him Cayley's expressions, in the simpler case, for the x's and ξ's in terms of an intermediary set of variables, and observing that any member of the intermediary set is the arithmetic mean of the corresponding members of the two given sets, begins by imagining merely "que les quantités x et ξ soient exprimés par des indéterminées auxiliaires θ, de sorte qu'on ait en général

$$x_r + \xi_r = 2\theta_r."$$

There is thus obtained

$$\begin{aligned} f(x_1, x_2, \ldots) &= f(2\theta_1 - \xi_1, 2\theta_2 - \xi_2, \ldots), \\ &= 4f(\theta_1, \theta_2, \ldots) - 2\left(\xi_1\frac{\partial f}{\partial\theta_1} + \xi_2\frac{\partial f}{\partial\theta_2} + \ldots\right) \\ &\qquad + f(\xi_1, \xi_2, \ldots), \end{aligned}$$

so that in order to have $f(x_1, x_2, \ldots) = f(\xi_1, \xi_2, \ldots)$ it is seen to be necessary that

$$\xi_1\frac{\partial f}{\partial\theta_1} + \xi_2\frac{\partial f}{\partial\theta_2} + \ldots = 2f(\theta_1, \theta_2, \ldots).$$

Now this condition is manifestly satisfied by putting $\xi_r = \theta_r$, but "la manière la plus générale de la vérifier en exprimant les quantités ξ en θ sera de faire

$$\xi_r = \theta_r + \tfrac{1}{2}\sum_{s=1}^{s=n} \lambda_{rs}\frac{\partial f}{\partial \theta_s},$$

les indéterminées λ étant assujetées à la condition $\lambda_{rs} = -\lambda_{sr}$." This of course implies that

$$x_r = \theta_r - \tfrac{1}{2}\sum_{s=1}^{s=n} \lambda_{rs}\frac{\partial f}{\partial \theta_s};$$

and there have thus been obtained in their general form the two sets of equations with which Cayley started in his special case.

For those who may wish to pursue farther this subject of "automorphic transformation" we may note that the actual expression of the x's in terms of the ξ's was given by Cayley in a paper dated 24th May 1854,* and that he extended his result to a *bipartite* quadric function in a paper dated 10th December 1857.†

Another problem, which in the early history of orthogonants we have seen to be of interest, namely, the simultaneous transformation of two quadrics, Cayley also dealt with, the first time in 1849 and the second in 1857.‡

Sylvester, J. J. (1853).

[The algebraical theory of the secular-inequality determinantive equation generalised. *Philos. Magazine*, vi. pp. 214–216: or *Collected Math. Papers*, i. pp. 634–636.]

The fundamental theorem here is that if

$$X_1 = ax+\alpha, \quad X_2 = \begin{vmatrix} ax+\alpha & bx+\beta \\ bx+\beta & cx+\gamma \end{vmatrix}, \quad X_3 = \begin{vmatrix} ax+\alpha & bx+\beta & dx+\delta \\ bx+\beta & cx+\gamma & ex+\epsilon \\ dx+\delta & ex+\epsilon & fx+\phi \end{vmatrix}, \ \ldots$$

and the coefficients of the highest powers of x in $X_1, X_2, X_3, \ldots$ have all the same sign, then the roots of X will be all real and will lie respectively in the intervals comprised between $+\infty$, the successive descending roots of X_{i-1}, and $-\infty$. The mode of proof is Cauchy's (1829).

* Cayley, A., "Sur la transformation d'une fonction quadratique en elle-même par des substitutions linéaires," *Crelle's Journ.*, l. pp. 288–299: or *Collected Math. Papers*, ii. pp. 192–201. See also Brioschi in *Annali di Sci. Mat. e Fis.*, v. pp. 201–206.

† Cayley, A., "A Memoir on the Automorphic Linear Transformation of a Bipartite Quadric Function," *Philos. Trans. Roy. Soc. London*, cxlviii. pp. 39–46: or *Collected Math. Papers*, ii. pp. 497–505.

‡ *Cambridge and Dublin Math. Journ.*, iv. pp. 47–50; and *Quart. Journ. of Math.*, ii. pp. 192–195: or *Collected Math. Papers*, i. pp. 428–431, and iii. pp. 129–131.

SPOTTISWOODE, W. (1853, August).

[Elementary theorems relating to determinants. Second edition, re-written and much enlarged by the author. *Crelle's Journ.*, li. pp. 209–271, 328–381.]

In trying to insert in his second edition an alternative process for establishing Cayley's result of 1846, Spottiswoode is very unfortunate. The place selected by him is immediately after the sentence defining *skew*, and therefore immediately preceding the former process; but in making the insertion (p. 260) the predicate of the important sentence in question has suffered excision, along with a very necessary explanation regarding the diagonal elements of the initial determinant. Further, at the utmost all that is established is the fact that the determinant of Cayley's substitution is equal to $+1$. Such neglect, however, can well be overlooked in view of certain deductions which he records, and which he says can be made from Cayley's result. These may be enunciated in more modern form as follows:—

If $|\mathrm{a}_{11}\,\mathrm{a}_{22}\,\ldots\,\mathrm{a}_{nn}|$ *or* Δ *be a unit-axial skew determinant,* $|\mathrm{A}_{11}\,\mathrm{A}_{22}\,\ldots\,\mathrm{A}_{nn}|$ *its adjugate, and* $|\omega_{11}\,\omega_{22}\,\ldots\,\omega_{nn}|$ *Cayley's orthogonant formed therefrom, then*

$$\left.\begin{aligned} \mathrm{A}_{1r}^2 + \mathrm{A}_{2r}^2 + \ldots + \mathrm{A}_{nr}^2 &= \mathrm{A}_{rr}\,.\,\Delta\,, \\ \mathrm{A}_{1r}\mathrm{A}_{1s} + \mathrm{A}_{2r}\mathrm{A}_{2s} + \ldots + \mathrm{A}_{nr}\mathrm{A}_{ns} &= \tfrac{1}{2}(\mathrm{A}_{rs} + \mathrm{A}_{sr})\,.\,\Delta \end{aligned}\right\} \quad . \quad . \quad . \quad (\alpha)$$

and

$$\left.\begin{aligned} a_{1r}\omega_{1r} + a_{2r}\omega_{2r} + \ldots + a_{nr}\omega_{nr} &= a_{rr} \\ a_{1r}\omega_{1s} + a_{2r}\omega_{2s} + \ldots + a_{nr}\omega_{ns} &= a_{rs} \end{aligned}\right\} \quad . \quad . \quad . \quad (\beta)$$

The former (α) belongs strictly to the theory of skew determinants, and has already been noted in its proper place.

CAYLEY, A. (1853, November).

[On the homographic transformation of a surface of the second order into itself. *Philos. Magazine,* vi. pp. 326–333: or *Collected Math. Papers,* ii. pp. 105–112.]

Here Cayley recalculates the general orthogonant of the 4th order, taking note on passing of the related identity

$$\begin{aligned} &(x + \nu y - \mu z + aw)^2 + (-\nu x + y + \lambda z + bw)^2 + (\mu x - \lambda y + z + cw)^2 + (-ax - by - cz + w)^2 \\ = \; &x^2 + y^2 + z^2 + w^2 + (\nu y - \mu z + aw)^2 + (-\nu x + \lambda z + bw)^2 + (\mu x - \lambda y + cw)^2 + (ax + by + cz)^2 \end{aligned}$$

Brioschi, Fr. (1854, March).

[La Teorica dei Determinanti, e le sue principali applicazioni; viii+116 pp., Pavia. Translation into French by Combescure; ix+216 pp., Paris, 1856. Translation into German by Schellbach; vii+102 pp., Berlin, 1856.]

In Brioschi's text-book, the paragraphs dealing with a "sostituzione ortogonale" are somewhat scattered, most of them appearing among the applications (pp. 24–26, 47–51, 62–69).

The first deserving of notice (p. 49) concerns the product $QP\bar{Q}$, where P and Q are determinants of the same order and $\bar{Q}$ is the conjugate of Q. Viewing the product as $Q\cdot(P\bar{Q})$ Brioschi first uses a result of Cauchy's to express any m-line minor of $Q\cdot(P\bar{Q})$ in terms of m-line minors of Q and $P\bar{Q}$: then for the said m-line minors of $P\bar{Q}$ he substitutes with the same assistance expressions involving m-line minors of P and Q: there is thus obtained for any m-line minor of $QP\bar{Q}$ an expression involving only m-line minors of P and Q. This result may be put in the form

$$(QP\bar{Q})^{(m)}_{\nu,s} = \sum\nolimits_r \Big[Q^{(m)}_{\nu,r}\{P^{(m)}_{r1}Q^{(m)}_{s1} + P^{(m)}_{r2}Q^{m}_{s2} + \ldots + P^{(m)}_{rz}Q^{(m)}_{sz}\}\Big],$$

if z be put for $n(n-1)\ldots(n-m+1)/1.2.\ldots m$, and if generally we use $A^{(m)}_{rs}$ to stand for an m-line minor of an n-line determinant A, the rows of A taken to form $A^{(m)}_{rs}$ being those whose numbers constitute the r^{th} combination of m of the integers $1, 2, \ldots, n$, and the columns those whose numbers constitute the s^{th} like combination. Putting $\nu = s$ we obtain the expression for an m-line *coaxial* minor of $QP\bar{Q}$, and thence for the sum of all such minors the expression

$$\sum\nolimits_r \sum\nolimits_s \Big[Q^{(m)}_{sr}\{P^{(m)}_{r1}Q^{(m)}_{s1} + P^{(m)}_{r2}Q^{(m)}_{s2} + \ldots + P^{(m)}_{rz}Q^{(m)}_{sz}\}\Big],$$

which changes into

$$\sum\nolimits_r \{P^{(m)}_{r1}M^{(m)}_{r1} + P^{(m)}_{r2}M^{(m)}_{r2} + \ldots + P^{(m)}_{rz}M^{(m)}_{rz}\},$$

if M be the determinant which equals Q^2. Specialising still further by making Q the determinant of an orthogonal substitution so that

$$M^{(m)}_{rr} = 1 \quad \text{and} \quad M^{(m)}_{rs} = 0,$$

Brioschi finally obtains the important "formula nota"

$$\sum\nolimits_s (QP\bar{Q})^{(m)}_{ss} = \sum\nolimits_s P^{(m)}_{ss},$$

which we may express in words for ourselves, thus:—*If* Q *be an orthogonant*

and P *any other determinant of the same order, then the sum of the* m-*line coaxial minors of* $QP\bar{Q}$ *is the same as the sum of the* m-*line coaxial minors of* P.

The other paragraph requiring notice concerns the determinant arising from Cayley's of 1846 by subtracting 1 from each diagonal element. The value of this is shown (p. 65) to be 0 when n is odd, and $2^n\Delta_0/\Delta$ when n is even, Δ being the basic determinant, and Δ_0 what Δ becomes on making all its diagonal elements zero. The result is easily reached on multiplying the given determinant by Δ and showing that the product is $(-1)^n 2^n \Delta_0$.

Brioschi, Fr. (1854, August).

[Note sur un théorème relatif aux déterminants gauches. *Journ.* (*de Liouville*) *de Math.*, xix. pp. 253–256: or in French translation of his *Teorica dei Determinanti*, pp. 144–147: or *Opere Mat.*, v. pp. 161–164.]

Brioschi's subject is really the equation

$$\begin{vmatrix} \omega_{11}-x & \omega_{12} & \cdots & \omega_{1n} \\ \omega_{21} & \omega_{22}-x & \cdots & \omega_{2n} \\ \cdot & \cdot & \cdot & \cdot \\ \omega_{n1} & \omega_{n2} & \cdots & \omega_{nn}-x \end{vmatrix} = 0,$$

in which the left-hand member is the determinant of Cayley's orthogonal substitution with $-x$ affixed to each diagonal element. He notes at once, of course, that if the basic determinant be $|a_{11}\,a_{22}\,\ldots\,a_{nn}|$, or Δ say, the equation may be changed into

$$\begin{vmatrix} A_{11}-y & A_{12} & \cdots & A_{1n} \\ A_{21} & A_{22}-y & \cdots & A_{2n} \\ \cdot & \cdot & \cdot & \cdot \\ A_{n1} & A_{n2} & \cdots & A_{nn}-y \end{vmatrix} = 0,$$

where y is put for $\frac{1}{2}(1+x)\Delta$. A further transformation is then effected by multiplying both sides by Δ and putting z for $1-\Delta/y$, the result being

$$\begin{vmatrix} z & a_{21} & \cdots & a_{n1} \\ a_{12} & z & \cdots & a_{n2} \\ \cdot & \cdot & \cdot & \cdot \\ a_{1n} & a_{2n} & \cdots & z \end{vmatrix} = 0.$$

Using Cayley's expansion (1847) for the determinant on the left, it is seen that when n is odd the equation resolves itself into $z=0$ and an equation

in z^2 with positive coefficients, and that when n is even it is already of the latter form. All values of z^2 thus obtainable must be negative, and consequently all the values of z save the value 0 must be imaginary and must occur in pairs whose sum is zero. But as

$$z = 1 - \frac{\Delta}{\frac{1}{2}(1+x)\Delta} = \frac{x-1}{x+1},$$

and

$$\therefore \quad x = \frac{1+z}{1-z},$$

it is clear that for every pair of values of z that differ only in sign there must be a pair of values of x that are reciprocals. The theorem reached by Brioschi we may thus enunciate for ourselves as follows:—*The roots of the equation*

$$\begin{vmatrix} \omega_{11}-x & \omega_{12} & \cdots & \omega_{1n} \\ \omega_{21} & \omega_{22}-x & \cdots & \omega_{2n} \\ \cdot & \cdot & \cdots & \cdot \\ \omega_{n1} & \omega_{n2} & \cdots & \omega_{nn}-x \end{vmatrix} = 0,$$

where $|\omega_{11}\,\omega_{22}\ldots\omega_{nn}|$ *is Cayley's orthogonant, are arrangeable in pairs of reciprocal imaginaries, save when* n *is odd, in which case there is the single real root* 1.

When instead of the ω's we take the coefficients of the substitution which transforms a *general* quadric into itself, the words "reciprocal imaginaries" need to be changed into "reciprocals." This generalisation Brioschi published a month or two sooner (see *Annali di sci. mat. e fis.*, v. pp. 201–206).

Bruno, Faà de (1854, September).

[Note sur un théorème de M. Brioschi. *Journ.* (*de Liouville*) *de Math.*, xix. p. 304.]

On multiplying both sides of Brioschi's equation (1854, August) by $|\omega_{11}\,\omega_{22}\ldots\omega_{nn}|$ and dividing by $(-x)^n$ an equation is obtained which differs from the original simply in having x^{-1} for x. The portion of the theorem which concerns "reciprocity" Faà de Bruno thus readily establishes.

Baltzer, R. (1857).

[Theorie und Anwendungen der Determinanten, . . . vi+129 pp., Leipzig. French translation by J. Houel; xii+235 pp., Paris, 1861.]

Baltzer devotes a whole section (§ 15) of seventeen pages (pp. 80–96) to the subject of "Die lineare, insbesondere die orthogonale Substitutionen." The section, like its fellows, is noteworthy, not for freshness of matter, but for good arrangement, clearness and compactness.

In treating of Cayley's orthogonant (§ 15, 6) he takes l, not 1, as the constant element of the basic determinant: and, when in the course of the proof he obtains the two values for each of Cayley's θ's, he does not equate them, but uses with each of them Hermite's observation

$$x_i + \xi_i \;=\; 2l\theta_i\,,$$

thus reaching the elements

$$\frac{2l\mathrm{L}_{rr}}{\Delta} - 1\,,\quad \frac{2l\mathrm{L}_{rs}}{\Delta}\,,$$

of the desired substitutions without more trouble. On the other hand, he fails to note that Cayley's θ's are so introduced as to ensure the equality of $x_1^2+x_2^2+\ldots$ and $\xi_1^2+\xi_2^2+\ldots$, and thus he is led to prove propositions already established (§ 15, 5).

Brioschi's equation of August 1854 being denoted (§ 15, 9) by $f(x)=0$, he multiplies $f(x)$ by $f(-x)$, and obtains for $f(x).f(-x)/x^n$ a skew determinant having each diagonal element equal to $1/x-x$. This determinant being therefore expressible as a sum of squares when n is even, and as $1/x-x$ times a sum of squares when n is odd, the part of Brioschi's proposition which asserts the unreality of the roots follows by a *reductio ad absurdum*.

Salmon, G. (1859).

[Lessons Introductory to the Modern Higher Algebra, . . . xii+147 pp., Dublin.]

In Salmon's treatment of the subject (§§ 118, 139, 142, 156–7, 163–4) only two points call for remark. In the first place, "orthogonal trans-

formation" with him is not as with his predecessors a transformation which merely changes

$$x^2+y^2+z^2+\ .\ .\ .\quad\text{into}\quad \xi^2+\eta^2+\zeta^2+\ .\ .\ .\ ,$$

but one which at the same time changes

$$ax^2+by^2+cz^2+\ .\ .\ .+2fyz+2gzx+2hxy+\ .\ .\ .\quad\text{into}\quad \mathrm{A}\xi^2+\mathrm{B}\eta^2+\mathrm{C}\zeta^2+\ .\ .\ .$$

In the second place, he has a fresh mode of arriving at the equation for determining A, B, C, . . . Calling the four quadrics just mentioned V, V′, U, U′, he forms the discriminant of U−λV, and asserts that the coefficient of all the several powers of λ in it must be invariants, and that, therefore, if the said discriminant be put equal to 0 and the equation so obtained be solved for λ, the roots resulting must be identical with the roots of the equation

$$\text{Discrim. }(\mathrm{U}'-\lambda\mathrm{V}')\ =\ 0\ ;$$

in other words, that we must have identically

$$\begin{vmatrix} a-\lambda & h & g & \dots \\ h & b-\lambda & f & \dots \\ g & f & c-\lambda & \dots \\ \cdot & \cdot & \cdot & \cdot \end{vmatrix} = \begin{vmatrix} \mathrm{A}-\lambda & 0 & 0 & \dots \\ 0 & \mathrm{B}-\lambda & 0 & \dots \\ 0 & 0 & \mathrm{C}-\lambda & \dots \\ \cdot & \cdot & \cdot & \cdot \end{vmatrix}$$

so that A, B, C, . . . are the values of λ in the equation

$$\text{Discrim. }(\mathrm{U}-\lambda\mathrm{V})\ =\ 0.$$

Hesse, O. (1859, October).

[Neue Eigenschaften der linearen Substitutionen welche gegebene homogene Functionen des zweiten Grades in andere transformiren die nur die Quadrate der Variabeln enthalten. *Crelle's Journ.*, lvii. pp. 175–182: or *Werke*, pp. 489–496.]

Hesse's object is that of Kummer (1843), Jacobi (1844, March), and Borchardt (1845, January), namely, to prove the reality of the roots of Lagrange's determinantal equation by showing that the product of their squared differences is essentially positive.

Taking the linear substitution

$$\xi_k\ =\ a_{k1}x_1+a_{k2}x_2+\ .\ .\ .\ +a_{kn}x_n\ \Big\}_{k=1}^{k=n}$$

we readily see that $\xi_1\xi_2\dots\xi_n$ is expressible as a sum of terms of the

form $Cx_1^{e_1}x_2^{e_2}\ldots x_n^{e_n}$, where $e_1+e_2+\ .\ .\ .\ +e_n=n$ and C is an integral function of a's,—a result which Hesse writes

$$\xi_1\xi_2\ldots\xi_n = \sum A_{e_1e_2\ldots e_n}x_1^{e_1}x_2^{e_2}\ldots x_n^{e_n},$$

the coefficient of any term being denoted by an A with n suffixes identical with the n exponents of the x's. Now let us suppose the substitution to be orthogonal, in which case we know that

$$x_k = a_{1k}\xi_1 + a_{2k}\xi_2 + \ .\ .\ .\ + a_{nk}\xi_n \left.\vphantom{\int}\right\}_{k=1}^{k=n};$$

and let us thereby transform $\sum A_{e_1e_2\ldots e_n}x_1^{e_1}x_2^{e_2}\ldots x_n^{e_n}$ so as to have it again in terms of the ξ's. In doing this Hesse pays attention only to the term in $\xi_1\xi_2\ldots\xi_n$, making the assertion that *the coefficient of* $\xi_1\xi_2\ldots\xi_n$ *in* $x_1^{e_1}x_2^{e_2}\ldots x_n^{e_n}$ *is either the same as the coefficient of* $x_1^{e_1}x_2^{e_2}\ldots x_n^{e_n}$ *in* $\xi_1\xi_2\ldots\xi_n$ *or differs from the latter coefficient by a merely arithmetical multiplier.* From this it follows that the coefficient of $\xi_1\xi_2\ .\ .\ .\ \xi_n$ in any term $A_{e_1e_2\ldots e_n}x_1^{e_1}x_2^{e_2}\ldots x_n^{e_n}$ is a merely arithmetical multiple of $A^2_{e_1e_2\ldots e_n}$; and, if the multiplier in question be denoted by $\Theta_{e_1e_2\ldots e_n}$, there results from the equatement of coefficients

$$1 = \sum \Theta_{e_1e_2\ldots e_n}A^2_{e_1e_2\ldots e_n}.$$

Next, let us suppose in addition that our substitution transforms an n-ary quadric

$$f_1(x_1, x_2, \ .\ .\ ., x_n) \quad \text{into} \quad g_1\xi_1^2 + g_2\xi_2^2 + \ .\ .\ .\ + g_n\xi_n^2,$$

a step which, as we know, introduces the quantities whose reality is in question. In regard to them Hesse first recalls Jacobi's proof (1833) that they are such that

$$g_1^2\xi_1^2 + g_2^2\xi_2^2 + \ .\ .\ .\ + g_n^2\xi_n^2, \quad g_1^3\xi_1^2 + g_2^3\xi_2^2 + \ .\ .\ .\ + g_n^3\xi_n^2, \quad .\ .\ .\ .$$

are also expressible as homogeneous quadric functions of the x's, and that the coefficients of these quadrics are rational integral functions of the coefficients of the original quadric f_1. It is seen to be not inappropriate therefore to use

$$f_p(x_1, x_2, \ .\ .\ .\ , x_n) \quad \text{for} \quad g_1^p\xi_1^2 + g_2^p\xi_2^2 + \ .\ .\ .\ + g_n^p\xi_n^2$$

and to denote the partial differential-quotient of $f_p(x_1, x_2, \ldots, x_n)$ with respect to x by $f'_p(x_k)$, thus giving

$$\tfrac{1}{2}f'_p(x_k) = a_{k1}g_1^p\xi_1 + a_{k2}g_2^p\xi_2 + \ .\ .\ .\ + a_{kn}g_n^p\xi_n.$$

The next step is the deduction of an important result from the consideration of the determinant

$$\begin{vmatrix} x_1 & x_2 & \cdots & x_n \\ \tfrac{1}{2}f'_1(x_1) & \tfrac{1}{2}f'_1(x_2) & \cdots & \tfrac{1}{2}f'_1(x_n) \\ \tfrac{1}{2}f'_2(x_1) & \tfrac{1}{2}f'_2(x_2) & \cdots & \tfrac{1}{2}f'_2(x_n) \\ \cdot & \cdot & \cdot & \cdot \\ \tfrac{1}{2}f'_{n-1}(x_1) & \tfrac{1}{2}f'_{n-1}(x_2) & \cdots & \tfrac{1}{2}f'_{n-1}(x_n) \end{vmatrix} \quad \text{or } \Delta \text{ say.}$$

Each element being linear in the x's, the determinant is of the n^{th} degree in those variables, and therefore

$$\Delta = \sum B_{e_1e_2\ldots e_n} x_1^{e_1} x_2^{e_2} \ldots x_n^{e_n}.$$

On the other hand, if we substitute for each element its expression in terms of the ξ's, the result is manifestly a product determinant, and we learn that

$$\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2n} \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ a_{n1} & a_{n2} & a_{n3} & \cdots & a_{nn} \end{vmatrix} \cdot \begin{vmatrix} \xi_1 & g_1\xi_1 & g_1^2\xi_1 & \cdots & g_1^{n-1}\xi_1 \\ \xi_2 & g_2\xi_2 & g_2^2\xi_2 & \cdots & g_2^{n-1}\xi_2 \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \xi_n & g_n\xi_n & g_n^2\xi_n & \cdots & g_n^{n-1}\xi_n \end{vmatrix}.$$

$$= (\pm 1) \cdot | g_1^0 g_2^1 \ldots g_n^{n-1} | \cdot \xi_1\xi_2 \ldots \xi_n.$$

Equating these two values and substituting the expression found at the outset for $\xi_1\xi_2 \ldots \xi_n$ we obtain

$$\sum B_{e_1e_2\ldots e_n} x_1^{e_1} x_2^{e_2} \ldots x_n^{e_n} = (\pm 1) \cdot | g_1^0 g_2^1 \ldots g_n^{n-1} | \cdot \sum A_{e_1e_2\ldots e_n} x_1^{e_1} x_2^{e_2} \ldots x_n^{e_n},$$

and thus see that

$$B_{e_1e_2\ldots e_n} = (\pm 1) \cdot | g_1^0 g_2^1 \ldots g_n^{n-1} | \cdot A_{e_1e_2\ldots e_n},$$

as Jacobi had shown in 1845 in the case of $n=3$. With the help of this, Hesse's first result at once becomes

$$| g_1^0 g_2^1 \ldots g_n^{n-1} |^2 = \sum \Theta_{e_1e_2\ldots e_n} B^2_{e_1e_2\ldots e_n},$$

and the desired result is reached.

[LIST OF AUTHORS.

LIST OF AUTHORS

whose writings are herein dealt with.

(*Issued separately February* 26, 1910.)

Andrews' Measurements of the Compression of Carbon Dioxide and of Mixtures of Carbon Dioxide and Nitrogen. Edited by Professor C. G. Knott.

(Supplementary Note.)

In the table (p. 17, Vol. XXX.) giving the data for the mixture of nitrogen and carbon dioxide in the ratio of 1 to 3·43, the volumes of the mixture are calculated from the column-lengths on the assumption that the tube used was Tube E as in the preceding experiments in the same notebook. This, however, is not absolutely certain, there being no distinct mention made in the notebook as to the tube used in the experiments with this particular mixture. Dr Andrews' immediate object was to find the critical temperature and pressure; and for this a knowledge of the absolute volume of the mixture was not needed.

XIV.—The Restoration of an Ancient British Race of Horses.

By **J. C. Ewart**, F.R.S., University of Edinburgh.

(Read December 20, 1909. MS. received January 12, 1910.)

In a work published in 1846,* Professor Owen figured two upper molars † of a small member of the Equidæ family which lived in the south of England along with the mammoth. A study of these and other molars led Owen to conclude that the small equine which lived in England in prehistoric times was either an ass or a zebra. Assuming that the small Oreston fossil equine had "callosities on the fore legs only, the tail furnished with a terminal brush, and a longitudinal dorsal line," Owen gave it the name *Asinus fossilis*. In support of the view that a "wild ass or quagga" as well as a wild horse and a wild boar entered "into the series of British Pliocene hoofed mammalia," ‡ Owen mentions that he had seen a fossil second phalanx or pastern bone of a small species of Equus about the size of the zebra from the Pliocene crag at Thorpe, and that Dr Mantell had described teeth and bones of "a small species about the size of a Shetland pony" § from the super-cretaceous drift deposit at Brighton—the deposit which, owing to the abundance of mammoth bones, is known as the "Elephant Bed."

I am unable to offer any opinion about the phalanx from Thorpe, but I am satisfied that there is no reason for assuming that the bones of the small species from the "Elephant Bed" belong either to an ass or a zebra. The small teeth referred to by Dr Mantell, like the cannon bones, indicate that the small equine of the "Elephant Bed" at Brighton was a horse about 12 hands high, allied to the stout race so abundant during the Mammoth age in the vicinity of Solutré. Evidence of this relationship is readily obtained by studying the cannon bones from Pleistocene deposits. In fossil, as in recent, Asiatic wild asses, the cannon bones are long and slender, the length of the metacarpal being at least eight times, and that of the metatarsal nine times, the width at the middle of the shaft. The small "Elephant Bed" cannon bones hitherto discovered are short and wide. A

* *A History of British Fossil Mammals and Birds*, figs. 157 and 158, p. 396.

† These molars (m. 2 and m. 3) were found in a cavernous fissure at Oreston near Plymouth. Similar molars came from the drift at Chatham and Kesingland in Suffolk.

‡ *Loc. cit.*, p. xxiv.

§ *Medals of Creation*, 1844, vol. ii. p. 40.

m. 2.

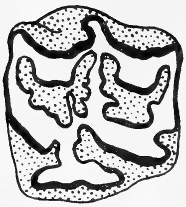

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Fig. 1.

m. 3.

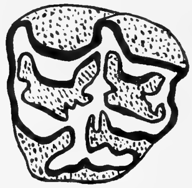

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Fig. 2.

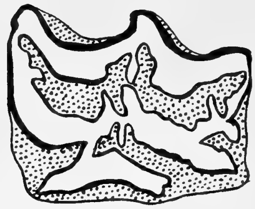

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Fig. 3.

m. 1.

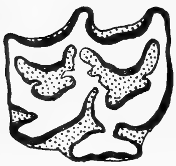

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Fig. 6.

p

Fig. 5.

m. 2.

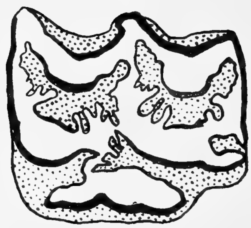

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Fig. 4.

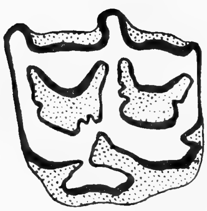

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Fig. 9.

p.m. 3.

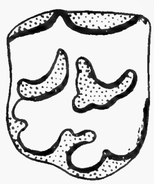

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Fig. 8.

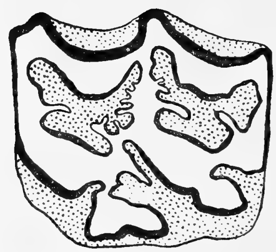

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Fig. 7.

p.m. 4.

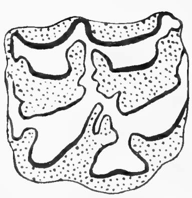

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Fig. 12.

m. 1.

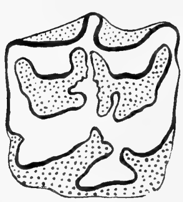

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Fig. 11.

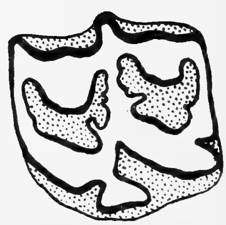

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Fig. 10.

metatarsal in the British Museum from the "Elephant Bed" is 235 mm. long and 33 mm. wide,* *i.e.* the length, instead of being, as in the Onager, over nine, is only seven times the width at the middle of the shaft. The molars (figs. 1 and 2) which formed the type for Owen's *Asinus fossilis,* differ, in size and shape and in the arrangement of the enamel folds, from the molars of *Equus fossilis* from Kent's Cave (fig. 3) and from the large complex fossil molars of Owen's *Equus plicidens* from Oreston (fig. 4), which seem to belong to the same stock as *Equus namadicus* of the Indian Pleistocene and *Equus complicatus* widely distributed in pre-glacial times in North America. The small Oreston molars (m. 2 and m. 3) also differ from the molars of the horse of Solutré (fig. 5), and from the molars of all the asses and zebras I have examined.

If only the teeth figured by Owen were available for study, it would be extremely difficult to determine the zoological position of the small equine which in Pleistocene times inhabited the south of England. Fortunately, the Oreston fissure has yielded a first molar (m. 1) as well as second and

DESCRIPTION OF FIGURES 1 TO 12.

FIG. 1.—Upper second molar (m. 2), nat. size. Oreston, Owen's *Asinus fossilis.* Crown, 1·83 times length of its pillar (*p*). After Owen.

FIG. 2.—Last upper molar (m. 3), nat. size. Oreston, Owen's *Asinus fossilis.* After Owen.

FIG. 3.—Upper molar (*Equus fossilis*), nat. size. Kent's Hole. Crown, 1·2 times length of pillar (*p*). After Owen.

FIG. 4.—Upper second molar (m. 2), nat. size. *Equus plicidens*, Owen, Oreston. Crown, 1·4 times length of pillar (*p*). After Owen.

FIG. 5.—Upper molar, nat. size. Solutré (*Equus robustus*). After Boule.

FIG. 6.—First upper molar (m. 1), nat. size. Oreston. From section 5 mm. below grinding surface of slightly worn crown. The crown is 2·4 times length of pillar (*p*). Brit. Museum.

FIG. 7.—Upper molar (*Equus stenonis*), nat. size. In this tooth the crown is nearly three times the length of the grinding surface of the pillar (*p*). It is commonly assumed domestic horses are descended from two or more varieties of *E. stenonis*, which acquired long-pillared molars.

FIG. 8.—Upper third premolar (nat. size) of *Merychippus*, a small three-toed Miocene horse. The pillar (*p*) is small and nearly circular. After Loch.

FIG. 9.—Upper molar (nat. size) from Pleistocene of Algiers. The pillar in this molar is intermediate between the Haute Loire (fig. 10) and the Oreston, m. 1 (fig. 6). After Boule.

FIG. 10.—Upper molar (nat. size) from the Pliocene (Coupet, Haute Loire). The pillar is shorter in this small Pliocene race than in m. 1 (fig. 6) from Oreston. After Boule.

FIG. 11.—First upper molar (m. 1) of a 36·5 inches six-year-old Shetland pony. The internal pillar (*p*), smaller than in the first molar (fig. 6), from Oreston, very closely agrees with the pillar of the molar (fig. 10) from the French Pliocene, and the outlines of the crescents (the pits which extend into the crown) are almost identical. The crown is nearly three times the length of its pillar.

FIG. 12.—Last premolar (p.m. 4) of the 36·5 inches Shetland pony. The internal pillar (*p*), one-third the length of the crown and shorter than the pillar in m. 1 (fig. 11), closely agrees with the internal pillar of the Newstead pony (fig. 22).

* This metatarsal belonged to a stout race about 12 hands high, now probably represented by thick-set Iceland ponies of the "forest" type.

third molars. The first molar from Oreston evidently belonged to a decidedly smaller individual than the molars figured by Owen.* The first molar (m. 1) from Oreston having only just come into use, was so slightly worn that it was impossible to make out the arrangement of the enamel folds. To admit of the pattern formed by the enamel being studied, a section was carried across the crown about 5 mm. from the grinding surface.† As fig. 6 shows, the grinding surface of the internal pillar of this small first molar is relatively, as well as absolutely, shorter than in the second and third molars (figs. 1 and 2); instead of being less than twice the length of its pillar as in molar 2, the crown of molar 1 is from before backwards nearly three times the length of the grinding surface of its pillar. In having a short crown and a small internal pillar, the first molar (m. 1) from Oreston differs from the corresponding tooth in *Equus fossilis* (fig. 3), from *Equus plicidens*, *Equus namadicus* and *Equus robustus* (fig. 5), but more or less closely resembles the first molar of *Pliohippus*, *Equus stenonis* (fig. 7), and *Equus sivalensis* (fig. 24).

In the three-toed Miocene horse *Merychippus* the internal pillar (fig. 8) is small and nearly circular, but in *Pliohippus* this internal fold of enamel is flattened and the grinding surface is about one-third the length of the crown measured from before backwards. During Pliocene and Pleistocene times the grinding surface of the internal pillar, in at least some of the cheek teeth of the Equidæ, increased considerably. In *Equus sivalensis* the internal pillar is decidedly long in molar 2 and molar 3 (fig. 24), usually somewhat elongated in premolars 3 and 4, but still short in molar 1; in *Equus namadicus* the internal pillar is long in all the six large upper cheek teeth. In having the grinding surface of the internal pillar short in molar 1 but long in molars 2 and 3, the small Oreston equine agrees with *Equus sivalensis* (fig. 24) of India. As it happens, small teeth resembling the small first molar (m. 1) from Oreston have been found in Pliocene or Pleistocene deposits in Italy, France, and North Africa. The small equine, with small short-pillared molars, which inhabited Auvergne and other parts of France in early Pleistocene times, has generally (like the small-pillared Italian race) been regarded as a member of the *Equus stenonis* group of species, or known as *Equus ligeris*. On the other hand, the small equine with short-pillared teeth which in Pleistocene times inhabited North

* As the animal to which the very small first molar from Oreston belonged either died or was killed when about a year old, it may have been an unusually poorly developed member of its race.

† For having the small first molar from Oreston sectioned, and for many other obligations, I am indebted to Dr Smith Woodward, F.R.S., Keeper of the Palæontological Department of the British (Natural History) Museum.

Africa was regarded by M. Thomas as a member of the ass tribe and named *Equus asinus atlanticus*.

M. Boule, who studied the small equine molars from the Pliocene and Pleistocene deposits of France (fig. 10) and the Pleistocene deposits of North Africa (fig. 9), arrived at the conclusion that the small French variety of *Equus stenonis* (sometimes known as *Equus ligeris*) and *Equus asinus atlanticus* of North Africa were intimately related, and might be regarded as the ancestors of the zebras now living in South Africa.*

It thus appears that the small equine teeth found in the south of England were regarded by Owen as belonging to an ass or a zebra, that the small-pillared teeth found in France were regarded as belonging to a variety of *Equus stenonis* (*i.e.* to *Equus ligeris*), that similar small teeth found in North Africa formed the type of M. Thomas' species *Equus asinus atlanticus*, and that *Equus ligeris* and *Equus asinus atlanticus* were looked upon by M. Boule as the direct ancestors of some of the species of striped horses now living in Africa.

From a study of the teeth alone it is difficult to decide whether the small equines which in Pleistocene times ranged from Algiers to England should be regarded as members of the ass, the zebra, or the horse section of the Equidæ. Fortunately, the Italian, French, and English deposits which yielded the small-pillared teeth have also yielded small limb bones. At first it was impossible to say whether some of these limb bones belonged to an ass or to a small horse; but by making numerous measurements it was eventually possible to distinguish ass from horse cannon bones, to say whether the horse cannon bones belonged to a slender-limbed or a coarse-limbed race, and from the length of the cannon bones to estimate approximately the height at the withers. In Arabs and other slender-limbed modern breeds the metacarpals are decidedly narrower at the middle of the shaft than at the ends (fig. 13), whereas in Shires and other coarse-limbed breeds the shaft has nearly the same width throughout (fig. 14). Moreover, while in Arabs the total length of the metacarpal is from 7·20 to 7·50 times the width of the middle of the shaft, in coarse-limbed breeds—small Shetland ponies as well as Shires—the total length of the metacarpal is only from 5·40 to 5·70 times the width at the middle of the shaft.

In the Onager (a variety of which lived in Europe during the Ice age), the cannon bones are decidedly more slender than in the finest desert Arab, and even in the Kiang of Tibet the length of the metacarpal may be equal to 8·50 times the width of the shaft.

* Marcellin Boule, "Equidés Fossiles." *Extrait du Bull. de la Soc. Géol. de France*, 3e série, tome xxvii., 1899.

It is generally assumed that slender-limbed domestic breeds are the descendants of a coarse-limbed wild species. Of this, however, there is no evidence. At the beginning of the Pliocene period there were slender-limbed species in America (*e.g.*, *Pliohippus*). At a somewhat later period a slender-limbed true horse (*Equus sivalensis*) made its appearance to the south of the Himalayas; towards the close of the Pliocene there were

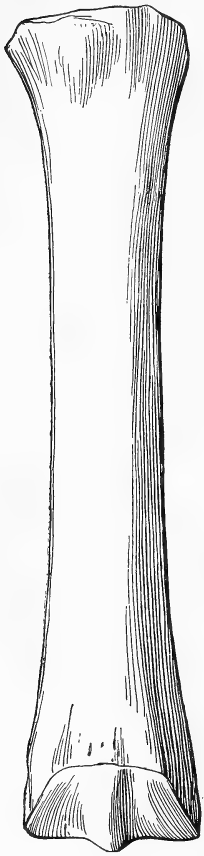


Fig. 13.

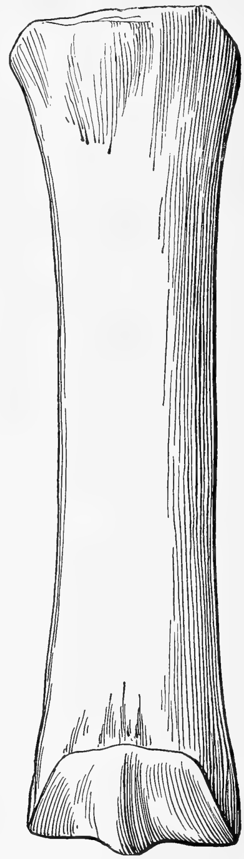


Fig. 14.

Fig. 13.—Metacarpal ($\frac{1}{2}$ nat. size) of a 12·2 hands slender-limbed horse of the "plateau" or *E. agilis* type. The total length is 7·5 times the width at middle of shaft.

Fig. 14.—Metacarpal ($\frac{1}{2}$ nat. size) of a 12·3 hands coarse-limbed horse of the "forest" or *E. robustus* type. The length is 5·5 times the width of shaft.

slender-limbed wild species in Italy and France, and slender-limbed species inhabited Europe during the Neolithic, Bronze, and La Tène periods.* Evidence of the existence of small fine-boned horses in Italy during Pliocene times we have in metacarpals from the valley of the Arno. One of these metacarpals is 220 mm. long by 30 mm. wide, another has a length of

* A metatarsal found at Spandau near Berlin measures 237 mm., and has a width at the middle of the shaft of 25 mm. In this Bronze age cannon bone (which belonged to a horse about 12·1 hands high) the length is 9·48 times the width, as in a very fine-boned small Arab which I received some years ago from Mr W. Scawen Blunt.

231 mm. and a width of 32 mm.—in the one the length is 7·30 times, in the other 7·43 times, the width.

That a similar but perhaps smaller race existed in France about the end of the Tertiary period is made evident by a metacarpal in the British Museum from Auvergne. This metacarpal, which measures 173 mm. by 24 mm. (*i.e.* in length is 7·20 times the width), belonged to a slender-limbed animal which probably measured under 11 hands at the withers.*

That slender-limbed varieties found their way to England at a remote period is proved by a metacarpal in the British Museum from Kent's Cave, Torquay. This metacarpal, which measures 220 mm. by 30·25 mm. (is in length 7·27 times the width), doubtless belonged to a fine-boned 13 hands horse allied to the Arab-like race which in Pliocene times frequented the valley of the Arno. Further, the bones and teeth of slender-limbed horses from 12 to 13 hands have been frequently met with during the exploration of Roman forts and settlements and of tumuli and crannogs. But notwithstanding the large amount of data collected, it has hitherto been impossible to feel absolutely certain that the small-pillared teeth and slender limb bones from Pleistocene and later deposits belonged to members of the same race, or that the Oreston teeth did not, as Owen believed, belong to an ass or a zebra.

Now, however, it has been proved that teeth of the Oreston type and cannon bones of the Kent's Cave type occur in the same animal. The proof has been provided by a nearly perfect skull (figs. 15 and 17) and an almost complete set of limb bones from the Roman fort of Newstead, near Melrose. The skull and limb bones are the remains of a five-year-old pony, Arab-like in make, which measured between 12 and 13 hands at the withers. In this pony the metacarpals are 214 mm. long and 28·8 mm. wide—the length is hence 7·42 times the width at the middle of the shaft. In the metacarpals this Newstead horse closely agrees with the small fine-boned fossil horses from the valley of the Arno, with the small fossil horse of Auvergne and the small fossil horse of Kent's Cave, Torquay. The two last molars (fig. 22) of the Newstead horse, though larger, very closely

* The examination of cannon bones of slender-limbed ponies of a known size indicates that in a 9 hands pony the metacarpal measures 140 mm. to 145 mm., and the metatarsal 175 mm. to 180 mm., and that as a rule each hand (4 inches) added to the height at the withers implies an increase of 20 mm. to the length of the cannon bones. Hence, when the metacarpal measures 160 mm. the height may be estimated at 10 hands, when 180 mm. at 11 hands, when 200 mm. at 12 hands, and when 220 mm. at 13 hands. But in a slender-limbed 14 hands horse the metacarpal may be only 235 mm., in a 15 hands horse 250 mm., and in a 16 hands horse 265 mm. In coarse-limbed horses and ponies the metacarpals are relatively shorter than in fine-limbed breeds, *e.g.* in a 15 hands horse of the "forest type" the metacarpal may only measure 240 mm.

Fig. 15.

Fig. 16.

Fig. 15.—Upper view of skull of a slender-limbed horse of the "plateau" type, about 12·2 hands high, from the Roman fort of Newstead. Length from occipital crest to alveolar point (*i.e.* point between central incisors), 494 mm.; length from line connecting supra-orbital foramina to alveolar point, 338 mm.; frontal (greatest) width, 185 mm. Owing to face being long and narrow, the frontal index (185 × 100 ÷ 338) is 54, as in high-caste Arabs.

Fig. 16.—Upper view of skull of coarse-limbed pony of the "forest" (Solutré) type, from Newstead. Owing to the great width of the face the frontal index is 61; in a broad-browed Shetland pony with a short dished face the frontal index may be 63; but in the very long-faced wild horse of Mongolia it may be only 50.

resemble the two molars figured by Owen (figs. 1 and 2), and the first molar (fig. 22) in all essential points agrees with the small first molar (fig. 6) from Oreston. Though all the teeth of the Newstead horse are smaller, they bear a general resemblance to the teeth of *Equus sivalensis* (fig. 24), the 15 hands Pliocene horse preserved in the Siwalik Hills of India. As in *Equus sivalensis* there is a first premolar (fig. 22, p.m. 1), but the pillar of the third and fourth premolars is less elongated, and hence more like the pillar in the still older Pliocene species *Pliohippus*. With the exception of the last molar all the six large cheek teeth have relatively shorter and narrower pillars than in *Equus przewalskii* (said to be the modern representative of *Equus fossilis*) and *Equus namadicus*; they are also shorter than in prehistoric horses of the "forest" type (fig. 23), which probably represent the stout "Elephant Bed" horse and the horse of Solutré (*Equus robustus*). The close resemblance between the teeth from Oreston and the teeth of the 12·2 hands Newstead horse strongly supports the view that the small equine which lived in the south of England along with the mammoth was a true horse, and not, as Owen believed, an ass or a zebra. Further, when the limbs as well as the teeth are considered, there are good grounds for believing that the 12·2 hands Newstead horse is a nearly pure descendant of the slender-limbed race which in Pliocene times inhabited Italy and France and in Pleistocene times ranged from North Africa to England.

For the fine-limbed horse with small-pillared molars which in Pleistocene times ranged from Algiers to England, I originally suggested the name *Equus gracilis*; but as this name is not available, I have adopted the name *Equus agilis*.

The question now arises, What part did *Equus agilis* play in the formation of modern breeds? Further inquiry will in all probability show that during the Ice age there were two varieties of *Equus agilis*: (1) a Northern variety with a coat, mane, and tail adapted for a cold, damp climate—a variety from which modern ponies of the Celtic type are in part descended; and (2) a Southern or North African variety with a fine coat, a thin lank mane, and a tail without a tail-lock—a variety which contributed largely in the making of the finer kinds of Barbs and Arabs. The Northern variety may be known as *Equus agilis celticus*, the Southern as *Equus agilis libycus*.*

In support of the view that ponies of the Celtic type have mainly sprung from *Equus agilis*, it may be mentioned that in a six-year-old Shetland pony of the riding or Celtic type, the first molar (fig. 11), in

* This Southern variety may be regarded as the ancestor of Prof. Ridgeway's "fine bay horse of North Africa," *Equus caballus libycus*.

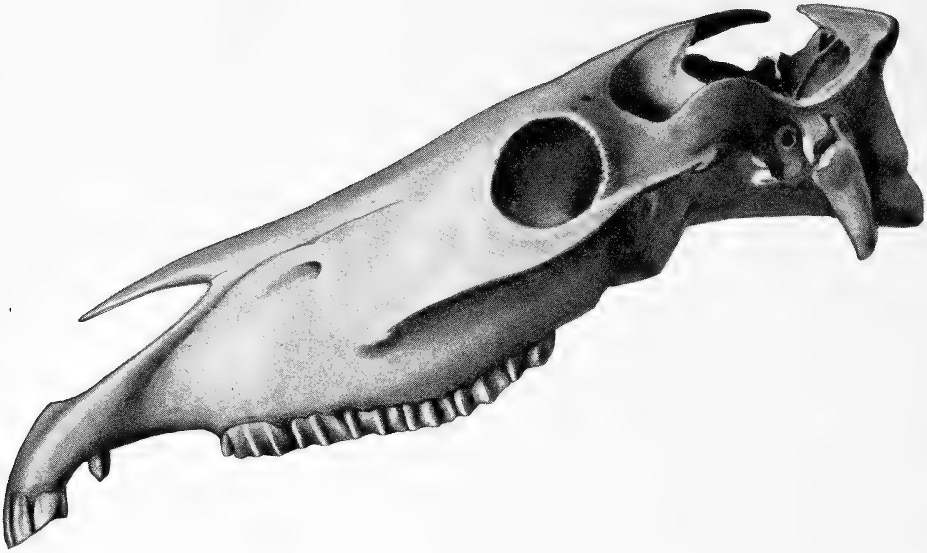


Fig. 17.

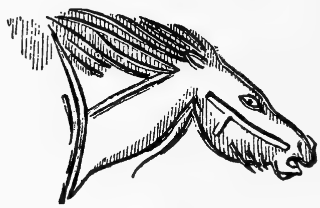


Fig. 19.

Fig. 20.

Fig. 21.

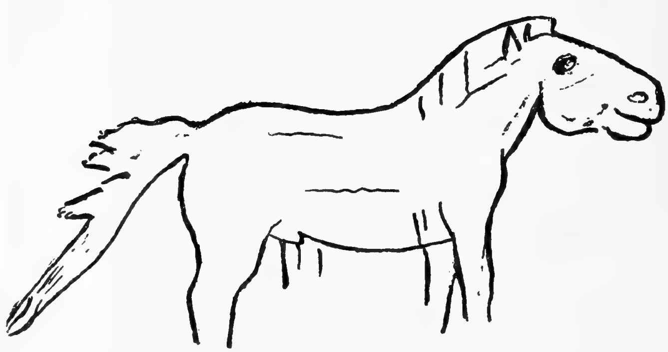


Fig. 18.

size and in the enamel foldings (including the internal pillar), closely resembles the small molar (m. 1) (fig. 10) from the French Pliocene, while the last premolar (fig. 12) resembles the last premolar (fig. 22) of the 12·2 hands Newstead horse, but has a smaller pillar.

The Shetland pony ("Eric"), to which these teeth belonged, measured 36·5 inches at the withers, and was characterised by a fine small head, fairly slender limbs, a tail-lock, and the absence of hind chestnuts.*

How many of the modern ponies of the Celtic type, in their teeth and limbs, agree with the small horse which in prehistoric times frequented the south of England it is impossible to say, for the simple reason that very few skeletons of ponies have been preserved. Hitherto, in addition to examining the teeth and limb bones of the pedigree Shetland pony "Eric," I have only had the opportunity of studying the teeth and limb bones of an Exmoor pony, a New Forest pony, a Barra (Hebridean) pony, and two Iceland ponies.

In the Exmoor pony (a bay-brown 11·1 hands pony with a light muzzle, the ergots absent and the hind chestnuts small), the length of the metacarpals is equal to 6·6 times their width; the pillar of the last premolar (p.m. 4) is as long as that of the second molar (m. 2), and the face is broad and nearly in a line with the cranium. These characters indicate that the Exmoor pony is a nearly equal blend of the "forest" (*Equus robustus*) and "plateau" (*Equus agilis*) types. In the New Forest pony (a 12·2 hands flea-bitten grey, with ergots but only one hind chestnut, with a fine head

DESCRIPTION OF FIGURES 17 TO 21.

FIG. 17.—Side view of skull of slender-limbed Newstead horse. The face is bent downwards, so that the palate forms an angle of 8° 10′ with the base of the cranium—in a Prjevalsky stallion the deflection is 16° 5′, but in an Iceland pony of the "forest" type it may be 2°. In the skull figured the premaxillæ have a length of 182 mm.; in a "forest" horse of the same size the premaxillæ measure 171 mm. The premaxilla-frontal index is 98 in the one (Newstead), but only 83 in the other.

FIG. 18.—Drawing of a horse of the "steppe" type, with an upright mane and the tail "roughened at the root." Madelaine Cave. Palæolithic age.

FIG. 19.—Drawing of head and neck of the "plateau" or *Equus agilis* type. The head is Arab-like in outline, and the mane seems to have been long enough to arch to one side of the neck. Combarelles Cave. Palæolithic age. In a drawing of a slender-limbed horse of the same period the tail has long hair up to the root, as in a modern Barb.

FIG. 20.—Leg of a pony with a broad dorsal band (eel-mark). The zebra-like bars on the leg are wide apart.

FIG. 21.—Leg of a pony with a narrow dorsal band such as occurs in the Celtic pony and Prjevalsky's horse. The leg bars are close together.

* For the opportunity of studying the skeleton of "Eric" I am indebted to Mr Charles M. Douglas of Auchlochan, Lesmahagow.

and long pasterns, and the face narrower and more deflected than in the Exmoor), the metacarpals are in length 7 times their width, and the pillar of the last premolar is shorter than that of the second molar. These traits indicate that in the grey New Forest pony the "plateau" type prevailed.

In the Barra pony (a 12 hands dark brown with slender limbs, only one small hind chestnut, but the ergots well developed), the face is longer and more deflected than in the Exmoor, the pillar of the first molar is shorter than in premolar 4 and molars 2 and 3, and the length of the metacarpal is 7·20 times the width. These characters point to the Barra pony being a blend of the "plateau" and "Siwalik" (*Equus sivalensis*) types. One of the Iceland ponies (a grey with a complete set of callosities) obviously belonged to the "forest" or *Equus robustus* type. Evidence of "forest" blood was afforded by the short broad dished face, by all six large cheek teeth having long pillars (fig. 23), by the metacarpals being in length only 5·6 times their width, and by the presence of 6 lumbar and 18 caudal vertebræ. The other Iceland (a yellow dun with a fine small head, a short back, and a well-developed tail-lock) closely approached the ideal "Celtic" type. The face is as long and fine and nearly as much deflected, and the pillars of the last premolar and first molar are nearly as short, as in the 12·2 hands Newstead pony; there are 5 lumbar and only 16 caudal vertebræ; but the cannon bones in length are only 6·8 times the width, and the hoof bones are relatively broad.

A study of these pony skeletons and the skeleton of a small Arab made it highly probable that a typical representative of the small slender-limbed horse which lived in England along with the mammoth no longer survives.

Having, by crossing Shetland, Jersey, and other breeds of cattle, produced a small ox with the horns, long frontal region, slender limbs, and probably also the colour of the Celtic shorthorn (*Bos longifrons*), it occurred to me that by crossing and selection I might re-create the Celtic pony of prehistoric times, perhaps even reproduce the small race represented by the molars from Oreston.

Ponies representing Exmoor, Welsh, Connemara, Barra, Shetland, Faroe, Iceland, Norse, Russian, Mongolian, Battak, Java, and Arab breeds were obtained and crossed in various ways. Of some forty crosses eventually produced, some belong to the robust "forest" type, some are a blend of the "forest" and "plateau" types, in others there is a suggestion of the Prjevalsky ("steppe") type, while several in their limbs, teeth, and skull closely agree with the 12·2 hands Newstead pony.

The results strongly suggest that the ponies of North-western Europe are mainly a blend of a coarse-limbed, broad-browed, short-faced race of the

"Elephant Bed" or Solutré type, and a fine-limbed race characterised by a fine muzzle and short-pillared molars, a race (like asses and zebras) without hind chestnuts and (unlike asses and zebras and the wild horse of Mongolia) without fetlock callosities or ergots.

That modern ponies have in part sprung from an almost wartless wild race akin to the small Oreston horse is supported by the following experiments. A black 11 hands Shetland pony from Unst,* with a complete set of callosities (four chestnuts and four ergots) and a mere vestige of a tail-lock, crossed with a bay 14 hands Arab, with all eight callosities but no vestige of a tail-lock, produced a black mare with a tail-lock and only minute vestiges of hind chestnuts.

This Arab-Shetland cross, bred in 1906 with a 14 hands chestnut Arab (Parakh) having large hind chestnuts and well-developed ergots, produced a black mare in which the hind chestnuts and all four ergots are absent. I can only account for the sudden disappearance of the ergots (never hitherto found absent in asses or zebras or in the wild horse of Mongolia) and of the hind chestnuts (present in all the wild horses of Mongolia, all the Shires and Clydesdales, and all but one of the thoroughbreds examined) by assuming they were absent or represented by minute vestiges in the wild ancestors from which the Shetland pony mare and the two Arab stallions had in great part descended.

In the Arab-Shetland first crosses the cannon bones are shorter and broader than in the 12·2 hands Newstead pony, and the neck is somewhat short; but in an Arab-Shetland-Connemara mixture (fig. 25) the teeth and cannon bones are practically identical with those of the fine-limbed Newstead horse, and the neck (which usually bears an intimate relation to the length of the fore limbs) is as long as in small Arabs.

It thus appears that by mixing the blood of Connemara, Shetland, and Arab ponies, animals are soon obtained which in the teeth and limbs are practically identical with the 12·2 hands Newstead horse—a horse which in its molars agrees with the small fossil Oreston race, and in its cannon bones with the fine-limbed fossil horse of Kent's Cave, Torquay.

It is of course impossible to ascertain the colour of the Oreston horse or to determine whether it had an upright mane like Prjevalsky's horse, or a long mane clinging to the neck as in Arabs.

It is generally assumed that in the wild horses of prehistoric times the mane was short and upright.

In the Arab-Shetland crosses the mane is long and fine, and it is also

* This pony, bred by the late Rev. J. Ingram, Unst, probably includes an Arab or a Barb amongst its ancestors.

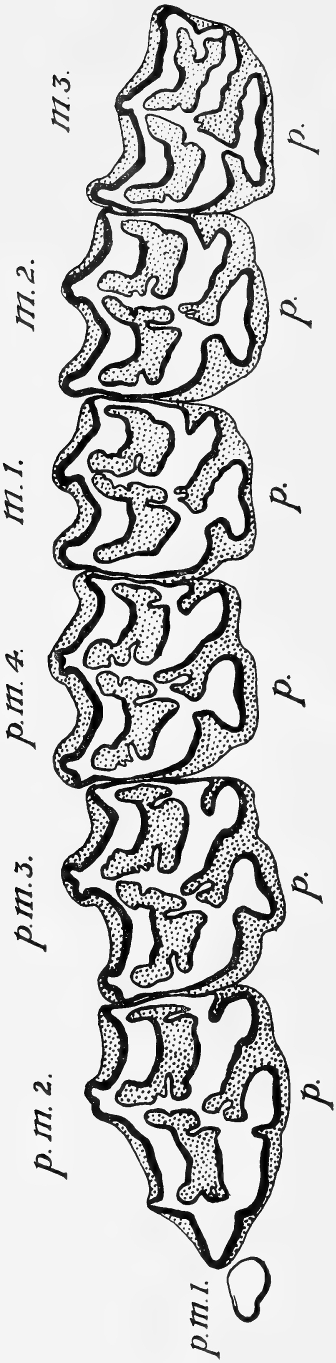


Fig. 22.

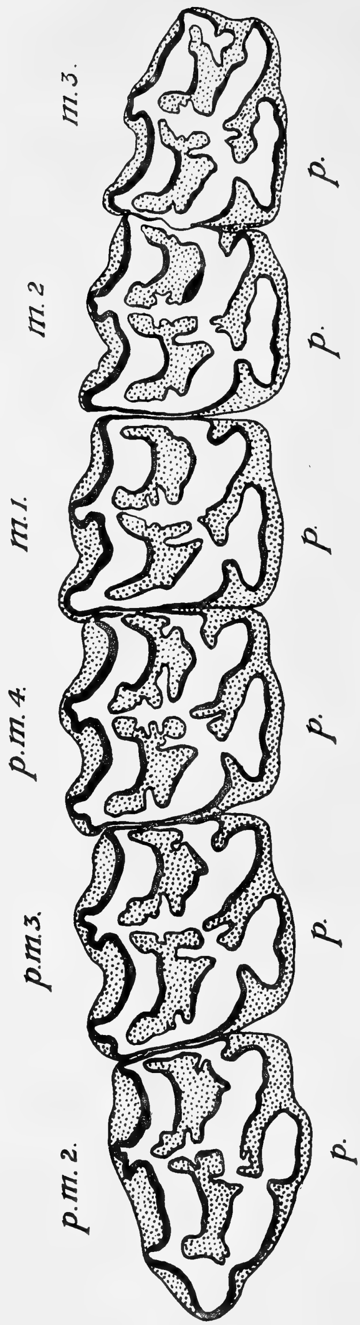


Fig. 23.

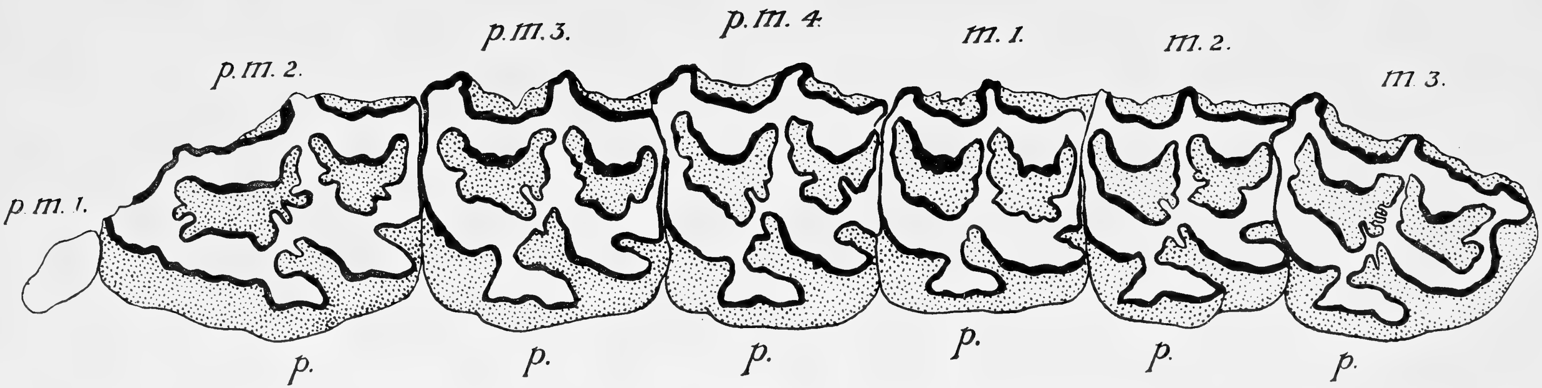


Fig. 24.

Fig. 22.—Upper cheek teeth (nat. size) of a Newstead horse ("plateau" type), about 12·2 hands high (for skull see figs 15 and 17). The two last molars (m. 2 and m. 3) closely resemble the two molars from Oreston (figs. 1 and 2), which formed type of Owen's *Asinus fossilis*; the first molar (m. 1) is slightly more complex than m. 1 from Oreston (fig. 6), and still more complex than the molar from Lake Karar, Algiers (fig. 9); in the fourth premolar (p.m. 4), which resembles p.m. 4 of the Shetland pony (fig. 12), and p.m. 4 of *Equus sivalensis* (fig. 24), the crown is nearly three times the length of its pillar instead of less than twice the length, as in *Equus fossilis* and modern cart-horses. The first premolar is about half the size of p.m. 1 in *Equus sivalensis* (fig. 24).

Fig. 23.—Upper cheek teeth (nat. size) of an Iceland pony of the "forest" type, with teeth practically identical with those of the Solutré horse (*Equus robustus*). The first premolar is absent; the internal pillar of the third and fourth premolars and the first molar is long; the crown of p.m. 4 is twice the length of its pillar, and the crown of m. 1 less than twice the length of its pillar. Hence p.m. 3, p.m. 4, and m. 1 of the Iceland pony are more specialised than the corresponding teeth in the Newstead pony.

Fig. 24.—Upper cheek teeth (nat. size) of *Equus sivalensis*. The first premolar (p.m. 1) is large and lying in front of p.m. 2; the pillar of p.m. 4 is shorter than the pillar of m. 2, but larger than the pillar of m. 1. Premolar 4, in having the pillar shorter than m. 2, agrees with the Newstead pony (fig. 22) and horses of the "plateau" type, but differs from the Iceland pony (fig. 23) and horses of the "forest" and "steppe" types. Though *Equus sivalensis* is the oldest true horse known, it has more highly specialised teeth than the Oreston and Newstead ponies. After Lydekker. *Palæontologia Indica*, Ser. x., vol. ii.

long and flowing in Welsh-hackney-Connemara crosses; but in a mixture of these five breeds (a cross between the black Arab-Shetland mare and a black Welsh-hackney-Connemara stallion) the mane is short and arches to one side of the neck, as in zebra-pony hybrids.

In drawings of horses of the steppe type copied from the Madelaine Cave the mane is represented as short and upright (fig. 18), but in a drawing of a fine-headed horse (fig. 19) in the Combarelles Cave there is a suggestion of a mane long enough to arch to one side of the neck. It is hence possible that in the Welsh-hackney-Connemara-Shetland-Arab pony we have reproduced the kind of mane worn by the fine-headed, slender-limbed horses which lived in Europe during the Palæolithic period.

Hitherto it has been generally assumed that the wild ancestors of the domestic horse were of a dun colour and more or less striped. Referring to the remote ancestors of horses, Prof. H. F. Osborn says we may imagine them "as resembling a lot of small fox-terriers, in size only eleven inches, or two and three-quarters hands, at the withers, covered with short hair which may have had a brownish colour with lighter spots resembling the sunbeams falling through the leaves of trees, and thus protecting the little animals from observation." *

Sir Harry Johnston believes that the dark stripes in zebras and other recent Equidæ represent the ground colour,† but as a rule it is assumed that the light stripes of zebras, the grey and reddish-brown colours of asses, and the yellow-dun colour of Prjevalsky's horse represent the original ground colour.

If the remote ancestor of the Equidæ, as Prof. Osborn imagines, was of a dark colour relieved by light spots (or longitudinal white stripes like the young tapir), it is conceivable that in course of time spots united to form light irregular transverse bands such as one gets in zebra-hybrids; that later the light bands formed well-defined stripes such as occur in zebras, or, in the case of desert forms, blended with each other to give rise to a coat almost free of stripes such as we now find in the kiang and the wild horse of Mongolia.

In Prjevalsky's horse there is a narrow, not very pronounced dorsal band, and sometimes a faint shoulder stripe and indistinct bars on the legs; but in domestic horses, bays as well as duns, there are sometimes vestiges of numerous stripes. In some domestic horses there is a broad dorsal band, in others a narrow dorsal band.‡ When the dorsal stripe is broad, the bars

* *The Century Magazine*, vol. lxix., No. 1, 1904.

† The Woburn Library, *British Mammals*, pp. 276–277, 1903.

‡ In zebra-hybrids there is often a narrow light stripe at each side of the dorsal band, continuous with light hairs at each side of the mane.

on the legs are wide apart (fig. 20); when the dorsal stripe is very narrow, the bars on the legs are usually also narrow and near each other (fig. 21).

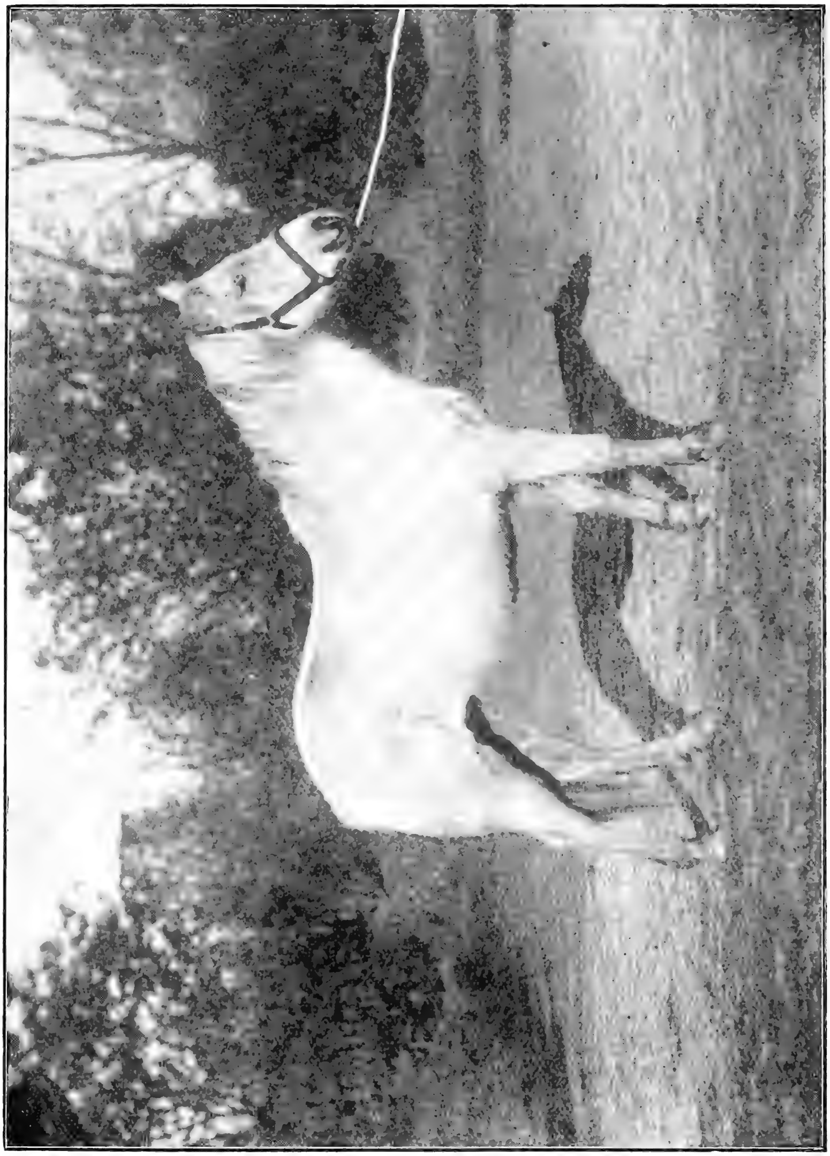


FIG. 25.—A Connemara-Shetland-Arab pony, which, in the size of the head and in the limbs and teeth, probably fairly accurately reproduces the slender-limbed Newstead 12–13 hands pony.

Horses of the "forest" type have a broad dorsal band, and sometimes vestiges of stripes on the face, neck, shoulders, trunk, and hind quarters; but in horses of the "plateau" (*Equus agilis*) type the dorsal band is narrow, and vestiges of shoulder and other stripes are rare.

The wild horse (*Equus przewalskii*) now found in Mongolia was probably as destitute of stripes in prehistoric times as it is to-day. That the wild species from which modern horses of the "forest" type are descended was at least as richly striped as some of the recently exterminated quagga is extremely probable; and it is also probable that the wild species which contributed to the making of the Kathiawar, Battak, Java, and other Oriental breeds was also more or less striped; but the numerous crosses between ponies of the Celtic and the Libyan or Arab types afford no support to the view that the slender-limbed horse which in prehistoric times ranged from North Africa to England had a richly striped coat.

The dun-coloured crosses between Arabs and native ponies I came across in Mexico some years ago, *i.e.* ponies with a fine head, slender limbs, and the hind chestnuts small or absent, had as a rule only a narrow dorsal band. On the other hand, dun-coloured crosses of the "forest" type—with a long body, and short limbs provided with a complete set of callosities—had as a rule a broad dorsal band, bars on the legs, and sometimes in addition shoulder and face stripes.

One of my Shetland-Arab crosses was a yellow dun with a fairly broad dorsal band and distinct bars on the legs. This cross was the offspring of the black Shetland pony from Unst and a bay Arab (Insaf) imported from India by Lord Arthur Cecil. But in this case the stripes were inherited direct from the sire, a very fleet Arab of the Siwalik or *Equus sivalensis* type, striped like a Kathiawar or Battak pony.

In the crosses mentioned above, with the hind chestnuts small or absent, stripes were conspicuous by their absence; but when the Welsh-hackney-Connemara-Shetland-Arab brown stallion with the short mane was mated with a bay-dun mare (the offspring of a brown Barra pony and a yellow-dun Iceland mare), a yellow-dun colt was obtained with a narrow dorsal band and an indistinct shoulder stripe. This dun colt (fig. 26) in make and temperament differs from all the other crosses bred. The mane, instead of falling to one side at the fourth month, only began to arch to one side at the sixth month, and, judging from the behaviour of the mane in the sire, the mane in this colt will probably always be short and clear of the neck. Notwithstanding the fact that this colt is a mixture of seven breeds, he is extremely well formed, has the tail set-on high, the hind chestnuts absent, and only minute vestiges of ergots. Unlike all the other members of the Equidæ family I have examined, this colt has each front chestnut made up of two pieces—a small circular piece about 10 mm. in diameter, and above this at a distance of 3 mm. a somewhat smaller triangular piece. Whether

this indicates that the front chestnut in the Equidæ has been derived from two callosities or pads it is impossible to say.

Though in the head, teeth, callosities, hoofs, tail, and temperament this mixture of seven breeds decidedly differs from a Prjevalsky colt of the same age, it very closely agrees in colour with some of the Prjevalsky colts bred at Woburn. The Prjevalsky horse has been specialised for a steppe

Fig. 26.—A six-months-old colt obtained by crossing Connemara, Welsh, hackney, Iceland, Barra, Shetland, and Arab ponies. In colour this colt resembles the wild horse of Mongolia. The mane is still upright, the tail is well set on, the hind chestnuts are absent, and there are only minute vestiges of ergots. When mature this mixture of seven breeds may fairly accurately represent the small horse of Oreston, to which Owen gave the name *Asinus fossilis*.

life, and in the process such stripes as may have existed in the more remote ancestors all but disappeared, presumably by an extension of the light and the gradual reduction of the dark areas. If the small horse which inhabited Western Europe along with the mammoth, as the limbs and hoofs suggest, was specialised for a plateau or desert life, it would, like the steppe horse, gradually get rid of its stripes, and each race as it was formed would acquire the colour best adapted for its special environment.

In addition to assuming that the small horse which lived in the south of England along with the mammoth (the equine named by Owen *Asinus fossilis*) had slender limbs and the grinding surface of the pillars of the fourth premolar and first molar shorter than in the last two molars, it may be said that the fine-boned contemporary of *Equus fossilis* was probably as destitute of stripes as the wild horse now living in Mongolia, but had a mane long enough to arch to one side of the neck.

About the origin of the small Oreston horse nothing is known. Perhaps it was a descendant of *Equus sivalensis* (the 15 hands horse with the face strongly deflected on the cranium which suddenly appeared in

Table showing Differences between "Steppe," "Plateau," and "Forest" Types.

	"Steppe," *E. przewalskii.*	"Plateau," *E. agilis.*	"Forest," *E. robustus.*
Skull.			
Length of vertex	518 mm.	494 mm.	506 mm.
Basilar length	460 "	442 "	451 "
Length of face*	371 "	338 "	336 "
Frontal width	187 "	185 "	206 "
Frontal index	50·4	54·8	61·3
Premaxilla length	180 mm.	182 mm.	171 mm.
Premaxilla-frontal index	96·2	98	83
Occipital foramen to vomer	119 mm.	112 mm.	112 mm.
" " to palatine	205 "	212 "	215 "
Palatal-frontal index	109	114	104
Deflection of face on cranium	16° 5′	8° 10′	2° 4′
Teeth.			
Length of six cheek teeth	180 mm.	162 mm.	169 mm.
" of premolar 1	3 "	7 "	absent
Size " 4	29 × 30 mm.	27 × 26 mm.	28 × 27 mm.
Length of pillar, premolar 4	14 "	9 "	14 "
Size of molar 1	26 × 29 "	24 × 25 "	25 × 26 "
Length of pillar, molar 1	14 "	9 "	14 "
Size of molar 2	26 × 27 "	24 × 25 "	25 × 25 "
Length of pillar, molar 2	13·5 "	13 "	15 "
Width across incisors	76 "	62 "	72 "
Cannon Bones.			
Metacarpal III. $\left\{\begin{array}{l}\text{length}\\ \text{width}\end{array}\right.$	$\frac{203}{31}=6{\cdot}6$	$\frac{214}{28{\cdot}5}=7{\cdot}5$	$\frac{204}{35}=5{\cdot}8$
Metatarsal III. $\left\{\begin{array}{l}\text{length}\\ \text{width}\end{array}\right.$	$\frac{249}{31}=8{\cdot}0$	$\frac{257}{28}=9{\cdot}2$	$\frac{237}{33}=7{\cdot}1$
Vertebræ.			
Lumbar vertebræ	5	5	6
Caudal "	18	16	18

* Distance from line connecting supra-orbital foramina to alveolar point.

India in Pliocene times along with the camel); but as the face is narrow and only forms an angle of about 8 degrees with the cranium, it is more likely to be a descendant of a late Miocene species allied to *Pliohippus gracilis*.

Towards the cost of the crossing experiments contributions were received from the Royal Society Government Grant for Scientific Investigations and from the Moray Fund of the University of Edinburgh.

Figures 1 to 10, 13, 14, 15, 17, and 22 to 24 were drawn by Miss G. M. Woodward; figures 16, 20, 21, and 26 are from drawings by Mr G. Taylor.

Fig. 27.—A pony of the Celtic type in winter coat, with a well-developed "tail-lock." This pony, imported from the north of Iceland in 1900, is the granddam of the colt represented in fig. 26.

(*Issued separately February* 26, 1910.)

XV.—Current Observations in Loch Garry. By E. M. Wedderburn, W.S.

(MS. received January 21, 1910. Read January 10, 1910.)

Following on the current observations made during 1908 in Loch Ness * by the author and Mr W. Watson, a few observations were made in Loch Garry (Ness basin) during August and September 1909. Mr Macdonald, Fort Augustus, again assisted in the work. The Loch Ness observations showed that the currents in the loch were very confused, but certain general conclusions were drawn; *e.g.* (1) that when a lake has become stratified and the temperature discontinuity has appeared, the return current is nearly always found above the discontinuity; (2) that towards the windward end of the loch the return current may take place close to the surface; (3) that slow currents are to be found below the discontinuity, and with the same direction as the surface current. All these conclusions are borne out by the Loch Garry observations, and some additional information has been obtained.

Loch Garry is a much more convenient size for observation than Loch Ness. The main basin of the loch, in which the observations were made, is about 4 miles long, with a maximum depth of about 220 feet. A description of the loch will be found in the Lake Survey Reports, *Geogr. Journ.*, vol. xxx. p. 401, 1907. Observations were made at two stations, viz. at the east end of the loch in about 100 feet of water, and at the deepest part of the loch. Buoys were moored in these positions as described in the paper on the Loch Ness observations. The observations were interrupted on several occasions by the breaking of the wire by means of which the buoys were moored, and it is thought that faulty wire must have been supplied, as the same method of anchoring answered well in Loch Ness.

The current meter used was the same instrument as was used in Loch Ness, and after being overhauled it worked very well indeed. It was noticed that during very stormy weather the propeller of the meter had a tendency to move backwards owing to the jolting movement of the waves, so that the velocity of the currents recorded was probably rather greater than the instrument indicated. If any work requiring great accuracy is to be done with this instrument, further investigation into the effect of the jolting

* "Observations with a Current Meter in Loch Ness," *Proc. Roy. Soc. Edin.*, xxix. p. 620.

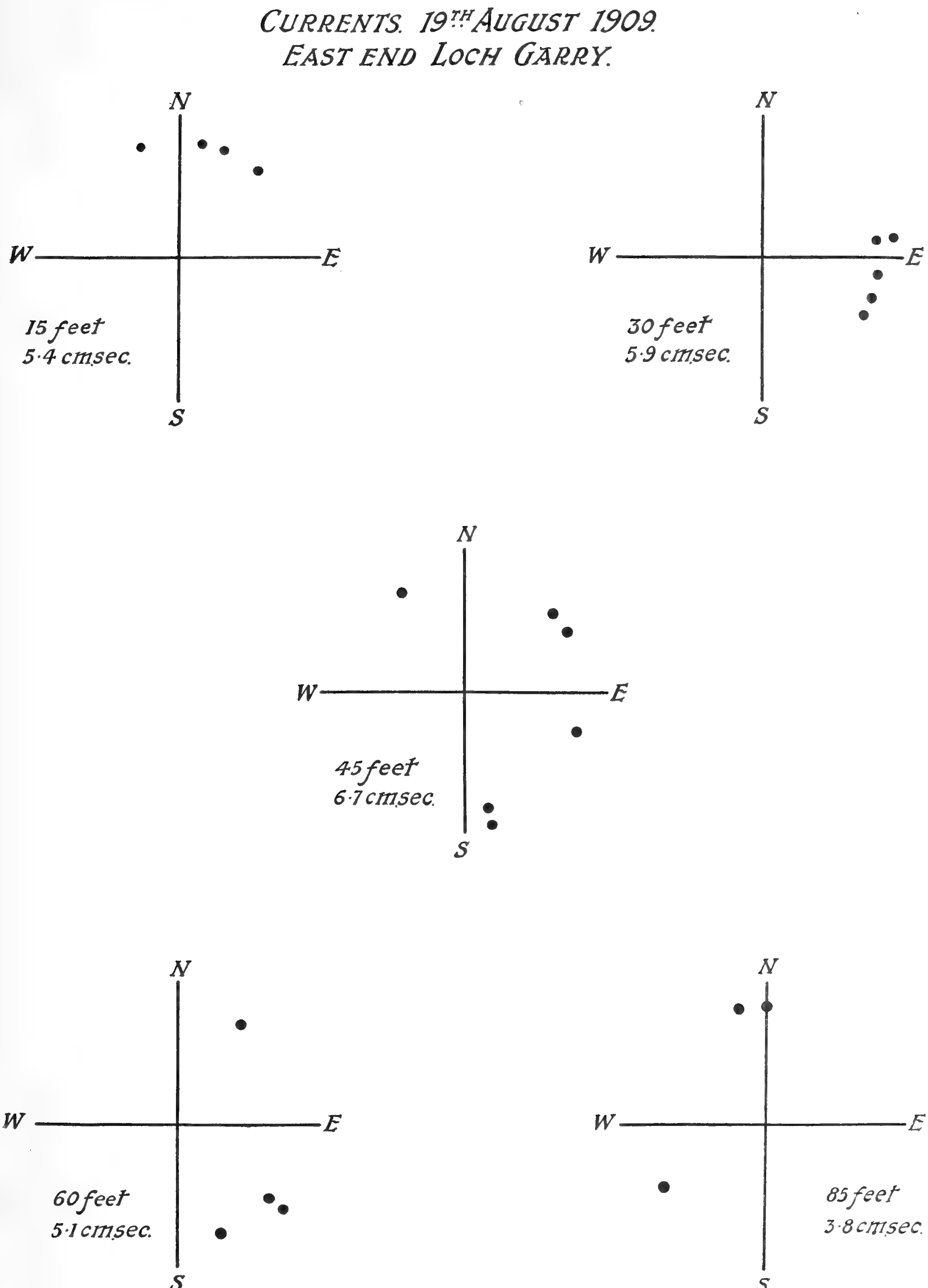

CURRENTS. 19TH AUGUST 1909.
EAST END LOCH GARRY.
N
W
E
S
15 feet
5·4 cm.sec.
N
W
E
S
30 feet
5·9 cm.sec.
N
W
E
S
45 feet
6·7 cm.sec.
N
W
E
S
60 feet
5·1 cm.sec.
N
W
E
S
85 feet
3·8 cm.sec.

FIG. 1

of the waves will be necessary. The present observations are, however, sufficiently accurate to show the general nature of currents in the loch.

An attempt was made to carry out our previous suggestion of numbering the balls which indicate the direction of the current, so as to determine how the direction of the current varied. This was found quite workable, and it showed that the variation in direction was very irregular, and was not due to a gradual change of direction. As an example, the following observation at 45 feet on 6th August may be cited. The observation lasted for one

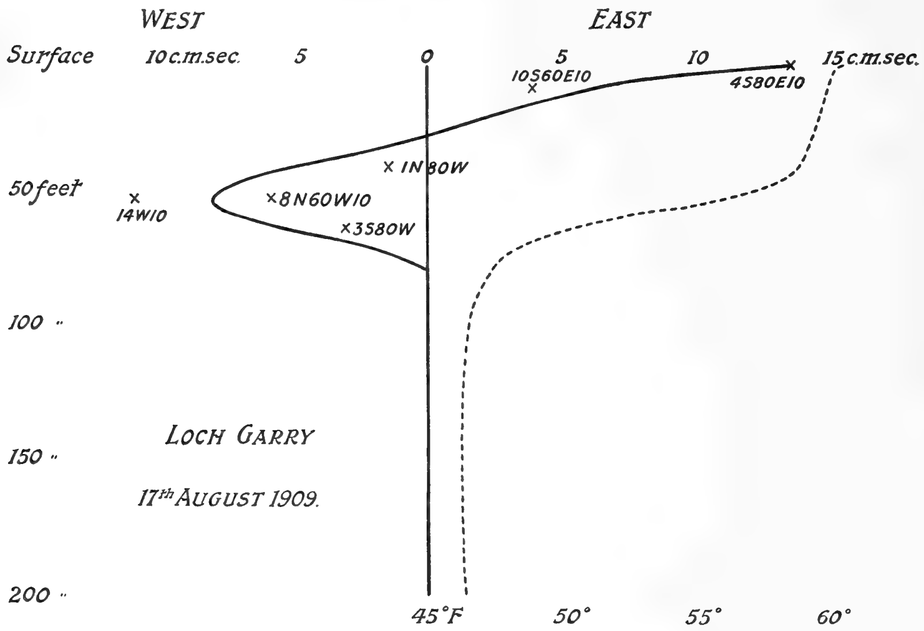


FIG. 2

hour, and during that time the average rate of the current was 4·5 centimetres per second. There were eleven indications of direction, and the following is the manner in which the direction varied:—

1st indication,	N. 60° W.	7th indication,	S. 80° W.
2nd "	N. 80° W.	8th "	S. 60° W.
3rd "	N. 70° W.	9th "	S. 50° W.
4th "	W°.	10th "	N. 80° W.
5th "	W°.	11th "	S. 40° W.
6th "	N. 60° W.		

The numbered balls were troublesome to work with in stormy weather and consumed a good deal of time, so that after the irregularity of the variation in direction had been ascertained, ordinary unnumbered balls were used.

There was a well-marked temperature discontinuity in the lake during

the period of observation, as is shown by the following typical series of temperatures taken at the deepest part of the lake :—

Depth.	17th Aug. 1909.	11th Sept. 1909.
Surface.	60·5° F.	55·0° F.
10 feet.	59·4	54·8
25 ,,	59·1	...
35 ,,	59·0	...
50 ,,	55·6	53·5
60 ,,	50·5	52·8
75 ,,	47·6	51 2
85 ,,	...	47·4
100 ,,	46·5	46·9
200 ,,	46·4	46·6

The first series of observations shows a discontinuity at a depth of about 50 feet, and the second, being later in the year, at about 75 feet.

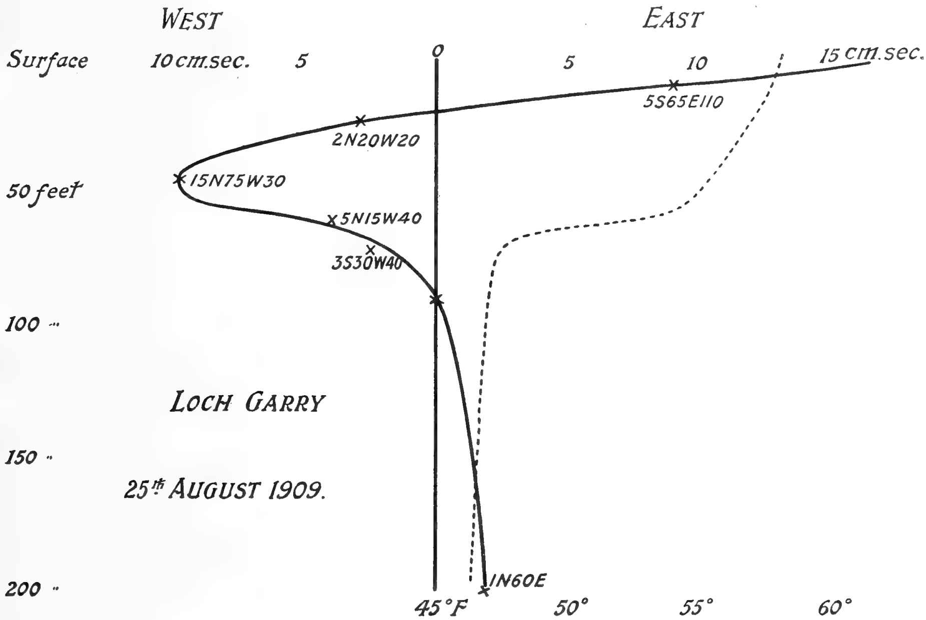


FIG. 3

Observations at the East End of the Loch.

Observations were made at the east end of the loch on sixteen days, and, neglecting observations at the surface, on nine of these days there were observed currents of greater velocity than 3 centimetres per second

(the limit of accuracy of the current meter). On 2nd August and on the morning of the 3rd it was calm, and currents were very slight, indicating, either that currents produced by the storm of the previous day had died away very rapidly, or that the storm had not continued long enough to produce steady currents. Probably the former is the correct explanation. A moderate but steady wind blew on the afternoon of the 5th and on the 6th. Currents in the same direction as the wind were recorded down to depths of 25 feet. At 45 feet there was a fairly strong current, but unfortunately no direction was recorded by the meter. We may assume,

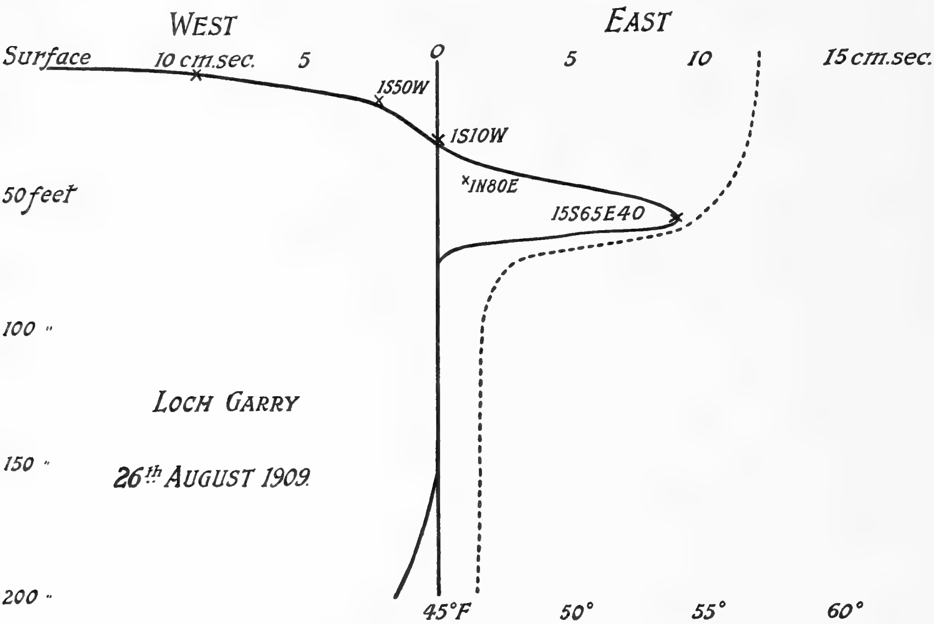


FIG. 4

however, with some certainty that it was a return current. On the 7th there was again a very strong current at 50 feet, with very variable direction, and also on the 9th. On 10th August there was a calm, and only very slight currents were recorded. On the 14th and 16th, though the wind was steady the currents were slight. On the 18th and 19th there were strong westerly winds, and on the 19th strong currents were recorded at all depths. Below 30 feet, *i.e.* below the direct surface current, the directions were very variable. The velocity of the current was strongest at 45 feet—6 centimetres per second—and the following were the recorded directions:—2 S. 10 E., 1 S. 70 E., 1 N. 50 E., 1 N. 60 E., 1 N. 30 W.* Strong winds continued on the 20th, and again the current was of greatest strength

* For explanation of the notation, see paper on "Loch Ness Observations," *sup. cit.*

at 45 feet, and was variable in direction:—2 S. 20 W., 1 S. 30 W., 1 S. 10 W., 1 S. 60 E., 2 N. 70 E. On the 21st, strong currents, again of variable direction, were observed from 75 to 100 feet. On the 28th moderate currents were observed down to 75 feet. The current was strongest at 60 feet, and variable in direction:—1 S., 1 S. 70 E., 1 E., 2 N. 80 E., 1 N. 80 W.

The most notable feature about the observations at the end of the loch was the great variability of the currents. The direct surface currents were fairly uniform, but no very definite return current was observed. Currents were strongest in the neighbourhood of the temperature discontinuity, but

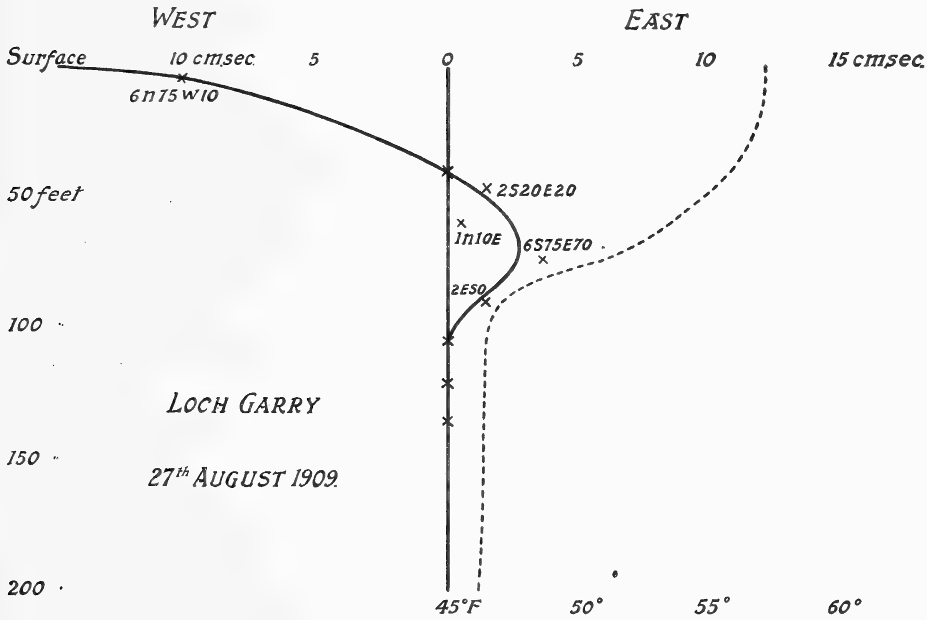


FIG. 5

they were nearly always very variable in direction. The diagrams on fig. 1 represent the observations on 19th August, the directions of the currents at the various depths being shown by dots on the compass diagrams.

Practically all the observations were made during a west wind, and so were made at the lee end of the loch. On the 19th a strong west wind was blowing, but the general trend of the currents for depths between 15 and 60 feet is easterly, and the variation in direction and the velocity are greatest near the discontinuity. Observations on the 19th will be referred to later.

Observations at the Centre of the Loch.

Series of observations were made at the centre of the loch on fourteen days, and some isolated observations were also made. The most notable

difference between the observations there and at the end of the loch was the greater strength of the currents and their greater uniformity in direction. The greatest velocity observed at the end of the loch (save at the surface) was 6·7 centimetres per second at a depth of 45 feet on 19th August. Greater velocities than this were observed on five days at the centre of the loch, the greatest being a velocity of 13·2 centimetres per second at 50 feet on 19th August. On the whole, too, there was greater uniformity in the direction of the currents at the centre. Thus on 17th August there was a current of 11·9 centimetres per second at 50 feet.

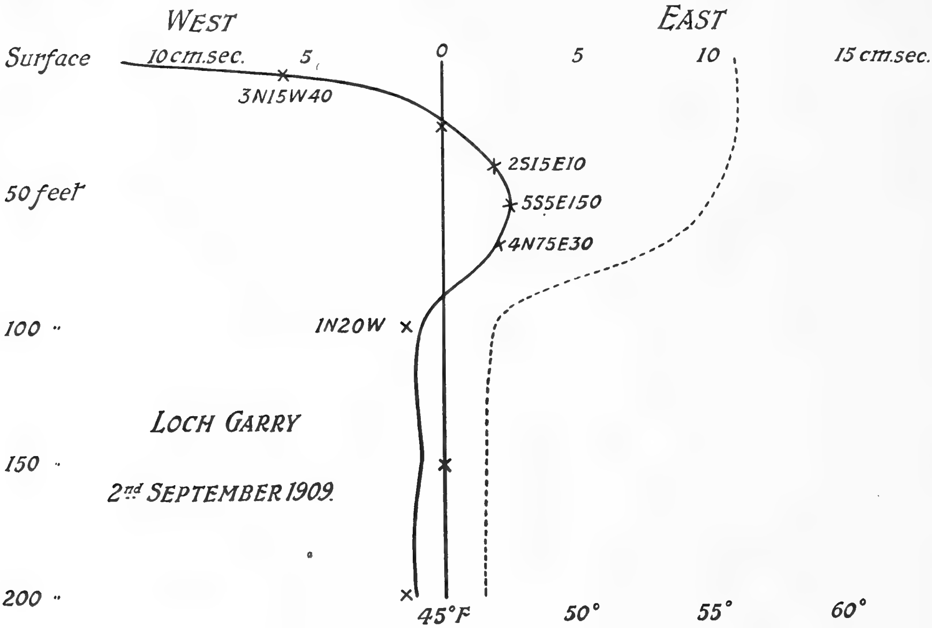


FIG. 6

Fourteen indications of direction were obtained, and they showed a variation in direction of only 10 degrees.

The most interesting observations are shown in figs. 2–7. In these diagrams the variation of the strength of the current with depth is shown by a continuous line. The portion of the curve to the right of the zero line represents easterly currents, and the portion to the left westerly currrents. Velocities are given in centimetres per second. The actual observations on which the curves are based are shown by crosses. The dotted lines represent the temperature distribution at the place of observations.

Fig. 2. Observations on 17th August. There was a light westerly

wind on the 16th, followed by a calm and easterly breezes. It is remarkable that the currents should be so pronounced as they were, in the absence

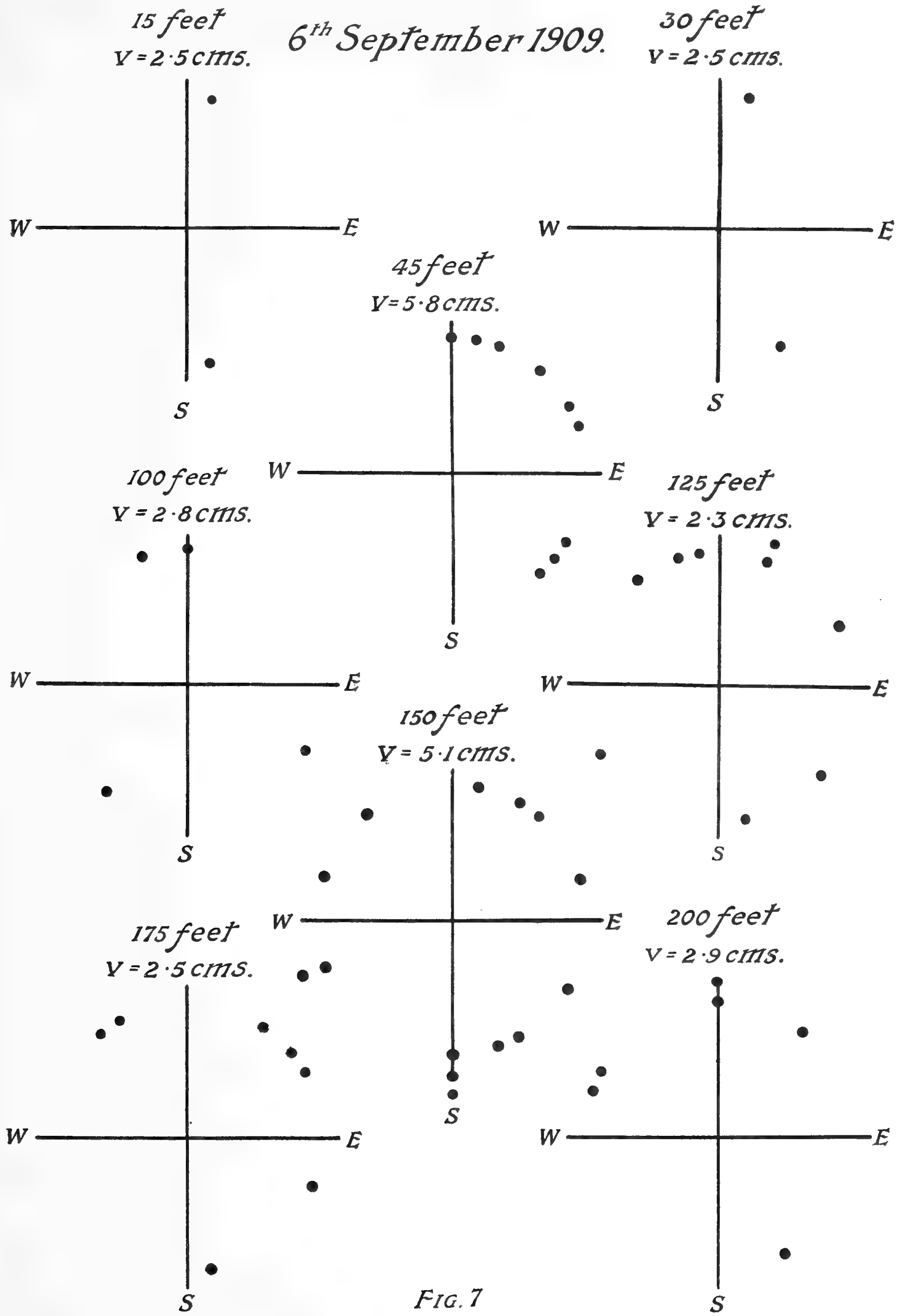


FIG. 7

of strong winds. The position of the return current is clearly seen, and its velocity is greatest in the neighbourhood of the temperature discontinuity. This relation between the return current and the discontinuity is

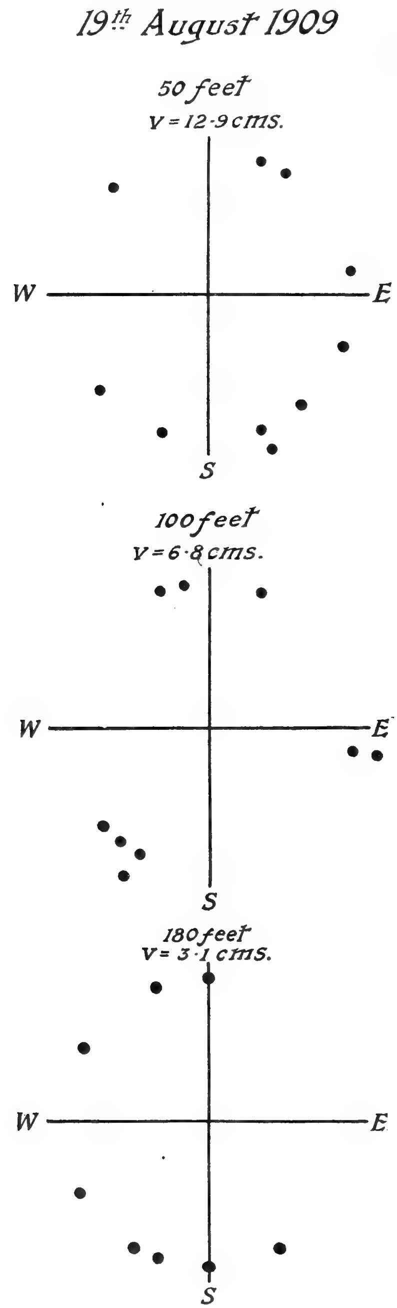


FIG. 8

seen in all the diagrams. All the currents are very uniform in direction.

Fig. 3. 25th August. The wind had been blowing steadily from the east since the 23rd. On the 25th it was of moderate strength. The currents were fairly uniform, and a current in the same direction as the wind is shown at the bottom of the lake.

Fig. 4. By 26th August the wind had changed to the west, and the current systems seem to have adapted themselves very quickly to the change of direction.

Fig. 5. On 27th August westerly winds continued. The temperature discontinuity is not nearly so marked as formerly, because the distribution of temperature has been disturbed by the high winds. Probably in consequence of this the return current is not so marked as usual, and is found in deeper water.

Fig. 6. 2nd September. On 31st August there were strong westerly winds, but on 1st September it was nearly calm. On 2nd September there was a moderate westerly wind, and notwithstanding the absence of strong winds there is found a deep current in the same direction as the wind. The surface currents are not strong, and it is surprising that any currents were found below the discontinuity.

It would appear from the observations that high winds do not necessarily mean strong currents, except, of course, at the surface. Thus the considerable velocity of 11·9 cm. per second was recorded on 17th August during light winds. On the other hand, the currents on 6th September during high west winds were comparatively slow. They were, however, very confused in direction, as is seen from the diagrams in fig. 7.

On 19th August the wind was so strong that only a few observations could be made at the centre of the loch. They are shown on fig. 8. Though strong, they are very variable in direction. The observations at the end of the loch on this date, which have already been referred to, also showed very confused currents. Apparently the strong winds had the effect of thoroughly mixing the loch. This was shown also by the temperature observations, for there was a lowering of the temperature of the surface layers and a deepening of the depth of the discontinuity.

Throughout the observations special attention was directed to currents occurring below the discontinuity. Currents were frequently observed at the very bottom of the lake, but they were usually variable in direction. The currents of 19th August have already been referred to, and though the direction of the currents at 180 feet is variable, the average direction is westerly, *i.e.* in the same direction as the wind. The diagrams in fig. 9

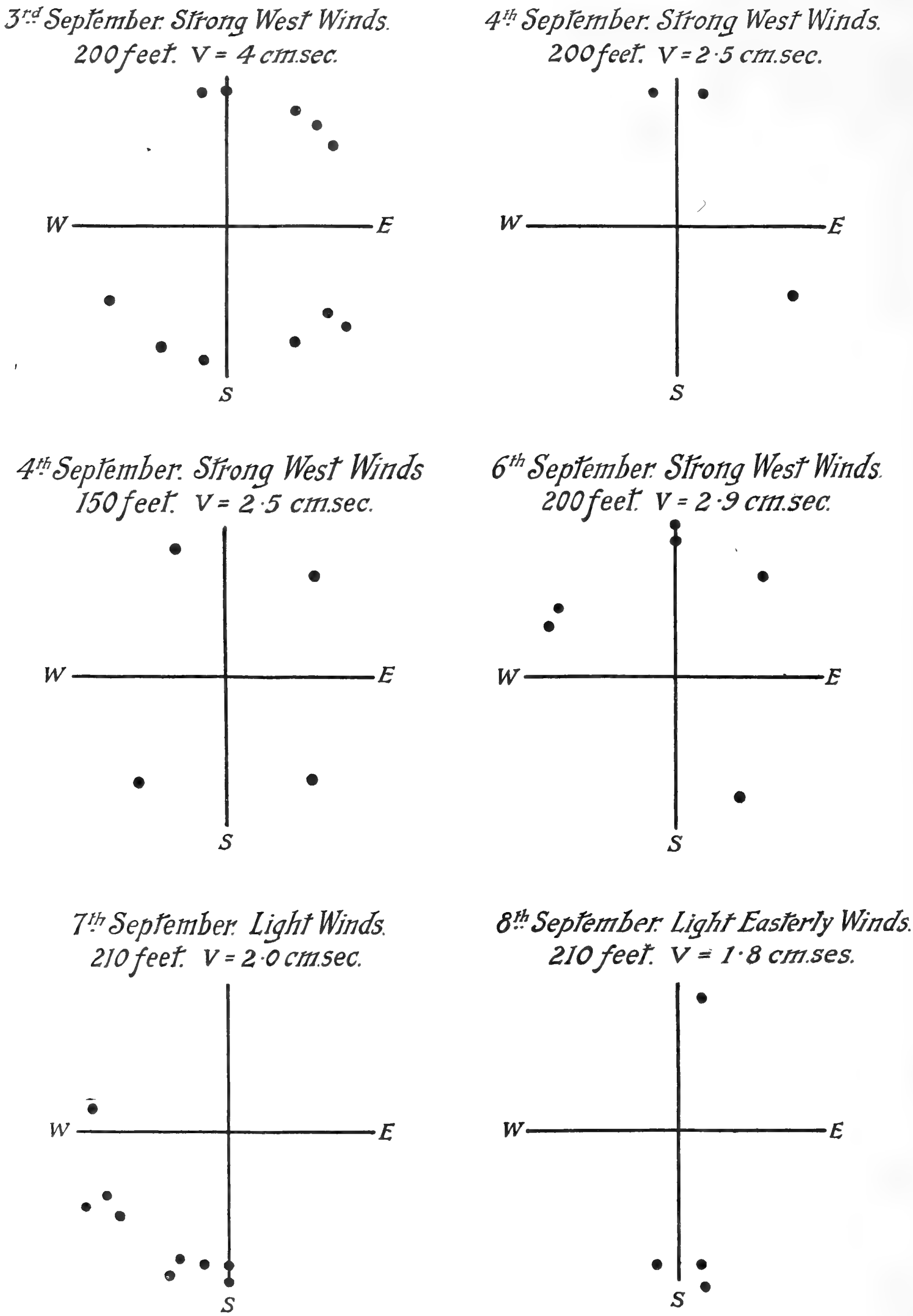

3rd September. Strong West Winds.
200 feet. V = 4 cm.sec.
W
E
S
4th September. Strong West Winds.
200 feet. V = 2·5 cm.sec.
W
E
S
4th September. Strong West Winds
150 feet. V = 2·5 cm.sec.
W
E
S
6th September. Strong West Winds.
200 feet. V = 2·9 cm.sec.
W
E
S
7th September. Light Winds.
210 feet. V = 2·0 cm.sec.
W
E
S
8th September. Light Easterly Winds.
210 feet. V = 1·8 cm.ses.
W
E
S

FIG. 9

give the principal deep-water observations. Little conclusion can be drawn from them, save that currents do exist at the bottom of the loch, and that they are at times of considerable strength. When the currents are strong they appear to be confused in direction, and are probably more of the nature of eddies. The slower currents appear to be steadier, and to be in the same direction as the surface currents.

Considerable stress has been laid on the variability of the direction of the currents, but it must not be forgotten that in certain conditions—continuous moderate winds—the currents may be very uniform. In all the observations shown in figs. 2 to 6 there was considerable uniformity. For example, on 17th August, at 50 feet, in an observation lasting for thirty minutes, there were fourteen indications of direction, and the variation in the direction of the current was only 10°. But even in this case, when the observation at 50 feet was repeated three hours later, and when eight indications of direction were obtained, with again a variation in direction of 10°, the mean direction of the current had varied from W. to N. 60° W.; the velocity of the current had also fallen from 11·9 to 6·5 cm. per second.

The variability of the strength of the current was almost as marked as the variability in its direction; *e.g.*, determinations at 40 feet on 18th August gave respectively 1·3, 1·9, and 3·8 cm. seconds for the velocity; at 200 feet on 6th September, ·8 and 3·0 cm. seconds; and at 200 feet on 4th September, 1·1, 2·5, and 4·3 cm. seconds.

The value and interest of the observations would have been greatly increased had it been possible to observe simultaneously with two or more current meters. It would have then been possible to follow more closely the changes in the direction and in the velocity of the currents. On several occasions doubts arose as to whether some of the variations observed were not instrumental, and doubts of this sort would have been put at rest if a second current meter had been available for control observations. It is to be hoped that more elaborate observations will be made on a future occasion.

(*Issued separately March* 8, 1910.)

XVI.—On a New Species of Cactogorgia. By **Jas. J. Simpson**, M.A., B.Sc., Carnegie Research Fellow, Natural History Department, University of Aberdeen. (With One Plate.)

(MS. received January 24, 1910. Read January 24, 1910.)

Amongst the unnamed Alcyonaria in the collection of the Royal Scottish Museum, Edinburgh, is a beautiful colony belonging to the genus *Cactogorgia*, which Mr Eagle Clarke has kindly handed me for identification and description.

In 1907 (*Trans. Roy. Soc. Edin.*) I established the genus *Cactogorgia* for several small colonies from the Indian Ocean, and referred these to three separate species, viz. *celosioides*, *alciformis*, and *expansa*. Thomson and M'Kinnon, in *Trans. Linn. Soc.* (*Zool.*), 1909, have described another species from the Seychelles under the name of *Cactogorgia lampas*, and the present colony must also be referred to a new species. This we propose to name *Cactogorgia agariciformis*, n. sp., on account of its very definite mushroom-shape.

It is interesting to note that the inclusion of these two new species has not necessitated any change in the original generic diagnosis.

Cactogorgia agariciformis, n. sp.

This species is represented by a single specimen of a slightly orange-yellow colour—that is, after prolonged preservation in alcohol. It has been attached to rock, and the basal disc is overgrown by an encrusting sponge. The colony (fig. 1) is 7·5 cm. in height, and consists of two very distinct parts: (1) a lower, almost cylindrical, stalk, 4·8 cm. long, 7 mm. in diameter at the base and 12 mm. at the top; and (2) an upper, polyp-bearing, part, elevated in the centre, circular in outline and expanded horizontally, giving the whole colony a very distinct mushroom appearance. The breadth of the capitulum is 31 mm., and its maximum height 12 mm.

The whole colony is very stiff and rigid, owing to the densely interlaced, large, warty spindles, which are quite visible to the naked eye. These are arranged for the most part longitudinally, and give the translucent appearance which was characteristic of *C. celosioides* to the whole colony.

The stalk contains several large canals (fig. 2). These are supported by

extremely thick non-collapsible walls which are densely packed with large warty spicules. The canals branch near the capitulum, and connect with the polyps by means of small solenia.

The polyps are situated all over the capitulum, few in number, of a large size, and arising like the disc florets in the Compositæ (fig. 1). Each consists of a very distinct verucca, which is supported by large longitudinally arranged spindles. The apices of these project, and form strong protection to the retracted anthocodia. The oral openings of the verrucæ are about 5 mm. apart, but the bases overlap slightly.

The anthocodiæ are all retracted within the verrucæ. They are moderately large, and have a dense armature. They are about 2 mm. in height and 1 mm. in diameter. The anthocodial armature (fig. 3) consists of a "crown" and eight distinct "points." The "crown" consists of about twenty-two to twenty-eight rows of slightly curved spicules, which are placed circumferentially and interlock closely. Surmounting this there are eight triangular "points," each consisting of about four pairs of slightly bent spicules which are arranged loosely *en chevron*. There are usually a few small scattered spicules between the "points." When at rest the tentacles are infolded and overlap one another, and when expanded are about 1 mm. in length. They are conical in shape, and have a few simple pinnules. They contain small scale-like spicules arranged *en chevron* on their aboral surface.

The spicules vary in shape and character in the various parts of the colony. Those of the stalk are for the most part large spindles, some of which are almost scale-like (fig. 4). They are covered with large papillose, irregular warts. In the verrucæ they are predominantly spindles, either straight or variously curved (fig. 5). These are also covered with warts, but are not so rugose as those of the stalk.

The spicules of the anthocodiæ are straight and curved spiny spindles. Some of these may bifurcate at one end (fig. 6).

On the aboral surface of the tentacles there are small scales, irregular in outline or with a slight constriction in the middle. The flattened surface of these is slightly papillose (fig. 7).

The following are some of the measurements of the chief types, length by breadth, in millimetres :—

(*a*) Stalk, $2{\cdot}5 \times 0{\cdot}4$; $2{\cdot}5 \times 0{\cdot}36$; $1{\cdot}2 \times 0{\cdot}2$; $0{\cdot}9 \times 0{\cdot}2$.
(*b*) Verrucæ, $2{\cdot}8 \times 0{\cdot}4$; $2{\cdot}5 \times 0{\cdot}3$; $1{\cdot}25 \times 0{\cdot}2$.
(*c*) Anthocodiæ, $0{\cdot}9 \times 0{\cdot}1$; $0{\cdot}85 \times 0{\cdot}07$; $0{\cdot}8 \times 0{\cdot}045$; $0{\cdot}5 \times 0{\cdot}05$.
(*d*) Tentacles, $0{\cdot}12 \times 0{\cdot}04$; $0{\cdot}04 \times 0{\cdot}02$.

The record of the locality of this specimen has been unfortunately lost. All the other species have been recorded from the Indian Ocean.

In some respects this species approaches *Cactogorgia expansa*, but it is easily distinguished by the characteristic shape of the colony and by the architecture of the anthocodial armature.

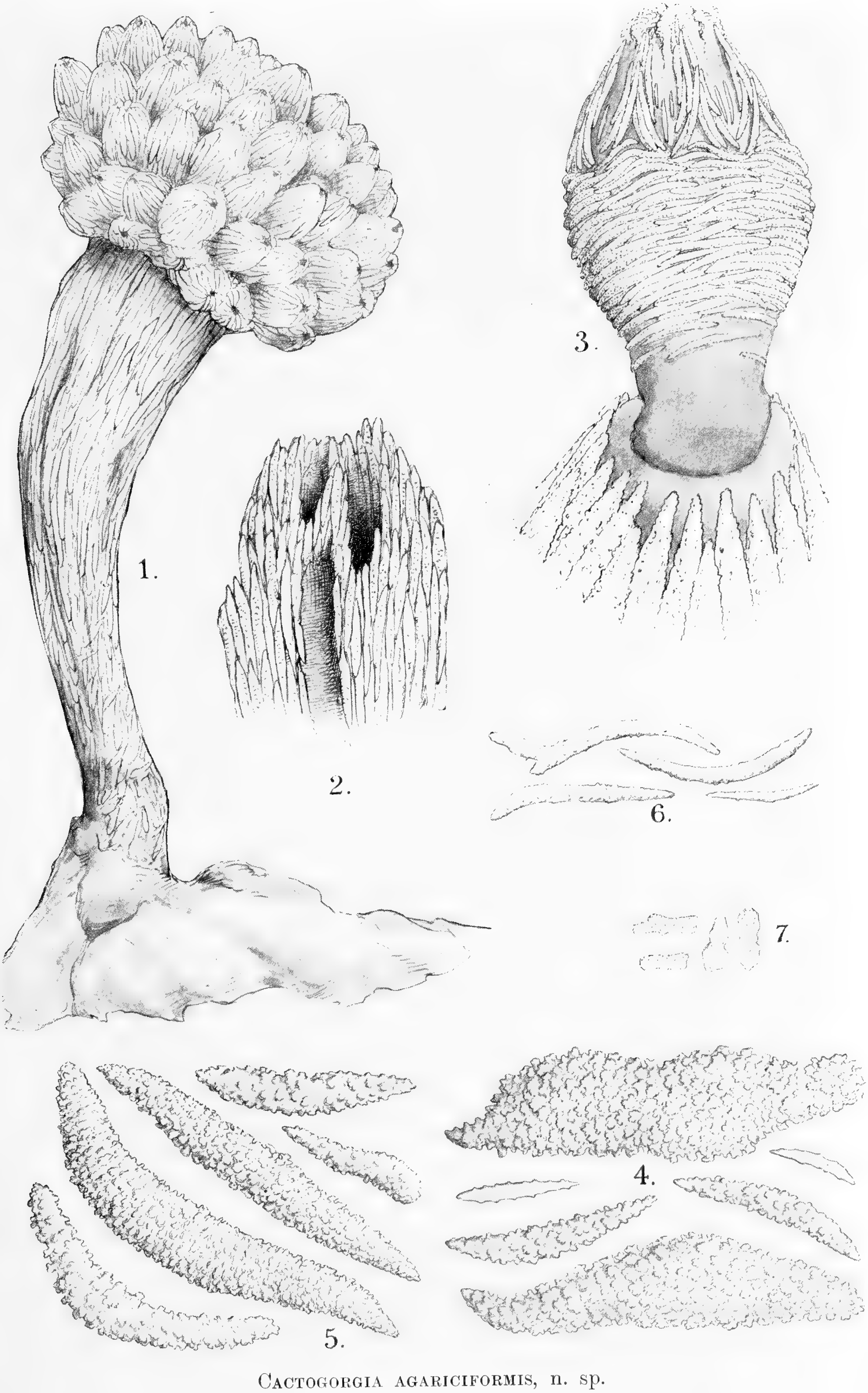
The following table gives a summary of the differences in the anthocodial armature for the different species of *Cactogorgia*:—

Species.	"Crown."	"Point."
C. celosioides, Simpson.	7–10 rows of curved spindles.	1 large pair, with occasionally 1 or 2 smaller ones between.
C. expansa, Simpson.	About 8 rows of curved spindles.	6–8 pairs arranged *en chevron.*
C. alciformis, Simpson.	10–14 rows of curved spindles.	10–15 spindles only *slightly en chevron.*
C. lampas, Thomson and M'Kinnon.	6 rows of horizontal spindles.	About 3 converging pairs of spindles, and between two "points" lies a single spindle.
C. agariciformis, n. sp.	22–28 rows of slightly curved spindles.	4 pairs of curved spindles *en chevron*, with a few scattered between the "points."

EXPLANATION OF PLATE.

Fig. 1. Colony enlarged almost twice natural size.
Fig. 2. Stalk broken across to show the large main canals (× 6).
Fig. 3. Polyps enlarged to show anthocodial armature (× 35).
Fig. 4. Spicules from the stalk (× 25).
Fig. 5. Spicules of the verrucæ (× 25).
Fig. 6. Spicules of the anthocodiæ (× 40).
Fig. 7. Spicules from the aboral surface of the tentacles (× 85).

(*Issued separately March* 10, 1910.)



CACTOGORGIA AGARICIFORMIS, n. sp.

Mr J. J. SIMPSON.

XVII.—The Development of the Autonomic Nervous Mechanism of the Alimentary Canal of the Bird.

By **Williamina Abel**, M.D., Carnegie Scholar. (From the Physiological Department, University of Glasgow.) (With Four Plates.)

(MS. received December 17, 1909. Read January 24, 1910.)

Introduction.

Concerning the development of the ganglia and plexuses in the intestine, various theories have from time to time been formulated. The older writers, among whom were Remak (1, 2), Götte (5), Balfour and Foster (6), assumed that they had a mesoblastic origin, were formed *in situ*, and became joined on to the main sympathetic chain at a later date.

Later, an ectodermic origin was generally recognised as belonging to the central nervous system, but writers were not agreed as to the origin of the sympathetic. A few, among whom were His (4), Birdsall and Schenk (8), maintained that its origin was, like the central nervous system, ectodermic, but many held the older view that it was a mesoblastic structure. Among those who held the latter view, some regarded the sympathetic chain as the basis of the visceral ganglia, while others held that the visceral ganglia were developed *in situ*, and joined later to the main sympathetic chain.

At the present time the work of His junior (12, 13, 14) on the embryonic chick affords valuable evidence that the sympathetic chain is an outgrowth from the intervertebral ganglia, and that the different ganglia in the intestine are derivatives of it. This work of His has borne out many points given in the earlier writings of His senior (11) and Onodi (9).

Viewing the question from the standpoint of histology, the balance is at present in favour of the outgrowth theory—or, in other words, of the theory that the visceral ganglia and plexuses of the intestine are outgrowths of the sympathetic chain, which is in turn derived from the intervertebral ganglia. But the evidence is by no means conclusive.

When the question is, however, considered from the standpoint of the physiological work of Bayliss and Starling (19) on the intestine, and of Elliot (21) on the innervation of the urethra and bladder, the peculiar independence of the peripheral ganglia from those of the central nervous system seems so marked as to justify Elliot in suggesting the possibility

of an independent origin, more especially as he regards the results of histological research on this point as equivocal.

The theory of Gaskell as to the homology existing between the central nervous system of vertebrates and the nervous system of invertebrates would lead to the expectation that the development of the sympathetic system in vertebrates must follow that of the central nervous system.

The dubiety which still existed as to the precise mode of development of the nerve elements forming the autonomic nervous system of the intestine, and the possibility of their independent origin suggested by physiological experiment, seemed to show the desirability of again investigating the question from a histological standpoint, with the aid of a more modern method.

One of the most complete histological investigations on the question was made by His junior (14), who used the hæmatoxylin-eosin method. Since this is not a stain peculiarly adapted for nerve work, the desirability of repeating the investigation is apparent.

The present paper is therefore an account of such an investigation made on the embryonic chick with the aid of the most modern of nerve stains—the silver-nitrate method of Ramon y Cajal. The results are illustrated partly by sepia drawings for the sake of clearness, but a sufficient number of photographs are given to fully illustrate the various points dealt with. These photographs are selected from a large series used in illustrating a thesis on this subject which was presented for the degree of M.D. at the University of Aberdeen.

Historical.

Histological Section.

Remak (1) published in 1843 the results of observations made by him on the development of the embryonic chick. He described the whole nervous system as being mesoblastic in origin. The nerve network in the intestinal wall he regarded as developed primarily *in situ* from the mesoblastic tissue, and joined secondarily to the main sympathetic chain.

Four years later, in a second paper (2) on this subject, Remak stated that the nerve network in the intestinal wall appeared at the sixth day, and became joined to the main sympathetic chain during the second week.

In 1864 Hensen (3) advanced the theory that the cerebro-spinal system was ectodermic in origin, and suggested that all nerve structures originated from the ectodermic layer.

His senior (4) in 1868 described the cerebro-spinal system as originating from the ectodermic layer. Further, he described the sympathetic chain as

an offshoot from the cerebro-spinal system, and the various sympathetic ganglia and plexuses as outgrowths from the main sympathetic chain.

Götte (5), writing in 1872, supported the older theory of Remak, but added nothing important as evidence for his conclusion.

Balfour and Foster (6), in their conjoint paper of 1876, described the mesoblastic layer as the origin of nerve tissue, but failed to produce fresh evidence in support of their theory.

In 1877 Balfour (7), in an article on the development of elasmobranch fishes, described very minutely short outgrowths from the first portion of the spinal nerves. From the position of these outgrowths, and from the appearance of small clusters of cells at their terminations, Balfour thought it probable that they formed the origin of the sympathetic chain.

In 1879 Schenk and Birdsall (8) described the results of observations made by them on human embryos, and embryonic chicks and rabbits. In all three embryos they found the sympathetic chain to originate as masses of cells migrating from the spinal ganglia. In one case of a human embryo they claimed to have successfully traced outgrowing nerve cells from the sympathetic chain to the wall of the intestine.

Onodi (9) in 1884 published the results of an extensive investigation on the development of the sympathetic nervous system in embryonic fish, amphibians, reptiles, birds, and mammals. Of these, he found embryonic fish to give the most satisfactory results, as the rate of development is comparatively slow.

From a consideration of these results Onodi described the sympathetic chain as originating by a process of proliferation and migration of cells from the spinal ganglia. The various sympathetic nerve elements in the intestinal wall and elsewhere were found to be outgrowths from the sympathetic chain.

In 1890 Paterson (10) made a similar investigation on human embryos, and embryonic rabbits, rats, and mice. His results may be briefly summarised as follows:—

1. The sympathetic system is developed in mammalia out of cellular tissue surrounding the aorta, and is at first quite independent of the cerebro-spinal system.
2. The connection between those two structures is effected by outgrowths from the spinal nerves.
3. The various plexuses, ganglia, and non-medullated nerves of the general sympathetic system are outgrowths from the sympathetic chain.

In the same year His senior (11) gave a minute description of the

origin of the sympathetic chain from the intervertebral ganglia. This description differed from that given by Onodi of the same structure, in the fact that His regarded the cells of the future sympathetic chain as existing in an immature condition in the intervertebral ganglia, and only attaining maturity when they reached the position of the future sympathetic chain. As regards the visceral ganglia, they were described in Onodi's paper as originating from the sympathetic chain.

In 1890 also appeared a paper by His junior and Romberg (12), in which an account was given of the ganglionic cells in the heart, and of their relation to the sympathetic chain.

In 1891 His junior (13), in a second paper on this subject, gave a detailed description of the route followed by the nerve elements in their migration from the sympathetic chain to the heart.

In 1897 His junior (14) published a detailed description of the development of the abdominal sympathetic in the chick. A description was given of observations made from the third to the tenth days of incubation.

At the end of the third day sympathetic nerve cells were recognised in the cervical portion of the chick lying behind the carotid, and in the thoracic and abdominal portions behind the aorta. The recognition of these cells was based on their size, which was described as larger than the mesoblasts, and on the fact that their reaction to hæmatoxylin and eosin (the stain used) was better marked than that of the mesoblasts. Between these nerve cells and the spinal ganglia loosely arranged groups of nerve cells were recognised. At this age a very fine band of nerve tissue was described in the lowest portion of the intestine. From its character and position it was recognised as the intestinal nerve of Remak, but at this stage no actual connection was demonstrated between it and the rudimentary sympathetic chain. But His regarded it as undoubtedly the result of a cell migration from that structure.

At the end of the fourth day the sympathetic chain was found to be well advanced in development, and well-marked cellular migration from it was seen at the level of the heart. The intestinal nerve of Remak was easily recognised in the lower intestine, while in the stomach the vagus branches appeared.

At the fifth day sympathetic nerve cells were found in the stomach and intestinal wall. At the pyloric end of the stomach the first signs of a double layer of nerve tissue were seen. At this age a connection was found well established between the intestinal nerve of Remak and the sympathetic chain.

At the sixth day the secondary sympathetic chain described by His

as superseding a primary sympathetic chain in the lower cervical and thoracic regions was recognised. In the abdominal region the sympathetic network in front of the aorta was well developed, while in the wall of the intestine the two layers of nerve plexuses were easily recognised.

At the tenth day the abdominal sympathetic was found well developed.

In this paper also a short account was given of the development of the abdominal sympathetic in the human embryo.

In 1899 Dogiel (15) published the results of an examination made by him on the minute anatomy of the plexuses in the intestinal wall. In this article he also gave an interesting résumé of the most important publications dealing with this subject. The staining methods with methylene blue used in his own examination are described in detail. As a result of his observations, Dogiel classified the nerve cells in the ganglia of the intestinal wall into three varieties. The first or motor type was found most abundantly in the Auerbach plexus, the second or sensory type lay mostly in Meissner's plexus, while the third type was not restricted to either plexus, and seemed to possess characteristics of both the first and second types. The nerve fibres in the ganglia were also classified into two groups, detailed descriptions of which are found in this article.

In 1904 Graham Kerr (16) described the early development of motor nerves in Lepidosiren. The development of a motor nerve was traced back to a protoplasmic bridge extending between the medullary tube and the myotome. That is, the motor nerve exists from the first as a protoplasmic band between the spinal cord and the end organ. At first this band is naked, but later it is covered with mesenchymatous protoplasm laden with yolk. As development proceeds the yolk is used up, and the protoplasm with its nuclei extends into the substance of the nerve trunk. This forms the protoplasmic sheath of the nerve.

Harrison (17), in an account of transplantation experiments made in connection with the problem of nerve development, described the following results:—

1. Limb ends of tadpoles transplanted to various parts of normal individuals develop normally, and are supplied by nerves from the host which supply the region in which the transplantation was effected.
2. Limb ends taken from "nerveless" tadpoles, when transplanted into a normal individual, are supplied in an exactly similar manner.

Reference was also made to a series of experiments made by Harrison and Lewis, in which they showed that transplantation of a ganglion to a nerveless region resulted in the development of radiating nerve fibres; and,

conversely, that destruction of ganglionic cells completely arrested nerve development.

As a result of these experiments, Harrison states that the essential part or axis cylinder of a nerve is an outgrowth from the central nervous system, since without the ganglionic cells no nerve fibres develop.

In 1908 Miss Meiklejohn (18) published a preliminary note on the development of the plexiform nerve mechanism in the alimentary canal of the chick. The stain used was the silver-nitrate method of Ramon y Cajal. Miss Meiklejohn's results may be tabulated as follows:—

1. At two and a half to three days nerve cells and fibres were stained in the spinal cord and retina.
2. At three to four days the staining of the nerve elements in the central nervous system was better marked.
3. At four and a half to five days vagal fibres were seen in the stomach, and a nerve plexus of a simple character was seen in both the stomach and upper intestinal walls.
4. At the fifth and sixth days the vagal fibres in the stomach were better marked, and the plexus in stomach and intestine more fully developed.

Experimental Physiology.

Bayliss and Starling (19) gave a full description of a series of experiments made by them on the movements and innervation of the small intestine. As a result of these experiments they recognised two movements, the pendulum and the peristaltic. The pendulum movements were described as rhythmic contractions affecting the longitudinal and circular muscle coats, myogenic in origin, and probably propagated by means of muscle fibres. The peristaltic movements were described as true co-ordinate reflexes, started by mechanical stimulation of the intestine, and carried out by means of Auerbach's plexus. Further, it was stated that they were independent of the connections of the gut with the central nervous system, that they travelled in one direction from above down, and that they were abolished by the use of cocaine or nicotine, which paralysed the local nerve mechanism.

The theory was also formulated that Auerbach's plexus was a nervous system with two reflexes—inhibition and augmentation—the object being the propulsion of food. Paralysis of this nervous system by the use of nicotine or curare abolished the peristaltic but not the pendulum movements; hence it was concluded that the latter were myogenic in origin.

A joint paper by Langley and Magnus (20) dealt with the question of

the movements of the intestine before and after degenerative section of the mesenteric nerves. Observations were made on the descending colon of the rabbit and the jejunum and ileum of the cat. It was found that in the case of three cats and two rabbits, in which almost complete isolation of the intestinal loops was secured, the reaction of the intestine to adrenalin and nicotine was normal, but subnormal to atropine, strychnine, and cocaine.

Elliot (21), in his paper on the innervation of the bladder and urethra, dealt with the question of the origin of the visceral ganglia. He pointed out that in the two varieties of efferent nerves from the anterior root, that going to striated muscle had no ganglionic station *en route*, while the other to non-striated muscle invariably had. This being the case, the protoplasmic nucleated mass at the nerve ending in striated muscle must be the homologue of the ganglionic cell, or the ganglionic cell must be developed peripherally. The various points of divergence between the ganglia of the central nervous system and those of the viscera as regards their reaction to nicotine and curare were emphasised, whilst the embryological evidence of the origin of the visceral ganglionic cells was described as equivocal.

In conclusion, Elliot regarded the balance of evidence to be in favour of the visceral ganglia being separate developmental units, and not outgrowths from the sympathetic chain.

Yanase (22), in a description of observations on the intestine of embryonic porpoises, made an interesting note to the effect that movement in the intestinal canal only occurred after the development of Auerbach's plexus.

In 1908 Magnus (23), from the results of experiments, described the pendulum movements of the intestine as due to the agency of Auerbach's plexus. In this connection he criticised the view of Bayliss and Starling that this movement was myogenic, since it persisted after the application of cocaine. Magnus pointed out that the pharmacological proof was not sufficiently convincing, as only one function of Auerbach's plexus might be affected by the drug. In proof of his theory Magnus separated the two muscle coats of the intestine, and found that in the longitudinal coat, to which Auerbach's plexus adhered, the pendulum movements continued, whereas they were quite wanting in the circular coat. Further, on stimulating a preparation free of Auerbach's plexus, either by continuous chemical or electrical stimuli, it was found that there was an absence of rhythmical movements, and of a refractory period, while the preparation might be tetanised. On the other hand, when an Auerbach plexus preparation was subjected to the same stimuli it showed rhythmical movements, a refractory period, and a complete absence of the tetanic condition. From this it was

argued that the rhythmical action and refractory period must depend on Auerbach's plexus. The question as to where the Auerbach plexus received the stimuli necessary for the causation of the pendulum movements was discussed.

In this connection the work of Roger (24) and Straschesko (25) was referred to. They suggested that the contents of the intestine might furnish the necessary stimuli. The fact that the pendulum movements persisted after separation of the longitudinal from the circular coats excluded this theory. Magnus suggested that the stimuli might be afforded by a material previously developed in the nerve centres of the plexus. A considerable portion of the paper dealt with "acting point" of poisons in the gut.

Present Investigation.

1. *Methods.*

In this investigation the silver-nitrate staining method of Ramon y Cajal was used. The method consists in impregnating the tissue with the silver salt, and subsequently exposing it to the action of a reducing agent. Three modifications of this method have been described by Ramon y Cajal (27). From experience with the method and its various modifications it was found that the original method gave the most satisfactory results with chicks at the very early stages of development. The first modification (27A) proved most successful with chicks from four days of incubation and onwards. The second modification (27A) also gave very good results with chicks after four days of incubation, but seemed on the whole more adapted for adult tissue. The third modification (27B) was introduced by Ramon y Cajal for work on the lumbricus, and with this material it certainly gives most beautiful results, but with embryonic tissue it was not found to be a success.

For some time the uncertainty of the results of this staining method gave rise to considerable trouble. Great care was taken with the technique, but nevertheless the number of unaccountable failures remained high. A number of small experiments were made, among which was that of embedding the embryo in a thin coating of agar previous to immersion in the silver solution. This was tried in the hope of preventing the deposition of the silver round the embryo, an accident which frequently occurred with embryos of two to three days' incubation. The embedding in agar was found to prevent this, and to in no way hinder the penetration of the silver salt. With chicks of more than three days' incubation the agar was found

to hinder penetration, and had to be abandoned. The percentage of failures, however, still continued high, and small variations introduced experimentally into the technique resulted in no improvement. Up to this time perfectly fresh tissue had been used, on the assumption that the fresher the tissue the greater were the chances of success. An experiment was, however, tried, in which a chick which had undergone a certain degree of post-mortem change was subjected to the stain. The result was so good that a series of experiments were made both on adult and embryonic tissues, and it was found that much more satisfactory results were got if post-mortem changes were unchecked for a certain number of hours. The results of these experiments may be tabulated as follows:—

A. *Embryonic Tissue.*

1. With chicks of two to three days' incubation the best results were got with perfectly fresh tissue.
2. With chicks after three and a half to four days' incubation the best results were got where post-mortem changes of not more than twelve hours had taken place.
3. With chicks after four and a half to five days' incubation excellent results were got where the tissue underwent from twelve to twenty-four hours' post-mortem change.
4. With chicks after six and seven days' incubation post-mortem changes of twenty-four hours may be allowed to elapse. In the case of chicks of just six days' incubation, however, the full twenty-four hours' change was rarely tried; eighteen to twenty hours' change was generally found most successful.
5. With embryonic dogfish, where the gut was sufficiently developed to allow of its being removed, changes of twenty-four hours' duration were found to be associated with a very high degree of success.

B. *Adult Tissue.*

1. Brain and spinal cord stained excellently if twenty-four to thirty-six hours were allowed to elapse between the death of the animal and the treatment of the tissue.
2. The plexiform nerve mechanism in the intestinal wall of an adult animal rarely showed a satisfactory reaction to the stain unless twelve to twenty-four hours elapsed between the death of the animal and the staining of the tissue.
3. An investigation which was made on the nerve supply of the ovary led to the observation being made that the best

results were got after the lapse of from twenty-four to thirty-six hours.

The tissues must be kept moist and in a cool atmosphere during the period allowed for post-mortem change. In the case of the tissues of adult animals they were generally left *in situ* until the necessary lapse of time had occurred.

The silver-nitrate method stains fully developed nerves a dark brown, almost black colour, the rest of the tissues being a golden yellow or light brown. As a general rule the nerve fibres stain better than the nerve cells. With embryonic nerve tissue the silver nitrate shows a curious selective action, in that it stains the developing nerves in accordance with their degree of development. The more advanced the development of a nerve is, the better is it stained. This property of the stain made it singularly well adapted for the investigation of nerve development.

Since the silver nitrate stains other tissues as well as nerves, some difficulty is at times met with in distinguishing nervous from non-nervous tissue. Several factors, such as the microscopic appearances, the relative size and thickness, and the position of the structures, serve as guides; but a peculiar sheen, recognised after a little practice, and belonging solely to nerve tissue stained by silver nitrate, forms the best distinguishing guide. Unfortunately, the stain is not always permanent; many sections show after the lapse of about two years either fading or granularity. On the whole, embryonic tissue seems to retain the stain better than adult. Some attempts have been made to fix the stain with hyposulphate of soda, but the success of this plan is as yet doubtful.

2. *Material.*

Embryonic chicks at stages varying from two to seven days' incubation have formed the material upon which this investigation has been carried out.

The accounts given by different workers of the successful utilisation of embryonic fish in similar investigations led to experiments being made with this material. Plaice eggs were procured from the Bay of Nigg Hatchery, Aberdeen, through the courtesy of Dr Williamson, but, owing to technical difficulties connected with the use of the silver-nitrate stain, the results were disappointing. It was found that immersion of the whole egg in the silver solution was followed by such a degree of hardening that the egg chipped under the microtomic razor. Attempts were made to

remove part of the yolk, but in this case the embryo suffered so much that it was practically useless.

Young plaice were procured from the same source, and when embedded in agar were found to cut easily, and reacted well to the silver-nitrate stain. For various reasons, however, this material was abandoned in favour of embryonic chicks.

3. *Results.*

The earliest age at which chick embryos were examined in connection with this investigation was at the end of the second day of incubation.

At this age the rudimentary spinal cord appeared as a hollow rod of cells, which were roughly divisible into three layers according to their shape. The innermost layer was comprised of fairly circular cells; the next of polygonal or pear-shaped cells; while the outermost layer was formed by loosely arranged pyriform cells. All the cells showed well-marked nuclei. At the postero-lateral part of the spinal cord a few nerve fibres were recognised, which evidently came from the cells of the middle layer. The rudiments of the posterior spinal ganglia were seen at this stage as a proliferation of cells situated at the postero-lateral position of the spinal cord (Pl. I. fig. 1). These cells were very similar to those comprising the two outer layers of the spinal cord, and, like them, showed well-marked nuclei. The cells in that part of a spinal ganglion nearest to the spinal cord were seen to be closely arranged, while between them lay a very few nerve fibres, evidently derived from the spinal cord. At the ventral extremity of a spinal ganglion the cells were, on the other hand, loosely arranged, while a very few were seen to have broken away from the ganglion, and lay among the mesoblastic cells at some little distance from the ganglion. These cells were distinguished from the mesoblasts by their size, which was somewhat larger, and by their pigmentation with the silver stain being darker. From the appearance and position of these nerve cells they seemed to form the beginning of a ventral-cell migration from the posterior spinal ganglia, similar to that described by His for the formation of the sympathetic chain. No trace of the anterior spinal nerve roots was seen at this stage, neither was any nerve structure found in the splanchnopleure.

At the *middle of the third day* of incubation the central nervous system was found to have advanced considerably in development. The posterior spinal ganglia were larger, better stained, and showed the presence of more nerve fibres between the cells. At the position of the anterior spinal nerve roots a few very delicate and faintly stained nerve fibres were

seen in some embryos, but it was not until some hours later that a satisfactory demonstration of these nerves was obtained.

In the tissue lying round the notochord, and between it and the aorta, a very few nerve cells were recognised in the thoracic and lower cervical regions. These cells closely resembled those described at the end of the second day as having broken away from the posterior ganglia. From their appearance and position they seemed to represent a stage in a ventral migration of nerve cells from the posterior spinal ganglia to form the sympathetic chain. At this stage no satisfactory chain of cells could be traced from the posterior spinal ganglia to the neighbourhood of the notochord or aorta, but as the number of cells found in this latter position was so small, it was highly probable that the relationship of the connecting cells was lost in the various sections. Careful examination of the tissue in front of the aorta and of the splanchnopleure failed to show the presence of either nerve cells or fibres.

At the *end of the third day* both anterior and posterior spinal nerves were readily recognised.

In the tissue round the notochord, and between it and the aorta, nerve cells were seen similar in appearance to those described in this position at the middle of the third day (Pl. I. fig. 2). These nerve cells, which are somewhat more numerous than at the middle of the third day, were found practically only in the thoracic and lower cervical regions. At several points in the thoracic region an almost complete chain of cells was traced from the ventral extremity of a posterior spinal ganglion to the neighbourhood of the notochord. None of these nerve cells was found in front of the aorta, and no nerve elements were found in any part of the alimentary canal.

After *four and a half days'* incubation the sympathetic system as a whole was found to have made a very marked advance in development. The sympathetic chain was represented by a fairly uniform chain of cells lying behind the aorta (Pl. III. fig. 1). In the upper abdominal region nerve cells were traced round the aorta to its ventral wall, in front of which a delicate plexiform arrangement of sympathetic nerve cells and their outgrowths was formed. This plexus was stained a fainter colour than the nerve cells in the sympathetic chain.

In the mesentery a thin chain of nerve cells and fibres led down to the gut wall, while in both the stomach and upper intestinal walls a very delicate plexiform arrangement of nerve cells and fibres was recognised (Pl. III. fig. 2). In the stomach wall the nerve cells were arranged in two layers, one on the outer margin of the wall, while the other was much more deeply situated.

At this stage of development, therefore, sympathetic nerve cells and fibres were seen in front of the aorta, in the mesentery, and in the wall of the alimentary canal. The vagus fibres (line drawing) also entered the stomach wall at this age.

The next stage of development examined was during the *fifth day* of incubation. At this age the sympathetic chain formed a more or less continuous line of cells behind the aorta. The migration of sympathetic

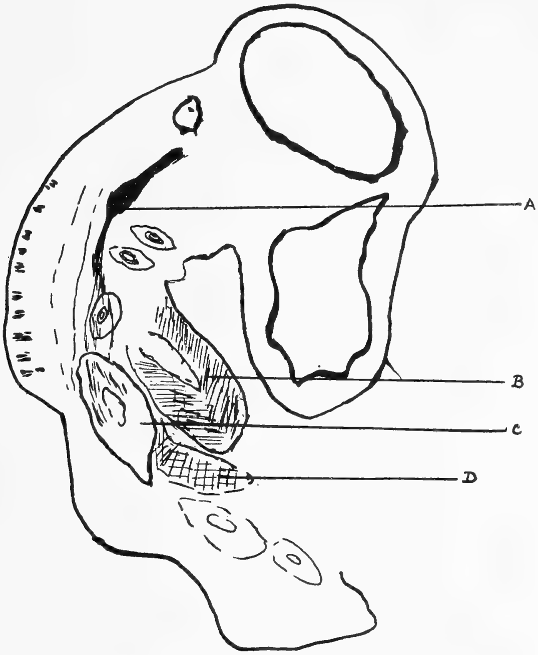


Fig. 1.—L. S. chick at $4\frac{1}{2}$ days. A, vagus; B, heart; C, stomach with vagus fibres; D, liver.

nerve elements round the aorta was better marked (Pl. I. fig. 3), while in the pelvic region a network of sympathetic nerve cells and fibres indicated the position of a future plexus. From the lower or sacral portion of the spinal cord nerve fibres were seen growing out towards this rudimentary plexus.

In the mesentery, numerous chains of nerve cells and fibres were found leading down to the gut wall (Pl. I. fig. 4), while some of the larger abdominal aortic branches were accompanied by chains of those cells. In the stomach and intestinal wall the double layers of nerve elements were better marked, the nerve elements being more numerous and reacting better to the stain. The outer layer showed the nerve cells arranged in

small clusters, with delicate, faintly-stained outgrowths. In the inner layer the nerve cells were not arranged in clusters, but formed a delicate network with their interlacing fibres. At several points a delicate chain of nerve cells could be traced from the outer to the inner layer. The vagus fibres were widely scattered and very numerous in the stomach wall. A small cluster of cells, similar in type to those forming the two nerve layers in the stomach wall, was found lying against the outer margin of the stomach wall, just where the vagus entered. A few were recognised further up the trunk of the vagus, and apparently represent a route followed by some sympathetic cells along the course of the vagus branches to the stomach (Pl. III. fig. 3). In the lower intestine a cylindrical band of nerve cells and fibres was recognised lying on the dorsal side of the gut. This corresponds to a similar band described as the intestinal nerve of Remak by His junior in his paper on the development of the abdominal sympathetic. At this stage no direct communication could be demonstrated between this band of nerve tissue and the nerve elements in the pelvic region. A very few nerve cells were found in the wall of this portion of the intestine, but no true plexus was recognised. This stage of development confirmed the observations made at the previous stage. It also showed the effect of the silver-nitrate stain on the various portions of the sympathetic system. The best stained part was the sympathetic chain, which was the oldest of the sympathetic nerve structures, while from it to the plexuses in the intestinal wall the nerve elements showed a decreasing degree of pigmentation. This fact and its significance will be dealt with fully in the next section.

At the *sixth day* of incubation the sympathetic chain was still more sharply defined, and delicate fibro-cellular connections were seen between it and the spinal nerves (line drawing). At the upper abdominal region and downwards a well-marked plexiform arrangement of sympathetic nerve cells and fibres was seen in front of the aorta (Pl. III. fig. 4). In the pelvic portion the plexiform arrangement of nerve cells described at the fifth day was better marked, while outgrowths from the lower portion of the spinal cord in the position of *nervi erigentes* were traced to the lower gut. The mesenteric bands of nerve cells and fibres were better developed (Pl. II. fig. 1), and at several points a complete scheme of sympathetic nerve cells and fibres was seen extending from the pre-aortic plexus to the gut wall. In the stomach and small intestine the double layers of nerve cells and fibres were better marked. The number of cells in both the inner and outer layers had increased, and with their outgrowths formed a more complicated network (Pl. IV. fig. 1). The

vagus fibres were found to be abundant in the stomach, while a few vagus fibres were found in the first portion of the small intestine. The cluster of sympathetic nerve cells described at the fifth day as lying at the point of entrance of the vagus to the stomach wall was not nearly so prominent, the cells having become, in all probability, dispersed in the stomach wall. Slight chains of cells were seen at several points connecting the two plexiform layers of nerve tissue in the gut wall. In the lower intestine a fairly well-developed plexus was seen lying deeply in the wall of the gut, but the cylindrical band of nerve cells on the dorsal side of the gut, *i.e.* the intestinal nerve of Remak, was by far the more prominent structure. It was found to extend higher up the gut than at the fifth

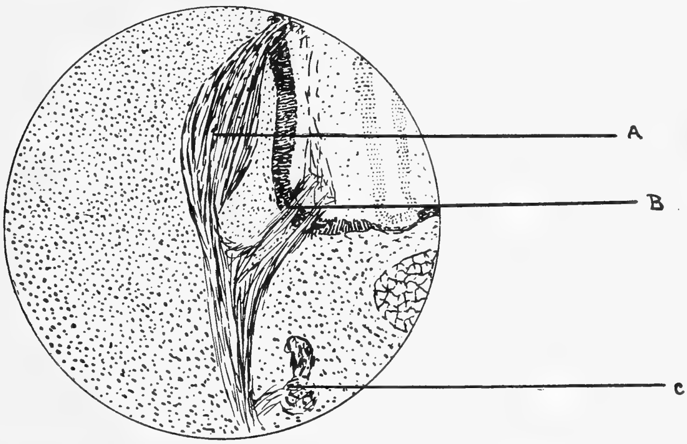


Fig. 2.—A, posterior root ; B, anterior root ; C, sympathetic ganglion.

day, but otherwise presented no new features. At this stage an unsatisfactory and doubtful connection appeared to exist between it and the sympathetic nerve elements in the pelvic plexus.

At this stage of development the growth of the various portions of the sympathetic chain was marked not only by an increase in number of the nerve elements but by the better staining of its parts.

At the *seventh day* of incubation the sympathetic chain and its connection with the spinal nerves were well developed. A complete and well-developed connection existed between the sympathetic chain and the intestine. The plexus in front of the aorta in the abdominal region was exceedingly well marked, while its reaction to the silver stain was considerably improved (Pl. II. fig. 2). At the lower level of the stomach this pre-aortic plexus showed comparatively large clusters of cells, which were apparently the precursors of abdominal ganglia in the adult. From the

pre-aortic plexus, chains of cells passed down the mesentery to the gut wall (Pl. II. fig. 3, Pl. IV. fig. 2). These mesenteric nerve chains showed a marked increase in development from the sixth day, besides a sharper definition with the silver stain. In the gut wall, both in stomach and intestine, the two layers of nerve cells were clearly seen (Pl. IV. fig. 3). The outermost layer, which was directly connected with chains of sympathetic cells in the mesentery, still showed the cells arranged in small clusters situated somewhat apart and connected by delicate nerve fibres. In the deeper layer the cells are arranged almost singly, but closely interwoven by their outgrowths. The vagus fibres were widely spread over

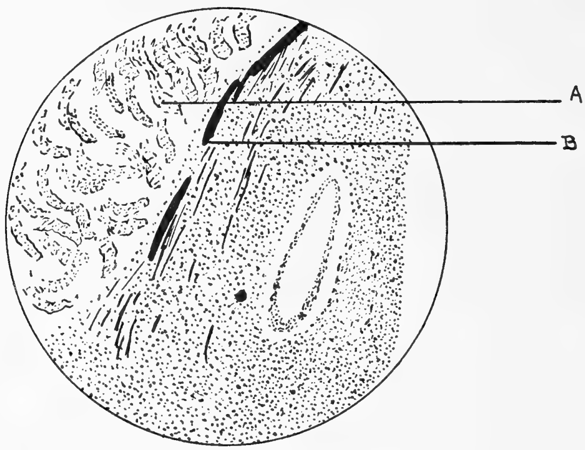


Fig. 3.—A, liver ; B, vagus fibres in wall of stomach.

the stomach wall (line drawing), a few extending to the first portion of the small intestine. These fibres form a fine interlacing network, and mix freely with the more lightly stained fibres belonging to the sympathetic. Nowhere was any actual connection found between the vagus fibres and the cells lying in either layer in the gut wall. In the lower intestine the plexiform nerve layer described at the sixth day was found better developed, while a few nerve cells were seen at the outer margin. The cylindrical band of nerve cells and fibres forming the intestinal nerve of Remak already referred to was found definitely connected with the lower portion of the sympathetic (Pl. II. fig. 4, Pl. IV. fig. 4). The outgrowths from the lower portion of the spinal cord, noted at earlier stages as extending to the pelvic plexus, were better developed, and from their position seemed to correspond to the *nervi erigentes*.

The chick embryo therefore showed at this age a complete chain of nerve cells and fibres from the spinal cord, *via* the *rami communicantes*

and the sympathetic chain to the plexiform nerve network in the gut wall. A difference in the degree of staining of the various nerve elements was also clearly evident. All were clearly stained, but the plexuses in the wall of the gut were distinctly lighter than the nerve cells in front of the aorta or in the mesentery.

The conclusions arrived at from the foregoing results, together with the special evidence derived from the peculiar property of the silver-nitrate stain of affecting tissue in accordance with their degree of development, will be dealt with in the next section.

Tabulation of Results.

1. At the end of the second day the posterior spinal ganglia were recognised, and from them a few nerve cells were seen to have broken away, and to lie somewhat ventral to the ganglia.

2. At the middle and end of the third day nerve cells similar in type to those described at the end of the second day were found lying in the tissue near the notochord and behind the aorta.

3. At four and a half days the sympathetic chain was readily recognised lying behind the aorta. Chains of cells were traced from it round the aorta down the mesentery to the gut wall, in which a plexiform arrangement was seen. The vagus nerve was found in the stomach wall at this stage of development.

4. At the fifth day the sympathetic chain and the connections between it and the plexiform arrangement in the gut wall were found better developed. The vagus branches in the stomach were well marked.

5. At the sixth day improvement was noted in the development of the structures mentioned at the fifth day, and in addition the *nervi erigentes* were recognised.

6. At the seventh day the sympathetic chain and its connections with the anterior and posterior spinal nerves, and with the visceral ganglia in the intestine, were easily traced. The vagus and *nervi erigentes* were found to be even better developed than at the sixth day.

Conclusions.

1. The descriptions of the various stages of development of the embryonic chick from the second to the seventh days show that the whole sympathetic system is secondary in formation to, and directly derived from, the central nervous system.

2. The abdominal sympathetic is produced by the migration of cells

from the spinal cord and intervertebral ganglia downwards through the mesentery to the gut. From these cells are formed the various divisions and synapses of the autonomic system. That these cells are not the sheath cells described by Harrison as growing from the posterior root in the tadpole is indicated by some of their number subsequently forming the cells from which the two plexuses in the intestinal wall develop.

In comparing my results with those of other histologists, I find that in many points they coincide with those described by His junior (14). The application of more modern methods of staining has, in my hands, confirmed the main facts recorded by him. As regards the fate of some portions of the sympathetic chain and mode of development of nerve fibres in the abdominal sympathetic, our results do not, however, agree.

1. His described the formation of a secondary sympathetic chain in the lower cervical and thoracic regions, the primary structure being wholly utilised in forming visceral ganglia. With this observation I do not agree, as I fail to find the dwindling of the sympathetic chain described in these positions, neither do I find that the reaction of the nerve elements of the sympathetic chain to the silver-nitrate stain in the cervical and thoracic regions showed any variation from the steady improvement seen in the abdominal portion of the structure. It would seem unlikely that an absolutely fresh formation, such as a secondary sympathetic chain occurring so late as the sixth day, could be overlooked, more especially with the Ramon y Cajal method, to which nerve structures show so much sensitiveness. The fact that His used a hæmatoxylin-eosin stain, which is not, like the silver nitrate, a peculiar nerve stain, seems to make his conclusion still more doubtful. Further, the fact that His illustrated the various stages of development of the sympathetic chain by diagrammatic drawings influences, I think, adversely the value of the evidence brought forward by him on this point.

2. As regards the character of the outgrowths from the sympathetic chain to the gut, His stated that a cellular chain of nerve cells was formed between the spinal cord and the gut, and that this was replaced about the sixth day by outgrowing nerve fibres from cells in the spinal cord. My results do not support the second part of this statement.

I find the first connection between the spinal cord and gut to be composed of chains of nerve cells. These chains are often very well developed, and form conspicuous structures in the mesentery (Pl. II. fig. 1) up to the fifth or sixth days of development. At the sixth, but more especially at the seventh, days definite nerve fibres appear and gradually replace the

cellular chains (Pl. II. fig. 3, Pl. III. fig. 4). In several chicks at the sixth day of incubation the arrangement, elongation, and union of the nerve cells of the primary nerve-cell chains in the mesentery suggest that some at least of the nerve fibres which appear in the mesentery about this time are formed by the union of these cells (Pl. III. fig. 4). In many sections, on the other hand, this appearance is not found, but in those cases the nerve fibres are not observed to be in direct relationship with cells in the spinal cord.

Several difficulties seem to me to oppose the complete acceptance of the outgrowth theory for these autonomic nerves as fibres. Firstly, the fact that at the fifth and sixth days well-marked chains of nerve cells are found in the mesentery, while at the seventh they are almost, if not wholly, replaced by nerve fibres (contrast Pl. II. fig. 1, Pl. II. fig. 3, and Pl. III. fig. 4), seems to me to offer a problem as to the fate of the nerve cells forming those chains. So many may be utilised for ganglia and plexuses in the gut wall, but the increase of the plexiform nerve mechanism in this latter position between the sixth and seventh days (Pl. IV. fig. 1 and Pl. IV. fig. 3) is not, in my opinion, sufficient to account for all the nerve cells. Secondly, if the nerve fibres are in reality outgrowths from the spinal cord, their rate of growth must be exceedingly rapid, for, as shown by Pl. II. fig. 3, they reach the gut at the seventh day, their first appearance in the mesentery being noted at the sixth day. Should they, therefore, be outgrowths, they form in a little over twelve hours a connection between the spinal cord and the gut. The migration of the nerve cells from the spinal cord to the gut was much slower, for they were found behind the aorta and in the region of the notochord at the end of the third day, while it was not until four and a half days that the gut wall was reached. Considering the increased density of the embryonic tissue at the seventh day contrasted with that on the third and fourth days, and the longer distance to be traversed by an outgrowth from the spinal cord to the gut, it seems improbable that the nerve fibres seen in the mesentery at the seventh day are outgrowths from cells in the spinal cord. I am inclined to attribute the appearances presented in sections of chicks at six and seven days' development, in which the mesenteric nerve fibres seem to be developed independently of the cellular nerve chains, to the fact that I have failed to hit the correct time at which the transformation of the cellular nerve chains to nerve fibres occurs. This point of time, I consider, I had the good fortune to hit in several cases (Pl. III. fig. 4), and in those cases I found an appearance, as seen in the figure referred to, of a "beaded" nerve fibre, each "bead" being formed by a nerve cell.

The conclusion stated at the beginning of this section, that the sympathetic system is secondary in its development to the central nervous system, is interesting in connection with Gaskell's view of the relationship of the central nervous system of vertebrates to that of annelids and arthropods. In his book, *The Origin of Vertebrates,* Gaskell describes the central nervous system of the vertebrate as the homologue of the nervous system of the annelid or arthropod. Reasoning from Gaskell's standpoint, the homologue of the invertebrate nervous system in the vertebrate would be the most primitive portion of the vertebral nervous system, and the first to develop. This theory is supported by my results, which, as already stated, point definitely to the sympathetic system being secondary in formation.

I wish to acknowledge my deep indebtedness to Professor Noël Paton, under whose guidance I made this research, both for suggestion and help during my work, and for helpful criticism of this paper.

DESCRIPTION OF ILLUSTRATIONS.

Plates I. and II. consist of sepia drawings; III. and IV. of photographs.

LIST OF REFERENCES.

(1) Remak, *Über die Entwickelung des Huhnchens im Ei,* 1843.

(2) Remak, *Über ein selbstandiges Darmnervensystem,* Berlin, 1847.

(3) Hensen, "Zur Entwickelung des Nervensystems," *Virchow's Archiv,* 1864.

(4) His (senior), *Untersuchungen über die erste Anlage des Wirbelthierliebes,* 1868.

(5) Götte, *Die Entwickelungsgeschichte der Unke,* 1872.

(6) Balfour and Foster, *Grundzuge der Entwickelungsgeschichte,* 1876.

(7) Balfour, "Development of Elasmobranch Fishes," *Journal of Anatomy and Physiology,* 1877.

(8) Schenk and Birdsall, "Die Entwickelung des sympathicus," *Mitth. aus d. embry. Inst. in Wien,* vol. T, 1879.

(9) Onodi, "Über die Entwickelung des sympathetischen Nervensystem," *Archiv für mik. Anat.,* vol. xxvi.

(10) Paterson, "The Development of the Sympathetic Nervous System in Mammals," *Proceedings of the Royal Society,* 1890.

(11) His (senior), "Histogenese und Zusammenhang der Nervenelemente," *Archiv für Anat.,* Abtheg. 1890.

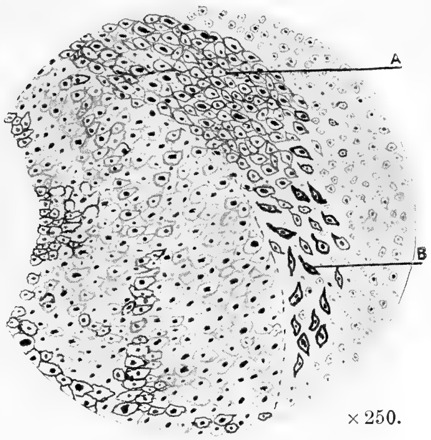


Fig. 1.—A, posterior ganglion spinal cord ; B, migrating nerve cells. (End of second day.)

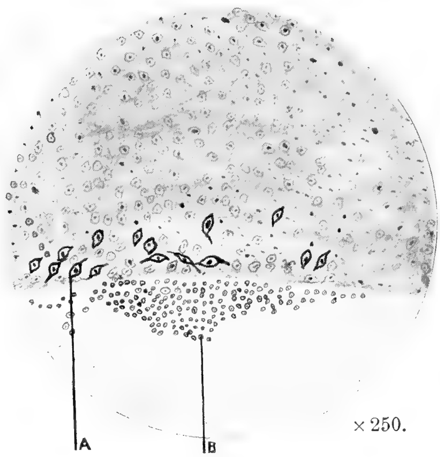


Fig. 2.—A, sympathetic nerve cells lying behind the aorta ; B, aorta. (End of third day.)

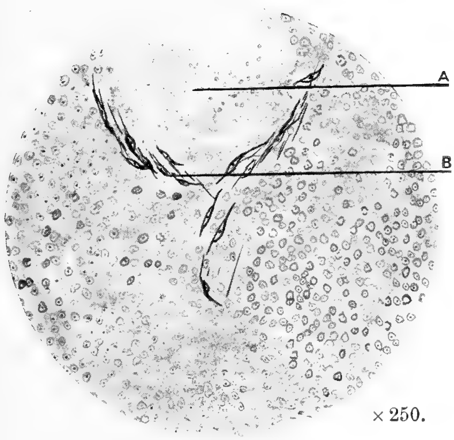


Fig. 3.—A, aorta ; B, sympathetic nerve cells and fibres in front of the aorta. (Fifth day of incubation.)

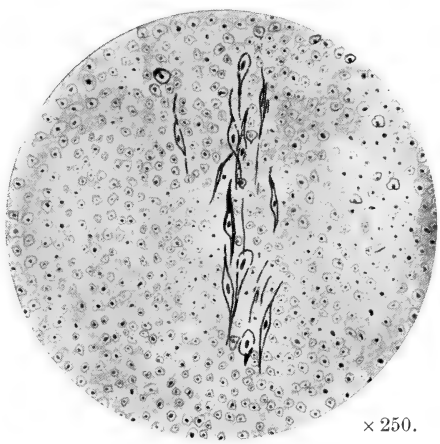


Fig. 4.—Sympathetic nerve cells and fibres in the mesentery. (Fifth day.)

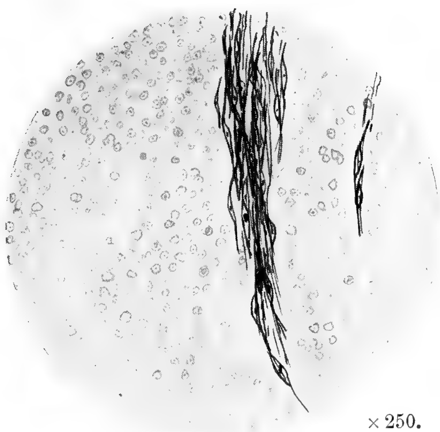


Fig. 1.—Sympathetic nerve cells and fibres lying in the mesentery. (Sixth day.)

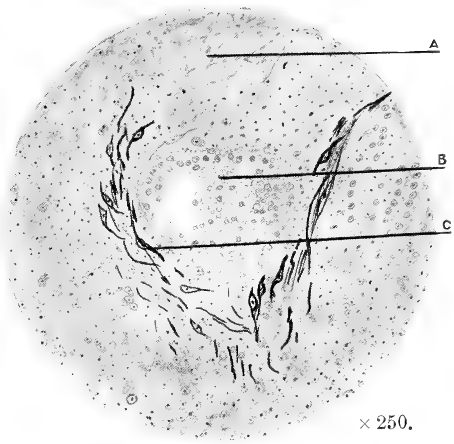


Fig. 2.—A, notochord; B, aorta; C, sympathetic nerve cells and fibres at the side and in front of aorta. (Seventh day.)

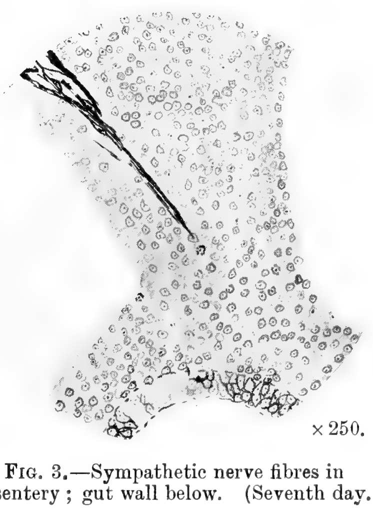


Fig. 3.—Sympathetic nerve fibres in mesentery; gut wall below. (Seventh day.)

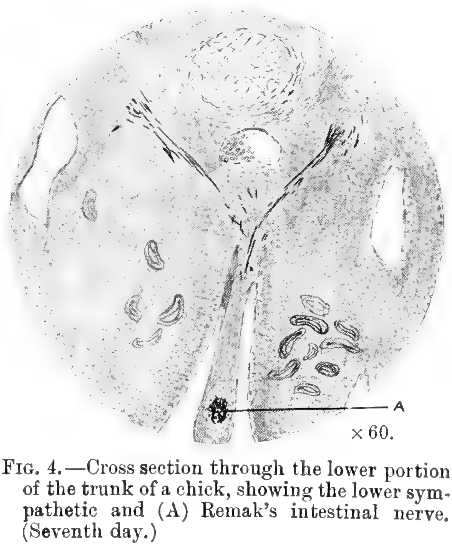


Fig. 4.—Cross section through the lower portion of the trunk of a chick, showing the lower sympathetic and (A) Remak's intestinal nerve. (Seventh day.)

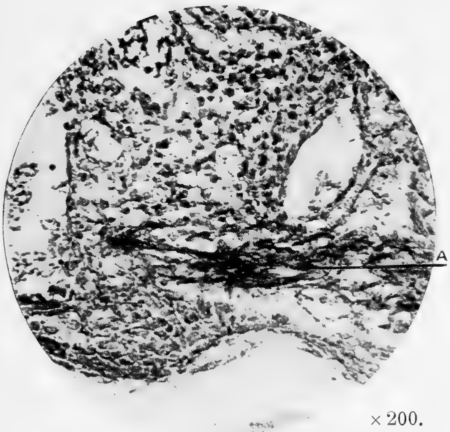


Fig. 1.—A, sympathetic nerve cells lying behind the aorta. (Four and a half days.)

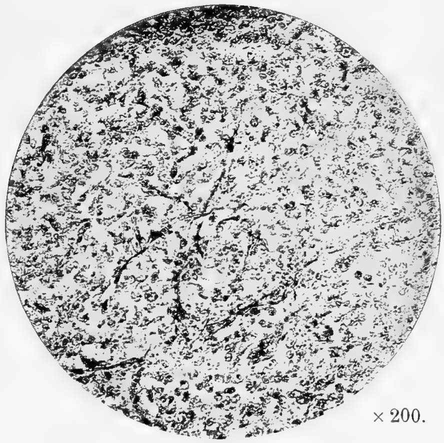


Fig. 2.—Sympathetic nerve plexus (dark lines) in the wall of the gut at four and a half days.

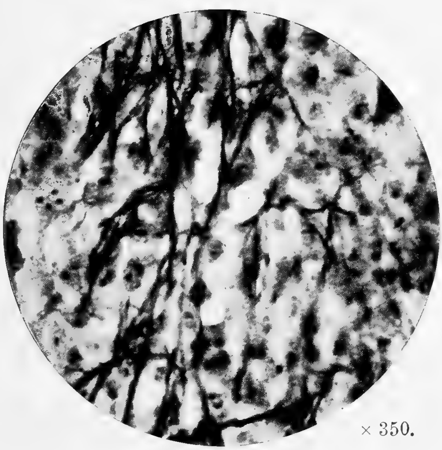


Fig. 3.—Vagus fibres in stomach wall, showing sympathetic nerve cells lying in the interstices. (Fifth day of incubation.)

Fig. 4.—Nerve cells and fibres in front of the abdominal aorta at sixth day incubation.

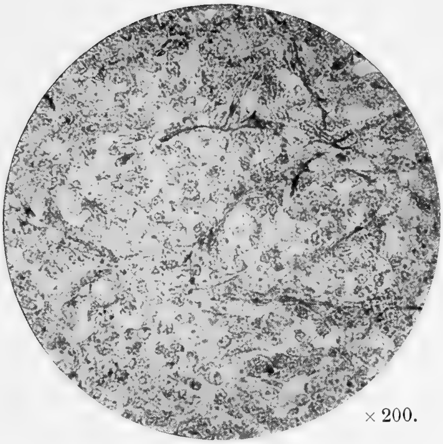


Fig. 1.—Sympathetic nerve plexus in the gut wall at the sixth day.

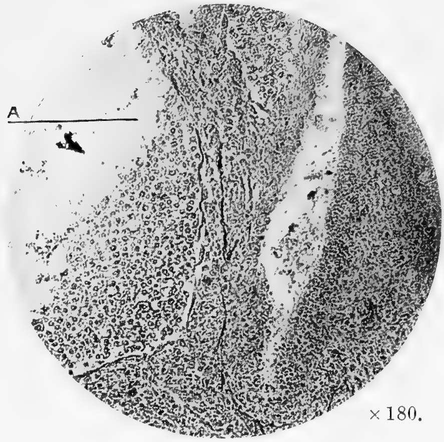


Fig. 2.—Chain of sympathetic nerve cells and fibres extending to the front of the aorta (A). (Seventh day incubation.)

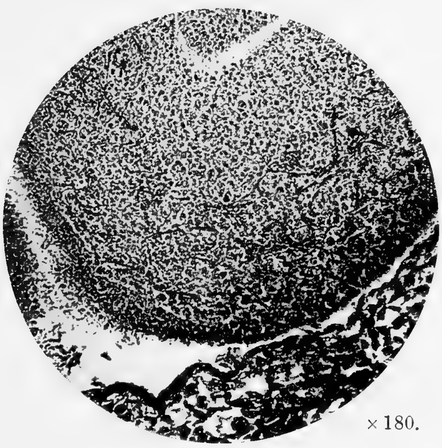


Fig. 3.—Sympathetic nerve plexus in the wall of the gut at the seventh day.

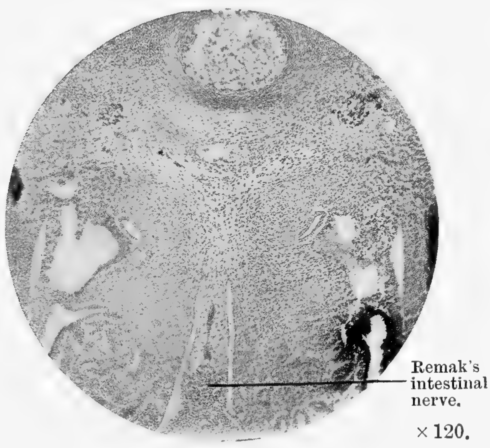


Fig. 4.—Connection between the wider sympathetic and "Remak's intestinal nerve." (Seventh day incubation.)

(12) His (junior) and Romberg, "Beiträge zur Herzinnervation," *Fortschnitte der Medicin*, 1890; *Verhandlungen d. Congresses f. innere Medicin*, 1890.

(13) His (junior), "Die Entwickelung des Herznervensystems bei Wirbelthieren," *Abhandlungen der k. Sachs. Gesellschaft der Wissenschaften*, Math.-phys. Classe, 1891.

(14) His (junior), "Über die Entwickelung des Bauchsympathicus," *Archiv für Anat.*, 1897.

(15) Dogiel, "Bau der Ganglien in dem Geflechten des Darmes und der Gallenblase des Menschen und der Saugethiere," *Archiv für Anat.*, 1899.

(16) Graham Kerr, "On Some Points in the Early Development of Motor Nerve Trunks and Myotomes in *Lepidosiren paradoxa*," *Transactions of the Royal Society of Edinburgh*, vol. xli., part i., 1903–4.

(17) Ross Granville Harrison, "Experiments in transplanting Limbs, and their bearing upon the Problem of the Development of Nerves," *Journal of Experimental Zoology*, vol. iv., 1907.

(18) Meiklejohn, "On the Development of the Plexiform Nerve Mechanism in the Alimentary Canal of the Chick," *Journal of Physiology*, 1908.

(19) Bayliss and Starling, "On the Movements and Innervation of the Small Intestine," *Journal of Physiology*, vol. xxiv.

(20) Langley and Magnus, "Some Observations of the Movements of the Intestine before and after Degenerative Section of the Mesenteric Nerves," *Journal of Physiology*, vol. xxxiii.

(21) Elliot, "The Innervation of the Bladder and Urethra," *Journal of Physiology*, vol. xxxv.

(22) Yanase, "Beiträge 3: Physiol. d. peristalt. Bew. d. embryonalen Darmes," *Pfluger's Archiv*, 1907.

(23) Magnus, "Die Bewegungen des Verdaŭüngskanals," *Ergebnisse der Physiologie*, 1908.

(24) Roger, "Les mouvements de l'intestin à l'état normal et dans les expér. occlusions," *Journal de physiol. et path.*, 1908.

(25) Straschesko, "Zur Physiologie des Darms," cit. nach Boldireff, *Zeitsch. f. Entwickelüngslehre*, 1907 (quoted from Magnus (22)).

(26) Gaskell, *Origin of Vertebrates*, 1908.

(27) Ramon y Cajal, *Trabajos del Laboratorio de Investigaciones Biologicas*, 1904.

(27a) Ramon y Cajal, *Algunos Metodos de Coloracion de los cilindros ejes neurofibrillas y Nidos nerviosos*, p. 1.

(27b) Ramon y Cajal, *Neuroglia y neurofibrillas del Lumbricus*, p. 277.

(*Issued separately March* 25, 1910.)

XVIII.—The Glenboig Fireclay.* By J. W. Gregory, D.Sc., F.R.S. (L. and Ed.), Professor of Geology in the University of Glasgow. (With One Plate.)

(MS. received February 24, 1909. Read March 15, 1909.)

CONTENTS.

1. The Nature of Fireclay.

According to Percy's definition,† "clays are termed fireclays or refractory clays when they resist exposure to a high temperature without melting or becoming in a sensible degree soft and pasty." The refractoriness of fireclay is due to its low proportions of fluxes. The best known British fireclays occur beneath the coal seams of the coal measures, and are therefore known as "under-clays" or "seat-clays." The explanation of their paucity of fluxes usually offered is that the alkalies have been withdrawn as food by the vegetation which formed the coal. Thus Professor Tarr ‡ remarks that fireclays "are particularly abundant in the Carboniferous rocks associated with coal beds, the plants having been instrumental in the withdrawal of the alkalies." This view has been widely accepted, and the death of the successive coal forests has been attributed to the exhaustion of the plant foods in the underlying soils. Caution is, however, necessary in the application of this theory; for plants can only withdraw soluble alkalies from the

* I must express my indebtedness for help during several visits to Glenboig to the late A. H. Dunnachie, the General Manager of Glenboig Union Fireclay Co. Ltd.; Mr J. Macintyre, the mine manager; also to Mr G. W. Tyrrell of the Glasgow University for the preparation of slides and determination of the specific gravity, and to Mr D. P. Macdonald, the Baxter Demonstrator in Geology, for the analysis of the sideroplesite. Also to Professor A. C. Seward of Cambridge and Dr J. W. Evans of the Imperial Institute for examining one of the slides, and to Mr Fingland for the photographs.

† J. Percy, *Metallurgy, Fuels*, 1875, p. 87.

‡ R. S. Tarr, *Economic Geology of the United States, with Briefer Mention of Foreign Mineral Products*, 1894, p. 400.

soil, and the alkalies are usually present in the form of silicates, which decompose slowly. Hence the total removal of alkalies from a soil by vegetation would be a very lengthy process. Moreover, alkalies are not the only, or indeed the most common of fluxes, for iron is usually the most potent. Further, some good fireclays are not associated with coal seams, while well-washed river muds may be poorer in alkali than an underclay, the alkalies having been washed out of the material. Thus the river muds from the Rhine near Bonn, analysed by Bischoff,* contain only ·89 per cent. of potash and ·39 per cent. of soda; or after deducting the water and organic material the percentages of potash and soda are 1·02 and 0·45 respectively. Another analysis by Bischoff of river mud from the Rhine collected above Lake Constance contained potash ·55 per cent. and soda ·54 per cent., or after deducting the water and carbonates the percentage of these two constituents only rose to ·91 per cent. of potash and ·90 per cent. of soda.

Nevertheless, as plants certainly collect potash in their leaves, which are readily carried away by the wind, and also in their fleshy parts which could be blown away after drying, the poverty in potash of the clays below coal seams must, no doubt, be to some extent due to the action of the forests that once grew over them. Ries † is, however, no doubt correct in his warning that solution has sometimes been the effective agent in the elimination of the alkalies, and their absorption by the plant roots is not the universal explanation of the conversion of what would have been a common clay into fireclay.

That many of the common British fireclays were once the actual subsoils of Carboniferous forests is shown by the abundance of fossil roots found in them. Thus the late Professor A. H. Green ‡ remarks of the Yorkshire fireclay "they always contain the fossil called Stigmaria, which is now known to be a root; and long black ribbon-shaped filaments, which are the rootlets given off by the larger Stigmaria root, ramify through them in every direction. Many instances have been observed where fossilised trunks of trees, still standing erect in the position in which they grew, and attached to their roots, rise out of an underclay. There can be no doubt, then, that the underclays are old vegetable soils, and that, unlike all the Carboniferous rocks we have hitherto noticed, they were formed not under water but on dry land. They mark, in fact, a succession of periods during

* F. Bischoff, *Elements of Chemical and Physical Geology*, vol. i., 1854, p. 123.

† H. Ries, *Clays*, 1906, p. 179.

‡ A. H. Green and others, "The Geology of the Yorkshire Coalfield," *Mem. Geol. Surv.*, 1878, p. 19.

which the area where they occur became in some way raised above water and converted into dry land.

"The underclays, as their name implies, usually form the floor of a seam of coal; but underclays do occur without any coal over them. There is usually, however, if coal be absent, a seam of highly carbonaceous black shale above an underclay."

2. The Glenboig Fireclay.

When I visited the Glenboig fireclay mines I expected to see Stigmaria in the fireclay in its usual abundance. To my surprise I could find none, and on inquiry of Mr A. H. Dunnachie, the general manager, and Mr Macintyre, the mine manager, they were unable to show me any tree roots in the clay or to refer to any certain evidence of their existence. This was the more striking as Stigmaria rootlets occurred in the overlying sandstones, and good specimens from this horizon are preserved in the mine laboratory. I have made repeated search for Stigmaria or other rootlets in the Glenboig fireclay, especially as I was told by Mr Hinxman that he has not been able to find any.*

The microscope shows the presence of some decomposed vegetable material in the clay, but no roots or rootlets *in situ* in it. The plant fragments shown in microscopic sections probably resulted from rotten vegetation floating in water. Neither have I seen any remains of other organisms in the Glenboig fireclay.

The absence of roots from the Glenboig fireclay, therefore, at once suggested that this clay had a different mode of origin from the typical English Carboniferous underclays; and the Glenboig fireclay is of especial economic interest, as I understand the fire-bricks from it are unequalled in quality.

The exact position of the Glenboig fireclay seam has been demonstrated by the recent careful survey of the Glenboig district by Mr Hinxman.†

The fireclay mined at Glenboig is the lower fireclay of the Millstone Grit series. Its exact horizon is shown by its occurrence from two to three fathoms above a band of limestone which is said to be locally known as "the Roman Cement," though this name is not known at the mine. This Roman Cement lies, according to Mr Hinxman, from 40 to 50 feet above the Castlecary limestone, so that the fireclay is near the base of the Millstone

* Professor Boyd Dawkins, on the other hand, has identified some markings as casts of rootlets; the evidence for this identification seems to me, however, inadequate.

† *Summary of Progress of the Geological Survey for* 1905 (pp. 119–121) *and for* 1907 (pp. 102–103).

Grit. Above the fireclay there is a series of sandstones, above which there is often a thin coal seam; above this seam is a band of dark shales in which Mr Tate discovered marine fossils. The exact sequence of beds is shown in the following record of a recent bore:—

	Fm.	ft.	in.
Made ground	0	4	0
Broken whin [quartz diabase]	0	3	6
Soft broken clay	0	4	6
Hard ironstone rib	0	0	4
Broken clay	0	1	0
Broken sandstone	5	2	0
Dark shale	0	2	6
Ironstone rib	0	0	2
Sandy shale	0	2	0
Coarse sandstone	0	0	8
Shale	0	1	0
White sandstone	1	4	3
Coarse sandy shale	2	0	0
Shale with plies of coal	0	5	10
Ironstone	0	0	3
Dark sandy shale	1	4	9
White ,,	0	4	0
Dark ,,	0	2	6
Compound	1	0	2
Fireclay	1	0	4
Dark variegated shale	0	3	0
Hard white sandstone	0	1	6
Dark shale	0	2	0
Roman Cement	0	0	10
	19	3	1

Examination in the mine gives no definite evidence as to the method of formation of this fireclay. It is usually jointed into small irregular angular lumps, the surfaces of which are abundantly slickensided, as in most thin seams of homogeneous clay which have undergone much contraction, and as is common in most English fireclay.* Bedding is only obscurely indicated in the mass, but is often clearly shown in microscopic sections. The character of the bedding indicates that the clay was deposited slowly in quiet water, and that the original arrangement of the particles in the middle of the clay lumps has not been disturbed; for flakes of quartz

* See, *e.g.*, A. H. Green, *op. cit.*, p. 19.

may be seen standing on edge, as if they had fallen through water into soft mud and remained imbedded in their original vertical position (Plate fig. 4).

Microscopic examination of the clay shows that its grains are often very fresh and angular. Many of the flakes of felspar have their cleavage angles quite sharp, and some of the felspar is surprisingly fresh.

3. The "Clay-Substance"—Halloysite not Kaolinite.

The chief constituent of the fireclay is a fine-grained clay substance, containing grains and flakes of various other minerals.

The chemical composition of the clay substance in the Glenboig fireclay has been determined through a careful investigation by Professor Fawsitt of Sydney University. He isolated from the clay by repeated washing a hydrous silicate of alumina, which is white in colour, and according to his analysis is practically identical in composition with kaolinite, though it may contain more combined water. But when the water has been driven from this mineral at 105° C. the remaining water is in the same proportion as in kaolinite, and there would be nothing in composition to show that this material when dried at 105° C. is not kaolinite. But microscopic examination shows that it is amorphous and not crystalline.

This material, though it could only be obtained from fireclay at an expenditure of labour that would make it a very costly commodity, could be used for making porcelain; so it may be called china clay or kaolin. I prefer not to call it kaolinite, since it occurs as amorphous granules, and not in crystalline scales. It occurs in minute rounded granules, of which the average diameter is about ·001 mm. It is one of those forms of clay-substance which is not kaolinite. If this material is to be referred to a particular mineral species, it may be included in halloysite, the amorphous hydrous bisilicate of alumina. The only difference in composition between kaolinite and halloysite is in the proportion of water: whereas kaolinite has 14·0 per cent., halloysite may have up to 20 per cent. But the water in halloysite in excess of the two molecules in kaolinite appears to be present in a much looser combination, and most of it may be driven off at practically the boiling-point. Thus Professor Liversidge * has described a halloysite from Berrima in New South Wales with silica 45 per cent., alumina 38·5 per cent., combined water 12·8 per cent., and water given off at 105° C. 3 per cent. This halloysite has practically no more water than samples of halloysite, of which analyses are quoted by Dana †: in one of them the

* A. Liversidge, *Minerals of New South Wales*, 1888, p. 195.

† Dana, *System of Mineralogy*, 6th edit., 1892, pp. 688–689.

proportion of water left at 100° C. was as low as 15·27 per cent.; in a variety of indianite dried at 100° C. it is 14 per cent., and in a series of analyses by Le Chatelier the water left after heating to 250° varies from 14·3 to 13·0 per cent. One of these samples contained only a total water of 16·5 per cent. It seems, therefore, allowable to define halloysite as an amorphous bisilicate of alumina with two molecules of combined water, and with a few per cent. of water less firmly united. If that definition of halloysite be accepted, there is no reason why much of the material called "clay substance," including the base of the Glenboig fireclay, should not be described as halloysite.

It must be borne in mind that all aluminous clay substance does not belong to one mineral species, as the material may belong to one of several allied species. The late A. H. Green,* for example, in his *Physical Geology*, remarked regarding pure clay that "it is likely that there are several varieties differing from one another in the proportion of silica and the amount of water they contain." Percy, in an earlier and careful discussion of the nature of clay substance, left the question unsettled, as he recognised that the proportions of water varied in different carefully analysed clays. Thus Forchhammer's analyses show that the china clay of Passau has the composition of—

Silica	46·23
Alumina	35·27
Water	18·50
	100·00

Its formula is $9SiO_2$, $4Al_2O_3$, $12H_2O$, which differs from kaolinite by an excess of both silica and water. The slight excess of silica is probably due to some mechanically included fine quartz flour; and if so, the formula of this Passau clay may be represented $2SiO_2$, Al_2O_3, $3H_2O$.

For the reasons already given, a variation of 4 per cent. of water may be allowed within the range of a mineral species; and if the Passau clay be amorphous, there is nothing in its composition to prevent its inclusion in halloysite.

The distinctions between kaolinite and halloysite can be readily recognised under the microscope. The kaolinite occurs in minute scales which resemble white mica, but have a much lower refractive index, which can be recognised when tilted flakes are examined under crossed nicols. They often grow, moreover, in piles, like heaps of cards of different sizes. The larger flakes sometimes show, in addition to the perfect basal cleavage, three intersecting

* A. H. Green, *Physical Geology*, 1876, p. 68.

cleavages parallel to their hexagonal edges. This hexagonal cleavage was described by Reusch * from the kaolinite of the National Belle Mine, Silverton, San Juan County, Colorado. The flakes of kaolinite appear to be less elastic than those of mica, and accordingly they frequently break along the cleavages; flakes are found with sectors broken out by fracture along these cleavages.

Halloysite, on the other hand, has no cleavage; it is granular and amorphous, and often has a faint yellowish tint. It agrees with the material described by Mr Hutchings † as the finest substance in the Seaton fireclay. He says: "The thinnest possible films of fireclay under $\frac{1}{8}$th-inch objective show a minutely granular substance, with usually a faint yellowish tinge and of such an extreme tenuity that in polarised light it is quite inactive, or depolarises only just perceptibly in a faint speckly manner." "This granular matter," he adds, "is, I suppose, the mixture usually spoken of as kaoline." This substance may be called kaolin if found in sufficient purity to be useable for making china clay; but it is not kaolinite, and when amorphous I should prefer to call it halloysite.‡

Scales of kaolinite occur in various clays. Dr Hatch,§ in his *Text-book of Petrology*, correctly stated that kaolinite "is the chief constituent of china clay." I had occasion early last year to examine the china clay from

* H. Reusch, "Krystallinische Kaolin von Denver in Colorado," *Neu. Jahrb.*, 1887, vol. ii. p. 71.

† W. M. Hutchings, "Notes on the Probable Origin of some Slates," *Geol. Mag.*, New Ser., decade 3, vol. vii., 1890, p. 271.

‡ The publication of this paper has been delayed to allow some chemical tests as to the solubility of the clay substance to be repeated. According to Lacroix * kaolinite is "inattaquable par les acides," while hydrochloric acid "décompose facilement" halloysite. This distinction between the two species seems reasonable, as an amorphous material such as halloysite might be expected to be more easily decomposed than a crystalline species such as kaolinite. A careful investigation by Mr D. P. Macdonald shows that the clay substance of Glenboig fireclay is readily decomposed by boiling in hydrochloric acid. Thus 6·5 per cent. of the total of 37·65 per cent. of alumina was dissolved out by boiling for two hours in hydrochloric acid, 23·2 per cent. by boiling for six hours, and practically the whole of it (36·6 per cent. out of 37·65 per cent.) by boiling for thirteen hours. Boiling in hydrochloric acid therefore completely decomposes the material; and if M. Lacroix's distinction be valid, it cannot be kaolinite.

As the Glenboig fireclay is of Carboniferous age and underlies an intrusive sill, it is not surprising that the percentage of water in its clay substance is smaller than in some of the more recent French halloysites, and it is therefore somewhat less readily decomposed and is nearer in physical properties to kaolinite than varieties of halloysite with a higher percentage of water.

§ F. G. Hatch, *Text-book of Petrology*, 1892, p. 103.

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* *Minéralogie de la France et de ses Colonies*, vol. i., Paris, 1895, p. 472.

the Carpalla Mine near St Austell, and recognised abundant kaolinite.* Scales of kaolinite also occur in the clays of Bovey Tracey and in a sample of the Stourbridge clay, though in each case they are mixed with halloysite. I have not, however, been able to find any kaolinite in the Glenboig fireclay or any Scotch clays. The clay substance in them, so far as I have examined them, is either an amorphous silicate or excessively fine particles of quartz. The hydrous silicate of alumina in this Glenboig firelay was probably formed by the decomposition of felspar by carbonic acid at or near the surface of the earth and at low temperatures; and thus it is not surprising that the mineral thus produced has taken the form of an amorphous and not of a crystalline silicate.

Clay substance in general, of course, includes much more than halloysite, for it is sometimes silica in the form of quartz flour, or it may be kaolinite or the dust of other silicates of alumina; or fine grained sericite or other white mica.† Halloysite, however, is probably the essential constituent of most common clays. It is the chief, but not, as implied by Senft, the universal clay substance. "Das Grundbildungsmittel für alle thonartigen Substanzen ist die Thonsubstanz," which, he continues, when pure, consists of two molecules of silica, one of alumina, and two of water.‡

That some fireclays are formed of sericite has been proved by Hutchings § for some near Newcastle-on-Tyne, and it is well known that much of the clayey material which miners have called talc and dolomite is only sericite. There is, however, comparatively little white mica in the Glenboig fircelay.

4. Materials included in the Clay.

The commonest inclusions in the clay are grains of quartz (fig. 3), which is sometimes so abundant that some thin layers become argillaceous sandstone. The quartz, however, is in large grains, and it thus acts as a refractory constituent, and the clay behaves better as a fireclay than might be inferred from a bulk chemical analysis.

Felspar is fairly common, and includes many grains and cleavage flakes of very fresh plagioclase, probably derived from some volcanic rocks that existed in the neighbourhood during the formation of the Millstone Grit. Hornblende and mica, including biotite, are both present; but the mica is scarce in comparison with the fireclays investigated by Hutchings; there

* This identification was subsequently fully confirmed by Mr G. Hickling, "China Clay; its Nature and Origin," *Trans. Fed. Inst. Min. Eng.*, vol. xxxvi., 1909, pp. 1–26, pl. 1.

† For the presence of quartz flour as a clay constituent, see, *e.g.*, F. Senft, *Die Thonsubstanzen*, Berlin, 1879 (published 1878), p. 15.

‡ F. Senft, *ibid.*, p. 16.

§ W. M. Hutchings, "Clays, Shales, and Slates," *Geol. Mag.*, decade 4, vol. iii., 1896, p. 315.

are occasional crystals of zircon and abundant excessively small needles and granules of rutile.

Hydrated iron oxide occurs in grains and in occasional segregations.

5. Sideroplesite.

The most interesting mineral included in the Glenboig fireclay is a hexagonal carbonate with a well-defined zonal structure and at first sight, in sections, a strikingly organic appearance. The zonal structure (figs. 1 and 2) combined with the radial arrangement of the particles gives the crystals a general resemblance to some calcareous algæ. I accordingly sent the first slide in which these crystals were found to Professor A. C. Seward, F.R.S. He could not accept them as calcareous algæ, and kindly referred me to Karpinsky's monograph on the Trochiliscidæ as containing photographs of the calcareous algæ most similar to these structures.*

The sections show that these bodies are zonal crystals of a hexagonal carbonate belonging to the series of dolomite and siderite. They are in the form of rhombohedra with curved faces. Dr J. W. Evans kindly examined one of the slides, and remarked that the regular coincidence of the axes of the lozenge-shaped sections with the optical axes was inconsistent with their organic origin. The material of a fossil may, of course, be crystallised; but the direction of the extinctions in these crystals shows that their external form was an original structure. I can see no evidence that their zonal structure is a later development.

These crystals occur in abundance on certain layers; as, for example, in the clay known as the "compound," above the valuable fireclay beside the Gartverrie Haulage in the Star Mine. A seam full of these crystals was passed through at a depth of 59 feet in bore No. 5, described in the bore journal as 5 feet of "coarse dark fakes."† These crystals are recognisable as coarse grains by the naked eye, and that these are not quartz grains can be recognised by their softness.

Some of them were separated by Mr G. W. Tyrrell, who determined their specific gravity as 3·63, which is appreciably lower than that of siderite, which is given by Miers as 3·86 and by Dana as 3·7 to 3·9. For the following analysis I am indebted to Mr D. P. Macdonald, the Baxter Demonstrator in Geology in the University.

* A. Karpinsky, "Die Trochilisken," *Mém. Comité Géol.*, new ser., No. xxvii., St Petersburg, 1906, viii + 172 pp.; 3 pl., 59 figs. (in Russian, with German summary); see, *e.g.*, fig. 7, p. 12; fig. 10, p. 18; fig. 74, p. 82; and pl. 3, fig. 2.

† Fakes is a local term for laminated sandy clay or sandy shale.

Insoluble in dil. HCl and H_2SO_4	8·06
CO_2	33·26
CaO	1·56
MgO	3·39
FeO	46·45
Fe_2O_3	6·49
H_2O (by difference)	·79
	100·00
$= CaCO_3$	2·78
$MgCO_3$	7·08
$FeCO_3$	74·97
	84·83

These determinations established the position of this mineral, in conjunction with the microscopic evidence, as a member of the hexagonal carbonates intermediate between siderite and dolomite. The mineral contains an insoluble residue of 8·06 per cent. and 6·49 per cent. of iron peroxide. Its essential constituents are the balance of 74·97 per cent. of carbonate of iron, 7·08 per cent. of carbonate of magnesia, and 2·78 per cent. of carbonate of lime. Of the minerals of the dolomite-siderite series it agrees most closely with the sideroplesite of Breithaupt, of which the typical material contained 12 per cent. of carbonate of magnesia. Dana quotes a sideroplesite from Salzberg with 10·5 per cent. of carbonate of magnesia. The insoluble residue and iron peroxide occur as mechanical impurities, and excluding them the proportion of magnesium carbonate in the Glenboig mineral is 8·3 per cent., of carbonate of lime 3·3 per cent., and of carbonate of iron 88·4 per cent. The material is, therefore, a sideroplesite in which part of the magnesia is replaced by lime.

Sideroplesite was first described by Breithaupt * in 1858. He described it as occurring as rhombohedral lens-shaped crystals with rounded faces in the Halber Mond Mine at Böhmsdorf, near Schleiz, on the eastern border of Saxony; it was found in veins containing quartz and stibnite.

Professor Louis of Newcastle has described sideroplesite from Londonderry, Nova Scotia,† where it is found in veins interlacing with those of ankerite. The material occurs there on the western branch of the Great Village river on the southern slope of the Cobequid Mountains, and the

* A. Breithaupt, "Beschreibung neuer Mineralien," *Berg- und huttenm.-Zeit.*, vol. xvii., 1858, p. 54.

† H. Louis, "On the Ankerite Veins of Londonderry, Nova Scotia," *Trans. Nova Scotia Inst. Nat. Sci.*, vol. v., 1882, pp. 47–53.

material is so abundant that it promised to be of "high economic importance." * The material was massive, and no crystals were found; its composition, according to Professor Louis' three analyses, is—

Calcium carbonate	1·92
Ferrous "	68·15
Manganous "	1·87
Magnesium "	28·06
	100·00

Excluding the carbonates of calcium and manganese as accidental impurities, Professor Louis calculated the essential constituents as

Ferrous carbonate	70·02
Magnesia "	29·98

and the formula as $5FeCO_3 + 3MgCO_3$. Professor Louis maintained that owing to the large quantities of this material the sideroplesite should "be classed as a well-defined mineral species rather than as a mere variety of siderite." †

Heddle has recorded ‡ sideroplesite from Scotland; he found it in the deeper parts of the Sandlodge mines on the eastern coast of Shetland, south of Lerwick, where it occurs in quartz associated with chalcopyrite. Heddle regarded it as a calcareous variety of mesitite or pistomesitite, of which the composition is as follows:—

Mesitite	$MgCO_3$	59·2
	$FeCO_3$	40·8
Pistomesitite	$MgCO_3$	42·0
	$FeCO_3$	58·0

Heddle's analysis of the mineral from the Sandlodge mines showed that it contained

Carbonate of iron	62·4
" " manganese	2·0
" " magnesia	24·9
" " lime	9·9
Silica	·8
	100·0

His material, therefore, is not a typical sideroplesite, as the proportion of the siderite molecule is too low.

* *Ibid.*, p. 50. † *Ibid.*, p. 52.

‡ M. F. Heddle, *Mineralogy of Scotland*, vol. i., 1901, p. 141.

6. The Formation of the Glenboig Fireclay.

The characters of the Glenboig sideroplesite show that this mineral grew *in situ* at the time of the deposition of the clay and not by subsequent segregation like the whinny boles of iron carbonate in our Carboniferous sandstones. The centres of the crystals are often quite transparent and water clear, and they were doubtless formed in the water of the lagoon; the outer layers were probably added after the crystals had fallen on to the floor, as they contain some mechanically-included mud.

This view is supported by the facts that no such crystals have ever been described from the English underclays, and that a somewhat similar occurrence of dolomite in the Keuper Marl, discovered by Dr C. G. Cullis,* has been attributed by him to precipitation from the waters of an inland sea or lake. He has described numerous minute crystals of dolomite in the Keuper Marl from Westbury-on-Severn. These dolomite crystals are perfect in shape and have the form of the fundamental rhombohedron, but they are far more minute than the sideroplesite of Glenboig. Dr Cullis attributes the formation of these dolomite crystals to precipitation from solution in the waters of an inland sea or lake while the marl was being precipitated from suspension.

The existence of the sideroplesite, the angularity of the included sand grains, and the vertical position of some of the flakes, all indicate that the Glenboig fireclay was formed in still water. The absence of marine organisms precludes the idea that the formation was marine, for had fossils ever been present, they should have been excellently preserved in this compact clay. There are no fresh-water shells in the deposit. It seems to me, therefore, most probable that this fireclay originated in a great lagoon. The "Roman cement" below was a marine formation, for it is often crowded with marine shells. Mr Hinxman records the fact that the cement stone to the east of Glenboig contains abundant *Orthotetes crenistria*.† Above this marine limestone occurs a clay-band ironstone, and then follow shales and sandstones, followed by the fireclay. The fireclay, therefore, probably marks a period during the deposition of these beds in which the progressive emergence of the land led to part of the sea being cut off as a wide lagoon. Into this lagoon water from the adjacent land carried soluble bicarbonates of iron, magnesia, and lime; the loss of part of the carbonic acid reduced the bicarbonates to the insoluble unicarbonates,

* C. G. Cullis, "On a Peculiarity in the Mineralogical Constitution of the Keuper Marl," *Rep. Brit. Assoc. Leicester*, 1907, p. 507.

† *Summary of Progress*, 1907, p. 103.

and thus led to their precipitation and growth into these crystals of sideroplesite.

Continued emergence of the land led to the site of the lagoon being covered by a sheet of sand, which is now consolidated into the sandstones overlying the fireclay. This sand was in time covered by soil and by trees, whose roots exist as the Stigmaria in the sandstone, and whose remains form the thin coal seam above the sandstone. During the filling of the lagoon with sand strong streams flowed across the locality, cutting channels in the clay. These stream valleys were subsequently filled by sand and thus gave rise to the "bellies" of sandstone that occasionally cut out the fireclay and bring the sandstone of the roof down to the floor of the clay seam.

7. Summary of Conclusions.

1. The Glenboig fireclay was a lagoon deposit in the lower part of the Millstone Grit Series.

2. Its clay substance is an amorphous hydrous bisilicate of alumina, and may be regarded as the mineral halloysite; much clay substance is referable to this material. Other clay substances are kaolinite, sericite, and quartz-flour.

3. The fireclays contain zonal, lenticular, and rhombohedral crystals of sideroplesite which grew in the water and on the floor of the lagoon.

(*Issued separately April* 8, 1910.)

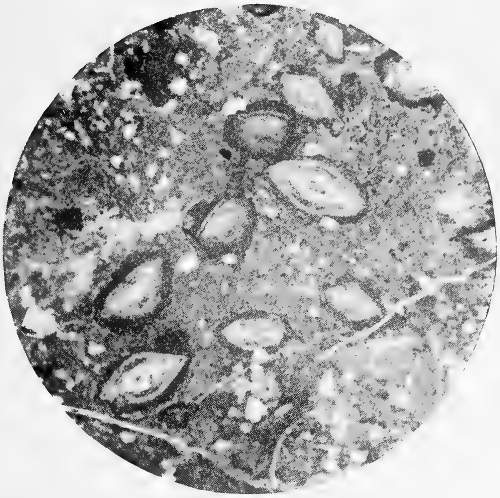

FIG. 1.—A thin section of the Glenboig fireclay, showing a layer with many crystals of sideroplesite. ×30 diameters (ordinary light).

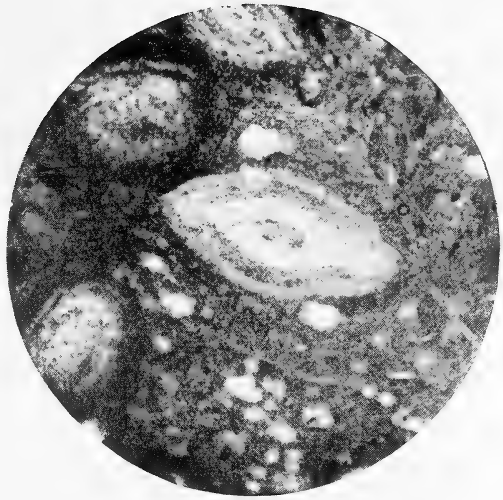

FIG. 2.—One of the same crystals. ×66 diameters (ordinary light).

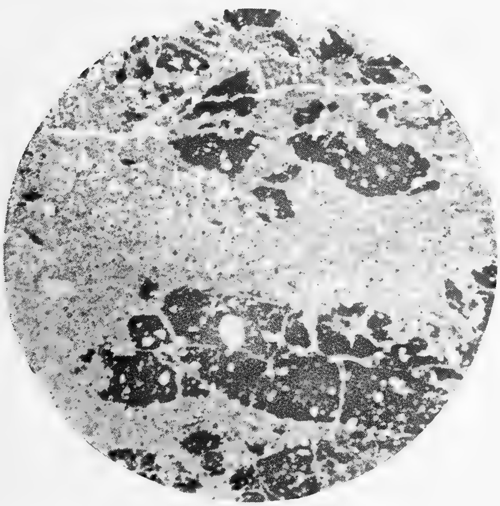

FIG. 3.—A section across the Glenboig fireclay, of a sample unusually rich in iron, and showing the bedding and large quartz grains. ×10 diameters (ordinary light).

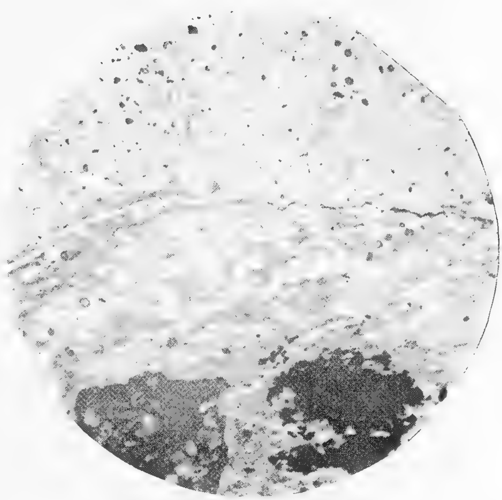

FIG. 4.—Section across Glenboig fireclay, showing the bedding and occasional vertical flakes of quartz. ×30 diameters (ordinary light).

J. FINGLAND, Phot.]

XIX.—Tuesite—A Scotch Variety of Halloysite. By J. W. Gregory, F.R.S., D.Sc., Professor of Geology in the University of Glasgow.

(MS. received February 24, 1909. Read March 15, 1909.)

The absence of china clay from Scotland has been used as a weighty argument in favour of the pneumatolytic or deep-seated origin of the great china clay masses of Cornwall and Devonshire, of southern Sweden, and of some of those in Germany. Kaolin, which is often regarded—I think correctly—as a synonym of china clay, and kaolinite, have, however, been occasionally recorded as occurring in Scotland. A reported occurrence at Troon may be easily dismissed. Thus it is only recorded doubtfully, with the remark that its composition has not been determined, by J. Sommerville and G. R. Thompson,* while John Smith † describes it as a bed of fine-grained volcanic dust deposited in water. Kaolinite has been recorded by Heddle ‡ from several Scottish localities; thus it is found in minute crystals in Shetland, and in a vein at the head of Glen Capel at Abington, Lanarkshire.

The existence of kaolinite in small quantities and in veins is of no special significance, but the statement that it occurs as a bed in the New Red Sandstone on the banks of the Tweed appeared, in the absence of any local kaolinite mass or veins, inconsistent with the pneumatolytic theory. Hence I thought this record worthy of investigation. The material is known as "Tuesite," and was named and described by Thomson.§ Thomson describes Tuesite as a milky-white, opaque, sectile mineral, with a composition which is shown by two analyses by R. D. Thomson and Richardson to be very similar to that of kaolinite.

There are many subsequent references to Tuesite, but none of them that I have seen add any correct information to that given by Thomson, and they afford an instance of the danger of unchecked quotation and the addition of probable, but inaccurate, inferences. The material is described by Thomson as occurring in "New Red Sandstone" on the banks of the Tweed. The identification of the sandstone as part of the New Red Sand-

* *Natural History of Glasgow and West of Scotland*, Brit. Assoc. Handbook, 1901, p. 552.

† J. Smith, "The 'China Clay' Mine and the Water of Ayr Hornstone Bed at Troon, Ayrshire," *Trans. Geol. Soc. Glasgow*, vol. xi., 1900, p. 238.

‡ M. F. Heddle, *Mineralogy of Scotland*, vol. ii., 1901, p. 148.

§ T. Thomson, *Outlines of Mineralogy*, 1836, vol. i. p. 244.

stone was in accordance with the view advocated a year later by Milne Home in his essay on the Geology of Berwickshire,* that the Red Sandstone of south-eastern Berwickshire and the adjacent parts of Roxburghshire was post-Carboniferous in age, and therefore belongs to the New Red Sandstone. This view was soon disproved, but Tuesite has still been assigned to the New Red Sandstone. One author is even more precise, as he refers it to the Bunter, for he says that it occurs "im bunten Sandstein."† Thomson remarked that Tuesite "makes excellent slate pencils." Dana‡ cautiously remarks that it is "used sometimes for slate pencils," while Bristow § in 1861 repeats the statement that it "makes excellent slate pencils," as if it were still quarried for that purpose.

As the natural mode of occurrence of material found in any quantity in the New Red Sandstone would be in beds, and as Dana described it as occurring in seams, I therefore visited the locality, expecting to find that Tuesite is a kaolinite occurring in beds in the New Red Sandstone, and that it had been worked to some extent between 1830 and 1860 for slate pencils. Tuesite, however, is not kaolinite. It is not found in beds; it does not occur in the New Red Sandstone, and it was apparently never worked for slate pencils. It is a variety of halloysite, which is found in veins in the Old Red Sandstone at the junction of the Upper Old Red Sandstone and some intrusive igneous rocks; and so far from being worked for slate pencils, the largest specimen of it that was ever found is reported to have been only the size of a man's fist; and though the material was collected at one time by children and used by them for slate pencils, it does not appear ever to have been actually worked for this purpose.

The only precise information as to its locality is given by Heddle, who said that it is found on the right bank of the Tweed about one mile below Drybridge. I accordingly visited this locality, and found some cliffs containing beds of red sandstone about a mile below the Drybridge Suspension Bridge, but could find there no trace of anything that would correspond to Tuesite. Searching up the river, I found in the ground of The Holmes, a little to the south of a rock beside the river known as Hare Craig, half a mile from Drybridge, some veins of white material which, though not Tuesite, represented an approximation to it. Subsequently, guided by Mr Thomson of St Boswell's, by the kind permission of Norman Ritchie, Esq., the proprietor of The Holmes, I was able to find some small fragments of

* D. Milne [Home], "A Geological Survey of Berwickshire," *Prize Essays* and *Trans. Highland and Agricultural Soc. Scotland*, new series, vol. v., 1837, pp. 206–211.

† Hintze, *Handbuch der Mineralogie*, vol. ii., 1897, p. 840.

‡ Dana, *System of Mineralogy*, 6th edit., 1892, p. 685.

§ H. W. Bristow, *A Glossary of Mineralogy*, p. 389.

Tuesite on the bank of a stream a few feet above the Tweed. It was on the slope below a place where Mr Thomson had obtained it when a boy.

The river banks here are covered by trees and scrub, and there is no clear exposure showing the exact mode of occurrence of the Tuesite. But it is certainly not in the Red Sandstone, for it occurs in some igneous rocks, which, according to the Geological Survey map (Sheet 25), is a "volcanic agglomerate in necks of Calciferous Sandstone age."

Tuesite does not occur as beds, and it was probably formed in veins by the action of hot ascending waters on the felspathic ash of the volcanic neck.

Microscopic sections show that the Tuesite cannot be kaolinite, for it is amorphous and practically isotropic, and it is natural, therefore, to compare the material with halloysite, the amorphous hydrous silicate of alumina. The percentage of water, according to the two oft-quoted analyses by Thomson, is 14·2 and 13·5 per cent.; and this amount would appear too low for halloysite unless the material had been dried at a temperature of above boiling-point. This proportion of water is that of kaolinite. Nevertheless, the material has been called halloysite by Dufrénoy * and Nicol,† though the chemical evidence on which they based their opinion would not alone justify this conclusion. The amorphous nature of the material as shown by the microscope, however, confirms their judgment. The fact that the material is found in small quantities, that it occurs as an alternation product in a volcanic neck, and not in beds in either the Old or New Red Sandstone, deprives it of the special interest it would have had if it had been a bed of kaolinite in a Palæozoic or Triassic Sandstone.

* A. Dufrénoy, *Traité de Minéralogie*, 2nd edit., vol. iii., 1856, p. 585.

† J. Nicol, *Manual of Mineralogy*, 1849, p. 222; *Elements of Mineralogy*, 1858, p. 170. Dana, on the other hand (*System of Mineralogy*, 6th edit., p. 685), calls Tuesite a lithomarge, and it is included by Greg and Lettsom in kaolin or as closely allied to kaolin, lithomarge, and halloysite (Greg and Lettsom, *Manual of Mineralogy of Great Britain and Ireland*, 1858 pp. 207, 448).

(*Issued separately April* 8, 1910.)

XX.—Contributions to the Chemistry of Submarine Glauconite.

By **W. A. Caspari**, B.Sc., Ph.D., F.I.C. (*Communicated by* Sir John Murray, K.C.B., F.R.S.)

(MS. received February 3, 1910. Read March 21, 1910.)

I. Granular Glauconite and its Purification.

The chemical composition of submarine glauconite is of considerable interest in view of the fact that glauconite is the only silicate which is synthesized at the bottom of the sea, and apparently nowhere else. Numerous analyses of this mineral have been published from time to time,* but the results are far from uniform, because the material almost always—certainly in most of the older analyses—was anything but pure. The analysis which inspires most confidence on this score is one which was recently carried out in the *Challenger* laboratory by Collet and Lee † on a purified granular glauconite dredged off the Californian coast by U.S.S. *Tuscarora* (1879).

If grains of glauconite could conveniently be removed out of a coarse greensand by hand-picking, the preparation of a pure sample would present no difficulties; but such is not the case. The method of isolation adopted by Collet and Lee depended on the use of a powerful electro-magnet, which extracts the highly ferruginous glauconite and leaves behind the accompanying quartzose and felspathic minerals. This process is very tedious, and a trifling contamination of the product by crystalline matter is inevitable. There can be no doubt, however, that the material analysed by Collet and Lee represents very pure glauconite.

Experiments in the *Challenger* laboratory have now led to a method by which glauconite can easily be isolated in a high degree of purity.

A greensand is first treated with dilute acid to dissolve out calcium carbonate, if present, and then washed to remove finely-divided and clayey matter. The residue, which now consists solely of glauconite grains plus crystalline sand, is digested with hydrochloric acid ($\frac{2}{1}$ n.) for ten minutes on the water-bath, and then for ten minutes with caustic soda (10 per cent.), also on the water-bath. Neither reagent attacks the glauconite appreciably, but the alkali takes up organic matter and acquires a dark-red colour and

* No less than 43 analyses are quoted by Leith, *U.S. Geol. Survey Mon.*, xliii. p. 239, 1903. Many of the specimens, however, are of continental origin, and some are not true glauconites at all.

† *Proc. Roy. Soc. Edin.*, xxvi. p. 259, 1906.

a sweetish odour. After another such treatment with acid, again followed by caustic soda, the glauconite is observed to disintegrate superficially. The alkaline liquor is now poured off and the residue shaken up with repeated doses of boiling distilled water, when the glauconite is rapidly and completely broken up and forms a bright-green suspension which does not readily settle. The process is repeated until only non-glauconite minerals remain behind, and the combined suspensions (which are slightly alkaline) are rendered faintly acid, when the glauconite is at once coagulated as a green flocculent precipitate. This is washed by decantation with hot water, filtered off, dried at 110°, and powdered.

The property thus possessed by glauconite of going into colloidal suspension affords a means of separating greensands into their constituents more quantitatively than was possible heretofore. Two specimens of submarine greensand placed at disposal by Sir John Murray were dealt with on these lines.

1. A highly glauconitic greensand dredged by U.S.S. *Albatross* (1904) in the Pacific, off Panama, lat. 6° 53′ N., long. 81° 42′ W., depth 556 fathoms.* It contains 5 per cent. of calcium carbonate and 9 per cent. of fine washings, which are green in colour and consist almost entirely of disintegrated glauconite. The glauconite grains are all of 0·25 to 0·6 mm. diameter, and approximate in shape to prolate ellipsoids: no obvious casts were observed. The grains have striæ of a light yellow non-calcareous incrustation wandering irregularly over the surface, whereby incipient decomposition is indicated; they are thus not such fine specimens as the *Tuscarora* grains, which are undoubtedly the purest glauconite hitherto brought up. From 12 gr. of washed and decalcified greensand, 0·36 gr. of mineral residue was obtained: the mineral species are mainly felspar (predominating) and quartz, with a few flakes of hornblende. The quantitative composition of the greensand may be summed up as follows:—

Calcium carbonate	5 per cent.
Glauconite grains	83 „
Minerals	3 „
Fine glauconite	9 „
	100

2. A very inhomogeneous, not predominantly glauconitic greensand dredged on the Agulhas Bank, lat. 34° 38′ S., long. 8° 33′ E., in 110 fathoms

* Murray and Lee, *Mem. Mus. Comp. Zoöl.*, xxxviii. p. 40, 1909. These authorities express the opinion (p. 41) that the sample may have been subjected to washing during or after the dredging.

by s.s. *Pieter Faure* (1898). The deposit contains 33 per cent. of calcium carbonate in the shape of foraminifera shells, many of which have a greenish-grey coating, possibly glauconitic; it also contains finely-divided dirt (largely organic), worm-tubes, chitinous fragments, etc. After the removal of calcareous and fine matter, the granular residue was divided into two portions by simple elutriation, viz.:—

a. (4 per cent. of deposit), dark greenish-grey, mainly glauconitic. The glauconite particles vary widely in size, from the smallest possible up to 0·3 mm. in mean diameter; they are mostly not rounded but very ragged in outline (suggesting exposure to wave or tidal action); there are a few well-defined casts. Residue after separation of glauconite = 12 per cent., quartz sand, sponge spicules, and brown organic matter.

b. (53 per cent. of deposit), white sand with an admixture of full-sized dark-green glauconite grains, well rounded. Mineral residue 88 per cent., almost pure quartz sand.

The dried greensand as a whole is composed, in round numbers, of

Calcium carbonate	33	per cent.
Glauconite grains	10	,,
Minerals	47	,,
Fine and miscellaneous . . .	10	,,
	100	

There is no appearance of incipient decomposition on the grains of fraction *a*; some of those of fraction *b*, however, have much the same superficial incrustation as *Albatross* grains.

The flocculent glauconite prepared from Agulhas Bank greensand is darker and more glaucous than that from the *Albatross* material, which is bright chrome-green in colour. *Tuscarora* glauconite is darker than the latter and more brilliant than the former. Chemical analysis of the purified glauconites gave the results stated below; Collet and Lee's analysis of *Tuscarora* glauconite is quoted in the third column:—

	Albatross.	Agulhas Bank.	*Tuscarora.*
Ignition loss	7·12	7·56	7·00
SiO_2	49 12	51·15	47·46
Al_2O_3	7·09	7·61	1·53
Fe_2O_3	25·95	18·83	30·83
FeO	0·89	2·78	3·10
MgO	3·10	4·54	2·41
K_2O	7·02	7·80	7·76
	100·29	100·27	100·09

It will be observed that the composition of glauconite is subject to small variations, especially in the alumina, magnesia, and ferrous iron.* On the whole, however, the general composition of glauconite may now be considered to be firmly established. If we regard Al_2O_3 as replacing Fe_2O_3, and CaO, MgO, and FeO as replacing K_2O, the formula $KFeSi_2O_6.H_2O$ agrees tolerably well with these analyses.†

To return to the colloidal suspension of glauconite, the formation of such a suspension is calculated to raise some doubts as to whether glauconite is really a crystalline and not rather a colloidal substance. The suspensions, whether prepared from granular or from purified glauconite, behave exactly like clay suspensions. Within a week or so they gradually deposit much of their load in progressively finer particles; but ultimately a true colloidal solution remains. This latter is transparent, opalescent, yellow by transmitted and green by reflected light, and is stable for months unless coagulated by a trace of acid or by a considerable addition of some neutral or alkaline electrolyte. It contains about $1\frac{1}{2}$ gr. of glauconite per litre.

Examined microscopically, flocculent glauconite presents the appearance of rounded particles, varying somewhat in size, with no suggestion whatever of crystalline outlines. A small proportion of the particles, especially the larger ones, retain the characteristic birefringence of granular glauconite. It is worthy of note that flocculent glauconite absorbs dyes (*e.g.* methylene blue) quite as greedily as clay.

II. Organic Matter in Glauconite.

When glauconite grains are made to yield a colloidal suspension by the method described above, it is evident that they undergo a drastic

* The determination of ferrous iron, which, in presence of organic matter, is the mineralogical chemist's despair, was carried out with special precautions in the present analyses, as follows:—One or two gr. of material are placed in a large platinum crucible with 15 cc. of 20 per cent. sulphuric acid and about 1 cc. of strong hydrofluoric acid. The crucible is closed off by a leaden block having two leaden tubulures, through which a current of carbon dioxide is sent through the apparatus. After 15 minutes' gentle boiling, the glauconite is dissolved, and the crucible is allowed to cool in carbon dioxide. The contents are transferred to a measuring-flask, made up to 50 cc., and allowed to stand in an atmosphere of carbon dioxide for 5–6 hours, by which time the organic matter will have settled to the bottom. An aliquot portion of the clear liquor is then pipetted off and titrated with permanganate. Organic matter would seriously impair the accuracy of this determination only in so far as it went into solution in dilute acid at boiling temperature; in the case of glauconite there does not seem to be much danger from this source.

† Clarke (*U.S. Geol. Survey Mon.*, xliii. p. 243, 1903) arrives at the same formula, $KSiFe_2O_6$, from a consideration of the older analyses, but leaves water of hydration out of account on the ground that it is "zeolitic." As a matter of fact, glauconite can take up several molecules of "zeolitic" water, but the one molecule which persists in the sharply dried mineral would rather seem to be water of constitution.

disintegration brought about by purely chemical means. This disintegration is accompanied, or preluded, by a disengagement of organic matter soluble in alkali. Now the most characteristic feature of glauconite under the microscope is its cellular or reticulated structure; it consists of a yellowish-green birefringent material enclosed in a dark isotropic network. From these data it seems legitimate to conclude that granular glauconite is composed of elementary particles of pure glauconite held together by a framework of some substance which is wholly or largely organic.

In point of quantity, the total organic matter in granular glauconite plays but a small part. Combustion-analysis of *Albatross* grains revealed the presence of 0·31 per cent. of carbon, which corresponds to about twice that amount of organic substance. No difficulty was experienced in isolating the organic matter from *Albatross* grains by dissolving away the glauconite with a dilute mixture of hydrochloric and hydrofluoric acids. The residue then takes the form of black flakes, which under high magnification have the appearance of very ragged non-fibrous tissue of a brown colour. It is only partially soluble in dilute alkali, giving an extract similar to the dark alkaline liquor obtained when granular glauconite is disintegrated; that is, there are at least two kinds of organic substance present, soluble and insoluble, respectively, in alkali. Conversely, flocculent glauconite, though it no longer yields anything to alkali, still contains organic matter, as is observed when it is acted upon by concentrated sulphuric acid.

From the dark-red alkaline solution referred to above, the organic solute is completely precipitated on acidification in brown flakes; these, when dried, form a dark-brown powder, which on heating chars without intumescence and leaves a ferruginous ash. A specimen containing 9·2 per cent. of ash (mainly Fe_2O_3 with a little silica) was subjected to combustion-analysis with the following results, calculated on ash-free material:—

Carbon	54·85 per cent.
Hydrogen	5·79 ,,

These figures indicate that in elementary composition the substance resembles that not very well-defined body, humic acid. The characteristic and quite distinct odours emitted by the alkaline solution before and after acidification are exactly like those emitted by alkali-soluble humus (from peat or lake-deposits) in similar circumstances. It is, indeed, difficult not to believe that the organic matter in glauconite is of the nature of humic acid, *i.e.* a product of the decay of vegetable matter. If this be really the

case, some colour is lent to the hypothesis long ago thrown out as a mere suggestion by Julien,* that humus acids play a part in the formation of glauconite; in agreement with which view is the fact that the characteristic occurrences of glauconite are not in the deep sea but rather at no great distance from the shore-line.

There is a further consideration which seems to bear on this point. According to Murray and Renard,† granular glauconite is formed from casts or moulds, within foraminifera shells, of infiltrated clayey silt. The principal changes involved in the transition from clay to glauconite are removal of alumina and silica and attachment of potash. It has been pointed out, however, by Leith,‡ that in order to form the highly ferruginous glauconite far more iron than was originally contained in a clay cast must have been imported from outside. Since there are 5–10 per cent. of Fe_2O_3 in submarine mud, and 20–30 per cent. of glauconite, and since a ballast of 10–15 per cent. of alumina has to be eliminated, not to mention silica, it must be admitted that the production of glauconite casts from clay *in situ* seems an uphill process. Such a process would be greatly facilitated if a substance having a specific solvent and therefore concentrative power for iron (*e.g.* humic acid) were at hand. Putting together, then, the existence of humus-like carbonaceous matter within glauconite grains, the presence, frequently reported, of vegetable refuse § in greensand deposits, and the well-known solvent power of humic acid for iron, the participation of decaying vegetable matter in the formation of glauconite, whether we start from clay casts or not, appears far from improbable; but it is impossible to suggest a precise rationale with the knowledge so far at disposal, the more so since in all likelihood bacterial activity is involved.

Collet and Lee draw attention to the absence of glauconite in lakes, and are disposed to account for it by the tendency of humic acid to keep iron and silica in solution. Where there is much dissolved humic acid, as in some lakes (by no means all, or even the majority), this solvent action may play a part in preventing the formation of glauconite; but after all, the most potent reason for its absence would seem to be the scarcity of one of its essential constituents, namely potash. In ordinary lake-waters K_2O seldom exceeds 5 parts per million, whilst in sea-water there are about 400 parts per million.

* *Proc. Amer. Assoc.*, xxviii. p. 363, 1879.

† *Challenger Reports*, "Deep-Sea Deposits," p. 389, 1891.

‡ *Loc. cit.*, p. 254.

§ *Challenger Reports*, "Deep-Sea Deposits," p. 380; Murray and Lee, *loc. cit.*, p. 20.

III. A Synthetic Silicate Resembling Glauconite.

Some attempts by Calderon and Chaves, quoted by Collet and Lee,* to synthesize glauconite-like silicates seem to have led to no very definite results. By means of the method now to be described, a substance very similar to glauconite may be prepared in the laboratory. The principle consists in allowing colloidal solutions of a complex ferric radical to react on an alkaline silicate solution. 100 cc. of a 10 per cent. solution of potassio-ferric tartrate (corresponding to 2·4 gr. Fe_2O_3) are added to 50 cc. of a solution of potassium silicate containing 1·2 gr. SiO_2. The mixed liquids immediately assume a greenish-blue colour, and in a very short time set to a clear stiff jelly. The jelly is broken up and heated under pressure to 180° for 6–8 hours, the result being a magma of green flocculent particles in a colourless mother-liquor. These particles are washed by decantation with dilute acid, dilute alkali, and water, then filtered off and dried.

The substance thus obtained is a double silicate of potassium and iron. A portion of the latter is in the ferrous state: the double tartrate was originally not quite free from ferrous iron, and a further partial reduction of ferric iron would seem to have taken place in the heating operation. Under the microscope the substance appears quite amorphous and, except for a slight variability in the colour of the particles, homogeneous. It is not readily attacked by alkalies or cold dilute acids; warm dilute acids, however, slowly decompose it, in which respect it is rather less resistant than glauconite. The colour of the substance is grass-green, but the tint fluctuates somewhat in different preparations. That the green colour is due to ferrous iron is indicated by the fact that solutions of a hypobromite, or of hydrogen peroxide, discharge the colour of the silicate, converting it into a straw-coloured substance.

Chemical analysis after drying at 110° gave the following figures:—

Ignition loss	4·19
SiO_2	56·80
Fe_2O_3	28·16
FeO	4·16
K_2O	7·27
	100·58

In composition, therefore, this silicate is not unlike natural glauconite, the chief differences being that it is more acid and contains much less water of hydration. Whilst it is not pretended that its formation and

* *Loc. cit.*, p. 263.

properties throw any direct light on the glauconite problem, evidence is now at hand that when ferric oxide held in solution by an organic acid acts on a hydrosol of silica in presence of potash (which may well be what occurs when glauconite is formed), a green potassic ferroso-ferric silicate is, under suitable conditions, produced. Further, since the green colour of the artificial silicate is bound up with the presence of ferrous iron it seems likely that the same may hold good for natural glauconite; nevertheless, experiments made with the aim of "bleaching" the latter by oxidising agents proved unsuccessful, because it does not readily enter into reaction with aqueous solutions.

IV. The State of Aggregation of Glauconite.

Owing to the birefringence and pleochroism exhibited by submarine glauconite when examined under the microscope, it has hitherto been usual to regard glauconite as a crystalline mineral; indeed Collet and Lee definitely relegate it to the monoclinic system.* Against this we have to set the fact that in the submarine mineral, whether granular or pulverulent, nothing in the least like a crystal-contour has ever been noted. It is true that fossil "glauconites," embedded in continental formations, have been from time to time described, which are morphologically as well as optically crystalline. These, however, cannot be accepted as identical with recent submarine glauconite, though they may perhaps be metamorphic derivatives of it; in the absence of analyses it is not unlawful to suspect that some of them may be a quite distinct mineral, possibly chlorite.

On the other hand, certain properties of glauconite indicating a state of aggregation differing from that of ordinary crystalline minerals have been referred to above. Some additional light is shed on this point by the behaviour of glauconite as regards hydration. It is a peculiarity of colloid minerals (*e.g.* clays) and of zeolites that they absorb somewhat large proportions of water, according to the moistness of the air with which they are in contact, without forming definite hydrates. In order to ascertain whether glauconite falls into this class, a series of experiments was made according to the method first described by Van Bemmelen † and extended to minerals by Tammann ‡ and Löwenstein.§ Purified *Albatross* glauconite, prepared as on p. 364, was spread in fine powder on glass dishes, which were suspended in closed jars over sulphuric acid of various concentrations; that is, they were exposed to various tensions of aqueous vapour. The jars were left in a cellar, at a temperature ranging from 9° to 11°, for eight months, after which equilibrium was thoroughly established. A specimen

* *Loc. cit.*, p. 243.

† *Zeitschr. anorg. Chem.*, xiii. p. 231, 1897.

‡ *Zeitschr. f. phys. Chem.*, xxvii. p. 323, 1898.

§ *Zeitschr. anorg. Chem.*, lxiii. p. 691, 1909.

of oceanic Red Clay * was treated side by side with the glauconite. The powders were then assayed for moisture by simple ignition. The results take the following form:—

Concentration of acid.	Tension of aqueous vapour.	Glauconite; percentage of moisture.	Red clay; percentage of moisture.
1 per cent.	9·2 mm.	30·35	24·19
30 „	6·8 „	18·29	13·29
45 „	4·3 „	13·22	11·43
55 „	2·3 „	11·31	10·53
65 „	1·2 „	9·47	9·28
75 „	0·5 „	8·45	8·36
Fully dried	...	7·12	6·94

The figures in the last line refer to the combined water remaining after the minerals have been dried to constancy at 110°, which may be regarded as water of constitution.

From the above table it will be perceived, firstly, that glauconite becomes a highly hydrated mineral in presence of moist air (whilst still remaining an apparently dry powder). No doubt this is the condition in which it exists in its native element, a third or more of its weight consisting of water. Secondly, it is evident, without drawing up a curve, that the hydration decreases continuously with the tension of aqueous vapour in equilibrium, whence it follows that there are no definite hydrates representing a series of distinct molecular species. Thirdly, there is a marked parallelism as regards hydration between glauconite and Red Clay.

It is well known that this kind of water absorption, which is especially characteristic of colloids, is also shared by the unquestionably crystalline zeolites. The inference, then, which we may now draw as to the nature of glauconite is that it is certainly not an ordinary crystalline silicate like felspar or mica, but that it must be either a zeolite or a colloid. As between these two alternatives, it is less easy to decide. The property possessed by glauconite of absorbing dyes is again common to both colloids and zeolites. In favour of the view that glauconite is a colloid, we have the absence of crystal-contours and the ease with which it forms colloidal suspensions and solutions. Evidence of crystalline habit, on the other hand, is afforded by the optical anisotropy of glauconite. To this, however, it may be rejoined that isotropic matter in a state of strain is equally capable of showing birefringence. That glauconite may exist under some such strain seems not unlikely when we consider the structure of glauconite

* Analysis under No. 1 in *Proc. Roy. Soc. Edin.*, xxx. p. 190, 1910.

grains, in which the glauconite proper would appear to be enclosed by a network of foreign substance; and the vehemence with which the grains fly into powder under the action of acid and alkaline solutions points in the same direction. On the whole, then, though it would be premature to regard the matter as settled, the probability is that glauconite is essentially a colloidal silicate.

(*Issued separately April* 8, 1910.)

XXI.—A Chemical Investigation into the Nature of the Clay Substance in the Glenboig Fireclay. By D. P. McDonald, M.A., B.Sc., Baxter Demonstrator in Geology in the University of Glasgow. *Communicated by* Professor J. W. Gregory.

(MS. received March 5, 1910. Read March 21, 1910.)

This investigation into the nature of the clay substance contained in the fireclay from Glenboig was undertaken at the suggestion of Professor J. W. Gregory. Professor Gregory thought that the fine material might be halloysite, and the results obtained tend to confirm this view.

The very finest material was separated by repeated washing from a sample of the best clay, and used for the analyses. Although apparently white when suspended in water, the material when dry had a decided buff colour. As the colour indicated the presence of some impurity, efforts were made by further washing to remove it; but with no good result. The sample was dried in the steam oven and analysed.

Analysis.

SiO_2	=46·67	per cent.
Al_2O_3 + small amount of Fe_2O_3	=37·65	,, ,,
H_2O at 105° C.	= 2·13	,, ,,
H_2O (combined)	=12·66	,, ,,
	99·11	

The analysis showed the presence of CaO and MgO in small amount—·16 per cent. of CaO. The MgO was not determined.

This result agrees with those obtained by Dr C. Fawsitt *—

	I.	II.
SiO_2 . . .	48·05	46·6
Al_2O_3 . .	36·53	37·2
H_2O − / H_2O + .	13·45	13·85
	98·03	97·65

* Proof *in causa* The Caledonian Railway Company *against* The Glenboig Union Fireclay Company, Ltd., p. 168.

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MODEL INDEX.

Schäfer, E. A.—On the Existence within the Liver Cells of Channels which can be directly injected from the Blood-vessels. Proc. Roy. Soc. Edin., vol. , 1902, pp. .

Cells, Liver,—Intra-cellular Canaliculi in.
E. A. Schäfer. Proc. Roy. Soc. Edin., vol. , 1902, pp. .

Liver,—Injection within Cells of.
E. A. Schäfer. Proc. Roy. Soc. Edin., vol. , 1902, pp. .

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PROCEEDINGS

OF THE

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SESSION 1909–10.

Part V.] VOL. XXX. [Pp. 375–438.

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[*Continued on page* iii *of Cover*.

The analysis of the clay substance points to its being either kaolinite or halloysite. A comparison of the results tabulated below leads to no very satisfactory conclusion.

ANALYSES OF KAOLINITE GIVEN BY LACROIX IN "MINÉRALOGIE DE LA FRANCE," vol. i., 1895, p. 463.

	I.	II.	III.	IV.	V.
SiO_2 . . .	47·0	45·97	48·68	46·5	46·67
Al_2O_3 . . .	39·4	40·12	36·92	39·5	37·65
			MgO ·52		CaO ·16
			Na_2O ·58		MgO not det.
H_2O . . .	14·4	13·91	13·13	14·0	14·79
	100·8	100·00	99·83	100·0	99·27

I. (Pholérite, Lodève); M. Pisani (*Comptes rendus*, liii. 1072, 1861).
II. (La Chartreuse près Liege); de Koninck (*Bull. Ac. Sc. Belg.*, lxiv. 734, 1877).
III. (Saint-Yrieix); Forchammer (*Pog. Ann.*, xxxv. 337, 1835).
IV. Calculated from the formula for kaolinite.
V. Present analysis.

ANALYSES OF HALLOYSITE GIVEN BY LACROIX IN "MINÉRALOGIE DE LA FRANCE," vol. i. p. 475.

	I.	II.	III.	IV.	V.	VI.
SiO_2 . . .	48·95	47·9	46·3	46·3	48·7	47·7
Al_2O_3 . . .	31·46	38·0	39·5	38·7	36·5	38·5
CaO . . .	3·88	...	...	...	...	·17
						MgO not det.
H_2O . . .	14·48	14·3	14·3	14·0	13·6	12·9
	98·77	100·2	100·1	99·0	98·8	99·27
Loosely combined H_2O	...	5·4	8·5	12·5	4·0	2·13

I. (Huelgoat); Dufrénoy (*Mém. pour servir à une descript. géol. de la France*, ii. 220, 1834).
II. (Huelgoat); M. Lechatelier (*Bull. Soc. Minér. de France*, x. 210).
III. (Angleur (Belgique)); M. Lechatelier (*Bull. Soc. Minér. de France*, x. 210, 1887).
IV. (Miglos, Ariège); *ibid.*
V. (Laumède, Dordogne); *ibid.*
VI. Present analysis calculated for comparison.

The microscopic examination of the sample showed that, in spite of the repeated washing, there was still a small amount of free quartz present. As the clay in the ordinary condition is highly siliceous, this might be expected to be the case. In order, therefore, to eliminate the free quartz, the Al_2O_3 was supposed to exist in combination as the hydrous silicate of alumina, kaolinite.

Results calculated on the Assumption that the Silicate is Kaolinite.

	I.	II.
SiO_2	44·3	43·8
Al_2O_3	37·7	37·2
H_2O	13·3	13·2
Excess H_2O . .	1·5	·65
Free SiO_2 . .	2·4	2·8
	99·2	97·7

I. Present analysis.
II. Analysis by Dr Fawsitt.*

It is important to notice that the percentage of water is greater than that required to satisfy the formula for kaolinite. The excess of water is not sufficiently large, however, to prove that the mineral in question is halloysite, although it must be remembered that in the published analyses of halloysite this loosely combined water varies within wide limits.

Effect of Boiling the Clay Substance with Concentrated Hydrochloric Acid.

A portion of the washed clay (2·5 gms.) was boiled with concentrated hydrochloric acid (160 c.c.) for two hours. The residue was filtered off and the filtrate evaporated to dryness. The Al_2O_3 was estimated after the small amount of Fe_2O_3 had been removed. The result showed that 6·5 per cent. of Al_2O_3 had been obtained from the clay by this treatment with the acid. The residue was again boiled with acid for four hours, the quantity of acid being also doubled. The percentage of Al_2O_3 obtained by this digestion amounted to 16·9, but the small amount of Fe_2O_3 was also weighed along with the Al_2O_3, the amount of Fe_2O_3 being so slight as not to affect the argument. The residue from the second estimation was boiled for seven hours, and 13·2 per cent. of Al_2O_3 obtained. Altogether, therefore, 36·6 per cent. of Al_2O_3 was got from the sample in this way. Since the analysis showed that the total Al_2O_3 present amounted to 37·65 per cent., it is obvious that practically all the silicate of alumina is decomposed by boiling hydrochloric acid.

That the Al_2O_3 exists as a silicate is proved by the microscopic examination. If the alumina is free, then the silica must be free; and, as stated before, only a very small amount of free silica was found in the sample.

* Proof *in causa* The Caledonian Railway Company *against* The Glenboig Union Fire-clay Company, Ltd., p. 168.

This decomposition by hydrochloric acid goes far to prove that the mineral is halloysite; for, according to Lacroix,* kaolinite is not affected by concentrated hydrochloric acid, whereas halloysite is readily attacked. Since the time taken to decompose the clay substance in the Glenboig fireclay is considerable, it can scarcely be said to be readily attacked. The greater resistance to decomposition may be found to depend on the much smaller content of loosely combined water which it contains. With a view to comparing the behaviour of other recognised halloysites towards hydrochloric acid, two samples were treated in exactly the same manner. A halloysite from the Dordogne yielded 2·2 per cent. of Al_2O_3 after two hours' boiling, while a sample of lenzinite from the Eifel, Rhine, gave 21·97 per cent. in the same time. It is interesting to note that the Dordogne halloysite contains only 4 per cent. of loosely held water, while lenzinite has 8·8 per cent.

The evidence in favour of the mineral being halloysite may be summed up:—

1. The percentage of water is higher than that required for kaolinite.
2. Although the percentage of water is rather lower than that in the typical halloysites, it must be remembered that a very great variation is shown in their analyses.
3. The mineral is practically completely decomposed by boiling with concentrated hydrochloric acid.

* *Minéralogie de la France*, vol. i., 1895, p. 472.

(*Issued separately April* 20, 1910.)

XXII.—Borel's Integral and q-Series. By Rev. F. H. Jackson, M.A.

(MS. received November 29, 1909. Read December 20, 1909.)

I.

In his *Theory of Infinite Series,* Dr Bromwich gives an account of the recently developed theory of non-convergent and asymptotic series, so far as the arithmetical side of the theory is concerned. The connection between Borel's integral "sum" and Euler's well-known transformation

$$\sum u_n x^n = u_1 y + (\Delta u_1)y^2 + (\Delta^2 u_2)y^3 + \ldots\ldots ; \qquad y = x/(1-x) \qquad (1)$$

is discussed. Now, if we apply this transformation to such series, for example, as

$$\frac{x}{1-b} + \frac{x^2}{1-qb} + \frac{x^3}{1-q^2 b} + \ldots\ldots = \frac{x}{1-x} + \frac{bx}{1-qx} + \ldots\ldots , \qquad (2)$$

$$1 + \frac{(q^a-1)}{(q-1)}x + \frac{(q^a-1)(q^{a+1}-1)}{(q-1)(q^2-1)}x^2 + \ldots\ldots , \qquad (3)$$

which are of great interest in the theory of Elliptic Functions, we obtain results which may be described as formless, or at least of such complexity, owing to the mixture of q-factorials $(1-q^n)!$ with ordinary factorials $n!$, that the resulting series are practically useless so far as the possibility of applying further transformations is concerned. In fact, when we are dealing with power series, in which the coefficients are q-numbers, we must use a modification of Euler's transformation if the results are to possess that quality of form which will make them interesting and useful. The required modification of Euler's transformation is

$$\sum u_n x^n = \frac{x}{(1-x)}u_1 + \frac{x^2}{(1-x)(1-qx)}\Delta u_1 + \frac{x^3}{(1-x)(1-qx)(1-q^2x)}\Delta^{(2)}u_1 + \ldots\ldots ,$$

in which

$$\Delta^n u_1 = (D-1)(D-q)\ldots(D-q^{n-1})u_1 . \qquad (4)$$
$$Du_n = u_{n+1} ,$$

reducing to Euler's series in case $q=1$. If we apply this transformation to the above series (2) and (3) we obtain from (2) the well-formed series

$$\frac{x}{(1-x)(1-b)} + \frac{bx^2(q-1)}{(1-x)(1-qx).(1-b)(1-qb)} + \ldots\ldots$$

in which the general term is

$$(-1)^n \frac{(1-q^n)!}{(1-q^n b)!\,(1-q^n x)!} b^n x^{n+1} q^{\frac{1}{2}n(n-1)}.$$

The series (3) gives the form

$$\frac{1}{(1-x)} + q\frac{[a-1]}{(1-x)(1-qx)} + q^4, \text{ etc.},$$

in which the general term is

$$q^{n^2}\frac{[a-1][a-2]\ .\ .\ [a-n]}{(1-q^n x)!}.$$

As in previous papers, $[a]$ denotes $(1-q^a)/(1-q)$.

What has been written above with respect to Euler's transformation may be written, *mutatis mutandis*, with respect to Borel's integral

$$\overset{\infty}{\underset{0}{\mathscr{S}}} u_n = \int_0^\infty e^{-x} u(x)\, dx,$$

where

$$u(x) = \underset{n\to\infty}{\text{limit}} \left\{ u_0 + u_1 x + u_2 \frac{x^2}{2!} + \ .\ .\ + u_n \frac{x_n}{n!} \right\},$$

also of the integral sum of an asymptotic series

$$a_0 + \frac{a_1}{x} + \frac{a_2}{x^2} + \ .\ .\ .\ .\ .\ ,$$

$$\mathrm{J}(x) = \int_0^\infty e^{-t} f\left(\frac{t}{x}\right) dx.$$

(*Cf.* Bromwich, *Theory of Infinite Series,* pp. 268, 339–40.)

II.

In this note I should like to indicate the necessary modification of Borel's method if it is to be available in the case of q-series. G. H. Hardy,* in his papers on non-convergent series, has formulated the following principle which lies at the root of Borel's method: "If two limiting processes performed in a definite order on a function of two variables lead to a definite value X, but when performed in reverse order lead to a meaningless expression Y, we may agree to interpret Y as meaning X." For example, suppose

$$f(x, n) = \phi_0(x) + \phi_1(x) + \ .\ .\ .\ + \phi_n(x),$$

and that

$$\Phi(x) = \underset{n\to\infty}{\text{limit}}\, f(x, n),$$

* *Trans. Cambridge Phil. Soc.*, vol. xix., 1904, p. 297.

then

$$\sum_0^\infty\left[\int_0^\infty \phi_n(x)dx\right] = \int_0^\infty \Phi(x)dx.$$

It may happen that the integral on the right is convergent when the series on the left is non-convergent. Borel's integral is the special case in which

$$\phi_n(x) = e^{-x}u_n\frac{x^n}{n!},$$

where u_n is independent of x, and

$$\Phi(x) = \underset{n\to\infty}{\text{limit}}\, e^{-x}\left\{u_0 + u_1x + \ . \ . \ . + u_n\frac{x^n}{n!}\right\}$$
$$= e^{-x}u(x).$$

We see that if Borel's method is to be available in the case of any specified series $u_0+u_1+u_2+\ . \ . \ . \ .$ we must have considerable prior knowledge of the form of

$$\underset{n\to\infty}{\text{limit}}\left\{u_0 + u_1x + \ . \ . \ . + u_n\frac{x^n}{n!}\right\}$$

(in this case supposed convergent, though the condition may subsequently be removed). I express this otherwise, by saying, that Borel's method enables us to make use of our knowledge of the form of the limit of an infinite series (convergent for only a finite limited range of values of the variable) to perform certain operations and transformations in the case where the variable has passed outside the limited region of convergence. We must keep in mind that such use of non-convergent series is always formal and symbolic. It is interesting in this connection to recall Professor Mittag-Leffler's remarks: * "Borel's idea that he has obtained by his summation expression the power series itself, in the case where it diverges, is a play upon words, and all the more intemperate because it gives rise to the illusion, entirely false, that he has been able to extend the limits of the theory of analytic functions beyond those fixed by classic theory."

"As one is able, since the expression of Borel is convergent, to perform the same operation on the divergent series as on the convergent series $p(x)$, Borel's scheme implies only the translation for this special case of Weierstrass's theorem, 'If an analytic relation, however general or however special, exists between several different power series or their derivatives, this same relation subsists also for the functions in their totality.'"

A consideration of Hardy's principle will show us why Borel's integral is not available in dealing with q-series. We have, generally speaking, no

* *Bulletin Amer. Math. Soc.*, vol. xiv., 1908, p. 485 (International Congress, Rome, 1908).

knowledge of the form of such a limit as

$$\underset{n\to\infty}{\text{limit}}\left\{ u_0+u_1x+\ .\ .\ +u_n\frac{x^n}{n\,!}\right\},$$

in which the coefficients of x^r are numbers $\frac{u_r}{r\,!}$ containing q-factorials $(1-q^n)\,!$ etc., mingled with factorials of the ordinary type $n(n-1)\,..\,(n-r+1)$. Although, generally speaking, we may be ignorant of the form of such a limit as described above, we may have perfectly definite knowledge of the nature of such a limit as

$$\underset{n\to\infty}{\text{limit}}\left\{ u_0+u_1x+u_2\frac{x^2}{[2]\,!}+\ .\ .\ +u_n\frac{x^n}{[n]\,!}\right\},$$

in which $[n]$ denotes the q-number $(q^n-1)/(q-1)$ and the coefficients u_0, u_1 . . . . are q-factorials.

Wherever in Borel's theory the exponential e^x appears, I propose, in the case of q-series, to substitute one or other of the following two series which reduce to e^x in case $q=1$.

$$\mathrm{E}_q(x)=\underset{n\to\infty}{\text{limit}}\left\{ 1+x+\frac{x^2}{1+q}+\ .\ .\ +\frac{x^n}{[n]\,!}\right\}$$

$$\mathrm{E}(ax)=\underset{n\to\infty}{\text{limit}}\left\{ 1+ax+\frac{a^2x^2}{[2]}q+\frac{a^3x^3}{[3]\,!}q^3+\ .\ .\ +\frac{a^nx^n}{[n]\,!}q^{n(n-1)/2}\right\}.$$

Both series are continuous for all values of x from 0 to ∞. The conditions for convergence are easily established. Wherever in Borel's formulæ the sign $\int$ of integration appears, we shall use $\mathbf{S}$, the sign of q-finite integration, which is the operation reversing the q-finite difference operation

$$\Delta\phi(x)=\frac{\phi(qx)-\phi(x)}{qx-x}$$
$$=\phi'(x)\,.$$

The successive differences are denoted $\phi''(x)$, $\phi'''(x)$, ... In case the functions operated upon are differentiable, it is easy to see that in the limit $q=1$, these differences become the successive differential coefficients, and $\mathbf{S}$ becomes $\int$. With this explanation of the notation we write

$$\mathscr{J}_0^{\infty} u_n=\mathbf{S}_0^{\infty}\mathrm{E}(-qx)u(x)d(qx)\,,$$

where

$$u(x)=u_0+u_1x+u_2\frac{x^2}{[2]\,!}+\ .\ .\ .\ .$$

and

$$\mathrm{E}(-qx)=1-qx+q^3\frac{x^2}{[2]\,!}-\ .\ .\ .\ .\ .$$

For the case of an asymptotic q-series

$$a_0 + a_1x^{-1} + a_2x^{-2} + \ldots\ldots, \quad (x>0)$$

we define the sum as

$$\mathop{\mathrm{S}}_0^\infty \mathrm{E}(-qx)f\left(\frac{t}{x}\right)d(qt).$$

I use the factor $d(qt)$ merely to denote with respect to what variable (t) we are operating.

III.

The reader may easily verify the following: I omit constants, also $d(qx)$ under the finite-integral sign, since all integrations are with respect to x.

$$\Delta x^n = [n]x^{n-1}, \quad \mathrm{S}x^{n-1}d(qx) = x^n/[n]$$

$$\Delta(x+1)(x+q).(x+q^{n-1}) = [n](x+1)(x+q).(x+q^{n-2})$$

$$\Delta\mathrm{E}(ax) = a\mathrm{E}(aqx)$$

$$\mathrm{S}\mathrm{E}(aqx) = \frac{1}{a}\mathrm{E}(ax)$$

$$\mathrm{S}\mathrm{E}(-qx) = -\mathrm{E}(-x)$$

$$\mathrm{E}_q(x)\mathrm{E}(ax) = 1 + (1+a)x + \frac{(1+a)(1+qa)}{[2]!}x^2 + \ldots\ldots$$

$$\mathrm{S}\mathrm{E}_q(x)\mathrm{E}(ax) = \frac{1}{1+q^{-1}a}\left\{1 + (1+q^{-1}a)x + \frac{(1+q^{-1}a)(1+a)x^2}{[2]!} + \ldots\right\}$$

$$\mathrm{S}\mathrm{E}_q(bx)\mathrm{E}(aqx) = \frac{1}{b+a}\mathrm{E}_q(bx)\mathrm{E}(ax)$$

all of which will be required in the following work

$$\mathop{\mathrm{S}}_0^\infty x^n\mathrm{E}(-qx)\mathrm{E}_q(-bx) = \frac{[n]!}{(1+b)(1+bq)\ \ldots\ (1+bq^{n-1})}$$

Since

$$\begin{aligned}\Delta u_xv_x &= (u_{qx}v_{qx} - u_xv_x)/(qx - x)\\ &= u_x'v_x + u_{qx}v_x'\\ \mathrm{S}u_x'v_x &= u_xv_x - \mathrm{S}u_{qx}v_x'.\end{aligned}$$

Noting the presence of u_{qx} in the last term, we have

$$\mathrm{S}\mathrm{E}(-qx)x^n = -\mathrm{E}(-x)x^n + \mathrm{S}\mathrm{E}(-qx)[n]x^{n-1}.$$

Taking between limits 0 and ∞, we have

$$\mathop{\mathrm{S}}_0^\infty x^n\mathrm{E}(-qx) = [n]\mathop{\mathrm{S}}_0^\infty x^{n-1}\mathrm{E}(-qx) = [n]!. \quad \ldots \quad (5)$$

for the first term on the right-hand side of (5) vanishes, both when $x=0$ and when $x=\infty$. We see that

$$\mathrm{S}u_n\frac{x^n}{[n]!}\mathrm{E}(-qx) = u_n,$$

and provided we have knowledge of the limit

$$u(x) = \underset{n\to\infty}{\mathrm{L}} \left\{ u_0 + u_1 x + \ . \ . \ + u_n \frac{x^n}{[n]\,!} \right\}$$

we write

$$\overset{\infty}{\underset{0}{\mathscr{S}}} u_n = \overset{\infty}{\underset{0}{\mathbf{S}}} \mathrm{E}(-qx)u(x)d(qx) \quad . \quad . \quad . \quad . \quad . \quad (6)$$

For illustration consider the following examples :

$$1 - [2]t + [3]t^2 - \ . \ . \ . \ .$$

Here

$$u(x) = 1 - [2]tx + \frac{[3]}{[2]\,!} t^2 x^2 - \ . \ . \ . \ .$$
$$= (1 - qxt)\mathrm{E}_q(-tx)$$

and the q-integral (6) gives us

$$\overset{\infty}{\underset{0}{\mathbf{S}}} \mathrm{E}(-qx)\mathrm{E}_q(-tx) - q\overset{\infty}{\underset{0}{\mathbf{S}}} \mathrm{E}(-qx)\mathrm{E}_q(-bx)tx.d(qx)$$

$$\frac{1}{1+t} - \frac{qt}{(1+t)(1+qt)}$$

$$= \frac{1}{(1+t)(1+qt)} \qquad q \leqq 1\,, \ t < 1$$

which is correct, as may be easily verified in other ways. If $t=1$ $q=1$ the value is $\frac{1}{4}$, agreeing with Borel's sum of $1-2+3-$ . . . .

Example (B)

$$1 - [2]^2 t + [3]^2 t^2 - \ . \ . \ . \ .$$

gives an integral

$$\mathbf{S}\mathrm{E}(-qx)\mathrm{E}_q(-xt)\{1 - (q^2 + 2q)tx + q^3t^2x^2\}d(qx)$$
$$= \frac{1}{t+1} - \frac{(q^2+2q)t}{(1+t)(1+qt)} + \frac{[2]q^3t^2}{(1+t)(1+qt)(1+q^2t)}$$
$$= \frac{1-qt}{(1+t)(1+qt)(1+q^2t)}$$

which is easily verified. In case $q=1, t=1$ we have Borel's sum of $1-2^2+3^2-$ . . . $=0$.

IV.

Asymptotic formula :—

In the case of an asymptotic series

$$a_0 + a_1/x + a_2/x^2 + \ . \ . \ . \ .$$

we suppose that $f(v)$ is an associated series

$$f(v) = a_0 + a_1 v + a_2 \frac{v^2}{[2]\,!} + \ . \ . \ . \ .$$

convergent for certain positive values of v, and that the function defined only for these positive values of v is continuous from $v=0$ to ∞. We, moreover, suppose that constants A and l can be found such that

$$f^n(v) < \mathrm{AE}_q(lv)\,, \qquad (\textit{cf.}\ \text{Bromwich, p. 340, } \textit{T.I.S.})$$

where n denotes the index of any q-difference. Writing $f'(v)$ to denote the difference $\dfrac{f(qv)-f(v)}{qv-v}$ and putting $v=\dfrac{t}{x}$, we see that

$$f'\left(\frac{t}{x}\right) = x\frac{f(qt/x)-f(t/x)}{qt-t}\,,$$

which is

$$\Delta_t f(t/x) = \frac{1}{x} f'\left(\frac{t}{x}\right)$$

$$\begin{aligned}
\mathrm{J}(x) &= \mathop{\mathrm{S}}_0^\infty \mathrm{E}(-qt) f\left(\frac{t}{x}\right) d(qt) \\
&= -\left[\mathrm{E}(-t) f\left(\frac{t}{x}\right)\right]_0^\infty + \frac{1}{x}\mathop{\mathrm{S}}_0^\infty \mathrm{E}(-qt) f\left(\frac{t}{x}\right) d(qt) \\
&= a_0 + \frac{1}{x}\mathop{\mathrm{S}}_0^\infty, \text{ etc.}
\end{aligned}$$

Repeating, we obtain after n integrations

$$a_0 + \frac{a_1}{x} + \frac{a_2}{x^2} + \ \ldots\ + \frac{a^n}{x^n} + \frac{1}{x^{n+1}}\mathrm{S}\,\mathrm{E}(-qt) f^{n+1}\left(\frac{t}{x}\right) d(qt) \quad . \quad . \ (7)$$

Now supposing, as stated above,

$$f^{n+1}\left(\frac{t}{x}\right) < \mathrm{AE}_q(lt/x)$$

the integral on the right side of (7)

$$\begin{aligned}
&< \frac{\mathrm{A}}{x^{n+1}}\mathrm{S}\,\mathrm{E}(-qt)\mathrm{E}_q(lt/x) d(qt) \\
&< \frac{\mathrm{A}}{x^{n+1}} \frac{x}{x+l} \\
&< \frac{\mathrm{A}}{x^n(x+l)}\,,
\end{aligned}$$

and the asymptotic nature of the expression is established for

$$\underset{n\to\infty}{\text{limit}} \left\{ \mathrm{J}(x) - a_0 - a_1/x - a_2/x^2 - \ \ldots\ a_n/x^n \right\} = 0\,.$$

At this point I conclude, for my object is not to work out in great detail the sums of special series. There are many points which require elaboration, such as the convergency of the infinite q-integrals. In the case of

the integral

$$\underset{\lambda \to \infty}{\text{limit}} \int_0^\lambda x^n \mathrm{E}(-qx)d(qx)$$

the reader can easily construct a proof of convergency on the lines of the usual proof for

$$\underset{\lambda \to \infty}{\text{limit}} \int_0^\lambda x^n e^{-x} dx .$$

Just as Bromwich deduces Euler's series from Borel's integral, so, on parallel lines, it is easy to deduce the modified form of Euler's transformation which is given at the beginning (4) of this note. This may be left as an exercise for the reader.

(*Issued separately April* 20, 1910.)

XXIII.—Proposals for an Anemometer and a Portable Barometer.

By **J. T. Morrison**, M.A., B.Sc., Professor of Applied Mathematics, Victoria College, Stellenbosch, Cape Colony.

(Read January 10, 1910. MS. received January 14, 1910.)

A. An Anemometer that will register separately the North-Southerly and East-Westerly Components of the Wind.

1. The Object of the Instrument.

For many important meteorological purposes it would be an advantage to be able to keep separate records of the components of the wind in two fixed rectangular directions, the north - southerly and east - westerly directions being those that would naturally be chosen. At present the registering Robinson anemometer records merely the total amount of wind that has passed any spot; a separate pen gives the direction; and a somewhat laborious process of calculation is needed in order to obtain even an approximate estimate of the wind-components. The object of the instrument described below is to resolve the wind into northerly and easterly components and to record their absolute values with two separate pens. The instrument is so arranged that the distance of the one pen from its zero line at any instant will be a measure of the excess of north wind over south wind which has flowed past up to that instant, and the curve traced by that pen will show all the variations of this north-southerly component. The slope of this curve at any point gives the velocity of the north component at the corresponding instant of time. The curve traced by the other pen gives similar information regarding the easterly component. The ratio of the two slopes gives the direction. In fact, as the recording cylindrical drum turns uniformly on its axis, the first pen is drawn along its surface in a direction parallel to its axis at a rate proportional to the northerly component of the wind, while the other pen travels at a rate proportional to the easterly component. If an easterly wind changes to a westerly, the corresponding pen travels backwards, and so on.

2. The Principle of the Instrument.

The distinctive part of the new instrument is that which analyses the wind-velocity, or rather which analyses a rotation proportional to, and

having its axis in the same direction as, the wind-velocity. It will perhaps be more easily understood by reference to the diagram (fig. 1).

A sphere C D E is mounted on a horizontal axis A′A so as to be free to rotate round it. By suitably connecting the sphere with a wind-wheel and the horizontal axis with a wind-vane, the sphere is caused to spin on the axis at a speed proportional to the speed of the wind, while the axis itself veers with the wind so as to be always parallel to it, and so that one end of the axis (say A′) is always at the quarter from which the wind comes. The sphere always spins one way (say, right-handedly) as seen from the end of A′ of the axis; hence, when the direction of the wind is reversed, the rotation of the sphere in space will also be reversed, since the end A′

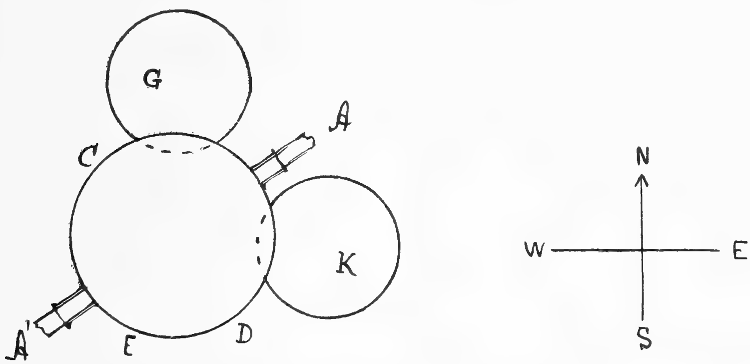


Fig. 1.

will be standing in the opposite quarter. The centre of the sphere remains at rest.

G and K are two small circular plates, horizontal, equal in size, with bevelled edges, and free to rotate round vertical axes through the respective centres. These plates are brought up below the sphere so as to touch it lightly with their bevelled edges. The centre of G is precisely north of the centre of the sphere, and the centre of K precisely east; also they are equidistant from the vertical line through the centre of the sphere. Hence the plates are on one level, and their points of contact with the sphere are at exactly the same distance below the level of the centre of the sphere.

Under these circumstances, whatever be the direction of the wind, the plate G will through its contact with the sphere turn at a speed proportional to the north-southerly component of the wind, while the plate K will turn at a speed proportional to the east-westerly component. Each plate will, of course, reverse its direction of rotation when the

direction of the wind is reversed. All that remains, therefore, is to cause each disc to record its rotation by making it draw a pen along a rotating cylinder.

The geometrical proof of the statement of the preceding paragraph is a simple one. We suppose ω, the angular velocity of the sphere, to be equal to k times V, the velocity of the wind, where k is assumed for the present to be a constant. Let r be the radius of a disc measured to its point of contact with the sphere, and d the distance of the point of contact below the centre of the sphere. Also suppose the axis A′A to make an angle θ with the west-easterly direction. Then if we direct our attention to the point where the sphere touches the bevelled edge of the disc, it will be clear that, whatever be the motion of that point of the sphere, no part of that motion will cause the disc to turn except that which is horizontal and tangential to the edge of the disc. The other components of the motion will merely give rise to a slide along the slope of the bevel. Now the total horizontal component of the motion of either point of contact is obviously ωd or $k\mathrm{V}d$, and this will be broken up into a component $k\mathrm{V}d\cos\theta$ tending to make only the east disc turn, and a component $k\mathrm{V}d\sin\theta$ tending to make only the north disc turn. But $\mathrm{V}\cos\theta$ and $\mathrm{V}\sin\theta$ are the easterly and northerly components of the wind. Hence the speeds of rotation of the discs are proportional to them. The speeds of rotation are actually $\dfrac{k\mathrm{V}d\cos\theta}{r}$ and $\dfrac{k\mathrm{V}d\sin\theta}{r}$.

3. Details of Construction.

The instrument may be regarded as consisting of two sections: the first, that which gives to the sphere the necessary rotation; the second, that whereby the discs actuate the recording pens. In designing both portions, but especially the former, the object has been to be able to calculate the speed of the wind from that of the sphere, on the basis of the geometrical connections, after making a small allowance for "slip." This allowance would, of course, be determined by experiment. Pains have therefore been taken to diminish friction as far as may be.

Section 1.—A wind-wheel consisting of four carefully made light screw-blades projecting outwards from a common horizontal axis is kept with its axis facing the wind by being mounted above a wind-vane in much the usual manner. Screw-blades of 2 metres pitch, 20 centimetres length from centre outwards, and 10 centimetres depth from front to back

appear likely to be suitable (see fig. 2). The horizontal axis passes back into a covered vessel B and there rests on ball-bearings at C and G, and has the thrust of the wind taken by a hard-steel end D. At E the axis carries a small "lantern" or toothed-wheel gearing which engages with the horizontal toothed wheel F. F is therefore made to rotate round its vertical axis by the spinning of the wind-wheel. The hollow vessel B is at the top of, and is rigidly attached to, the hollow vertical axis of the wind-vane, which is prolonged upwards above the vane. Through this hollow axis of the vane the vertical axis of the toothed wheel passes right down till both axes enter the room in which the registering apparatus is. Here the lower end of the axis of F drives a spur-wheel gearing which causes the sphere to turn.

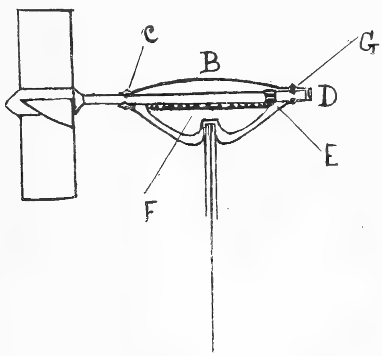


Fig. 2.

It is of importance to diminish, as far as may be, any frictional resistance which may hinder either the spinning of the wind-wheel or the setting of the vane. Fluid friction is no great disadvantage, as it may be taken to be proportional to the speed for the small speeds involved; and it can be used to damp the oscillations to which a wind-vane is liable even in steady winds.

To diminish the frictional resistance to the spinning of the toothed wheel F and its axis, the bottom of the vessel B is hollowed out into the shape of a surface of revolution, which contains oil; while the wheel F bulges downwards into the oil and is buoyed up by it. The vertical axis of F need be only a fairly stout wire. The whole weight of F and its axis can easily be supported in this way.

In the case of the axis of the vane, the following device is employed. To give due exposure to the vane and wheel it is proposed to mount the

whole on a vertical iron pipe L (fig. 3), of about 10 cm. bore, firmly fixed to the roof and rising at least $1\frac{1}{2}$ metres above it. The upper end of this iron pipe is closed except for the tubular opening M, which transmits the hollow axis of the vane and which forms a bearing for the

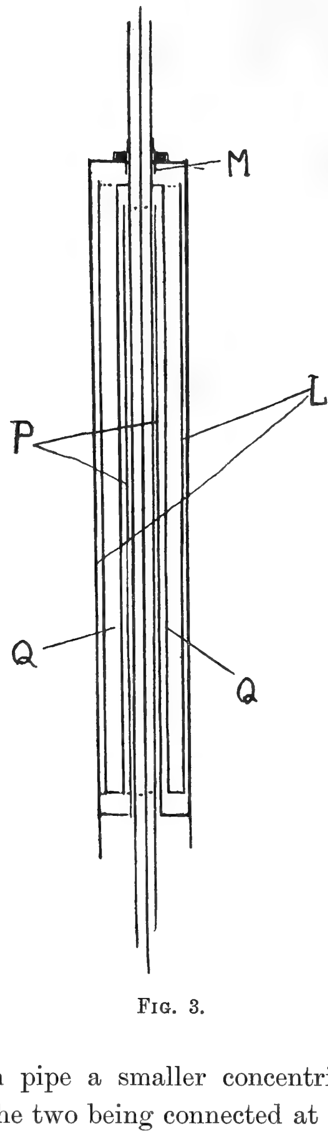


Fig. 3.

latter. Inside the iron pipe a smaller concentric pipe P is fixed for a considerable distance, the two being connected at the bottom of the latter so that the space between can be filled with water. To the vane axis a concentric double-walled tubular float Q is attached which dips in the water so that the weight of the vane and all that it carries may as far as possible be supported thereby.

Fig. 4 shows the arrangement by which the sphere is driven and its axis caused to veer so as to be always parallel to the wind. The vertical rectangular framework R R is continuous with the hollow axis S of the vane, and therefore turns with it; and its plane is the plane of the vane. T is the lower end of the axis of the toothed wheel that is driven by the wind-wheel. Its connection with the sphere is by a flexible part and spur-wheel gearing capable of giving either of two speeds to the sphere. The axis of the sphere must be capable of accurate adjustment so that it shall be truly level, and so that the centre of the sphere is exactly in the prolongation of the vertical axis of the wind-vane.

Section 2.—The second part of the instrument is that by which the

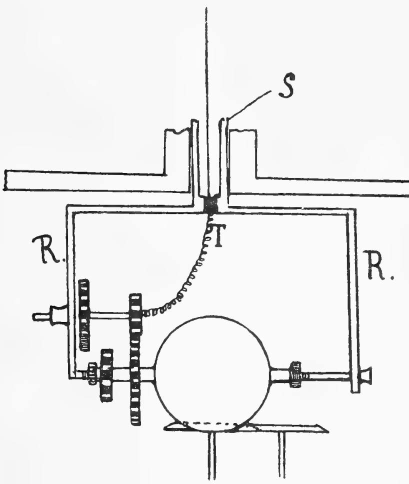


Fig. 4.

horizontal discs in the course of their rotation actuate the pens that make the final record on the uniformly turning recording cylinders.

Each disc is mounted on a thin vertical axis, which is rather more than supported at its lower end by a spring, the excess of the pressure of the spring over the weight of the disc and axis being used to keep the bevelled edge of the disc in gentle contact with the rotating sphere. The axis of the disc has on it a "lantern" or toothed-wheel gearing which drives a train of clockwork such as that of a cheap clock. The final or most slowly moving axis of the clockwork causes the pen to move over the recording cylinder.

Continuous winds from one direction will, of course, carry the pens to the ends of the cylinders. By a simple device, however, it is easy to contrive that they shall, when this happens, be released so as to return to zero, and there be once more caught by the moving clockwork.

It is hardly necessary to enter in detail into the methods proposed for the driving of the pens. Either of two methods is feasible. In the first, the pen is carried round with the final axis to a maximum angular distance of 30° from its zero position ere it is released. In this case the ordinates of wind-flux on the recording cylinder will be approximately arcs of a circle. In the other method, the pen is dragged longitudinally along the cylinder by a thread which is kept taut by weights, and which at one part of its length passes round a pulley driven by the clockwork of the instrument.

Experience will be the best guide as to the scale on which the angular motion of the wind-wheels should be reduced to give the angular motion of the pen. At present, I purpose giving release to the pen every time it has recorded a total of 120 kilometres of wind. With a pitch of 2 metres for the wind-wheel and a maximum angular motion for the pen of 30°, this would give a reduction ratio of 720,000 : 1.

B. A Portable Barometer on a New Principle.

The object aimed at in designing the instrument described below is to be able to effect in the field a determination of the barometric pressure free from the grave uncertainty which attaches to the readings of most aneroids, and having an accuracy not far short of that of an ordinary mercurial barometer. As the author has only recently arrived in Britain after a year of travel, he has been unable to have the instrument constructed so as to show it to the Society.

The instrument is essentially a constant-pressure air thermometer, alongside of which is placed a Six's thermometer for the purpose of enabling the user to bring the air contained in the air thermometer to constant pressure at any temperature. It will be most easily described by reference to the diagram (fig. 5).

The air thermometer consists of an inverted bulb A, of special construction; a descending tube B, the lower portion of which contains mercury; a cistern C, containing mercury, and of capacity adjustable by a screw D; and an ascending tube E, up which the mercury rises to a height depending on the pressure of the external atmosphere, E being open to the air at its upper end. A scale F, graduated downwards, slides alongside of the tubes D and E. If the air in the thermometer is always to be brought to the standard pressure of (say) 80 cms., then the lower end F of the scale would be marked 80, and the readings upwards would be 79, 78, etc. If now the air in the bulb be brought to 80 cm. pressure by the adjusting screw of the cistern, and if the lower end of the scale be brought to the top of the mercury in

the inner tube B, then the reading of the scale opposite the top of the mercury in the outer tube E will be directly the atmospheric pressure.

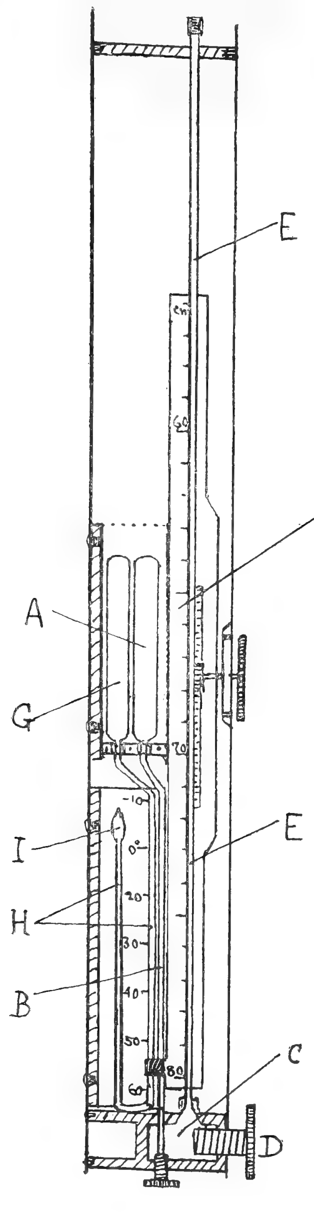


Fig. 5.

It remains, therefore, to provide a means of knowing when the inner air is brought to the standard pressure of 80 cms. This is effected by the

Six's thermometer. It consists, as usual, of an inverted bulb G, filled with an organic liquid (in this case having very uniform expansion); a U-tube H, the bend of which contains mercury; and the bulb I, partly filled with liquid. When the temperature rises, the mercury in the inner limb of the U-tube is depressed by the expansion of the liquid in the bulb G. Now it is possible, by proper adjustment of the bulbs A and G and of the stems B and H, to make the expansion of the liquid along H equal to that of the air along B, under the constant pressure of 80 cms., for equal rises of temperature; and not only so, but the level of the mercury in H can be made the same as that of the mercury in B at 0° C. When these adjustments are made once by the instrument-maker, then all that is needed at any field temperature to bring the air in A to the required standard pressure is to work the cistern screw till the mercury in the stem B comes to precisely the same height as the mercury in the inner tube of the Six's thermometer. The mark 80 on the scale is then brought to this same point, and then the atmospheric pressure can be read off directly by looking at the top of the mercury in E.

To facilitate the accurate adjustment of the menisci in H and B to the same level, I propose to attach to the scale a small blackened plate with sharp, straight lower edge, exactly on the level of the 80-cm. mark. This plate lies behind the stems B and H, which have otherwise a transparent background. It will first be brought down to touch the meniscus of the Six's therometer, the scale being thus at the same time put in proper position; the mercury in B is then brought up to its lower edge; and finally the reading of the atmospheric pressure is taken. In instruments where high accuracy is aimed at, a small reading microscope travelling up and down along the front of the instrument will be provided, to be used both for the accurate adjustment of the mercury in B and for the accurate reading of the mercury in E.

It is absolutely necessary that the instrument should always retain the air in the bulb A and stem B in whatever position it may be carried. To effect this a short part of the bore of B at its lower end is made capillary, and the end of the stem is ground flat and can be closed by a small screw working from below. This closing must always be done when the instrument is to be carried. The upper end of E also has a small screw-cap, and an arrangement is proposed for the removal from the cistern C of any air which may lodge in it from the stem E.

The preliminary adjustment of the bulb A so that the air thermometer and Six's thermometer may have degrees of exactly equal length is the most difficult point in connection with the instrument. But it is to be noted that after the first rough adjustment to equality is made, a very

delicate adjustment can be effected by sliding the Six's thermometer a little up or down, thereby altering the zero point on the air thermometer and the standard pressure under which the instrument is issued. It is of no importance what the exact value of that pressure shall be.

The other arrangements are simply for the purpose of mounting the various parts in rigidly fixed positions, for proper illumination, and for protection from damage in travel, and need not be described in detail. An instrument that will read from 31 inches to 22 inches pressure between 10° and 120° Fahr. will be about 15 inches in total length.

It need hardly be pointed out that the Six's thermometer can be used in the usual way as a maximum and minimum thermometer when a station is occupied for twenty-four hours. The two bulbs are enclosed in a thin metal case polished outside and blackened inside.

It is proposed to call the instrument a thermobarometer.

PS., January 24, 1910.—The usefulness of the barometer in field practice will be determined very largely by its portability; and it is realised that the Six's thermometer, and probably also the air thermometer, may require to be modified in the light of actual experience.

There is a superficial resemblance between this instrument and the sympiezometer invented by Adie and described by Dr Buchan in the article "Barometer" in the ninth edition of the *Encyclopædia Britannica.* Both take advantage of the expansion and contraction of air under the combined influence of changing pressure and temperature. By means of an attached thermometer and sliding scale, Adie applies a correction for temperature, and thereby deduces the barometric pressure. In the instrument described in this paper the mass of air is brought to a definite pressure, and the mercury column, which partly supports this pressure, at once gives by its length the acting pressure of the atmosphere. The novelty in the construction of the instrument is the device for securing that the air has been brought to the chosen definite pressure. In the principle of its action as well as in the arrangement for carrying this principle into effect the "thermobarometer" differs fundamentally from the sympiezometer.

(*Issued separately May* 7, 1910.)

XXIV.—The Theory of Bigradients in the Historical Order of Development up to 1860. By Thomas Muir, LL.D.

(MS. received January 22, 1910. Read February 7, 1910.)

As we have already pointed out (*Hist.*, i. p. 487), bigradients were first brought to light by Sylvester in 1840 in the paper in which he made known his so-called "dialytic" method of eliminating the unknown from two equations of the same or different degrees. Shortly afterwards Richelot and Cauchy recalled attention to Euler's and Bezout's method of 1764, as giving substantially the same result as Sylvester's, the fact being that the determinant obtained by Sylvester differs from that obtainable in the other case merely by being its conjugate. The details of these papers and of others related to them have already been given.

Cayley, A. (1844).

[Note sur deux formules données par MM. Eisenstein et Hesse. *Crelle's Journ.*, xxix. pp. 54–57; or *Collected Math. Papers,* i. pp. 113–116.]

Although Eisenstein's property * of the discriminant

$$a^2d^2 - 3b^2c^2 + 4ac^3 + 4b^3d - 6abcd\,, \text{ or } \Delta \text{ say,}$$

of the binary cubic $ax^3+3bx^2y+3cxy^2+dy^3$,—namely, the property that

$$A^2D^2 - 3B^2C^2 + 4AC^3 + 4B^3D - 6ABCD \;=\; (a^2d^2 - 3b^2c^2 + 4ac^3 + 4b^3d - 6abcd)^3$$

when

$$A, B, C, D \;=\; -\tfrac{1}{2}\frac{\partial\Delta}{\partial d}\,,\; \tfrac{1}{6}\frac{\partial\Delta}{\partial c}\,,\; -\tfrac{1}{6}\frac{\partial\Delta}{\partial b}\,,\; \tfrac{1}{2}\frac{\partial\Delta}{\partial a}\,,$$

—can be expressed in the form

$$\begin{vmatrix} A & 2B & C & . \\ . & A & 2B & C \\ B & 2C & D & . \\ . & B & 2C & D \end{vmatrix} = \begin{vmatrix} a & 2b & c & . \\ . & a & 2b & c \\ b & 2c & d & . \\ . & b & 2c & d \end{vmatrix}^3,$$

it is not as a relation between two four-line determinants that it has been studied.

Cayley in effect says that if we wish to find substitutes A, B, C, . . . for $a, b, c, \ldots$ so that

$$\phi(A, B, C, \ldots) \;=\; \{\phi(a, b, c, \ldots)\}^p\,,$$

* *Crelle's Journ.*, xxvii. pp. 105–106, 319–321.

we must (1) find a quantic u of which $\phi(a, b, c, \ldots)$ is an invariant; (2) express $\phi(a, b, c, \ldots)$ as a determinant of the same number of lines as u has facients; and (3) transform u into U by a linear substitution of which the said determinant is the modulus.

The coefficients of U will then be the substitutes required. For example, Δ being an invariant of the binary cubic and being expressible in the form

$$\begin{vmatrix} bc-ad & 2(c^2-bd) \\ 2(b^2-ac) & bc-ad \end{vmatrix},$$

we should have to transform the said cubic by the substitution

$$\left.\begin{array}{rcl} x &=& (bc-ad)\xi + 2(c^2-bd)\eta \\ y &=& 2(b^2-ac)\xi + (bc-ad)\eta \end{array}\right\},$$

and the discriminant of the new cubic thus obtained being Δ multiplied by a power of the modulus must be a power of Δ. Unfortunately, in this case it would be Δ^7, whereas in Eisenstein's case the power-index is 3. Instead of the binary cubic, therefore, Cayley takes the binary trilinear

$$ax_1y_1z_1 + bx_1y_1z_2 + cx_1y_2z_1 + dx_1y_2z_2 + ex_2y_1z_1 + fx_2y_1z_2 + gx_2y_2z_1 + hx_2y_2z_2,$$

of which a generalisation of Δ, namely,

$$\left.\begin{array}{r} a^2h^2 + b^2g^2 + c^2f^2 + d^2e^2 + 4adfg + 4bceh \\ -2ahbg - 2ahcf - 2ahde - 2bgcf - 2bgde - 2cfde \end{array}\right\} \text{ or Q say,}$$

is an invariant; expresses Q as a two-line determinant

$$\begin{vmatrix} ah-bg-cf+de & -2(eh-fg) \\ -2(ad-bc) & ah-bg-cf+de \end{vmatrix};$$

makes the substitution

$$\left.\begin{array}{rcl} x_1 &=& (ah-bg-cf+de)\xi_1 - 2(eh-fg)\xi_2 \\ y_1 &=& -2(ad-bc)\xi_1 + (ah-bg-cf+de)\xi_2 \end{array}\right\};$$

and, as the multiplier connecting the new Q and the old is now the *second* power of the modulus, he obtains what was wanted.*

The substitutes found turn out to be

$$\tfrac{1}{2}\frac{\partial Q}{\partial a}, \quad \tfrac{1}{2}\frac{\partial Q}{\partial b}, \quad \tfrac{1}{2}\frac{\partial Q}{\partial c}, \quad \ldots .$$

but no explanation of this is vouchsafed.

Eisenstein's case is the degeneration made by putting

$$\begin{array}{rl} & a,\ b,\ c,\ d,\ e,\ f,\ g,\ h \\ = & a,\ b,\ b,\ c,\ b,\ c,\ c,\ d\,. \end{array}$$

* This short paper of Cayley's teems with misprints, both in the original and in the *Collected Math. Papers.*

Had the two other sets of variables been at the same time transformed with the same determinant for modulus, we should have had the new Q equal to Q^7.

HEILERMANN, [H.] (1845).

[Ueber die Verwandlung der Reihen in Kettenbrüche. *Crelle's Journ.*, xxxiii. pp. 174–188.]

The determinant which here appears for the first time is different from but resembles those to which Sylvester's dialytic method of elimination leads, being exemplified for the 4th and 5th orders by

$$\begin{vmatrix} a_3 & a_2 & b_2 & b_3 \\ a_2 & a_1 & b_1 & b_2 \\ a_1 & a_0 & b_0 & b_1 \\ a_0 & . & . & b_0 \end{vmatrix}, \quad \begin{vmatrix} a_4 & a_3 & a_2 & b_3 & b_4 \\ a_3 & a_2 & a_1 & b_2 & b_3 \\ a_2 & a_1 & a_0 & b_1 & b_2 \\ a_1 & a_0 & . & b_0 & b_1 \\ a_0 & . & . & . & b_0 \end{vmatrix}.$$

Calling these Δ_3, Δ_4 Heilermann writes his main result in the form

$$\frac{a_0 + a_1x + \ldots + a_nx^n}{b_0 + b_1x + \ldots + b_mx^m} = \frac{\Delta_0}{b_0} - \cfrac{\Delta_1 x}{\Delta_0 - \cfrac{\Delta_2 x}{\Delta_1 - \cfrac{\Delta_0\Delta_3 x}{\Delta_2 - \cfrac{\Delta_1\Delta_4 x}{\Delta_3 - \ldots}}}} \ldots$$

the ending on the right being

$$-\frac{\Delta_{2n-3}\Delta_{2n}x}{\Delta_{2n-1}} \quad \text{or} \quad -\frac{\Delta_{2m-4}\Delta_{2m-1}}{\Delta_{2m-2}}$$

according as $m <$ or $> n$.

CAYLEY, A. (1848, August).

[Nouvelles recherches sur les fonctions de M. Sturm. *Journ.* (*de Liouville*) *de Math.*, xiii. pp. 269–274; or *Collected Math. Papers*, i. pp. 392–396.]

Recalling his former paper on the same subject in *Liouville's Journal*, xi. (1846), pp. 297–299, where Sturm's functions had been expressed in terms of sums of powers of the roots of the original function, he intimates now the discovery of more simple expressions in terms of the *coefficients* of the said function. At the same time he draws attention to the fact that his result may be viewed as unconnected with Sturm's division-process, and it is in this general light that he prefers to state it. Beginning with two functions V and V′ of the n^{th} degree, namely,

$$ax^n + bx^{n-1} + \ldots, \qquad a'x^n + b'x^{n-1} + \ldots.$$

and forming therefrom the series of functions

$$\begin{vmatrix} V & V' \\ a & a' \end{vmatrix}, \quad \begin{vmatrix} xV & V & xV' & V' \\ a & . & a' & . \\ b & a & b' & a' \\ c & b & c' & b' \end{vmatrix}, \quad \begin{vmatrix} x^2V & xV & V & x^2V' & xV' & V' \\ a & . & . & a' & . & . \\ b & a & . & b' & a' & . \\ c & b & a & c' & b' & a' \\ d & c & b & d' & c' & b' \\ e & d & c & e' & d' & c' \end{vmatrix},$$

etc., which he denotes by $-F_1$, F_2, $-F_3$, ... , he affirms that there is a homogeneous linear relation connecting every consecutive three of the latter functions,* namely,

$$\begin{aligned} P_1{}^2F_3 \quad &+ \quad (xP_1P_2 + P_1P'_2 + P'_1P_2)F_2 \quad + \quad P_2{}^2F_1 = 0\,, \\ P_2{}^2F_4 \quad &+ \quad (xP_2P_3 + P_2P'_3 + P'_2P_3)F_3 \quad + \quad P_3{}^2F_2 = 0\,, \\ &\cdots\cdots\cdots\cdots\cdots\cdots \end{aligned}$$

where by P_1, P_2, P_3, ... are meant the determinants

$$\begin{vmatrix} a & a' \\ b & b' \end{vmatrix}, \quad \begin{vmatrix} a & . & a' & . \\ b & a & b' & a' \\ c & b & c' & b' \\ d & c & d' & c' \end{vmatrix}, \quad \begin{vmatrix} a & . & . & a' & . & . \\ b & a & . & b' & a' & . \\ c & b & a & c' & b' & a' \\ d & c & b & d' & c' & b' \\ e & d & c & e' & d' & c' \\ f & e & d & f' & e' & d' \end{vmatrix}, \ldots .$$

and by P'_1, P'_2, P'_3, ... the determinants got from P_1, P_2, P_3, ... by altering the last rows into

$$c\ c'\,; \quad e\ d\ e'\ d'\,; \quad g\ f\ e\ g'\ f'\ e'\,; \quad \ldots .$$

No proof is given of the relations; indeed, after pointing out that they involve the proposition that the first and last of three consecutive functions are of opposite sign for every value of x that makes the intermediate function vanish, Cayley adds: "Je n'ai pas encore réussi à démontrer dans toute la généralité l'équation identique d'où dépend cette propriété."

The case which brings him into closer contact with Sturm, namely, where V' is the differential-quotient of V, is dealt with in some detail.

HEILERMANN, [H.] (1852, December).

[Independente Berechnung der Sturm'schen Reste. *Crelle's Journ.*, xlviii. pp. 190–206.]

The subject of this paper is of course closely connected with that of the author's previous work (1845). Like Cayley, he begins with two functions that are unrelated, and subsequently passes to the special case where the

* The signs require verification.

one is the derivate of the other; but, unlike Cayley, he makes the said functions of different degrees, namely,

$$c_{00}x^n + c_{10}x^{n-1} + c_{20}x^{n-2} + \ldots\ldots + c_{n0}$$
$$c_{01}x^{n-1} + c_{11}x^{n-2} + c_{21}x^{n-3} + \ldots\ldots + c_{n-1,1}.$$

Following the ordinary division-process for expressing the ratio of the second function to the first as a continued fraction of the form

$$\frac{p_0}{x+}\frac{p_1}{1+}\frac{p_2}{x+}\frac{p_3}{1+}\cdots$$

and denoting the remainders in order by

$$\frac{1}{c_{01}}\left\{c_{02}x^{n-1} + c_{12}x^{n-2} + c_{22}x^{n-3} + \ldots\right\},$$
$$\frac{1}{c_{02}}\left\{c_{03}x^{n-2} + c_{13}x^{n-3} + c_{23}x^{n-4} + \ldots\right\},$$
$$\frac{1}{c_{01}c_{03}}\left\{c_{04}x^{n-3} + c_{14}x^{n-4} + c_{24}x^{n-5} + \ldots\right\},$$
$$\cdots\cdots\cdots\cdots\cdots$$

he finds that

$$p_0 = \frac{c_{01}}{c_{00}}, \quad p_{r+1} = \frac{c_{0,r+2}}{c_{0,r}c_{0,r+1}}, \quad p_{2n-1} = \frac{c_{1,2n-2}}{c_{0,2n-2}}.$$

The second suffix of any one of the new c's is seen to indicate the remainder-function to which the c belongs, and the first suffix the position of the c in that remainder. To obtain expressions for these in terms of the original two sets of c's it is taken for granted, and with reason, that as a result of the process we have generally

$$c_{r,s} = c_{0,s-1}c_{r+1,s-2} - c_{0,s-2}c_{r+1,s-1} = \begin{vmatrix} c_{0,s-1} & c_{0,s-2} \\ c_{r+1,s-1} & c_{r+1,s-2} \end{vmatrix}. \qquad (1)$$

By using this twice upon itself, so as to lower the second suffixes of the first column, there is found

$$c_{r,s} = \begin{vmatrix} c_{0,s-2} & c_{0,s-3} & \cdot \\ c_{1,s-2} & c_{1,s-3} & c_{0,s-2} \\ c_{r+2,s-2} & c_{r+2,s-3} & c_{r+1,s-2} \end{vmatrix}, \qquad (2)$$

where the second suffixes are now $s-2$, $s-3$. A page is then occupied in ridding (2) in the same way of the elements which have $s-2$ for a suffix, the result being

$$c_{r,s} = c_{0,s-3}\begin{vmatrix} c_{0,s-3} & c_{0,s-4} & \cdot & \cdot \\ c_{1,s-3} & c_{1,s-4} & c_{0,s-3} & c_{0,s-4} \\ c_{2,s-3} & c_{2,s-4} & c_{1\ s-3} & c_{1,s-4} \\ c_{r+3,s-3} & c_{r+3,s-4} & c_{r+2,s-3} & c_{r+2,s-4} \end{vmatrix}. \qquad (3)$$

With increasing tediousness a five-line determinant is reached having elements with $s-4$ and $s-5$ for second suffixes, a six-line determinant having elements with $s-5$ and $s-6$ for second suffixes, and so on. The form of the determinant of the $(2q+2)^{\text{th}}$ order is thus deduced, the factor preceding it being said to be

$$(c_{0,s-3}c_{0,s-4})(c_{0,s-5}c_{0,s-6})^2(c_{0,s-7}c_{0,s-8})^3\ldots(c_{0,s-2q+1}c_{0,s-2q})^{q-1}(c_{0,s-2q-1})^q.$$

Sturm's division-process, in which each remainder is of a lower degree than the remainder preceding it, and for which, therefore, the corresponding continued fraction has each partial denominator a linear function of x, has close relationship with the above division-process, because the continued fraction already obtained is identical with *

$$\frac{p_0}{x+p_1-}\,\frac{p_1p_2}{x+p_2+p_3-}\,\frac{p_3p_4}{x+p_4+p_5-}\cdots$$

Expressions for the remainders corresponding to this continued fraction are thus readily obtainable, and a section (§ 3) is devoted to finding simplified substitutes for them. The remaining section concerns the strictly Sturmian case, where one of the original functions is the derivate of the other.

Bruno, Faà di (1855, July).

[Sulle funzioni simmetriche delle radici di un' equazione. *Annali di sci. mat. e fis.*, vi. pp. 412–419.]

As evidence of the value of a certain theorem Bruno adduces the ease with which the expansion of the resultant of a pair of equations may be calculated, and he prints at full length the resultants $R_{2,2}$, $R_{3,3}$, $R_{4,4}$, that is to say, the final expansions of

$$\begin{vmatrix} \cdot & a & b & c \\ a & b & c & \cdot \\ \cdot & p & q & r \\ p & q & r & \cdot \end{vmatrix}, \text{ etc.,}$$

the arrangement of the terms being such as to make evident the fact that each resultant is unaltered by reversing the order of the two sets of coefficients of which it is a function: for example :— †

$$R_{2,2} = (a^2r^2+c^2p^2)-(abqr+bcpq)-2acpr+acq^2+b^2pr.$$

* This identity Heilerman published again separately in 1860 (see *Zeitschrift f. Math. u. Phys.*, v. pp. 262–263). It is included, however, in a result given by Stern in 1883 (see *Crelle's Journ.*, x. p. 156); and a still more general identity will be found in the *Proc. Edin. Math. Soc.*, xxiii. p. 37.

† The expression for $R_{4,4}$ is full of inaccuracies.

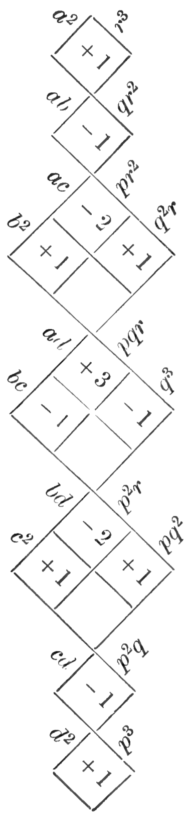
CAYLEY, A. (1856, December).

[Memoir on the resultant of a system of two equations. *Philos. Trans. R. Soc.* (London), cxlvii. pp. 703–715; or *Collected Math. Papers*, ii. pp. 440–453.]

As the resultant, $R_{3,2}$ say, of the pair of equations

$$ax^3 + bx^2y + cxy^2 + dy^3 = 0, \qquad px^2 + qxy + ry^2 = 0,$$

is homogeneous and of the 3rd degree in the coefficients of the second equation, and at the same time homogeneous and of the 2nd degree in the coefficients of the first equation, Cayley seeks a convenient form of representation in which this double homogeneity will be prominent. What he obtains is *



where the first square represents a^2r^3, the second $-abqr^2$, the third $-2acpr^2+b^2pr+acq^2r$, and so on. The result is reached in two ways, the second being by developing the dialytic eliminant

* Three misprints being corrected.

$$\left|\begin{array}{ccccc} \cdot & a & b & c & d \\ a & b & c & d & \cdot \\ \cdot & \cdot & p & q & r \\ \cdot & p & q & r & \cdot \\ p & q & r & \cdot & \cdot \end{array}\right| .$$

The two-line minors of the first two rows of this determinant being denoted by 12, 13, . . . , 45, and the three-line minors of the remaining rows by 123, 124, . . . , 345, it is seen that

$$\left.\begin{array}{rl} R_{3,2} = & 12\cdot345 - 13\cdot245 + 14\cdot235 - 15\cdot234 \\ & \qquad + 23\cdot145 - 24\cdot135 + 25\cdot134 \\ & \qquad\qquad + 34\cdot125 - 35\cdot124 \\ & \qquad\qquad\qquad - 45\cdot123 \end{array}\right\}$$

and it will be found that

$$12\cdot345, \quad -13\cdot245, \quad 14\cdot235 + 23\cdot145, \quad -(15\cdot234 + 24\cdot135), \ldots$$

correspond to the 1st, 2nd, 3rd, 4th, . . . squares of Cayley's expression.

The paper closes with the six resultants $R_{2,2}$, $R_{3,2}$, $R_{4,2}$, $R_{3,3}$, $R_{4,3}$, $R_{4,4}$ printed each in the new form as a chain of squares;* they occupy four quarto pages.

Zeipel, V. v. (1858, June).

[Demonstration of a theorem of Mr Cayley's in relation to Sturm's functions. *Quart. Journ. of Math.*, iii. pp. 108–117, or *Nouv. Annales de Math.*, xix. pp. 220–224.]

The theorem referred to is that of August 1848. Zeipel's proof (pp. 108–114) is lengthy and unattractive, and scarcely warrants reproduction. The remaining pages are occupied with the curious identities

$$\begin{vmatrix} P_{r-1} & P_r & \cdot \\ P'_{r-1} & P'_r & P_r \\ P''_{r-1} & P''_r & P'_r \end{vmatrix} = -P^2_{r-1}\mathbf{P}_{r+1}, \qquad \begin{vmatrix} P_{r-1} & P_r & \cdot \\ P'_{r-1} & P'_r & P_r \\ P'''_{r-1} & P'''_r & P''_r \end{vmatrix} = -P^2_{r-1}P'_{r+1},$$

and the corresponding identities in which the left-hand members are of a higher odd order than the third.

Hesse, O. (1858, October).

[Il determinante di Sylvester ed il risultante di Eulero. *Annali di Mat.* . . . ii. pp. 5–8; or *Werke*, 475–480.]

The determinant referred to is that to which Sylvester was led in 1840 by his so-called "dialytic" method, and which, as we have already seen,

* In the expression for $R_{4,4}$ there is at least one misprint, namely, a^2c^2 for a^2b^2 outside the third square of the chain.

Hesse himself arrived at in 1843; and the resultant coupled with it is Euler's of 1748, which takes the form of a product of differences of the roots of the two given equations. Both forms, as well as others, are treated of by Cauchy in his paper of 1840, already dealt with.

The equations being

$$\left.\begin{array}{rl} a_3x^3 + a_2x^2 + a_1x^1 + a_0 &= 0 \\ b_2x^2 + b_1x^1 + b_0 &= 0 \end{array}\right\} \text{ or say } \left\{\begin{array}{rl} \phi(x) &= 0 \\ \psi(x) &= 0, \end{array}\right.$$

Hesse multiplies Sylvester's eliminant by $s_0s_2 - s_1^2$, the squared difference-product of the roots β_1, β_2 of the second equation, with the result

$$\begin{vmatrix} a_0 & a_1 & a_2 & a_3 & \cdot \\ \cdot & a_0 & a_1 & a_2 & a_3 \\ b_0 & b_1 & b_2 & \cdot & \cdot \\ \cdot & b_0 & b_1 & b_2 & \cdot \\ \cdot & \cdot & b_0 & b_1 & b_2 \end{vmatrix} \cdot \begin{vmatrix} s_0 & s_1 & s_2 & s_3 & s_4 \\ s_1 & s_2 & s_3 & s_4 & s_5 \\ \cdot & \cdot & 1 & \cdot & \cdot \\ \cdot & \cdot & \cdot & 1 & \cdot \\ \cdot & \cdot & \cdot & \cdot & 1 \end{vmatrix}$$

$$= \begin{vmatrix} a_0s_0 + a_1s_1 + \ldots + a_3s_3 & a_0s_1 + a_1s_2 + \ldots + a_3s_4 & a_2 & a_3 & \cdot \\ a_0s_1 + a_1s_2 + \ldots + a_3s_4 & a_0s_2 + a_1s_3 + \ldots + a_3s_5 & a_1 & a_2 & a_3 \\ b_0s_0 + b_1s_1 + b_2s_2 & b_0s_1 + b_1s_2 + b_2s_3 & b_2 & \cdot & \cdot \\ b_0s_1 + b_1s_2 + b_2s_3 & b_0s_2 + b_1s_3 + b_2s_4 & b_1 & b_2 & \cdot \\ b_0s_2 + b_1s_3 + b_2s_4 & b_0s_3 + b_1s_4 + b_2s_5 & b_0 & b_1 & b_2 \end{vmatrix},$$

where $s_p = \beta_1^p + \beta_2^p$, and where, therefore, the elements in the places 31, 32, 41, 42, 51, 52 of the right-hand member all vanish. There is thus obtained, if we denote Sylvester's eliminant by S,

$$S \cdot \begin{vmatrix} s_0 & s_1 \\ s_1 & s_2 \end{vmatrix} = b_2^3 \begin{vmatrix} a_0s_0 + a_1s_1 + \ldots + a_3s_3 & a_0s_1 + a_1s_2 + \ldots + a_3s_4 \\ a_0s_1 + a_1s_2 + \ldots + a_3s_4 & a_0s_2 + a_1s_3 + \ldots + a_3s_5 \end{vmatrix}.$$

But the determinant on the right is resolvable into

$$\begin{vmatrix} 1 & 1 \\ \beta_1 & \beta_2 \end{vmatrix} \cdot \begin{vmatrix} a_0 + a_1\beta_1 + a_2\beta_1^2 + a_3\beta_1^3 & a_0 + a_1\beta_2 + a_2\beta_2^2 + a_3\beta_2^3 \\ a_0\beta_1 + a_1\beta_1^2 + a_2\beta_1^3 + a_3\beta_1^4 & a_0\beta_2 + a_1\beta_2^2 + a_2\beta_2^3 + a_3\beta_2^4 \end{vmatrix},$$

that is, into

$$\begin{vmatrix} 1 & 1 \\ \beta_1 & \beta_2 \end{vmatrix} \cdot \begin{vmatrix} \phi(\beta_1) & \phi(\beta_2) \\ \beta_1\phi(\beta_1) & \beta_2\phi(\beta_2) \end{vmatrix}$$

and therefore into

$$\begin{vmatrix} 1 & 1 \\ \beta_1 & \beta_2 \end{vmatrix}^2 \cdot \phi(\beta_1) \cdot \phi(\beta_2).$$

We thus have

$$S = b_2^3\phi(\beta_1)\phi(\beta_2),$$

and, consequently, if a_1, a_2, a_3 be the roots of the first given equation,

$$\begin{aligned} S = b_2^3a_3^2(\beta_1 - \alpha_1)(\beta_1 - \alpha_2)(\beta_1 - \alpha_3) \\ (\beta_2 - \alpha_1)(\beta_2 - \alpha_2)(\beta_2 - \alpha_3), \end{aligned}$$

which is what was to be shown, the co-factor of $b_2^3a_3^2$ being Euler's product of differences.

Bruno, Faà di (1859).

[Théorie Générale de l'Elimination. Par le Chevalier François Faà di Bruno. . . . x+224 pp. Paris.]

In his section (pp. 32–40) dealing with the dialytic eliminant, Bruno, besides reprinting $R_{3,3}$, $R_{4,4}$, gives the full expansion of the resultant of

$$\left.\begin{array}{l} ax^3 + 3bx^2y + 3cxy^2 + dy^3 \;=\; 0 \\ bx^3 + 3cx^2y + 3dxy^2 + ey^3 \;=\; 0 \end{array}\right\}$$

and of the resultant of

$$\left.\begin{array}{l} ax^4 + 4bx^3y + 6cx^2y^2 + 4dxy^3 + ey^4 \;=\; 0 \\ bx^4 + 4cx^3y + 6dx^2y^2 + 4exy^3 + fy^4 \;=\; 0 \end{array}\right\}$$

—that is to say, the full expansion of the discriminant of the equation

$$ax^4 + 4bx^3 + 6cx^2 + 4dx + e \;=\; 0$$

and of the discriminant (with at least seven mistakes) of

$$ax^5 + 5bx^4 + 10cx^3 + 10dx^2 + 5ex + f \;=\; 0\,.$$

In the next section (pp. 40–46) he seeks to improve on what we have called Cayley's "chain of squares" by combining the last square with the first, the second from the end with the second from the beginning, and so on. For example, his expression for $R_{3,3}$, that is to say, for

$$(a^3s^3 - d^3p^3) + (-a^2brs^2 + cd^2p^2q) + \{2(-a^2cqs^2 + bd^2p^2r) + \ldots\} + \ldots$$

is

$$\begin{array}{c|c|} & \begin{smallmatrix} s^3 \\ d^3 \end{smallmatrix} \\ \hline \begin{smallmatrix} a^3 \\ p^3 \end{smallmatrix} & \pm 1 \\ \hline \end{array} + \begin{array}{c|c|} & \begin{smallmatrix} rs^2 \\ cd^2 \end{smallmatrix} \\ \hline \begin{smallmatrix} a^2b \\ p^2q \end{smallmatrix} & \mp 1 \\ \hline \end{array} + \begin{array}{c|c|c|} & \begin{smallmatrix} qs^2 \\ bd^2 \end{smallmatrix} & \begin{smallmatrix} r^2s \\ c^2d \end{smallmatrix} \\ \hline \begin{smallmatrix} a^2c \\ p^2r \end{smallmatrix} & \mp 2 & \pm 1 \\ \hline \begin{smallmatrix} ab^2 \\ pq^2 \end{smallmatrix} & \pm 1 & \\ \hline \end{array} + \begin{array}{c|c|c|c|} & \begin{smallmatrix} ps^2 \\ ad^2 \end{smallmatrix} & \begin{smallmatrix} qrs \\ bcd \end{smallmatrix} & \begin{smallmatrix} r^3 \\ c^3 \end{smallmatrix} \\ \hline \begin{smallmatrix} a^2d \\ p^2s \end{smallmatrix} & \mp 3 & \pm 3 & \mp 1 \\ \hline \begin{smallmatrix} abc \\ pqr \end{smallmatrix} & \pm 3 & \mp 1 & \\ \hline \begin{smallmatrix} b^3 \\ q^3 \end{smallmatrix} & \mp 1 & & \\ \hline \end{array} + \begin{array}{c|c|c|c|} & \begin{smallmatrix} prs \\ acd \end{smallmatrix} & \begin{smallmatrix} q^2s \\ b^2d \end{smallmatrix} & \begin{smallmatrix} qr^2 \\ bc^2 \end{smallmatrix} \\ \hline \begin{smallmatrix} abd \\ pqs \end{smallmatrix} & \mp 1 & \mp 2 & \pm 1 \\ \hline \begin{smallmatrix} ac^2 \\ pr^2 \end{smallmatrix} & \mp 2 & \pm 1 & \\ \hline \begin{smallmatrix} b^2c \\ q^2r \end{smallmatrix} & \pm 1 & & \\ \hline \end{array}\,,$$

a marked improvement on which would be

$$\begin{array}{c|c|} & \begin{smallmatrix} s^3 \\ p^3 \end{smallmatrix} \\ \hline \begin{smallmatrix} a^3 \\ d^3 \end{smallmatrix} & \pm 1 \\ \hline \end{array} + \begin{array}{c|c|} & \begin{smallmatrix} rs^2 \\ p^2q \end{smallmatrix} \\ \hline \begin{smallmatrix} a^2b \\ cd^2 \end{smallmatrix} & \mp 1 \\ \hline \end{array} + \ldots\,.$$

the second term in each binomial being derived from the first term by the change of a, b, c, d, p, q, r, s, into d, c, b, a, s, r, q, p,—that is to say, by the interchange

$$\begin{pmatrix} a & b & p & q \\ d & c & s & r \end{pmatrix}.$$

Further, he improves upon Cayley's squares by so transposing their rows,

where necessary, as to bring about axisymmetry.* Lastly, he tries (pp. 43–46) to justify Cayley's rule for calculating the coefficients placed inside any square of the chain.

Borchardt, C. W. (1859, November).

[Vergleichung zweier Formen der Eliminations-Resultante. *Crelle's Journ.*, lvii. pp. 183–186; or *Gesammelte Werke*, pp. 145–150.]

The problem here is exactly the same as Hesse's of the previous year. Instead, however, of multiplying S by $\zeta(\beta_1,\beta_2)$, he preferably multiplies $\zeta^{\frac12}(\beta_1, \beta_2, \alpha_1, \alpha_2, \alpha_3)$ by S, thus obtaining

$$\begin{vmatrix} 1 & \beta_1 & \beta_1^2 & \beta_1^3 & \beta_1^4 \\ 1 & \beta_2 & \beta_2^2 & \beta_2^3 & \beta_2^4 \\ 1 & \alpha_1 & \alpha_1^2 & \alpha_1^3 & \alpha_1^4 \\ 1 & \alpha_2 & \alpha_2^2 & \alpha_2^3 & \alpha_2^4 \\ 1 & \alpha_3 & \alpha_3^2 & \alpha_3^3 & \alpha_3^4 \end{vmatrix} \cdot \begin{vmatrix} a_0 & a_1 & a_2 & a_3 & \cdot \\ \cdot & a_0 & a_1 & a_2 & a_3 \\ b_0 & b_1 & b_2 & \cdot & \cdot \\ \cdot & b_0 & b_1 & b_2 & \cdot \\ \cdot & \cdot & b_0 & b_1 & b_2 \end{vmatrix} = \begin{vmatrix} \phi(\beta_1) & \beta_1\phi(\beta_1) & \cdot & \cdot & \cdot \\ \phi(\beta_2) & \beta_2\phi(\beta_2) & \cdot & \cdot & \cdot \\ \cdot & \cdot & \psi(\alpha_1) & \alpha_1\psi(\alpha_1) & \alpha_1^2\psi(\alpha_1) \\ \cdot & \cdot & \psi(\alpha_2) & \alpha_2\psi(\alpha_2) & \alpha_2^2\psi(\alpha_2) \\ \cdot & \cdot & \psi(\alpha_3) & \alpha_3\psi(\alpha_3) & \alpha_3^2\psi(\alpha_3) \end{vmatrix}$$

Now, E being Euler's product of differences, the first determinant on the left is resolvable into

$$\zeta^{\frac12}(\beta_1,\beta_2) \cdot \zeta^{\frac12}(\alpha_1,\alpha_2,\alpha_3) \cdot \mathrm{E},$$

as was first observed by Rosenhain in 1845 (Sept.); and the determinant on the right is resolvable into

$$\left\{ \zeta^{\frac12}(\beta_1,\beta_2) \cdot \phi(\beta_1) \cdot \phi(\beta_2) \right\} \left\{ \zeta^{\frac12}(\alpha_1,\alpha_2,\alpha_3) \cdot \psi(\alpha_1) \cdot \psi(\alpha_2) \cdot \psi(\alpha_3) \right\}.$$

We thus have

$$\begin{aligned} \mathrm{ES} &= \left\{ \phi(\beta_1) \cdot \phi(\beta_2) \right\} \left\{ \psi(\alpha_1) \cdot \psi(\alpha_2) \cdot \psi(\alpha_3) \right\}, \\ &= a_3{}^2\mathrm{E} \cdot b_2{}^3\mathrm{E}, \end{aligned}$$

and ∴

$$\mathrm{S} = a_3{}^2 b_2{}^3\mathrm{E}, \text{ as before.}$$

* The expression for $R_{4,4}$, though now given more accurately than before, is still disfigured by at least ten misprints.

LIST OF AUTHORS

whose writings are herein dealt with.

(*Issued separately May* 7, 1910.)

XXV.—The Theory of Persymmetric Determinants in the Historical Order of Development up to 1860. By Thomas Muir, LL.D.

(MS. received January 22, 1910. Read February 7, 1910.)

As has already been pointed out (*Hist.*, i. pp. 485–487 *), the special form of determinant named "persymmetric" in 1853 by Sylvester came first to light in 1835 in a paper of Jacobi's on the elimination of the unknown from two equations of the n^{th} degree, the fact being that the adjugate of Bezout's condensed eliminant—in other words, the adjugate of the determinant resulting from Bezout's "abridged method" of elimination—is there shown to be such that the elements of it whose place-numbers have the same sum are equal.

Rosenhain, G. (1844).

[Exercitationes analyticæ in theorema Abelianum de integralibus functionum algebraicarum. *Crelle's Journ.*, xxviii. pp. 249–278.]

What concerns our subject here is a digression (§§ 5–10, pp. 263–278) on the elimination of the unknown from two equations of the n^{th} degree. The first two sections (pp. 263–268) are little else than a reproduction of part of Jacobi's paper of 1835 dealing with Bezout's so-called "abridged method," and the remainder contains a discussion of other methods. In subject, therefore, the digression resembles Cauchy's paper of 1840.

At this point we have to recall the fact already reported, † that in Borchardt's paper of 1845 (January) a determinant of the special form we are now considering appeared as an expression for the square of the difference-product, and that a generalisation of this result was given by Cayley the year following.

Jacobi, C. G. J. (1845, August).

[Ueber die Darstellung einer Reihe gegebner Werthe durch eine gebrochne rationale Function. *Crelle's Journ.*, xxx. pp. 127–156: or *Gesammelte Werke*, iii. pp. 479–511.]

The subject here dealt with by Jacobi is that first considered by Cauchy in the fifth note to the *Analyse Algébrique* of 1821, namely, the extension

* The 7th and 8th lines of p. 486 have unfortunately been transposed by the printer. Also, in the first determinant of the footnote on the same page the first b_1 should be b_0.

† *Proc. Roy. Soc. Edin.*, xxvi. p. 362, pp. 364–366.

of Lagrange's interpolation-formula, or the finding of a function u of the form $N(x)/M(x)$ which shall have the values $u_1, u_2, \ldots, u_{n+m+1}$ when x has the values $x_1, x_2, \ldots, x_{n+m+1}$, it being understood that N and M are respectively of the n^{th} and m^{th} degrees in x.

The given $n+m+1$ equations

$$u_1 M(x_1) = N(x_1), \quad u_2 M(x_2) = N(x_2), \quad \ldots\ldots$$

are first used to eliminate the $n+1$ coefficients of $N(x)$, and thereby obtain m equations for the determination of the ratios of the coefficients of $M(x)$. This is interestingly accomplished by using the multipliers $x_1^p/f'(x_1)$, $x_2^p/f'(x_2)$ $\ldots\ldots$, where $f(x)=(x-x_1)(x-x_2)\ldots(x-x_{n+m+1})$, then performing addition, and finally utilising a known theorem regarding "partial fractions." The result is that for any one value of p we have

$$\sum_{i=1}^{i=n+m+1} \frac{u_i x_i^p M(x_i)}{f'(x_i)} = \sum_{i=1}^{i=n+m+1} \frac{x_i^p N(x_i)}{f'(x_i)} ;$$

and that therefore when p has any one of the values $0, 1, 2, \ldots, m-1$ we have

$$\sum_{i=1}^{i=n+m+1} \frac{u_i x_i^p M(x_i)}{f'(x_i)} = 0 .$$

By putting

$$v_p \quad \text{for} \quad \frac{x_1^p u_1}{f'(x_1)} + \frac{x_2^p u_2}{f'(x_2)} + \ldots + \frac{x_{n+m+1}^p u_{n+m+1}}{f'(x_{n+m+1})}$$

and

$$a + a_1 x + a_2 x^2 + \ldots + a_m x^m \quad \text{for} \quad M(x)$$

these last m equations become

$$\left.\begin{array}{l} v_0 a + v_1 a_1 + v_2 a_2 + \ldots + v_m a_m = 0 \\ v_1 a + v_2 a_1 + v_3 a_2 + \ldots + v_{m+1} a_m = 0 \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ v_{m-1} a + v_m a_1 + v_{m+1} a_2 + \ldots + v_{2m-1} a_m = 0 \end{array}\right\}$$

whence for $M(x)$ there is obtained the expression *

* By making the observation that the v's are neatly expressible as determinants the whole matter may be put much more simply. Thus, taking the case where $u=(\beta_0+\beta_1 x)/(a_0+a_1 x+a_2 x^2)$, we see at a glance that

$$\begin{vmatrix} 1 & x_1 & x_1^2 & x_1^p(\beta_0+\beta_1 x_1) \\ 1 & x_2 & x_2^2 & x_2^p(\beta_0+\beta_1 x_2) \\ 1 & x_3 & x_3^2 & x_3^p(\beta_0+\beta_1 x_3) \\ 1 & x_4 & x_4^2 & x_4^p(\beta_0+\beta_1 x_4) \end{vmatrix} = 0 \text{ when } p = 0 \text{ or } 1,$$

[*continued on next page.*

$$\begin{vmatrix} 1 & x & x^2 & \cdots & x^m \\ v_0 & v_1 & v_2 & \cdots & v_m \\ v_1 & v_2 & v_3 & \cdots & v_{m+1} \\ \cdot & \cdot & \cdot & \cdots & \cdot \\ v_{m-1} & v_m & v_{m+1} & \cdots & v_{2m-1} \end{vmatrix},$$

or, by further putting $w = v_{p+1} - xv_p$,

$$\begin{vmatrix} w_0 & w_1 & \cdots & w_{m-1} \\ w_1 & w_2 & \cdots & w_m \\ \cdot & \cdot & \cdots & \cdot \\ w_{m-1} & w_m & \cdots & w_{2m-2} \end{vmatrix}.$$

After finding other forms for $M(x)$, and varying (§ 2) the mode of finding them, Jacobi proceeds (§ 3, pp. 140–146) to deal with $N(x)$, first remarking, of course, that the one function is immediately determinable from the other, because the problem of representing $u_1, u_2, \ldots$ by $N(x)/M(x)$ is the same as the problem of representing $u_1^{-1}, u_2^{-1}, \ldots$ by $M(x)/N(x)$. Instead of utilising this, however, he takes from the theory of "partial fractions" the result

$$-\frac{N(x)}{f(x)} = \sum_{i=1}^{i=n+m+1} \frac{N(x_i)}{(x_i - x)f'(x_i)},$$

whence follows

$$-\frac{N(x)}{f(x)} = \sum_{i=1}^{i=n+m+1} \frac{u_i M(x_i)}{(x_i - x)f'(x_i)};$$

---

and therefore from the data that

$$\begin{vmatrix} 1 & x_1 & x_1^2 & u_1x_1^p(\alpha_0+\alpha_1x_1+\alpha_2x_1^2) \\ 1 & x_2 & x_2^2 & u_2x_2^p(\alpha_0+\alpha_1x_2+\alpha_2x_2^2) \\ 1 & x_3 & x_3^2 & u_3x_3^p(\alpha_0+\alpha_1x_3+\alpha_2x_3^2) \\ 1 & x_4 & x_4^2 & u_4x_4^p(\alpha_0+\alpha_1x_4+\alpha_2x_4^2) \end{vmatrix} = 0 \text{ when } p = 0 \text{ or } 1,$$

or, what is the same thing,

$$\begin{vmatrix} 1 & x_1 & x_1^2 & u_1x_1^p \\ 1 & x_2 & x_2^2 & u_2x_2^p \\ 1 & x_3 & x_3^2 & u_3x_3^p \\ 1 & x_4 & x_4^2 & u_4x_4^p \end{vmatrix}\alpha_0 + \begin{vmatrix} 1 & x_1 & x_1^2 & u_1x_1^{p+1} \\ 1 & x_2 & x_2^2 & u_2x_2^{p+1} \\ 1 & x_3 & x_3^2 & u_3x_3^{p+1} \\ 1 & x_4 & x_4^2 & u_4x_4^{p+1} \end{vmatrix}\alpha_1 + \begin{vmatrix} 1 & x_1 & x_1^2 & u_1x_1^{p+2} \\ 1 & x_2 & x_2^2 & u_2x_2^{p+2} \\ 1 & x_3 & x_3^2 & u_3x_3^{p+2} \\ 1 & x_4 & x_4^2 & u_4x_4^{p+2} \end{vmatrix}\alpha_2 = 0.$$

From these two equations on solving for $\alpha_0 : \alpha_1 : \alpha_2$ and substituting in $\alpha_0+\alpha_1x+\alpha_2x^2$ we obtain

$$M(x) = \begin{vmatrix} 1 & x & x^2 \\ v_0 & v_1 & v_2 \\ v_1 & v_2 & v_3 \end{vmatrix}$$

where $v^p = | x_1^0 \, x_2^1 \, x_3^2 \, x_4^p u_4 |$, or

$$M(x) = \begin{vmatrix} \omega_0 & \omega_1 \\ \omega_1 & \omega_2 \end{vmatrix}$$

where $\omega_p = v_{p+1} - xv_p = | x_1^0 \, x_2^1 \, x_3^2 \, x_4^p u_4(x_4 - x) |$.

so that if we put

$$\mathrm{R}_p \quad \text{for} \quad \sum_{i=1}^{i=n+m+1} \frac{x_i^p u_1}{(x_i - x)f'(x_i)},$$

we have

$$-\frac{\mathrm{N}(x)}{f(x)} = a\mathrm{R}_0 + a_1\mathrm{R}_1 + \ . \ . \ . \ + a_m\mathrm{R}_m ,$$

and therefore, by substituting the already found values of $a : a_1 : a_2 : \ldots : a_m$,

$$-\frac{\mathrm{N}(x)}{f(x)} = \begin{vmatrix} \mathrm{R}_0 & \mathrm{R}_1 & \ldots & \mathrm{R}_m \\ v_0 & v_1 & \ldots & v_m \\ v_1 & v_2 & \ldots & v_{m+1} \\ \cdot & \cdot & \cdot & \cdot \\ v_{m-1} & v_m & \ldots & v_{2m-1} \end{vmatrix} .$$

As, however, $x\mathrm{R}_p + v_p = \mathrm{R}_{p+1}$, we can change the elements of the second row here into $\mathrm{R}_1, \mathrm{R}_2, \ldots, \mathrm{R}_{m+1}$, and then the elements of the third row into $\mathrm{R}_2, \mathrm{R}_3, \ldots, \mathrm{R}_{m+2}$, and so on, thus arriving at a determinant of the same special form as in the case of $\mathrm{M}(x)$.

Combining the two results, Jacobi is thus led to the theorem that

$$-\frac{1}{f(x)} \cdot \frac{\mathrm{N}(x)}{\mathrm{M}(x)} = \begin{vmatrix} \mathrm{R}_0 & \mathrm{R}_1 & \ldots & \mathrm{R}_m \\ \mathrm{R}_1 & \mathrm{R}_2 & \ldots & \mathrm{R}_{m+1} \\ \mathrm{R}_2 & \mathrm{R}_3 & \ldots & \mathrm{R}_{m+2} \\ \cdot & \cdot & \cdot & \cdot \\ \mathrm{R}_m & \mathrm{R}_{m+1} & \ldots & \mathrm{R}_{2m} \end{vmatrix} \div \begin{vmatrix} w_0 & w_1 & \ldots & w_{m-1} \\ w_1 & w_2 & \ldots & w_m \\ \cdot & \cdot & \cdot & \cdot \\ w_{m-1} & w_m & \ldots & w_{2m-1} \end{vmatrix},$$

—a result not easily verifiable by giving x one of its $n+m+1$ values.*

* Continuing the case of the previous footnote we should prefer to begin with

$$\frac{\mathrm{N}(x)}{f(x)} | x_1^0 x_2^1 x_3^2 x_4^3 | = \begin{vmatrix} 1 & x_1 & x_1^2 & \mathrm{N}(x_1)/(x-x_1) \\ 1 & x_2 & x_2^2 & \mathrm{N}(x_2)/(x-x_2) \\ 1 & x_3 & x_3^2 & \mathrm{N}(x_3)/(x-x_3) \\ 1 & x_4 & x_4^2 & \mathrm{N}(x_4)/(x-x_4) \end{vmatrix},$$

and then proceeding exactly as before we should arrive at

$$\frac{\mathrm{N}(x)}{f(x)} | x_1^0 x_2^1 x_3^2 x_4^3 | = \begin{vmatrix} \rho_0 & \rho_1 & \rho_2 \\ \rho_1 & \rho_2 & \rho_3 \\ \rho_2 & \rho_3 & \rho_4 \end{vmatrix},$$

$$\text{where } \rho_p = | x_1^0 \, x_2^1 \, x_3^2 \, x_4^p u_4/(x-x_4) | .$$

The function sought would then be

$$\frac{(x-x_1)(x-x_2)(x-x_3)(x-x_4) \begin{vmatrix} \rho_0 & \rho_1 & \rho_2 \\ \rho_1 & \rho_2 & \rho_3 \\ \rho_2 & \rho_3 & \rho \end{vmatrix}}{| x_1^0 x_2^1 x_3^2 x_4^3 | \cdot \begin{vmatrix} \omega_0 & \omega_1 \\ \omega_1 & \omega_2 \end{vmatrix}}$$

For ourselves we may add that the theorem becomes still more interesting when it is pointed out that, by reason of the identity

$$| y_1^0 y_2^1 \ldots y_{n-1}^{n-2} \mathrm{Y}_n | \div | y_1^0 y_2^1 \ldots y_{n-1}^{n-2} y_n^{n-1} | = \frac{\mathrm{Y}_1}{\phi'(y_1)} + \frac{\mathrm{Y}_2}{\phi'(y_2)} + \ldots + \frac{\mathrm{Y}_n}{\phi'(y_n)}$$

where $\phi(y) = (y-y_1)(y-y_2) \ldots (y-y_n)$, the R's like the w's are all expressible as determinants of the order $n+m+1$, that the determinants in both cases belong to the special type known as alternants, and that R_p differs from w_p in the last column only;—in fact, that

$$\mathrm{R}_p = \begin{vmatrix} 1 & x_1 & x_1^2 & \ldots & x_1^{n+m-1} & x_1^p u_1/(x_1 = x) \\ 1 & x_2 & x_2^2 & \ldots & x_2^{n+m-1} & x_2^p u_2/(x_2 - x) \\ 1 & x_3 & x_3^2 & \ldots & x_3^{n+m-1} & x_3^p u_3/(x_3 - x) \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \end{vmatrix} \div \zeta^{\frac{1}{2}}$$

$$w_p = \begin{vmatrix} 1 & x_1 & x_1^2 & \ldots & x_1^{n+m-1} & x_1^p u_1(x_1 - x) \\ 1 & x_2 & x_2^2 & \ldots & x_2^{n+m-1} & x_2^p u_2(x_2 - x) \\ 1 & x_3 & x_3^2 & \ldots & x_3^{n+m-1} & x_3^p u_3(x_3 - x) \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \end{vmatrix} \div \zeta^{\frac{1}{2}}$$

where $\zeta^{\frac{1}{2}}$ is the difference-product of $x_1, x_2, \ldots, x_{n+m+1}$.

BORCHARDT, C. W. (1847, February).

[Développements sur l'équation à l'aide de laquelle on détermine les inégalités séculaires du mouvement des planètes. *Journ.* (*de Liouville*) *de Math.*, xii. pp. 50–67: *Gesammelte Werke*, pp. 15–30.]

The new section of this paper, which is an extension of Borchardt's of 1845 (January), is the third (pp. 54–60), and explains at length how, for the purpose of ascertaining the total number of real roots of the equation of the n^{th} degree $f(x)=0$, the coefficients of highest powers in the series of Sturm's functions $f(x), f_1(x), f_2(x), \ldots$ may be replaced, according to Sylvester, by

$$1, \quad n, \quad \sum (x_2-x_1)^2, \quad \sum (x_2-x_1)^2(x_3-x_1)^2(x_2-x_1)^2, \quad \ldots .$$

where $x_1, x_2, \ldots$ are the roots, and therefore by

$$1, \quad s_0, \quad \begin{vmatrix} s_0 & s_1 \\ s_1 & s_2 \end{vmatrix}, \quad \begin{vmatrix} s_0 & s_1 & s_2 \\ s_1 & s_2 & s_3 \\ s_2 & s_3 & s_4 \end{vmatrix}, \ldots .$$

where $s_r = x_1^r + x_2^r + \ldots + s_n^r$. All this, however, is practically implied in Cayley's paper of 1846 (August).*

* The proposition Borchardt is concerned with is of course that *The equation* f(x)=0 *has as many pairs of imaginary roots as there are changes of sign in any one of the three series mentioned.*

SYLVESTER, J. J. (1851, May).

[ESSAY ON CANONICAL FORMS: Supplement to a "Sketch of a Memoir on Elimination, Transformation, and Canonical Forms," 36 pp., London. Or *Collected Math. Papers*, i. pp. 203–216.]

In giving a preliminary notice of his general method for reducing odd-degreed functions to their canonical form, Sylvester says he based his method on the proposition that every one of the n-line minor determinants of the array

$$\begin{array}{ccccc} T_1 & T_2 & T_3 & \ldots & T_{n+1} \\ T_2 & T_3 & T_4 & \ldots & T_{n+2} \\ T_3 & T_4 & T_5 & \ldots & T_{n+3} \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ T_n & T_{n+1} & T_{n+2} & \ldots & T_{2n} \end{array}$$

vanishes if

$$T_i = a_1^{r-i}b_1^{s+1} + a_2^{r-i}b_2^{s+i} + \ldots + a_{n-1}^{r-i}b_{n-1}^{s+i}.$$

This, which he hastily calls "a beautiful and striking theorem," and which he generalises in Note B of an Appendix, arises from the simple fact that each determinant is the product of two zeros, T_i being

$$\left(a_1^{r-i},\ a_2^{r-i},\ \ldots,\ a_{n-1}^{r-i},\ 0 \between b_1^{s+i},\ b_2^{s+i},\ \ldots,\ b_{n-1}^{s+1},\ 0\right).$$

It is of more importance, therefore, to recall that it was in this year that Sylvester made the fruitful observation, already chronicled,* that the determinants $ac-b^2$, $ace+2bcd-ae^2-bd^2-c^3$, ... are expressible as "commutants," or rather that these special determinants could be represented in the umbral notation by using umbræ not wholly unconnected with one another. Thus, while

$$\begin{vmatrix} 0 & 1 & 2 \\ 0 & 1 & 2 \end{vmatrix} \text{ stands for the general determinant } \begin{vmatrix} 00 & 01 & 02 \\ 10 & 11 & 12 \\ 20 & 21 & 22 \end{vmatrix}$$

so long as the umbræ are understood to be entirely independent, it might also be used to stand for the special determinant

$$\begin{vmatrix} 00 & 01 & 02 \\ 01 & 02 & 03 \\ 02 & 03 & 04 \end{vmatrix}$$

if some mark were added to indicate that in the development 01 is to be put for 10, 02 for 20 or 11, 03 for 12 or 21, and 04 for 22.

* *Proc. Roy. Soc. Edin.*, xxv. pp. 939–942.

Sylvester, J. J. (1851, October).

[On a remarkable discovery in the theory of canonical forms and of hyperdeterminants. *Philos. Magazine*, ii. pp. 391–410: or *Collected Math. Papers*, i. pp. 265–283.]

The consideration of the problem of the canonisation of the binary quintic led Sylvester to the more general problem of determining the p's and q's in

$$(p_1x+q_1y)^{2n+1}+(p_2x+q_2y)^{2n+1}+\ldots.+(p_{n+1}x+q_{n+1}y)^{2n+1}$$

so as to make this expression identical with

$$a_0x^{2n+1}+(2n+1)a_1x^{2n}y^1+\tfrac{1}{2}(2n+1)2na_2x^{2n-1}y^2+\ldots.+a_{2n+1}y^{2n+1}.$$

This is at once seen to depend on the solution of the peculiar set of $2n+2$ equations

$$\left.\begin{array}{lllllll}
\pi_1 & + & \pi_2 & + \ldots. + & \pi_{n+1} & = & a_0 \\
\pi_1\lambda_1 & + & \pi_2\lambda_2 & + \ldots. + & \pi_{n+1}\lambda_{n+1} & = & a_1 \\
\pi_1\lambda_1^2 & + & \pi_2\lambda_2^2 & + \ldots. + & \pi_{n+1}\lambda_{n+1}^2 & = & a_2 \\
\cdot & \cdot & \cdot & \cdot\cdot\cdot\cdot & \cdot & \cdot & \cdot \\
\pi_1\lambda_1^{2n+1} & + & \pi_2\lambda_2^{2n+1} & + \ldots. + & \pi_{n+1}\lambda_{n+1}^{2n+1} & = & a_{2n+1}
\end{array}\right\}$$

where the new unknowns $\pi_1, \pi_2, \ldots., \pi_{n+1}, \lambda_1, \lambda_2, \ldots., \lambda_{n+1}$ are introduced merely for shortness' sake, namely

$$\pi_r \text{ for } p_r^{2n+1} \quad\text{and}\quad \lambda_r \text{ for } q_r \div p_r.$$

Taking $n+2$ consecutive equations beginning with the first, and eliminating the π's, there is obtained

$$\begin{vmatrix}
1 & 1 & \ldots. & 1 & a_0 \\
\lambda_1 & \lambda_2 & \ldots. & \lambda_{n+1} & a_1 \\
\lambda_1^2 & \lambda_2^2 & \ldots. & \lambda_{n+1}^2 & a_2 \\
\cdot & \cdot & \cdot\cdot\cdot\cdot & \cdot & \cdot \\
\lambda_1^{n+1} & \lambda_2^{n+1} & \ldots. & \lambda_{n+1}^{n+1} & a_{n+1}
\end{vmatrix} = 0,$$

which, if division by the difference-product of the λ's be effected, gives

$$a_{n+1}-a_n\sum\lambda_1+a_{n-1}\sum\lambda_1\lambda_2-\ldots. = 0.$$

A similar result is evidently reached by taking *any* $n+2$ consecutive equations, so that altogether we shall have

$$\left.\begin{array}{llllll}
a_{n+1} & - a_n\sum\lambda_1 & + a_{n-1}\sum\lambda_1\lambda_2 & - \ldots. & = & 0 \\
a_{n+2} & - a_{n+1}\sum\lambda_1 & + a_n\sum\lambda_1\lambda_2 & - \ldots. & = & 0 \\
a_{n+3} & - a_{n+2}\sum\lambda_1 & + a_{n+1}\sum\lambda_1\lambda_2 & - \ldots. & = & 0 \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
a_{2n+1} & - a_{2n}\sum\lambda_1 & + a_{2n-1}\sum\lambda_1\lambda_2 & - \ldots. & = & 0
\end{array}\right\}$$

—that is to say, a set of $n+1$ equations in the $n+1$ unknowns $\Sigma\lambda_1$, $\Sigma\lambda_1\lambda_2$, $\ldots\ldots$, $\lambda_1\lambda_2\ldots\lambda_{n+1}$, the solution of which is

$$\frac{1}{A_0} = \frac{\Sigma\lambda_1}{A_1} = \frac{\Sigma\lambda_1\lambda_2}{A_2} = \ldots\ldots$$

where A_r is the determinant whose array is got by deleting the $(r+1)^{\text{th}}$ column from the array

$$\begin{array}{ccccc} a_{n+1} & a_n & a_{n-1} & \ldots & a_0 \\ a_{n+2} & a_{n+1} & a_n & \ldots & a_1 \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ a_{2n+1} & a_{2n} & a_{2n-1} & \ldots & a_n . \end{array}$$

From this it follows that the λ's are the roots of the equation

$$A_0\lambda^{n+1} - A_1\lambda^n + A_2\lambda^{n-1} - \ldots = 0 ,$$

i.e.

$$\begin{vmatrix} \lambda^{n+1} & \lambda^n & \lambda^{n-1} & \ldots & \lambda^0 \\ a_{n+1} & a_n & a_{n-1} & \ldots & a_0 \\ a_{n+2} & a_{n+1} & a_n & \ldots & a_1 \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ a_{2n+1} & a_{2n} & a_{2n-1} & \ldots & a_n \end{vmatrix} = 0 ,$$

i.e.

$$\begin{vmatrix} a_{n+1} - a_n\lambda & a_n - a_{n-1}\lambda & \ldots & a_1 - a_0\lambda \\ a_{n+2} - a_{n+1}\lambda & a_{n+1} - a_n\lambda & \ldots & a_2 - a_1 \\ \cdot & \cdot & \cdot & \cdot \\ a_{2n+1} - a_{2n}\lambda & a_{2n} - a_{2n-1}\lambda & \ldots & a_{n+1} - a_n\lambda \end{vmatrix} = 0 .$$

On substituting in the first $n+1$ equations of the original set the values of $\lambda_1, \lambda_2, \ldots, \lambda_{n+1}$ thus found, the values of $\pi_1, \pi_2, \ldots, \pi_{n+1}$ are obtainable from a set of linear equations of the type associated with the name of Lagrange.

The latter part of this procedure is not given by Sylvester, who on reaching the set of equations in $\Sigma\lambda_1$, $\Sigma\lambda_1\lambda_2$, $\ldots$ suddenly draws the seemingly irrelevant conclusion "that

$$(x+\lambda_1 y)(x+\lambda_2 y)\ldots(x+\lambda_{n+1} y)$$

is a constant multiple of the determinant

$$\begin{vmatrix} x^{n+1} & -x^n y & x^{n+1}y^2 & \ldots \\ a_{n+1} & a_n & a_{n-1} & \ldots \\ a_{n+2} & a_{n+1} & a_n & \ldots \\ \cdot & \cdot & \cdot & \cdot \\ a_{2n+1} & a_{2n} & a_{2n-1} & \ldots \end{vmatrix}," \quad \text{or } \Delta \text{ say.}$$

As a matter of fact $(p_1x+q_1y)(p_2x+q_2y)\;\ldots\;(p_{n+1}x+q_{n+1}y)$

$$
\begin{aligned}
&= p_1p_2\ldots p_{n+1}(x+\lambda_1y)(x+\lambda_2y)\;\ldots\;(x+\lambda_{n+1}y)\,,\\
&= p_1p_2\ldots p_{n+1}(x^{n+1}+\textstyle\sum\lambda_1\,.\,x^ny+\sum\lambda_1\lambda_2\,.\,x^{n-1}y^2+\;\ldots\;)\,,\\
&= \frac{p_1p_2\ldots p_{n+1}}{\mathrm{A}_0}(\mathrm{A}_0x^{n+1}+\mathrm{A}_1x^ny+\mathrm{A}_2x^{n-1}y^2+\;\ldots\;)\,,\\
&= \frac{p_1p_2\ldots p_{n+1}}{\mathrm{A}_0}\,.\,\Delta\,,\\
&= \frac{p_1p_2\ldots p_{n+1}}{\mathrm{A}_0}\begin{vmatrix} a_{n+1}y+a_nx & a_ny+a_{n-1}x & \ldots \\ a_{n+2}y+a_{n+1}x & a_{n+1}y+a_nx & \ldots \\ \ldots & \ldots & \ldots \\ a_{2n+1}y+a_{2n}x & a_{2n}y+a_{2n-1}x & \ldots \end{vmatrix},
\end{aligned}
$$

from which we see (1) the point which Sylvester wished to make, namely, that $p_1x+q_1y\,,\,p_2x+q_2y\,,\;\ldots\;$ being viewed as the original unknowns, it is important to know that their values are multiples of the linear factors of Δ, and (2) that $\Delta=\mathrm{A}_0(x+\lambda_1y)(x+\lambda_2y)\;\ldots\;.$

Of course the conclusion drawn is that the transformation of a binary $(2n+1)$-ic into the sum of $n+1$ powers depends on the solution of a determinantal equation of the $(n+1)^{\text{th}}$ degree. As examples, the quintic and septimic are taken, the latter mainly for the purpose of drawing attention to the fact that the conditions of "catalecticism," that is, of $(a\,,b\,,\ldots,h\,\between\,x\,,\,y)^7$ being expressible in the form of the sum of three seventh powers—instead of four, as the general rule provides—require that the cofactors of the elements of the first row of the determinant

$$
\begin{vmatrix} y^4 & -y^3x & y^2x^2 & -yx^3 & x^4 \\ a & b & c & d & e \\ b & c & d & e & f \\ c & d & e & f & g \\ d & e & f & g & h \end{vmatrix}
$$

must all vanish, or, what by the homaloidal law is the same thing, that *two* of them vanish.

The analogous problem for even-degreed functions is next taken up, a beginning being made with the transformation of the quartic $(a\,,b\,,\ldots,\,e\,\between\,x\,,\,y)^4$ into the form

$$(p_1x+q_1y)^4+(p_2x+q_2y)^4+6\epsilon(p_1x+q_1y)^2(p_2x+q_2y)^2\,.$$

On putting

$$q_1\;=\;p_1\lambda_1\,,\quad p_2\;=\;q_2\lambda_2\,,\quad \epsilon p_1{}^2p_2{}^2\;=\;\mu\,,\quad \lambda_1+\lambda_2\;=\;s_1\,,\quad \lambda_1\lambda_2\;=\;s_2$$

there is obtained by equatement of like powers of x and y

$$\left.\begin{aligned}
a &= p_1{}^4 + p_2{}^4 + 6\mu\,,\\
b &= p_1{}^4\lambda_1 + p_2{}^4\lambda_2 + 3\mu s_1\,,\\
c &= p_1{}^4\lambda_1{}^2 + p_2{}^4\lambda_2{}^2 + \mu s_1{}^2 + 2\mu s_2\,,\\
d &= p_1{}^4\lambda_1{}^3 + p_2{}^4\lambda_2{}^3 + 3\mu s_1 s_2\,,\\
e &= p_1{}^4\lambda_1{}^4 + p_2{}^4\lambda_2{}^4 + 6\mu s_2{}^2\,,
\end{aligned}\right\}$$

and from these by operations which lead to the elimination of $p_1{}^4, p_2{}^4$ from every consecutive triad of equations

$$\left.\begin{aligned}
as_2 - bs_1 + c - \mu(8s_2 - 2s_1{}^2) &= 0\,,\\
bs_2 - cs_1 + d - \mu(4s_2 - s_1{}^2)s_1 &= 0\,,\\
cs_2 - ds_1 + e - \mu(8s_2 - 2s_1{}^2)s_2 &= 0\,,
\end{aligned}\right\}$$

or, if we put ν for $-\mu(8s_2 - 2s_1{}^2)$,

$$\left.\begin{aligned}
as_2 - bs_1 + (c+\nu) &= 0\\
bs_2 - (c - \tfrac{1}{2}\nu)s_1 + d &= 0\\
(c+\nu)s_2 - ds_1 + e &= 0
\end{aligned}\right\}.$$

From the resulting cubic equation

$$\begin{vmatrix} a & b & c+\nu \\ b & c-\tfrac{1}{2}\nu & d \\ c+\nu & d & e \end{vmatrix} = 0$$

ν can be determined, and thence in backward order s_1, s_2; μ; λ_1, λ_2; p_1, p_2; q_1, q_2, m.

In passing, note is taken of the fact that the said cubic when arranged according to powers of ν is

$$\nu^3 - (ae - 4bd + 3c^2)\nu + 2\begin{vmatrix} a & b & c \\ b & c & d \\ c & d & e \end{vmatrix} = 0$$

and that $ae - 4bd + 3c^2$ and the determinant here appearing are the two invariants * of the quartic under investigation.

The reduction of the octavic $(a_0, a_1, \ldots, a_8 \between x, y)^8$ to the form

$$u_1{}^8 + u_2{}^8 + u_3{}^8 + u_4{}^8 + 70\epsilon u_1{}^2u_2{}^2u_3{}^2u_4{}^2$$

where $u_r = p_r x + q_r y$ is shown in similar fashion to depend on the solution of the quintic equation

$$\begin{vmatrix}
a_0 & a_1 & a_2 & a_3 & a_4 - \nu \\
a_1 & a_2 & a_3 & a_4 + \tfrac{1}{4}\nu & a_5 \\
a_2 & a_3 & a_4 - \tfrac{1}{6}\nu & a_5 & a_6 \\
a_3 & a_4 + \tfrac{1}{4}\nu & a_5 & a_6 & a_7 \\
a_4 - \nu & a_5 & a_6 & a_7 & a_8
\end{vmatrix} = 0\,,$$

* The term "invariant" is first used in this paper.

where $\nu = 72\epsilon p_1^2 p_2^2 p_3^2 p_4^2 \mathrm{I}$ and I is the quadratic invariant $s_4 - \frac{1}{4}s_1 s_3 + \frac{1}{12}s_2^2$ of

$$x^4 + s_1 x^3 y + s_2 x^2 y^2 + s_3 x y^3 + s_4 y^4 \quad \text{or} \quad (x+\lambda_1 y)(x+\lambda_2 y)(x+\lambda_3 y)(x+\lambda_4 y).$$

The fact that the coefficients of ν^3, ν^2, ν^1, ν^0 are invariants of the octavic is insisted on, and generalisations effected for functions of the degree $4m$ and the degree $4m+2$.

Further, it is pointed out that when the said even-degreed functions after transformation are without the last (or unique) term,—that is to say, are in Sylvester's phraseology "meio-catalectic,"—the last of the series of invariants must vanish: for example, the condition that $(a_0, a_1, \ldots, a_6 \between x, y)^6$ may be expressible as the sum of three sixth powers is

$$\begin{vmatrix} a_0 & a_1 & a_2 & a_3 \\ a_1 & a_2 & a_3 & a_4 \\ a_2 & a_3 & a_4 & a_5 \\ a_3 & a_4 & a_5 & a_6 \end{vmatrix} = 0.$$

This, of course, may be proved independently, but is seen to be a conclusion from putting $\epsilon = 0$ in the foregoing.

SYLVESTER, J. J. (1852, April).

[On the principles of the calculus of forms. *Cambridge and Dub. Math. Journ.*, vii. pp. 62–97, 179–217 : or *Collected Math. Papers*, i. pp. 284–327, 328–363.]

Here the same subjects and the same special determinants are dealt with as in the preceding; and the determinant whose vanishing has been seen to be the condition for "meio-catalecticism" is denominated (p. 62) the *catalecticant* * of the even-degreed function in question, while the determinant whose resolution into linear factors furnishes Sylvester's canonical form of an odd-degreed function is called the *canonizant* † of the said function. As the former is an invariant of its function, so the latter is a covariant.

BRUNO, FAÀ DE (1852, May).

[Démonstration d'un théorème relatif à la réduction des fonctions homogènes à deux lettres à leur forme canonique. *Journ. (de Liouville) de Math.* . . . xvii. pp. 193–201.]

The subject of the whole of this paper is simply the solution of the set of equations dealt with in Sylvester's paper of 1851 (October). The process

* "Meicatalecticizant," Sylvester truly says, would have been the more correct word, but even he took alarm sometimes.

† The name would have been equally appropriate for the determinants of the preceding paper which have ν in their diagonal.

is lengthy and uninviting, the sole point of interest being that the equation in λ comes out in the form

$$\begin{vmatrix} a_0\lambda - a_1 & a_1\lambda - a_2 & \dots & a_n\lambda - a_{n+1} \\ a_0\lambda^2 - a_2 & a_1\lambda^2 - a_3 & \dots & a_n\lambda^2 - a_{n+2} \\ \cdot & \cdot & \cdot & \cdot \\ a_0\lambda^{n+1} - a_{n+1} & a_1\lambda^{n+1} - a_{n+2} & \dots & a_n\lambda^{n+1} - a_{2n+1} \end{vmatrix} = 0,$$

where the determinant is easily shown to be the same as one of Sylvester's forms by diminishing each row in order, beginning with the last, by λ times the row immediately preceding.

Chio, F. (1853, June).

[Mémoire sur les fonctions connues sous le nom de résultantes ou de déterminans. 32 pp., Turin.]

The second part (pp. 23–32) of Chio's memoir, which is headed "Exemples," mainly concerns Sylvester's set of equations of 1851 (October). His procedure is much more interesting than Faà de Bruno's. Using any multipliers $A_0, A_1, \dots$ with the first $n+2$ equations he obtains by addition

$$\begin{aligned} & x_0(A_0 + A_1\lambda_0 + A_2\lambda_0{}^2 + \dots + A_{n+1}\lambda_0^{n+1}) \\ + & x_1(A_0 + A_1\lambda_1 + A_2\lambda_1{}^2 + \dots + A_{n+1}\lambda_1^{n+1}) \\ + & \dots \\ + & x_n(A_0 + A_1\lambda_n + A_2\lambda_n{}^2 + \dots + A_{n+1}\lambda_n^{n+1}) = A_0a_0 + A_1a_1 + \dots + A_{n+1}a_{n+1}; \end{aligned}$$

and, the ratios of $A_0, A_1, \dots$ being supposed to be determined so as to make the coefficients of $x_0, x_1, \dots, x_n$ vanish, there results

$$A_0a_0 + A_1a_1 + \dots + A_{n+1}a_{n+1} = 0.$$

If each succeeding set of $n+2$ consecutive equations be treated in the same manner, the multipliers $A_0, A_1, \dots$ now being supposed to be partially determined, it follows that

$$\begin{aligned} A_0a_1 + A_1a_2 + \dots + A_{n+1}a_{n+2} &= 0, \\ A_0a_2 + A_1a_3 + \dots + A_{n+1}a_{n+3} &= 0, \\ \dots & \\ A_0a_n + A_1a_{n+1} + \dots + A_{n+1}a_{2n+1} &= 0. \end{aligned}$$

This derived set of $n+1$ equations suffices to give the values of the ratios of $A_0, A_1, \dots, A_{n+1}$ in terms of the a's, and the substitution of the said values in

$$A_0 + A_1\lambda + A_2\lambda^2 + \dots + A_{n+1}\lambda^{n+1} = 0$$

gives the equation for the determination of the λ's.

It is not noted by the author that having $n+2$ equations linear and homogeneous in the A's he could at once deduce

$$\begin{vmatrix} 1 & \lambda & \lambda^2 & \dots & \lambda^{n+} \\ a_0 & a_1 & a_2 & \dots & a_{n+1} \\ a_1 & a_2 & a_3 & \dots & a_{n+2} \\ \cdot & \cdot & \cdot & \cdots & \cdot \\ a_n & a_{n+1} & a_{n+2} & \dots & a_{2n+1} \end{vmatrix} = 0 .$$

The other forms of the equation, however, he gives full attention to.

Sylvester, J. J. (1853, June).

[On a theory of the syzygetic relations of two rational integral functions, ... *Philos. Trans. Roy. Soc., Lond.*, cxliii. pp. 407–548: or *Collected Math. Papers*, i. pp. 429–586.]

When dealing in art. 7 with Bezout's condensed eliminant of two equations of the n^{th} degree, Sylvester illustrates by the case of $n=5$, that is to say, where the equations are

$$\left.\begin{aligned} a_0x^5+a_1x^4+a_2x^3+a_3x^2+a_4x+a_5 &= 0 \\ b_0x^5+b_1x^4+b_2x^3+b_3x^2+b_4x+b_5 &= 0 \end{aligned}\right\},$$

pointing out that the eliminant may be constructed by first forming the array

$$\begin{matrix} |a_0b_1| & |a_0b_2| & |a_0b_3| & |a_0b_4| & |a_0b_5| \\ |a_0b_2| & |a_0b_3| & |a_0b_4| & |a_0b_5| & |a_1b_5| \\ |a_0b_3| & |a_0b_4| & |a_0b_5| & |a_1b_5| & |a_2b_5| \\ |a_0b_4| & |a_0b_5| & |a_1b_5| & |a_2b_5| & |a_3b_5| \\ |a_0b_5| & |a_1b_5| & |a_2b_5| & |a_3b_5| & |a_4b_5| \end{matrix}$$

and then, as it were, superposing the array

$$\begin{matrix} |a_1b_2| & |a_1b_3| & |a_1b_4| \\ |a_1b_3| & |a_1b_4| & |a_2b_4| \\ |a_1b_4| & |a_2b_4| & |a_3b_4| \end{matrix}$$

and next the array

$$|a_2b_3| .$$

In regard to these arrays he says in a footnote (p. 424), "A square arrangement having this kind of symmetry, namely, such as obtains in the so-called Pythagorean addition-table as distinguished from that which obtains in the multiplication-table, may be universally called *persymmetric*." This is apparently the first use of the word.

SPOTTISWOODE, W. (1853, August).

[Elementary theorems relating to determinants. Second edition, ... *Crelle's Journ.*, li. pp. 209–271, 328–381.]

Just as Spottiswoode viewed an axisymmetric determinant as the determinant of an n-ary quadric, so he closely associated a persymmetric determinant with an even-ordered binary quantic. Taking, for example, the binary quartic which Cayley would a year later have denoted by $(a_0, a_1, \ldots, a_4 \between x, y)^4$, namely,

$$a_0x^4 + 4a_1x^3y + 6a_2x^2y^2 + 4a_3xy^3 + a_4y^4 ,$$

Spottiswoode writes it in the form *

$$\begin{aligned} &(a_0x^2 + 2a_1xy + a_2y^2)x^2 \\ &+ 2(a_1x^2 + 2a_2xy + a_3y^2)xy \\ &+ \ (a_2x^2 + 2a_3xy + a_4y^2)y^2 ; \end{aligned}$$

and calling it U points out that

$$\left.\begin{aligned} \frac{\partial^2 U}{\partial x^2} &= 12(a_0x^2 + 2a_1xy + a_2y^2) \\ \frac{\partial^2 U}{\partial x\,\partial y} &= 12(a_1x^2 + 2a_2xy + a_3y^2) \\ \frac{\partial^2 U}{\partial y^2} &= 12(a_2x^2 + 2a_3xy + a_4y^2) \end{aligned}\right\} ,$$

and thus like Sylvester concludes that the evanescence of

$$\begin{vmatrix} a_0 & a_1 & a_2 \\ a_1 & a_2 & a_3 \\ a_2 & a_3 & a_4 \end{vmatrix}$$

is the condition that the second differential-quotients of U shall simul-

* A preferable form, because making the "catalecticant" still more prominent, is

$$\begin{array}{ccc|c} x^2 & 2xy & y^2 & \\ \hline a_0 & a_1 & a_2 & x^2 \\ a_1 & a_2 & a_3 & 2xy \\ a_2 & a_3 & a_4 & y^2. \end{array}$$

Similarly an odd-degreed function may be represented so as to bring the "canonizant" into prominence: for example $(a, b, \ldots, f \between x, y)^5$ may be written $(ax+by,\ bx+cy, \ldots \ldots, ex+fy \between x, y)^4$, or

$$\begin{array}{ccc|c} x^2 & 2xy & y^2 & \\ \hline ax+by & bx+cy & cx+dy & x^2 \\ bx+cy & cx+dy & dx+ey & 2xy \\ cx+dy & dx+ey & ex+fy & y^2. \end{array}$$

taneously vanish, or, say, that we shall have

$$\frac{\partial^2 U}{\partial(x,y)^2} = 0 .$$

BRIOSCHI, FR. (1854, March).

[LA TEORICA DEI DETERMINANTI, e le sue principali applicazioni; viii+116 pp., Pavia. French translation by Combescure, ix+216 pp., Paris, 1856. German translation by Schellbach, vii+102 pp., Berlin, 1856.]

Denoting by s_r the sum of the r^{th} powers of the roots of the equation

$$a_n + a_{n-1}x + \ldots + a_1x^{n-1} + x^n = 0$$

Brioschi recalls the n known relations

$$\begin{array}{l} a_ns_0 \;+ a_{n-1}s_1 + \ldots + a_1s_{n-1} + s_n \;\;= 0 \\ a_ns_1 \;+ a_{n-1}s_2 + \ldots + a_1s_n \;\;+ s_{n+1} = 0 \\ \cdots\cdots\cdots\cdots\cdots\cdots \\ a_ns_{n-1} + a_{n-1}s_n + \ldots + a_1s_{2n-2} + s_{2n-1} = 0 , \end{array}$$

and thus derives by elimination

$$\begin{vmatrix} s_0 & s_1 & \ldots & s_n \\ s_1 & s_2 & \ldots & s_{n+1} \\ \cdot & \cdot & \cdots & \cdot \\ s_{n-1} & s_n & \ldots & s_{2n-1} \\ 1 & x & \ldots & x^n \end{vmatrix} = 0 ,$$

and therefore

$$\begin{vmatrix} s_1 - s_0x & s_2 - s_1x & \ldots & s_n - s_{n-1}x \\ s_2 - s_1x & s_3 - s_2x & \ldots & s_{n+1} - s_nx \\ \cdot & \cdot & \cdots & \cdot \\ s_n - s_{n-1}x & s_{n+1} - s_nx & \ldots & s_{2n-1} - s_{2n-2}x \end{vmatrix} = 0 .$$

Further, he points out that if the last determinant be denoted by V_n, and the cofactor of its last element by V_{n-1}, and so on, then V_n being axisymmetric it follows from Cauchy's theorem of 1829 that $V_n, V_{n-1}, \ldots, V_1, 1$ possess the characteristic property of Sturm's remainders.

It is not noted that the set of n relations used gives each of the a's in terms of the s's, and that substitution in the original equation then gives

$$\begin{vmatrix} s_0 & s_1 & \ldots & s_n \\ s_1 & s_2 & \ldots & s_{n+1} \\ \cdot & \cdot & \cdots & \cdot \\ s_{n-1} & s_n & \ldots & s_{2n-1} \\ 1 & x & \ldots & x^n \end{vmatrix} = (x^n + a_1x^{n-1} + \ldots + a_n)\begin{vmatrix} s_0 & s_1 & \ldots & s_{n-1} \\ s_1 & s_2 & \ldots & s_n \\ \cdot & \cdot & \cdots & \cdot \\ s_{n-1} & s_n & \ldots & s_{2n-2} \end{vmatrix}$$

as may be otherwise seen.

BRIOSCHI, F. (1854, February).

[Sur les fonctions de Sturm. *Nouv. Annales de Math.*, xiii. pp. 71–80; or *Opere Mat.*, v. pp. 89–97.]

Brioschi in effect here recalls that if $f, f_1, f_2, \ldots$ be the series of Sturm's functions originating in the consideration of the equation

$$x^n + a_1x^{n-1} + a_2x^{n-2} + \ldots + a^n = 0, \text{ or, say, } f(x) = 0,$$

and $q_1, q_2, \ldots$ be the linear functions of x which are the quotients obtained in the process of finding $f_2, f_3, \ldots$ then

(1) $f = q_1f_1 - f_2,\quad f_1 = q_2f_2 - f_3,\quad \ldots\ldots,\quad f_{r-2} = q_{r-1}f_{r-1} - f_r$.

(2) f_r is of the $(n-r)^{\text{th}}$ degree in x.

(3) From (1) $\dfrac{f_1}{f} = \dfrac{1}{q_1-}\,\dfrac{1}{q_2-}\,\dfrac{1}{q_3-}\ldots$

(4) The successive convergents $\dfrac{1}{q_1}, \dfrac{q_2}{q_1q_2-1}, \ldots$ to this continued fraction being $N_1/D_1, N_2/D_2, \ldots$

$$N_r = \begin{vmatrix} q_2 & 1 & . & . & . & . & . \\ 1 & q_3 & 1 & . & . & . & . \\ . & 1 & q_4 & . & . & . & . \\ . & . & . & . & . & . & . \\ . & . & . & . & . & . & q_r \end{vmatrix}, \quad D_r = \begin{vmatrix} q_1 & 1 & . & . & . & . & . \\ 1 & q_2 & 1 & . & . & . & . \\ . & 1 & q_3 & . & . & . & . \\ . & . & . & . & . & . & . \\ . & . & . & . & . & . & q_r \end{vmatrix}.$$

(5) From (1) after eliminating $f_2, f_3, \ldots, f_{r-1}$ by repeated substitution or otherwise

$$f_r = D_{r-1}f_1 - N_{r-1}f.$$

(6) From (1) after eliminating $f_1, f_2, \ldots, f_{r-2}$

$$f = D_{r-1}f_{r-1} - D_{r-2}f_r.$$

With these facts before him he seeks to find expressions for f_r, D_r, N_r, or, say, for the coefficients in

$$\begin{array}{lll} A_{r,1}x^{n-r} + A_{r,2}x^{n-r-1} & + \ldots & , \\ B_{r,1}x^{r} + B_{r,2}x^{r-1} & + \ldots & , \\ C_{r,1}x^{r-1} + C_{r,2}x^{r-2} & + \ldots & , \end{array}$$

failing to note that, Cayley having in 1846 found such an expression for Sylvester's substitute for f_r, the annexure of a known multiplier to Cayley's result would have given him the most important of the three expressions sought.

In the first place he deduces from (4) that the coefficient of the highest

power of x in D_{r-1} is always $\frac{1}{n}$ of the coefficient of the highest power of x in N_{r-1}, because $q_1 = \frac{1}{n}x + \frac{a_1}{n^2}$; and from (6), by equating coefficients of x^n, that the coefficient of the highest power of x in f_{r-1} is the reciprocal of the highest power of x in D_{r-1}: in other words, that

$$C_{r,1} = nB_{r,1} = n/A_{r,1}\,.$$

In the next place, denoting the roots of the given equation by $x_1, x_2, \ldots, x_n$ he has from the theory of "partial fractions"

$$\sum_{i=1}^{i=n}\frac{f_r(x_i)}{f_1(x_i)} = 0\,, \quad \sum_{i=1}^{i=n}\frac{x_i f_r(x_i)}{f_1(x_i)} = 0\,, \quad \sum_{i=1}^{i=n}\frac{x_i^2 f_r(x_i)}{f_1(x_i)} = 0\,,$$
$$\ldots\ldots\,, \quad \sum_{i=1}^{i=n}\frac{x_i^{r-2} f_r(x_i)}{f_1(x_i)} = 0\,, \quad \sum_{i=1}^{i=n}\frac{x_i^{r-1} f_r(x_i)}{f_1(x_i)} = A_{r,1}\,;$$

and therefore from (5)

$$\sum_{i=1}^{i=n} D_{r-1}(x_i) = 0\,, \quad \sum_{i=1}^{i=n} x_i D_{r-1}(x_i) = 0\,, \quad \sum_{i=1}^{i=n} x_i^2 D_{r-1}(x_i) = 0\,,$$
$$\ldots\ldots\,, \quad \sum_{i=1}^{i=n} x_i^{r-2} D_{r-1}(x_i) = 0\,, \quad \sum_{i=1}^{i=n} x_i^{r-1} D_{r-1}(x_i) = A_{r,1}\,;$$

and consequently on putting

$$B_{r-1,1}x^{r-1} + B_{r-1,2}x^{r-2} + \ldots + B_{r-1,r} \quad \text{for} \quad D_{r-1}(x)$$
$$\text{and} \quad s_m \quad \text{for} \quad x_1^m + x_2^m + \ldots + x_n^m$$

there results

$$\left.\begin{array}{l} B_{r-1,r}\,s_0 + B_{r-1,r-1}\,s_1 + \ldots + B_{r-1,1}\,s_{r-1} = 0 \\ B_{r-1,r}\,s_1 + B_{r-1,r-1}\,s_2 + \ldots + B_{r-1,1}\,s_r = 0 \\ \ldots\ldots\ldots\ldots\ldots\ldots \\ B_{r-1,r}\,s_{r-2} + B_{r-1,1-1}\,s_{r-1} + \ldots + B_{r-1,1}\,s_{2r-3} = 0 \\ B_{r-1,r}\,s_{r-1} + B_{r-1,r-1}\,s_r + \ldots + B_{r-1,1}\,s_{2r-2} = A_{r,1} \end{array}\right\}.$$

The solution of this set of equations gives the B's in terms of $A_{r,1}$ and the s's, so that the finding of $A_{r,1}$ in terms of the s's is the next desideratum. This with the help of the relation $A_{r,1}\,B_{r,1} = 1$ is easily obtained, for from the set of equations it is seen that Δ_r being written for the persymmetric determinant of s's

$$B_{r-1,1} = \frac{A_{r,1}\Delta_{r-1}}{\Delta_r}$$

and therefore

$$A_{r,1} = \frac{\Delta_r}{\Delta_{r-1}} \cdot \frac{1}{A_{r-1,1}}\,;$$

whence,

$$\text{for } r \text{ even} \qquad A_{r,1} = \left(\frac{\Delta_2\Delta_4 \;.\;.\;.\; \Delta_{r-2}}{\Delta_1\Delta_3 \;.\;.\;.\; \Delta_{r-1}}\right)^2 \Delta_r$$

and

$$\text{for } r \text{ odd} \qquad A_{r,1} = \left(\frac{\Delta_1\Delta_3 \;.\;.\;.\; \Delta_{r-2}}{\Delta_2\Delta_4 \;.\;.\;.\; \Delta_{r-1}}\right)^2 \Delta_r$$

—a result in agreement, as far as it goes, with Sturm's of 1842, Sturm's non-determinant p_r being the equivalent of Brioschi's Δ_r.

The obtaining of Δ_r in terms of the coefficients of $f(x)$ is next illustrated by changing Δ_4 into the form *

$$\begin{vmatrix} 1 & . & . & . & . & . \\ . & 1 & . & . & . & . \\ . & . & s_0 & s_1 & s_2 & s_3 \\ . & s_0 & s_1 & s_2 & s_3 & s_4 \\ s_0 & s_1 & s_2 & s_3 & s_4 & s_5 \\ s_1 & s_2 & s_3 & s_4 & s_5 & s_6 \end{vmatrix}$$

and performing operations which we may denote by

$$\begin{aligned} &\mathrm{col}_6 + a_1\,\mathrm{col}_5 + a_2\,\mathrm{col}_4 + \;.\;.\;.\; + a_5\,\mathrm{col}_1 , \\ &\mathrm{col}_5 + a_1\,\mathrm{col}_4 + \;.\;.\;.\; + a_4\,\mathrm{col}_1 , \\ &.\;\;.\;\;.\;\;.\;\;.\;\;.\;\;.\;\;.\;\;.\;\;.\;\;.\;\;. \\ &\mathrm{col}_2 + a_1\,\mathrm{col}_1 , \end{aligned}$$

the result being

$$\Delta_4 = \begin{vmatrix} a_1 & 2a_2 & 3a_3 & 4a_4 & 5a_5 & 6a_6 \\ 1 & a_1 & a_2 & a_3 & a_4 & a_5 \\ . & 1 & a_1 & a_2 & a_3 & a_4 \\ . & . & n & (n-1)a_1 & (n-2)a_2 & (n-3)a_3 \\ . & n & (n-1)a_1 & (n-2)a_2 & (n-3)a_3 & (n-4)a_4 \\ n & (n-1)a_1 & (n-2)a_2 & (n-3)a_3 & (n-4)a_4 & (n-5)a_5 \end{vmatrix} .$$

To obtain the desired expression for $D_{r-1}(x)$ Brioschi takes the set of

* Brioschi (and afterwards Baltzer, § 12, 9) would have done better to change into the similar form of the *seventh* order, for then the result would have been

$$\Delta_4 = \begin{vmatrix} 1 & a_1 & a_2 & a_3 & a_4 & a_5 & a_6 \\ . & 1 & a_1 & a_2 & a_3 & a_4 & a_5 \\ . & . & 1 & a_1 & a_2 & a_3 & a_4 \\ . & . & . & n & (n-1)a_1 & (n-2)a_2 & (n-3)a_3 \\ . & . & n & (n-1)a_1 & (n-2)a_2 & (n-3)a_3 & (n-4)a_4 \\ . & n & (n-1)a_1 & (n-2)a_2 & (n-3)a_3 & (n-4)a_4 & (n-5)a_5 \\ n & (n-1)a_1 & (n-2)a_2 & (n-3)a_3 & (n-4)a_4 & (n-5)a_5 & (n-6)a_6 \end{vmatrix} ,$$

in which the determinant on the right has a simpler law of formation than Brioschi's and yet is readily reducible to the latter, and which, as we see on putting $n=4$, has the further merit of showing that Δ_n equals the dialytic eliminant of $f(x)=0$, $f'(x)=0$.

equations in the x's together with the equation from which the set was derived, and eliminates the B's, the result in the case of $r=3$ being

$$\begin{vmatrix} 1 & x & x^2 & -D_2 \\ s_0 & s_1 & s_2 & . \\ s_1 & s_2 & s_3 & . \\ s_2 & s_3 & s_4 & A_{3,1} \end{vmatrix} = 0,$$

whence of course he deduces for D_2 the expression

$$\frac{A_{3,1}}{\Delta_3}\begin{vmatrix} 1 & x & x^2 \\ s_0 & s_1 & s_2 \\ s_1 & s_2 & s_3 \end{vmatrix} \quad \text{or} \quad \left(\frac{\Delta_1}{\Delta_2}\right)^2 \begin{vmatrix} 1 & x & x^2 \\ s_0 & s_1 & s_2 \\ s_1 & s_2 & s_3 \end{vmatrix}$$

and the alternative form

$$\frac{n^2}{\begin{vmatrix} a_1 & 2a_2 \\ n & (n-1)a_1 \end{vmatrix}^2} \cdot \begin{vmatrix} a_1 & 2a_2 & 3a_3 \\ n & (n-1)a_1 & (n-2)a_2 \\ 1 & x+a_1 & x^2+a_1x+a_2 \end{vmatrix}.$$

The process of finding f_r is quite similar to this but much more troublesome, the equation taken along with the set of equations in the s's preparatory for elimination being

$$\frac{f_r(x)}{f(x)} = B_{r-1,r}\,u_0 + B_{r-1,r-1}\,u_1 + \ldots\ldots + B_{r-1,1}u_{r-1}$$

where

$$u_m = \frac{x_1^m}{x-x_1} + \frac{x_2^m}{x-x_2} + \ldots + \frac{x_n^m}{x-x_n}.$$

The two previous steps necessary to reach this are

$$\frac{f_r(x)}{f(x)} = \sum_{i=1}^{i=n} \frac{f_r(x_i)}{(x-x_i)f_1(x_i)} = \sum_{i=1}^{i=n} \frac{D_{r-1}(x_i)}{x-x_i};$$

and the result of the elimination is

$$\frac{f_r(x)}{f(x)} = \frac{A_r}{\Delta_r}\begin{vmatrix} s_0 & s_1 & \ldots & s_{r-1} \\ s_1 & s_2 & \ldots & s_r \\ \ldots & \ldots & \ldots & \ldots \\ s_{r-2} & s_{r-1} & \ldots & s_{2r-3} \\ u_0 & u_1 & \ldots & u_{r-1} \end{vmatrix}$$

which in the case of $r=4$ is changed by means of the substitutions

$$u_m = xu_{m-1} - s_{m-1}, \quad u_0 = f_1(x) \div f(x),$$

into

$$f_4(x) = \left(\frac{\Delta_2}{\Delta_1\Delta_3}\right)^2 \begin{vmatrix} a_1 & 2a_2 & 3a_3 & 4a_4 & 5a_5 \\ 1 & a_1 & a_2 & a_3 & a_4 \\ . & n & (n-1)a_1 & (n-2)a_2 & (n-3)a_3 \\ n & (n-1)a_1 & (n-2)a_2 & (n-3)a_3 & (n-4)a_4 \\ . & f_1(x) & Z_1 & Z_2 & Z_3 \end{vmatrix}$$

where

$$\begin{aligned} Z_1 &= (x+a_1)f_1(x) - n\,.f(x)\,, \\ Z_2 &= (x^2+a_1x+a_2)f_1(x) - [nx+(n-1)a_1]f(x)\,, \\ Z_3 &= (x^3+a_1x^2+a_2x+a_3)f_1(x) - [nx^2+(n-1)a_1x+(n-2)a_2]f(x)\,. \end{aligned}$$

The worthlessness of this in itself is apparent as soon as we note the presence of $f_1(x)$ and $f(x)$: when, however, the determinant is partitioned into two, one having $f_1(x)$ for a factor and the other $f(x)$, and the result compared with $f_4 = D_3f_1 - N_3f$, we obtain for N_3 an expression similar to that for D_3.

Brioschi, F. (1854, August).

[Intorno ad alcune questioni d' algebra superiore. *Annali di sci. mat. e fis.*, v. pp. 301–312: or French translation of Brioschi's *Teorica dei Determinanti*, pp. 151–170: or *Opere Mat.*, i. pp. 127–142.]

The questions referred to are much the same as those of his paper on Sturm's functions (1854, February), the first function

$$a_0x^n + a_1x^{n-1} + \ . \ . \ . \ + a_n \quad \text{or} \quad a_0(x-x_1)(x-x_2)\ldots(x-x_n) \quad \text{or} \quad f(x)$$

being, however, no longer connected with the second

$$b_0x^m + b_1x^{m-1} + \ . \ . \ . \ + b_m \quad \text{or} \quad \phi(x) \quad \text{where} \quad m<n\,.$$

Putting

$$S_r \quad \text{for} \quad \frac{x_1^r\phi(x_1)}{f'(x_1)} + \frac{x_2^r\phi(x_2)}{f'(x_2)} + \ . \ . \ . \ + \frac{x_n^r\phi(x_n)}{f'(x_n)}$$

he first proves the interesting theorem

$$a_0S_r + a_1S_{r-1} + \ . \ . \ . \ + a_rS_0 = b_{r+m-n+1}(r<n)\,.$$

Then temporarily denoting

$$\{\phi(x_r) \div f'(x_r)\}^{\frac{1}{2}} \quad \text{by} \quad A_r$$

he squares in two ways the determinant

$$\begin{vmatrix} A_1 & A_2 & \ldots & A_n \\ A_1x_1 & A_2x_2 & \ldots & A_nx_n \\ \ldots & \ldots & \ldots & \ldots \\ A_1x_1^{n-1} & A_2x_2^{n-1} & \ldots & A_nx_n^{n-1} \end{vmatrix}$$

thus obtaining

$$\begin{vmatrix} S_0 & S_1 & \cdots & S_{n-1} \\ S_1 & S_2 & \cdots & S_n \\ \cdot & \cdot & \cdots & \cdot \\ S_{n-1} & S_n & \cdots & S_{2n-2} \end{vmatrix} = A_1^2 A_2^2 \ldots A_n^2 \begin{vmatrix} s_0 & s_1 & \cdots & s_{n-1} \\ s_1 & s_2 & \cdots & s_n \\ \cdot & \cdot & \cdots & \cdot \\ s_{n-1} & s_n & \cdots & s_{2n-2} \end{vmatrix}$$

from which, on putting $(-1)^{\frac{1}{2}n(n-1)} f'(x_1) . f'(x_2) \ldots f'(x_n)$ for the determinant on the right * and substituting for the A's, he deduces

$$\begin{vmatrix} S_0 & S_1 & \cdots & S_{n-1} \\ S_1 & S_2 & \cdots & S_n \\ \cdot & \cdot & \cdots & \cdot \\ S_{n-1} & S_n & \cdots & S_{2n-2} \end{vmatrix} = (-1)^{\frac{1}{2}n(n-1)} \phi(x_1) . \phi(x_2) \ldots \phi(x_n) .$$

This result, be it noted, is not given in the original paper, but appears first in Combescure's translation (1856), which contains six pages (pp. 153–159) more than the original. Brioschi does not point out its significance in connection with Euler's first form of the resultant of $f(x)=0$, $\phi(x)=0$.

The remainder of the paper is of little interest in the present connection.

Brioschi, F. (1855, January).

[Sur les questions 241 et 141. *Nouv. Annales de Math.*, xiv. pp. 20–24: or *Opere Mat.*, v. pp. 107–111.]

If for all positive integral values of r and s we have

$$A_{r+s} = a_1 A_{r+s-1} + a_2 A_{r+s-2} + \ldots + a_s A_r ,$$

—in other words, if this last be a "recurrence-formula,"—it is readily seen that the last column of the persymmetric determinant

$$\begin{vmatrix} A_r & A_{r+1} & \cdots & A_{r+s-1} \\ A_{r+1} & A_{r+2} & \cdots & A_{r+s} \\ \cdot & \cdot & \cdots & \cdot \\ A_{r+s-1} & A_{r+s} & \cdots & A_{r+2s-2} \end{vmatrix} \quad \text{or } \Delta_{r,} \text{ say}$$

may be legitimately changed into

$$a_s A_{r-1} , \; a_s A_r , \; \ldots , \; a_s A_{r+s-2}$$

so that there is deducible

$$\Delta_{r,s} = (-1)^{s-1} a_s \Delta_{r-1,s} ,$$

and thence

$$\Delta_{r,s} = (-1)^{r(s-1)} a_s^r \Delta_{0,s} ,$$

* Brioschi unfortunately neglects the sign-factor. See *Proc. Roy. Soc. Edinburgh*, xxiii. p. 132, where the footnote might have made mention of the fact that the identity had already appeared in one of Cauchy's own memoirs of the year 1813. (See *Journ. de l'ec. polyt.*, x. cah. 17, p. 485.)

thus implying that $\Delta_{r,s}/(-1)^{r(s-1)}a_s^r$ is independent of r,—a result suggested to Brioschi by an old proposition of Euler's which is referred to under Continuants.

CAYLEY, A. (1856, March).

[A third memoir on quantics. *Philos. Trans. R. Soc.* (*Lond.*), cxlvi. pp. 627–647: or *Collected Math. Papers*, ii. pp. 310–335.]

Among Cayley's tables of invariants there naturally appear the catalecticants of the binary quartic, sextic, and octavic; so that we have from him the final expansions of the persymmetric determinants of the 3rd, 4th, 5th orders. Of canonizants only that of the quintic is given. The four results are those which he numbers 10, 34, 35, 16. The first an last we need not reproduce. The second is

$$\begin{vmatrix} a & b & c & d \\ b & c & d & e \\ c & d & e & f \\ d & e & f & g \end{vmatrix} = \begin{cases} aceg - acf^2 - ad^2g + 2adef - ae^3 - b^2eg + b^2f^2 \\ + 2bcdg - 2bcef - 2bd^2f + 2bde^2 - c^3g + 2c^2df \\ + c^2e^2 - 3cd^2e + d^4, \end{cases}$$

where the terms are arranged, as with Cayley, in alphabetical order. The third, altered in form, is

$$\begin{vmatrix} a & b & c & d & e \\ b & c & d & e & f \\ c & d & e & f & g \\ d & e & f & g & h \\ e & f & g & h & i \end{vmatrix} = \begin{aligned} & e^5 - e^3(ai + 3cg + 4df) \\ & \quad + e^2\{2(afh + bdi) + (ag^2 + c^2i) + 4(bfg + cdh) + 3(cf^2 + d^2g)\} \\ & \quad - e\{(ach^2 + b^2gi) + 2(adgh + bcfi) + 3(af^2g + cd^2i) \\ & \qquad + 4(bdg^2 + c^2fh) + 2(bf^3 + d^3h) - acgi \\ & \qquad - 2adfi - b^2h^2 - 2bcgh - 2c^2g^2 + 2cdfg + 3d^2f^2\} \\ & \quad + \{-(acf^2i + ad^2gi) + 2(acfgh + bcdgi) - (acg^3 + c^3gi) \\ & \qquad + (ad^2h^2 + b^2f^2i) - 2(adf^2h + bd^2fi) + 2(adfg^2 + c^2dfi) \\ & \qquad + (af^4 + d^4i) - 2(b^2fgh + bcdh^2) + (b^2g^3 + c^3h^2) \\ & \qquad + 2(bcf^2h + bd^2gh) - 2(bcfg^2 + c^2dgh) + 2(bdf^2g + cd^2fh) \\ & \qquad + (c^2f^2g + cd^2g^2) - 2(cdf^3 + d^3fg)\}. \end{aligned}$$

It is the term, L, independent of λ in the almost persymmetric determinant which Cayley * on Sylvester's suggestion calls the *lamdaic*, namely,

$$\begin{vmatrix} a & b & c & d & e-12\lambda \\ b & c & d & e+3\lambda & f \\ c & d & e-2\lambda & f & g \\ d & e+3\lambda & f & g & h \\ e-12\lambda & f & g & h & i \end{vmatrix}.$$

* CAYLEY, A. Mémoire sur la forme canonique des fonctions binaires. *Crelle's Journ.*, liv. pp. 48–58, 292: or *Collected Math. Papers*, iv. pp. 43–52. If "lamdaic" be not used as a noun, "lamdaic canonizant" would be better than "lamdaic determinant."

and which, if I, J, K be the other invariants of the octavic, is equal to

$$-2592\lambda^5 + 18I\lambda^3 + 3J\lambda^2 + 2K\lambda + L.$$

The expansions of I, J, K are those numbered 39, 43, 44 in Cayley's collection.

Bruno, Faà di (1856, August).

[Sopra i resti di Sturm. *Annali di sci. mat. e fis.*, vii. pp. 313–317.]

Beginning with two unrelated functions, P, Q, of the n^{th} and $(n-1)^{\text{th}}$ degrees, Bruno gives an expression for any one of the Sturmian series of functions thence derived, the coefficients of x in this expression being determinants which resemble in outward form those of Cayley's analogous expression of 1846 (August), but which have for elements the coefficients of $x^{-1}, x^{-2}, \ldots$ in Q/P. He says that the expression "salvo qualche modificazione" was found by Cauchy, but gives no reference.

Bellavitis, G. (1857, June).

[Sposizione elementare della teorica dei determinanti. *Mem. . . . Istituto Veneto* . . . . vii. pp. 67–144.]

Although the persymmetric determinant

$$\begin{vmatrix} s_0 & s_1 & s_2 & \cdots \\ s_1 & s_2 & s_3 & \cdots \\ s_2 & s_3 & s_4 & \cdots \\ \cdot & \cdot & \cdot & \cdots \end{vmatrix}$$

where s_r is the sum of the rth powers of the roots of the equation $x^n + a_1x^{n-1} + \ldots + a_n = 0$ has repeatedly come before us, it has always been with the understanding that no two of the roots were equal, and the order of the determinant has never been greater than the n^{th}. Bellavitis takes up the subject (§§ 45–50) with these conditions removed. He affirms that *when the number of rows exceeds the number of different roots, the persymmetric determinant*

$$\begin{vmatrix} s_{0+r} & s_{1+r} & s_{2+r} & \cdots \\ s_{1+r} & s_{2+r} & s_{3+r} & \cdots \\ s_{2+r} & s_{3+r} & s_{4+r} & \cdots \\ \cdot & \cdot & \cdot & \cdots \end{vmatrix}$$

vanishes, and when the numbers are the same the determinant is a multiple of the square of the difference-product of the said roots. By way of proof of the second part of the proposition the special case is taken

where the roots are a, a, a, b, b, c, and where, since

$$s_0 = 6, \quad s_1 = 3a+2b+c, \quad s_2 = 3a^2+2b^2+c^2, \quad \ldots\ldots$$

we have

$$\begin{vmatrix} s_5 & s_6 & s_7 \\ s_6 & s_7 & s_8 \\ s_7 & s_8 & s_9 \end{vmatrix} = \begin{vmatrix} 3 & 2 & 1 \\ 3a & 2b & c \\ 3a^2 & 2b^2 & c^2 \end{vmatrix} \cdot \begin{vmatrix} a^5 & b^5 & c^5 \\ a^6 & b^6 & c^6 \\ a^7 & b^7 & c^7 \end{vmatrix},$$

$$= 6\,|\,a^0b^1c^2\,|\,.\,a^5b^5c^5\,|\,a^0b^1c^2\,|\,,$$

$$= 6a^5b^5c^5\,|\,a^0b^1c^2\,|^2.$$

The other part of the proposition rests on the statement that a similar procedure leads in that case to factors having at least one column of zeros.

The case where the number of rows is *less* than the number of different roots is not considered; but the first part of the proposition is used to obtain the modification which it is possible to make in the relation

$$s_{n+r} + a_1 s_{n+r-1} + a_2 s_{n+r-2} + \ldots + a_n s_r = 0$$

when the roots cease to be all different.

Salmon, G. (1859).

[Lessons Introductory to the Modern Higher Algebra, ... xii+147 pp., Dublin.]

In Salmon's treatment of the foregoing subjects (p. 14, §§ 119–126, 162–165, p. 146) there are several points of freshness. His proof, for example, that *if*

$$(a,b,c,d,e,f\between x,y)^5 = (l_1x+m_1y)^5+(l_2x+m_2y)^5+(l_3x+m_3y)^5$$

the persymmetric form of the canonizant is equal to

$$|\,l_1m_2\,|^2\,|\,l_1m_3\,|^2\,|l_2m_3\,|^2\,.\,(l_1x+m_1y)(l_2x+m_2y)(l_3x+m_3y)\,,$$

is accomplished (§ 119) by using four differentiations to show that

$$\begin{aligned} ax+by &= l_1{}^4(l_1x+m_1y)+l_2{}^4(l_2x+m_2y)+l_3{}^4(l_3x+m_3y) \\ &= l_1{}^4u+l_2{}^4v+l_3{}^4w\,, \quad \text{say,} \\ bx+cy &= l_1{}^3m_1u+l_2{}^3m_2v+l_3{}^3m_3w\,, \\ &\ldots\ldots\ldots\ldots\ldots\ldots \end{aligned}$$

substituting these trinomials for $ax+by$, $bx+cy$, . . . in the canonizant, and then examining * the twenty-seven determinants into which the latter

* It is better to note at this stage that the determinant is the product of

$$\begin{vmatrix} l_1{}^2u & l_2{}^2v & l_3{}^2w \\ l_1m_1u & l_2m_2v & l_3m_3w \\ m_1{}^2u & m_2{}^2v & m_3{}^2w \end{vmatrix} \quad \text{and} \quad \begin{vmatrix} l_1{}^2 & l_2{}^2 & l_3{}^2 \\ l_1m_1 & l_2m_2 & l_3m_3 \\ m_1{}^2 & m_2{}^2 & m_3{}^2 \end{vmatrix}$$

and therefore is equal to $|\,l_1{}^2\ l_2m_2\ m_3{}^2|^2\,.\,uvw$.

can be partitioned. He also gives (§ 121) a new mode of arriving at the canonizant in the form

$$\left(\left\| \begin{matrix} a & b & c & d \\ b & c & d & e \\ c & d & e & f \end{matrix} \right\| \between x , y \right)^3 .$$

In the course of a short "Note on Commutants" he suggests (p. 146) that the two rows of umbræ

$$\left| \begin{matrix} 0 & 1 & 2 \\ 0 & 1 & 2 \end{matrix} \right| \text{ should stand for } \left| \begin{matrix} a_0 & a_1 & a_2 \\ a_1 & a_2 & a_3 \\ a_2 & a_3 & a_4 \end{matrix} \right|$$

in which case the suffixes of any term of the determinant are got by *adding* the first row of umbræ to a permutation of the second row.

LIST OF AUTHORS

whose writings are herein dealt with.

(*Issued separately June* 16, 1910)

XXVI.—A Method for determining Boiling-Points under Constant Conditions. By Alexander Smith and Alan W. C. Menzies.

(MS. received April 13, 1910. Read May 16, 1910.)

(Abstract.)

A method for determining the boiling-points of solids as well as liquids, such that little material is required, and only constant, easily reproducible conditions are involved, is much needed. None of these qualities is possessed by the distilling flask method.

The apparatus suggested consists of a bulblet with a bent capillary not less than 1 mm. in diameter. This is attached to a thermometer and suspended in a beaker (fig. 1). The latter contains water (for substances boiling below 100°), sulphuric acid (up to 200°), melted paraffin (up to 280°), or a mixture of sodium and potassium nitrates (45·5, 54·5 parts) for use from 220° upwards. A stirrer is provided.

After charging (with 0·03–0·1 g. of the substance), attachment, and submergence of the bulblet, the bath is heated. When the temperature has remained at the boiling-point for a few moments, dissolved and occluded gases and moisture have all been expelled, and a rapid stream of pure vapour issues. In case the substance is insoluble in the bath-liquid, the bubbles of vapour rise to the surface. With falling temperature and vigorous stirring, the point at which the bubbles suddenly cease can be read accurately: this is the boiling-point. In contrary case of mutual solubility of the substance and bath-liquid, the point at which boiling ceases cannot be ascertained sharply. Here the bath-liquid is allowed to recede into the capillary to a point 5–10 mm. from the opening, opposite to some mark on the thermometer or binding thread. The temperature at which this point is reached is the boiling-point.

The vapour pressure of the bath-liquid has no influence on the result, and chemical interaction of the substance and bath-liquid will not usually interfere with the observation.

The expulsion of foreign gases and vapours is repeated until constant values are obtained, or the impurity of the substance has been demonstrated by the absence of a constant boiling-point.

As in all boiling-point observations, the barometric height (at 0°) must be recorded. A correction for the head of liquid above the opening of the

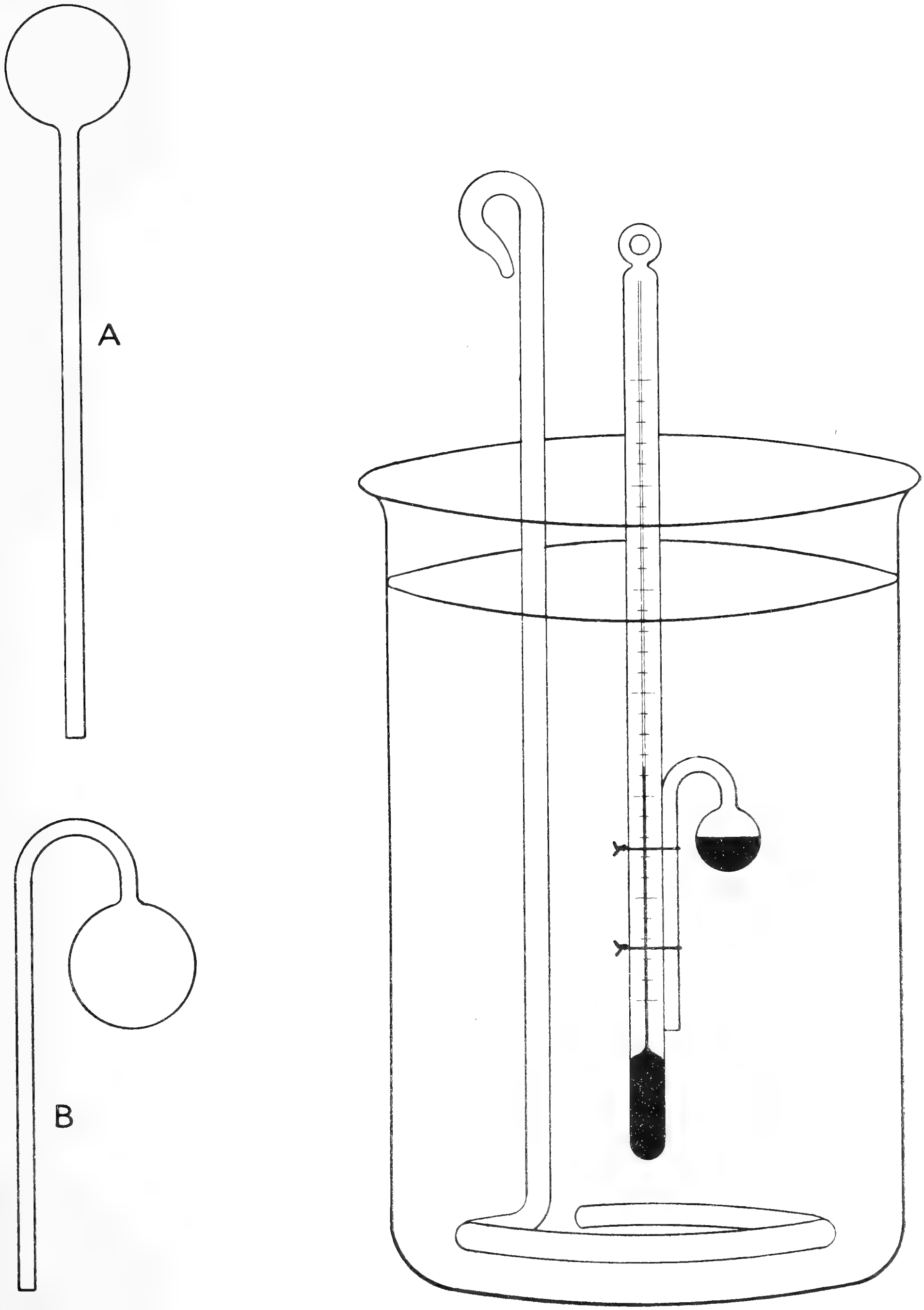


Fig. 1.

capillary, or the mark to which the bath-liquid ascends, as the case may be, is required. The immersed depth is measured ± 2 mm. and, taking into account the specific gravity * of the bath-liquid at the temperature concerned, is converted into mercury-height and added to the barometric reading.

The correction for the additional pressure caused by capillary ascension in the tube is usually of the order of 1 mm., and will be employed only when the temperature is determined more exactly than ± 1/25°.

The following are boiling-points of the same sample as obtained with the distilling flask (col. 7) and with the submerged bulblet (col. 6). For comparison, the latter boiling-points are reduced to the existing atmospheric pressure by the data in col. 5. The examples illustrate different miscible and chemically active pairs of substances and bath-liquids. The values for calomel and ammonium carbonate (col. 7) are taken from known vapour-pressure data.

Substance.	Bath-liquid.	Immersed Depth.	Density of Bath-liquid.	V.P. Increase for 1°.	B.P. Subm. Bulblet.	B.P. Dist. Flask.
Benzene . .	Water . .	113	0·97	24	79·01°	79·00°
" .	H_2SO_4 . .	104	1·77	...	79·04	79·00
" .	Min. oil .	60	0·9	...	79·35 [1]	79·33 [1]
Ether . .	$CaCl_2$ sol. .	70	1·47	27	33·87	33·85
" . .	Water . .	73	0·99	...	33·91	33·85
Alcohol . .	" . .	60	0·97	31	77·56	77·55
Camphor .	Paraffin .	45	0·69	20	208·00	207·65
Naphthalene .	2 nitrates .	68	1·93	17	217·68	217·73
Calomel . .	2 " .	59	1·85	...	383·2	[382·5]
Coml. am. carbonate	Olive oil .	57	0·88	...	58·5	[58·5]

[1] These observations were made at a different barometric pressure from the preceding ones.

Characteristics of the Submerged Bulblet Method.—The chief features of the method are :—

(1) That it is applicable to non-fusing solids, for the determination of the boiling-points of which no simple and accurate method has been known.

(2) That a minimum of material (at most 0·1 g.) and a minimum of time are consumed.

* We are indebted to Mr F. B. Plummer for making a series of determinations of the specific gravities of the bath-liquids at various temperatures. The following formulæ summarise the results, and give values correct to the second decimal place within the limits of temperature specified :—

Sulphuric acid (92·75 per cent., 30–200°), $1·818–0·000906(t–30)$.
Paraffin (m.-p. 53°, 60–230°), $0·778–0·000612(t–60)$.
Two nitrates (230–390°), $1·968–0·00075(t-230)$.

(3) That the conditions are definite and easily reproducible with exactness, and that, in particular, it is impossible for the liquid, the vapour, and the thermometer to differ in temperature. The boiling-points ascertained are therefore more accurate, and more exactly comparable than are those obtained by the usual method.

(4) That with impure or decomposing liquids a fractional distillation in miniature may quickly be carried out and the impureness recognised.

(5) That when the dissolved or occluded gases or volatile substances which are always present can be removed by boiling, the removal may be accomplished and a satisfactory boiling-point secured.

(6) That by taking the boiling-point of a mixture of the two, the identity or non-identity of two liquids of almost identical boiling-points may often be ascertained without sacrifice of an appreciable amount of material. This method will apply, however, only when the two substances are of chemically dissimilar natures, and not usually to very similar substances.*

(7) That the method may be adapted to finding boiling-points at normal pressure (760 mm.), and under reduced pressure (see next two papers).

* Young, *Stoichiometry* (London, 1908), 264.

(*Issued separately June* 16, 1910.)

XXVII.—A Common Thermometric Error in the Determination of Boiling-Points under Reduced Pressure. By Alexander Smith and Alan W. C. Menzies.

(MS. received April 13, 1910. Read May 16, 1910.)

(Abstract.)

When the bulb of a thermometer is enclosed in an evacuated vessel, the dilatation of the bulb introduces a considerable error in the temperature readings. This fact may be well known, but in the literature of boiling-point and vapour-pressure determinations we have observed no reference to it, and no corrections on account of it.* Yet, except in the roughest work, this effect cannot be ignored. Thus, a test carried out with eleven thermometers showed that when the pressure round the bulb was lowered from 748 mm. to 20 mm. and thermal equilibrium with the bath had been recovered, the readings were from 0·10° to 0·17° lower. In all but one case, when there was a slight permanent dilatation, the change was constant and was a linear function of the change in pressure. The change bore no relation to the sizes of the bulbs. The thickness of the glass varied considerably, but could not, of course, be measured.

Apparently, excepting in the roughest work, every thermometer to be used in vacuum distillation, or for measuring vapour pressures by methods which involve reducing the pressure round the bulb, should have its constant determined separately. The absence of corrections on this account vitiates many of the published data.

* Since the above was written, a single, inconspicuous instance has come to our notice.

Chicago, *February* 1910.

(*Issued separately June* 16, 1910.)

XXVIII.—A Simple Dynamic Method for determining Vapour Pressures. By Alexander Smith and Alan W. C. Menzies.

(MS. received April 13, 1910. Read May 16, 1910.)

(Abstract.)

Vapour Pressures.—The submerged bulblet apparatus described in a preceding paper may readily be adapted to determining vapour pressures. The apparatus is unchanged, excepting that the lower part of the thermometer is enclosed in a test-tube containing a portion of the bath-liquid (fig. 1). The interior of the test-tube communicates, through the L-tube, with a gauge, with a pump, and with the atmosphere. The bath is brought to constancy at the required temperature with the pressure in the apparatus above the vapour pressure of the substance. The pressure is next lowered gradually until a continuous stream of bubbles issues from the capillary. Then the pressure is allowed to rise until the stream ceases. The details of manipulation and correction may be understood from the former paper without special description.

To illustrate the adaptability of this method to securing rapidly a series of measurements of considerable accuracy, the following results with water are given. The bath-liquid was a heavy paraffin oil. The gauge consisted simply of a tube tied to a meter-rule and dipping into a vessel of mercury. Corrections for capillarity in the gauge and for immersed depth were made, and mercury heights were reduced to 0°. The thermometer showed no errors over 0·05°, and its thread was completely immersed.

Temp.	V. P. S. & M.	V. P. H. & H.	Diff. mm.	Temp.	V. P. S. & M.	V. P. H. & H.	Diff. mm.
49·0	88·0	87·8	+0·2	80·0	355·4	355·1	+0·3
60·7	155·3	154·1	+1·2	85·5	441·5	442·2	−0·7
67·8	213·8	212·2	+1·6	90·0	526·5	525·8	+0·7
73·6	273·5	272·5	+1·0	95·3	640·5	641·1	−0·6

The column H. & H. contains the values found by Holborn and Henning,* and in the fourth column appear the differences. The average divergence from our values is 0·46 mm., corresponding to an average temperature error

* *Ann. d. Physik*, [4], 26, 882 (1908).

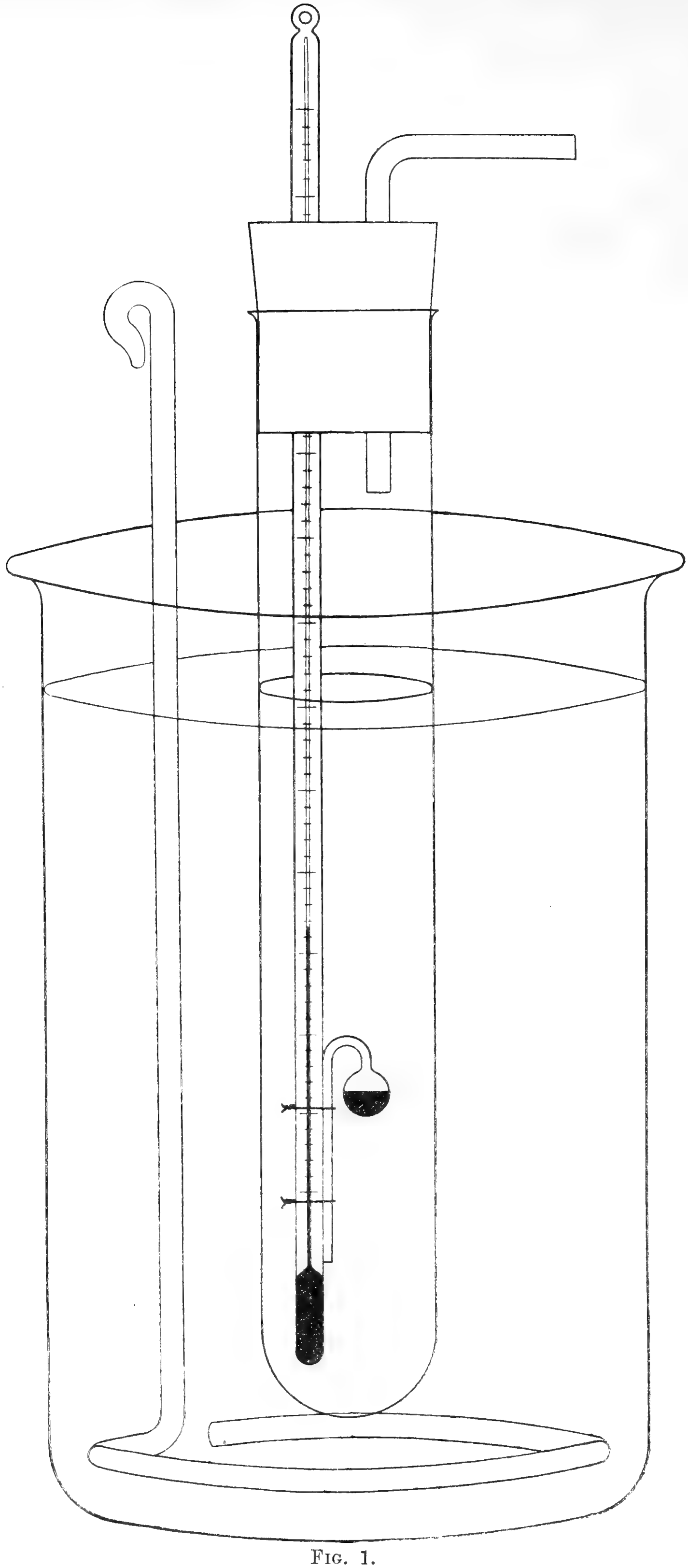


Fig. 1.

INSTRUCTIONS TO AUTHORS.

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Cells, Liver,—Intra-cellular Canaliculi in.
E. A. Schäfer. Proc. Roy. Soc. Edin., vol. , 1902, pp. .

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OF THE

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SESSION 1909–10.

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[*Continued on page* iii *of Cover.*

of 0·07°. The irregularity of the individual observations is due to the crudeness of the gauge and the lack of special precautions in ascertaining the temperatures—both intentional.

There are three sources of error. That due to capillary ascension of the bath-liquid (+0·5 to +0·6 mm.) and that due to dilatation of the bulb of the thermometer (equivalent to −0·5 to −1·2 mm.) act in opposite directions. When correction on these accounts has been made, the average divergence from the standard values becomes 0·16 mm. or 0·03°. In the third place, correction may be made for the local value of the gravity constant (here −0·25 mm. per 760 mm.). Taking account of this reduces the average divergence to 0·06 mm. or 0·01°.

Boiling-Points at exactly Normal Pressure and at Definite Reduced Pressures.—It is very desirable that the chemist, instead of publishing boiling-points as observed, and often omitting to give the exact pressure, should ascertain the boiling-point in each case at 760 mm.

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CHICAGO, *February* 1910.

(*Issued separately June* 16, 1910.)

XXIX.—Equilibrium in the Ternary System: Water, Potassium carbonate, Potassium ethyl di-propyl-malonate. By J. W. M'David, B.Sc., Carnegie Research Scholar. *Communicated by* Professor James Walker.

(MS. received April 13, 1910. Read May 16, 1910.)

During the electrolysis of a solution of potassium ethyl di-propyl-malonate it was discovered by Crichton * that when a concentrated aqueous solution of the salt was shaken up with a concentrated aqueous solution of potassium carbonate two distinct layers were formed. The peculiarity of this case lies in the fact that both substances are salts of the same metal, though cases of the non-miscibility of aqueous solutions of two totally different substances are common, as, for example, that of potassium carbonate and alcohol.

The object of the following investigation is to show how the miscibility of the two solutions depends on their concentrations, temperature, etc.

The potassium ethyl salt was prepared from di-propyl-malonic ester by half saponification with a solution of potassium hydroxide in methyl alcohol. The saponification was done in four stages, one-fourth of the amount of alkali necessary for half saponification being used each time. After addition of water the unsaponified ester which separated was removed by means of ether and the aqueous solution evaporated to dryness. The potassium ethyl salt was freed from di-potassium salt by dissolving in hot absolute alcohol in which the latter salt is practically insoluble. The ester salt so obtained was analysed, but was still found to contain a small quantity of di-potassium salt. A fresh sample was made as above described, but the saponification was only carried out to the third stage. The ester salt, purified as before, was found on analysis to be free from di-potassium salt.

0·2164 gm. potassium ethyl salt yielded 0·0744 gm. K_2SO_4;
i.e. 100 parts ,, ,, ,, 34·37 parts ,,

Theory—

100 parts $KC_{11}H_{19}O_4$ yield 34·27 parts K_2SO_4.

An attempt was made to determine the solubility of the pure potassium

* *Journal Chem. Soc.*, vol. lxxxix. p. 929 (1906).

ethyl salt. It was, however, found impossible to obtain a saturated solution, the liquid gradually assuming the consistency of a thick syrup in which solid particles were suspended in a fine state of division. A concentrated solution of the ester salt was then made and shaken up with a saturated solution of potassium carbonate. Mixture only took place to a limited extent, two distinct layers separating when the liquid was allowed to stand. Water was added drop by drop from a burette to the liquid, with constant stirring, until the two layers disappeared and one homogeneous solution was left. The disappearance of the two layers was quite sharply defined. The above experiment was then repeated quantitatively. 5·5 gms. potassium ethyl di-propyl-malonate were dissolved in 2 gms. water and 1·05 c.cs. of this solution were put into a graduated tube. 1·05 c.cs. of a saturated solution (about 52 per cent.) of potassium carbonate were added, and thereafter water drop by drop.

The following readings were taken:—

Total Volume of Liquid.	Vol. of K_2CO_3 Solution.	Vol. of KEt. Salt Solution.
2·10	1·05	1·05
3·05	1·55	1·50
3·45	1·65	1·80
3·80	1·45	2·35
3·95	1·15	2·80
4·05	homo	geneous.

From the above table it is seen that, while at first the volumes of both layers increase on addition of water, the potassium carbonate layer soon begins to diminish in volume while the volume of the ester salt solution increases steadily. This seems to indicate that it is the carbonate solution which gradually passes into that of the ester salt, one homogeneous liquid being obtained when the whole of the potassium carbonate solution disappears into the other layer. As will be seen later, on more accurate analysis this was found to be the case.

The mixtures to be investigated were made up by putting weighed quantities of potassium ethyl di-propyl-malonate, potassium carbonate and water into a stoppered tube and thoroughly mixing in a thermostat at 25° C. The mixture was then allowed to settle into two layers at the temperature of the bath.

Methods of analysing the two layers had next to be devised. The first method attempted was to find the total percentage of potassium in each layer, and then the percentage of carbon dioxide in the layers was found by

adding hydrochloric acid to a weighed quantity of solution and collecting the dried carbon dioxide in two weighed soda-lime tubes. From the amount of carbon dioxide obtained the percentage of potassium carbonate could be found, and hence the amount of potassium due to the carbonate could be calculated. This being subtracted from the total potassium gave the amount due to the potassium ethyl salt, and from this amount the

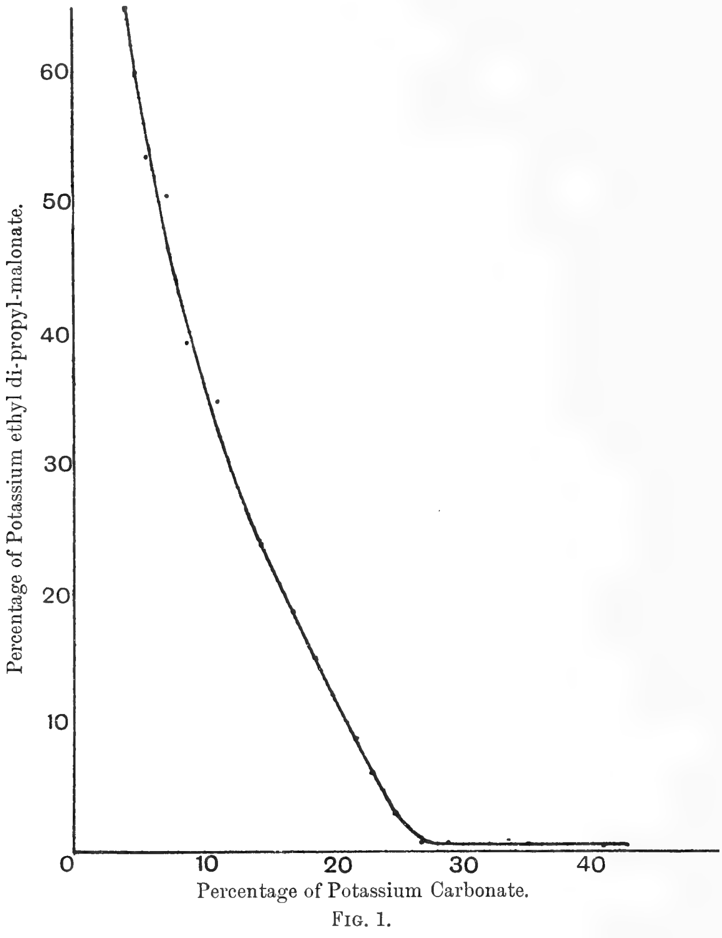


Fig. 1.

percentage of the ester salt could be ascertained. This method, however, failed to give concordant results. Finally the following methods were adopted. To determine the percentage of potassium ethyl salt, a weighed quantity of the solution was acidified with dilute sulphuric acid, when hydrogen ethyl di-propyl-malonate was precipitated. The ester acid was extracted with ether and washed to get rid of sulphuric acid. The ether was then evaporated off, and after addition of a little water the residue was titrated with a standard solution of baryta. To determine the percentage

of potassium carbonate present, the fact that the colour of phenolphthalein disappears when the potassium carbonate is changed to bicarbonate was made use of. A weighed quantity of solution was therefore titrated with an acid of known strength, phenolphthalein being used as indicator, and the percentage of potassium carbonate obtained. In order to check these results, in most cases the total potassium was determined gravimetrically. The results obtained were concordant.

A series of mixtures were made up as follows:—

No. of Mixture.	Weight of KEt. Salt (gms.).	Weight of K_2CO_3 (gms.).	Weight of Water (gms.).
1	3·081	3·081	5·115
2	3·011	3·010	6·025
3	1·953	4·333	9·210
4	3·059	3·048	8·158
5	3·058	3·143	9·654
6	3·017	3·125	10·673
7	3·027	3·203	12 032
8, 9	unknown.		

Nos. 1, 2, 4, 5, 6 and 7 were made up by taking approximately equal weights of the two salts. No. 3 was made up by taking totally different proportions of the salts, while the last two mixtures were obtained by diluting two of the more concentrated with water to nearly the point at which complete mixture took place.

The solutions were analysed, after equilibrium had been attained, by the methods explained above, with the following results:—

No. of Mixture.	Upper Layer.			Lower Layer.		
	Per Cent. KEt. Salt.	Per Cent. K_2CO_3.	Per cent. H_2O.	Per Cent. KEt. Salt.	Per Cent. K_2CO_3.	Per cent. H_2O.
1	65·1	4·05	30·85	0·4	42·6	57·0
2	59·8	4·9	35·3	0·4	40·7	58·9
3	53·5	5·6	40·9	0·5	35·0	64·5
4	50·5	7·2	42·3	0·9	33·5	65·6
5	39·2	8·7	52·1	0·7	28·9	70·4
6	34·6	11·0	54·4	0·8	26·8	72·4
7	23·5	14·5	62·0	3·0	24·8	72·2
8	18·6	17·0	64·4	6·05	23·1	70·85
9	15·0	18·6	66·4	8·7	21·7	69·6

As will be seen from the above table, even in the case of concentrated solutions there is a considerable quantity of potassium carbonate in the

upper layer. On the other hand, the amount of potassium ethyl di-propyl-malonate in the potassium carbonate layer is scarcely appreciable in the first six mixtures, and it is only when the solutions are comparatively dilute that there is any appreciable increase in the amount of the ester salt in the lower layer. This confirms the results of the rough experiment previously referred to, viz. that in order to produce equilibrium when a mixture is

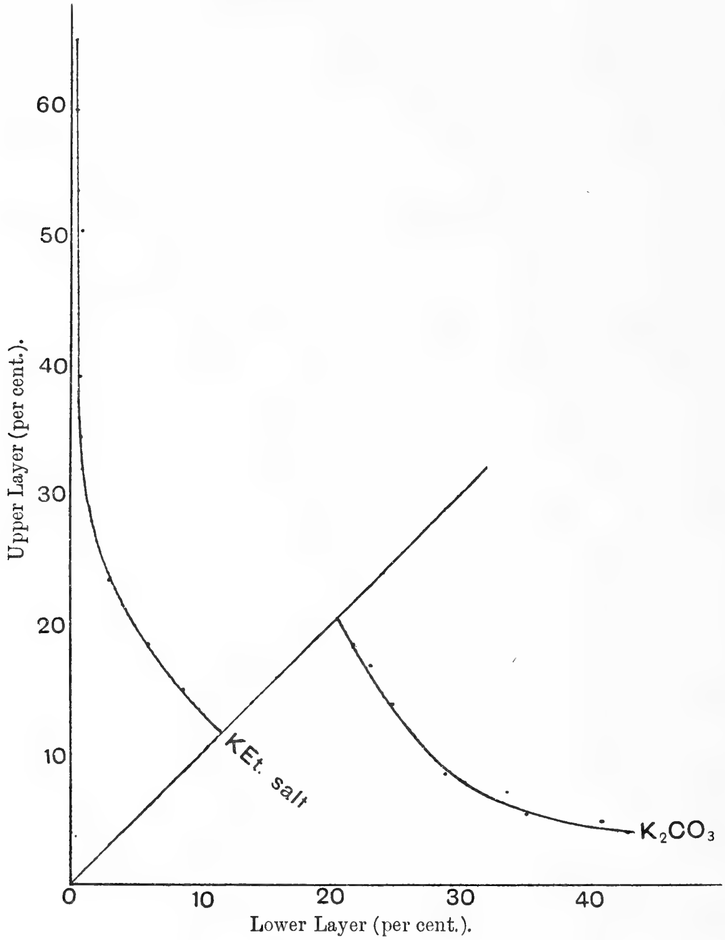


Fig. 2.

diluted, some potassium carbonate passes from the lower to the upper layer. It will also be observed that the percentage of water in the lower layer at first increases and then diminishes as the mixtures become more dilute.

The results shown in the above table may be represented graphically. By plotting the percentage of potassium ethyl di-propyl-malonate against that of potassium carbonate in each layer in the series 1 to 7, two curves (fig. 1) are obtained representing the upper and lower layers respectively.

The points for Nos. 8 and 9 lie on these curves produced. If the two curves are produced still further, they meet and form one continuous curve at the point where the two layers become homogeneous. This occurs when the composition of the two layers is identical. The percentage of each salt in the solution at this point can be determined from the curves in

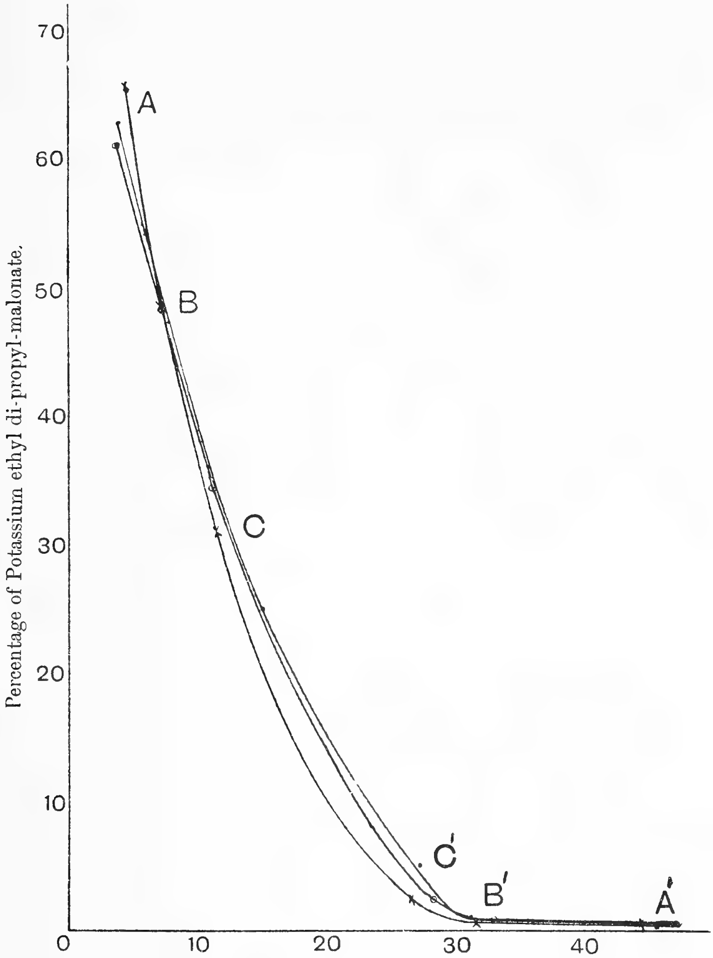


FIG. 3.

fig. 2. The curve to the left shows the connection between the percentage of potassium ethyl salt in the upper and lower layers respectively, while that to the right represents similarly the percentages of potassium carbonate in the two layers. A line drawn through the origin at an angle of 45° to the axes cuts the curves at points which give the percentage of potassium ethyl salt, and of potassium carbonate when the two layers become identical. Thus when the solution becomes homogeneous it contains 11·6 per cent.

potassium ethyl di-propyl-malonate, 20·6 per cent. potassium carbonate, and 67·8 per cent. water.

The above investigations all were carried out at 25° C. To ascertain roughly the effect of temperature, a mixture was made up which just gave a homogeneous solution at the above temperature. This solution was then heated slowly in a water-bath. A slight opalescence occurred at 25°·4 C., indicating that the solution was separating into two layers. Another portion of the same mixture was cooled to zero, but there was no apparent change.

Some of No. 2 mixture was put into a graduated tube and heated slowly in a water-bath, the volumes of the two layers being noted as follows:—

Temperature °C.	Vol. of Upper Layer (c.cs.).	Vol. of Lower Layer (c.cs.).
0·2	1·90	2·90
16·1	1·91	2·92
25·3	1·92	2·94
44·0	1·93	2·96
66·0	1·96	2·98
82·0	1·97	2·99

In order to observe more accurately the effect of temperature, the following mixtures were made up and brought into equilibrium at three different temperatures:—

No. of Mixture.	Weight of KEt. salt (gms.).	Weight of K_2CO_3 (gms.).	Weight of Water (gms.).
10	6·063	6·161	10·014
11	6·372	6·265	23·030
12	obtained by dilution.		

The following results were obtained:—

No. of Mixture.	Temperature °C.	Upper Layer.		Lower Layer.	
		Per Cent. KEt. Salt.	Per Cent. K_2CO_3.	Per Cent. KEt. Salt.	Per Cent. K_2CO_3.
10	2°	25·0	15·0	5·1	27·3
	25	31·0	11·35	2·6	26·55
	56	34·35	11·25	2·45	28·35
11	2	62·9	3·95	0·55	45·5
	25	65·9	4·35	0·45	44·2
	56	60·9	3·65	0·65	47·2
12	2	47·2	7·8	0·7	31·2
	25	48·6	7·35	0·4	31·6
	56	48·35	7·45	0·75	33·0

The diagram fig. 3 represents the above results. When the mixture is such that the upper layer contains about 48 per cent. of potassium ethyl di-propyl-malonate, the three iso-thermals are very close together, and hence at this concentration it is evident that the effect of temperature on the mixture is very small.

An attempt was made to find out the temperature at which the two solid phases and the two liquid phases were present at the same time, but owing to the extreme viscosity of the liquid this was not successful, though it was found to be probably in the vicinity of 46° C.

A sample of sodium ethyl di-propyl-malonate was also made, and its solutions were found to have the same properties with regard to a solution of sodium or potassium carbonate as the potassium ethyl salt had to either of these solutions.

Chemistry Department,
University of Edinburgh.

(*Issued separately July* 14, 1910.)

XXX.—On a New Method for differentiating between Overlapping Orders when mapping Grating Spectra. By Alexander D. Ross, M.A., B.Sc., Assistant to the Professor of Natural Philosophy in the University of Glasgow.

(MS. received May 19, 1910. Read June 6, 1910.)

Some time ago the author commenced an investigation of the Zeeman effect in the spark spectrum of certain rare elements. The research promised results of considerable interest, as the properties and relationships of these substances are as yet very imperfectly known, and certain of the elements show indication of high magnetic quality.* The following elements were examined: uranium, rubidium, gadolinium, samarium, scandium, dysprosium, neo-ytterbium, and lutecium. The author is greatly indebted to Sir William Crookes for his kindness in supplying the scandium, and to Professor Urbain of the University of Paris for the samples of the three last-mentioned elements.

The photographic work was carried out in the Physikalisches Institut of the University of Göttingen. An electric spark was used situated in the narrow pole gap of a Du Bois half-ring electro-magnet which had been lent for this purpose by the Berlin Akademie der Wissenschaften. The spectra were produced by a 21-foot radius Rowland grating having a ruled area of 6×2 inches, with 20,000 lines to the inch.

It was thought probable that difficulties might arise through inaccuracies in the published tables of the spectra of the rare elements. No such trouble has arisen in the cases of uranium, rubidium, gadolinium, samarium, and scandium. In one or two instances it has been found that lines ascribed to these elements have evidently been due to impurities in the salts used in the investigations of the spectra. A considerable number of hitherto unmapped lines have been photographed and their wave-lengths determined by measurement from neighbouring known lines. The position is, however, very different with regard to dysprosium, neo-ytterbium, and lutecium. The separate existence of the two last-mentioned elements was only discovered in 1907 by Professor Urbain as the result of a laborious fractional crystallisation of ytterbium salts.† Dysprosium, too, was first obtained in

* S. Meyer, *Sitz. Ber. der königl. Akad. zu Wien*, cx. p. 492 (1901), and G. Urbain, *Comptes Rendus*, cxlvi. p. 922 (May 4, 1908).

† G. Urbain, *Comptes Rendus*, cxlv. p. 759 (1907).

a comparatively pure state by the same chemist in 1906.* Their spectra are accordingly not yet fully known. The chief lines of the ultra-violet spark spectra of dysprosium,† neo-ytterbium, and lutecium ‡ have been measured by Urbain, while the arc-spectrum of dysprosium § has been investigated by Eberhard. As Urbain's tables of dysprosium, for example, contain only some 96 lines of the spark spectrum, while the Zeeman effect photographs obtained at Göttingen show over 3000 measurable lines, it became necessary to map the ordinary spectrum before proceeding with the research on the Zeeman effect.

The procedure followed in mapping is to photograph the spectrum of the substance under examination, and that of iron, on the same plates, the

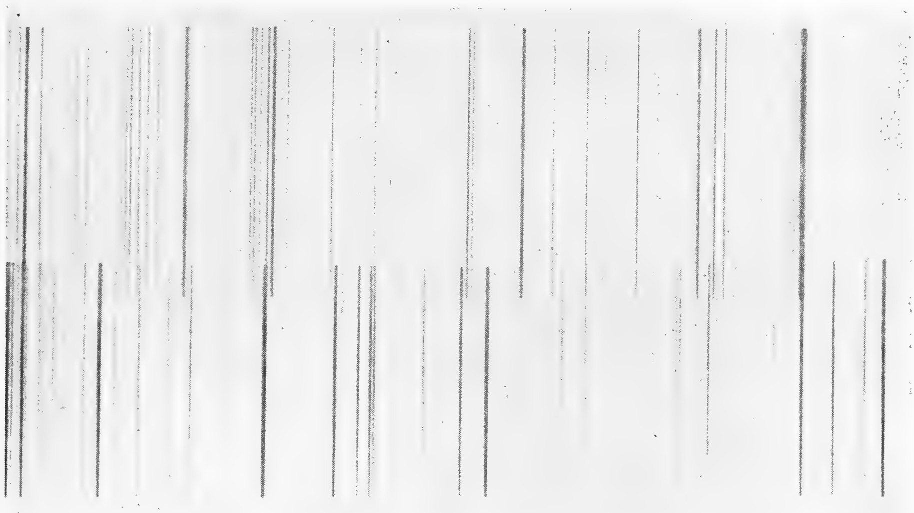


Fig. 1.—Spectra of Iron and Dysprosium.

two spectral bands overlapping one another to a greater or less extent. Fig. 1 shows the portion of the spectrum of dysprosium lying between wave-lengths 3930 and 4060, with the comparison spectrum of iron above. The wave-lengths of a large number of normal iron lines have been very accurately measured by Kayser ‖ and by Exner and Haschek.¶ These being taken as standards, the wave-lengths of the unknown lines on the plate can be determined by interpolation from measurements made with a comparator microscope.

* G. Urbain, *Comptes Rendus*, cxlii. p. 785 (1906).

† G. Urbain, *ibid.*, cxlvi. p. 922 (1908). ‡ G. Urbain, *ibid.*, cxlv. p. 759 (1907).

§ G. Eberhard, *Publik. des astrophysikalischen Observatoriums zu Potsdam*, Nr. 60 (1909).

‖ H. Kayser, *Drude's Ann.*, iii. p. 195 (1900).

¶ F. Exner and E. Haschek, *Wellenlängen-Tabellen*, Vienna (1904).

Grating spectroscopes, on account of the simple linear relation which exists between their dispersion and wave-length, are almost exclusively employed for this work. Owing, however, to the overlapping which occurs with the spectra of different orders, special precautions have to be taken to prevent lines being ascribed to a wrong part of the spectrum. The ordinary dry plate is fairly sensitive between the limits $\lambda = 2100$ and $\lambda = 5200$. Even beyond $\lambda = 2100$ bright lines may be photographed, although the absorption due to the gelatine film somewhat rapidly reduces the sensitiveness of the plate. The range of action is equally ill-defined

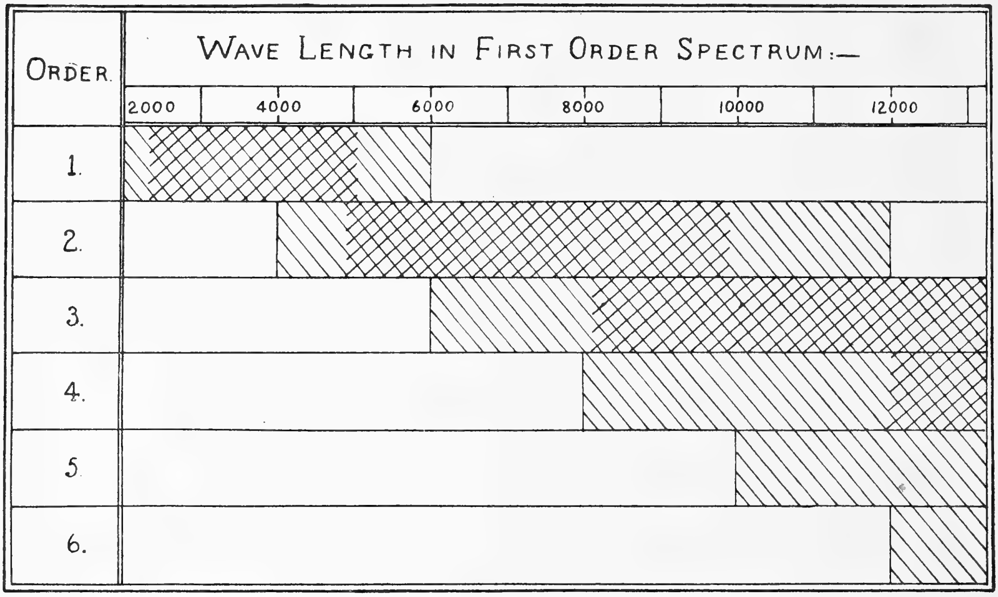


Fig. 2.—Overlapping Spectra.

on the less refrangible side. Ordinary plates will photograph the green and yellow regions of the spectrum if given sufficient exposure. Accordingly, the appearance of a spectrum line on an ordinary photographic plate is no evidence whatever that the wave-length lies between 2100 and 5200.

Fig. 2 shows the overlapping of spectra in the working range of the Göttingen instrument. The shaded portion of each spectrum shows the region within which photographs of lines may be obtained on ordinary plates with reasonable exposures. A deeper shading indicates the parts where the light is of comparatively high photographic activity. From the figure it will be seen that if one desired to photograph the pure second order spectrum from $\lambda = 2000$ to $\lambda = 6000$, precautions would require to be taken to cut out the first order spectrum from $\lambda = 4000$ to $\lambda = 6000$, the

third order from $\lambda = 2000$ to $\lambda = 4000$, and also bright lines in the fourth order from $\lambda = 2000$ to $\lambda = 3000$, and possibly in the fifth order from $\lambda = 2000$ to $\lambda = 2400$. The method adopted would probably be somewhat as follows:—

$\lambda = 4300$ to $\lambda = 6000$.—In photographing this region a glass absorption cell would be placed before the slit containing a freshly prepared $\frac{1}{4}$ per cent. aqueous solution of æsculin to which a trace of ammonia had been added.

$\lambda = 3550$ to $\lambda = 4300$.—A glass absorption cell with cobalt chloride solution would be most suitable here.

$\lambda = 3300$ to $\lambda = 3550$.—Employ a quartz cell with an aqueous solution of malachite green or of chrome alum.

$\lambda = 2000$ to $\lambda = 2300$.—A very narrow quartz cell containing a fresh solution of nitrosodimethylaniline in glycerine would cut out the visible end of the first order. The ultra-violet second order could then be photographed by a very prolonged exposure.

$\lambda = 2300$ to $\lambda = 3300$.—There is no suitable absorbent for removing the yellow of the first order which overlaps this region.

To map an unknown spectrum using the second order, one would therefore take, at least, the following five photographs:—(1) the ordinary spectrum, first, second, and third orders; (2) the second order spectrum with æsculin absorption; (3) the same with cobalt chloride; (4) the same with malachite green; and (5) the same with nitrosodimethylaniline. Even with these numerous photographs there would still be very considerable uncertainty in mapping the second order from $\lambda = 2300$ to $\lambda = 3300$. This would require to be cleared up as far as possible by reference to the plates belonging to the first and third orders in the first photograph. The work would not be simple, as the reduced dispersion in the first order causes difficulty when dealing with lines whose wave-lengths differ but little. The third order plates, too, are contaminated both with the second order blue-violet and the fourth order ultra-violet. The second order contamination is serious, as it is at the most photographically active part of the spectrum, and will include the wide-extending and intense cyanogen bands if carbon electrodes are used.

The difficulty of carrying out the procedure described above is the more important in view of the great cost it entails when one is investigating rare elements. As a result, the work hitherto carried out on these substances has frequently been confined in the main to first order photographs taken between the limits $\lambda = 2700$ and $\lambda = 4700$. Little has been done in photo-

graphing farther into the visible spectrum and into the ultra-violet, or in making the more accurate measurements which are possible in spectra of higher order.

It is proposed to explain here a new method which has been employed by the author for working in the higher orders without the necessity for taking so many photographs as were detailed above. The method depends on a study of the relative intensities of the components into which the spectral lines are resolved when the source of light is situated in a powerful magnetic field.

Zeeman has shown * that in general a single line breaks up into a triplet when the light is examined in a direction at right angles to the lines of force of the magnetic field. Thus a line of oscillation-frequency n gives rise to three lines whose frequencies are n and $n \pm \frac{eH}{4\pi m}$, where e is the charge of the electron, m its mass, and H the intensity of the magnetic field. This symmetrical triplet has each component line plane-polarised; the two outer components are polarised with their vibration directions perpendicular to the lines of magnetic force, and the central component is polarised parallel to the lines of force. According to the elementary theory of Lorentz,† if I_v, I_c, I_r be the intensities of the components taken in order, beginning with the most refrangible, then $I_v = \frac{1}{2} I_c = I_r$. When Zeeman effect observations are made with a diffraction grating, it is generally found that there is a greater or less departure from the above relationship. In some cases the three components may be of almost equal intensity, and in others the central component may be much more than twice as strong as the outer lines. Zeeman was the first to show that this apparent anomaly was produced by the selective action of the grating.‡ When the plane of polarisation of the incident light is parallel to the rulings of the grating the intensity of the reflected light is a maximum, and when the plane is at right angles to the rulings the intensity is a minimum. Hence it is only when the planes of polarisation of all the three components are inclined at the same angle of 45° to the grating lines that the polarising effect of the grating is eliminated and the true relative intensities of the components are apparent.

In many cases the action of the magnetic field is to produce more complicated types of resolution than the triplet. The elementary theory assumed a single electron, free from constraint, showing three degrees of

* P. Zeeman, *Phil. Mag.*, xliv. pp. 55 and 255 (1897).

† H. A. Lorentz, *Rapp. prés. au Congrès Intern. de Phys.*, *Paris*, iii. p. 29 (1900).

‡ P. Zeeman, *Zittingsversl. Kon. Akad. v. Wetensch.*, Amsterdam, Oct. 1907.

freedom; but to account for the complex cases which arise, couplings between the electrons are postulated. A fairly common type of resolution is the quartet in which the vibrations parallel to the lines of force do not preserve their period, but change into two vibrations, one of higher and one of lower frequency. In this case the vibration along the lines of magnetic force is associated with changes in directions at right angles, and the magnetogyric effect of the latter reacts on the original vibration. In this instance we should expect all four components to be of the same intensity.

Referring now to fig. 3, the first column shows a simple spectrum line and its resolution into a triplet or quartet with normal intensities of the

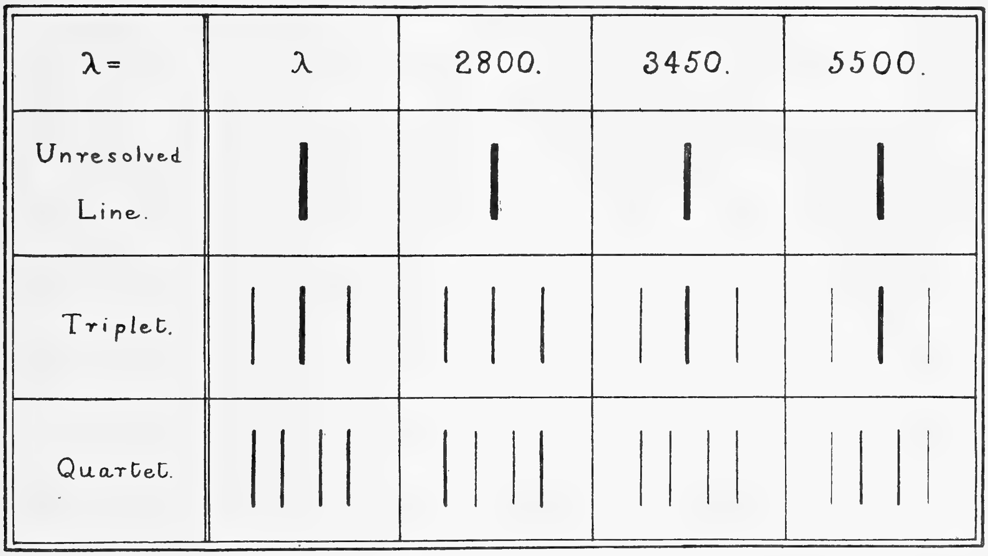


Fig. 3.—Intensities of Zeeman Effect Components.

components. In Zeeman effect investigations it is usual to employ a quartz lens for focussing the light on the slit of the spectroscope. Unless this lens is constructed in a special manner it will be found to have a dextro- or lævogyric action. Suppose that, in consequence of the optical activity of the lens, the polarisation planes of all the components of a resolved line are brought to an angle of 45° with the grating rulings when the wave-length of the line has some definite value, say 3450. The intensities of the components (as is shown in the third column of fig. 3) will have their normal ratios, which were illustrated in the first column. Now if the lens has a rotary action equivalent to that of a millimetre quartz plate which has been cut with its faces perpendicular to the crystalline axis, it will cause rotation of the polarisation planes for rays of light of wave-lengths 2800

and 5500 through angles of 45° more or less than the angle for light of wave-length 3450.* Thus, with lines of wave-lengths 2800 and 5500 we shall have in the one case a maximum intensity of the components polarised perpendicularly to the lines of force and a minimum intensity of the parallel polarised components, and in the other case the reverse effect. This is shown in the second and fourth columns of fig. 3 for the resolution of the simple triplet and quartet. Throughout the spectrum we shall have a gradual variation in the relative intensities of the components, each wave-length having a characteristic intensity ratio. Hence it is evident that if we had occurring close together on a photographic plate a line of wave-length about 2750 in the second order and one of wave-length about 5500 in the first order, the intensity ratios in a Zeeman effect photograph would at once enable one to assign the lines to their proper order.

The author has employed this method throughout in mapping the spark spectrum of dysprosium, and has found it to work satisfactorily. The spark was produced by a large Max Kohl induction coil worked by a motor mercury interrupter. Six large Leyden jars were placed in parallel with the spark, and a self-inductance connected in the spark circuit increased the sharpness of the metal lines while removing those due to the atmosphere. The electrodes were of specially purified carbon, and were impregnated before use with dysprosium chloride. The light was focussed on the slit by a quartz lens having a maximum thickness of about 6 mms., and—when used centrally—giving a gyratory effect equivalent to a millimetre plate of optically active quartz. Two photographs were taken for the purpose of mapping the spectrum, viz. (1) with the ordinary spectrum and having the iron spectrum partially superposed for comparison (see fig. 1), and (2) with the Zeeman effect. In the second case the source of light was situated in the 4·5 mm. pole gap of the electro-magnet in a field of about 25,500 c.g.s. units. Forty-seven plates, measuring 12×5 cms., manufactured by Schleussner of Frankfurt a/M., were employed in each photograph, and gave the entire spectral band between the points $\lambda = 2100$ and $\lambda = 13{,}000$ in the first order. The plates have been measured with an instrument specially built to the author's requirements by Zeiss of Jena, and modelled on their Abbé comparator microscope. The positions on the plates of all the chief dysprosium lines and of a large number of iron lines were measured. The wave-lengths of the former were then calculated, the spectrum order being determined from the Zeeman effect photographs in all cases where doubt might arise. Fig. 4 shows a part of a Zeeman effect plate, magnified about three times. The left-hand line is in the blue-green part of the first order, and the other line

* Gumlich, *Wied. Ann.*, lxiv. p. 349 (1898).

lies in the second order ultra-violet. The intensity ratios of the components at once enables these orders to be ascertained.

When employing the method herein described, it is advantageous to have the total effect of the quartz in the optical path about that indicated. In this way lines whose vibrations are about octaves show the greatest dissimilarity of intensity ratio in the photographs. The differentiation of orders is then most easily accomplished where the first and second orders overlap, and where the need for differentiation is greatest. There is also

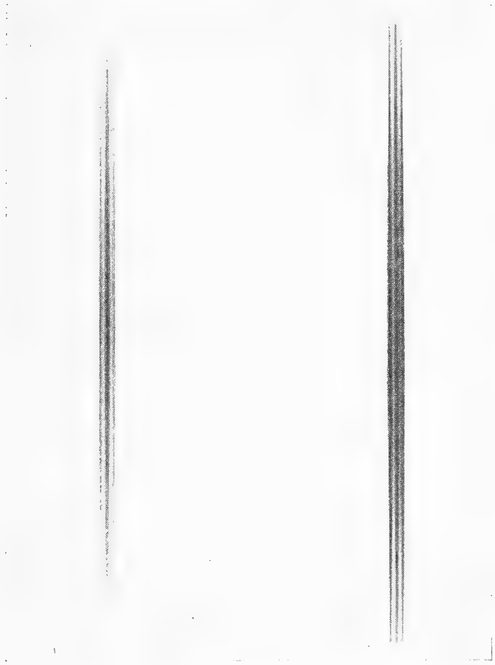


Fig. 4.—Differentiation of Orders.

a sufficiently great difference in the intensity ratios at the part of overlapping of the second and third orders. As a rule, there is no call for differentiation tests between higher orders.

It is of course well known that some lines show abnormalities in the distribution of light intensity in the various components. It is seldom, however, that the intensity ratios are so abnormal as to cause a risk of error in employing the method which has been described.

The method seems well adapted for use with gratings of large dispersion. Not only does it effect an enormous saving in expenditure when work is being carried out on the rarer elements, but the second photograph obtained with the Zeeman effect is of considerable intrinsic importance, and may be

the means of revealing series among the spectrum lines, or of showing relationships between the element under examination and others which have been similarly investigated.

The photographic part of the experiments made on this subject were carried out in the University of Göttingen, and the author is indebted to Professor Voigt for the use of the Rowland grating and its equipment. A research grant from the Carnegie Trust met the cost of the photographic materials and of the Zeiss comparator.

(*Issued separately July* 28, 1910.)

XXXI.—On Two Relations in Magnetism. By R. A. Houstoun,

M.A., D.Sc., Ph.D., Lecturer in Physical Optics in the University of Glasgow. *Communicated by* Professor A. Gray, F.R.S.

(MS. received April 27, 1910. Read June 6, 1910.)

The object of this paper is to derive two relations in magnetism. In substance they are not new, but in method of statement they are, and the derivation presented here is shorter and simpler than other methods.

Consider a ferro-magnetic wire hanging vertically inside a vertical solenoid with heating jacket, a pan being attached to the lower end of the wire for holding weights. Then, if hysteresis be neglected, the state of the wire may be regarded at any time as a function of the three independent variables T, F, and H,—temperature, stretching force, and magnetic field intensity. If T, F, and H suffer small changes, then the heat received by the whole wire is given by

$$dq = c dT + b dF + a dH.$$

Let B denote the induction in the wire, v its volume, and x the vertical displacement of its lower end. Then the work done on the wire when F and H are increased is $F dx + vH dB/4\pi$. Let U be the intrinsic energy of the wire and S its entropy. Then—

$$dU = dq + F dx + \frac{vH dB}{4\pi}$$

$$= \left(c + F\frac{\partial x}{\partial T} + \frac{vH}{4\pi}\frac{\partial B}{\partial T}\right)dT + \left(b + F\frac{\partial x}{\partial F} + \frac{vH}{4\pi}\frac{\partial B}{\partial F}\right)dF + \left(a + F\frac{\partial x}{\partial H} + \frac{vH}{4\pi}\frac{\partial B}{\partial H}\right)dH,$$

and

$$dS = \frac{c}{T}dT + \frac{b}{T}dF + \frac{a}{T}dH.$$

Since these are perfect differentials, we have the following six independent relations :—

$$\frac{\partial c}{\partial F} + \frac{\partial x}{\partial T} = \frac{\partial b}{\partial T} \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

$$\frac{\partial c}{\partial H} + \frac{v}{4\pi}\frac{\partial B}{\partial T} = \frac{\partial a}{\partial T} \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

$$\frac{\partial b}{\partial H} + \frac{v}{4\pi}\frac{\partial B}{\partial F} = \frac{\partial a}{\partial F} + \frac{\partial x}{\partial H} \quad . \quad . \quad . \quad . \quad . \quad (3)$$

$$\frac{1}{T}\frac{\partial c}{\partial F} = \frac{\partial}{\partial T}\left(\frac{b}{T}\right) = \frac{1}{T}\frac{\partial b}{\partial T} - \frac{b}{T^2} \quad . \quad . \quad . \quad . \quad . \quad (4)$$

$$\frac{1}{T}\frac{\partial c}{\partial H} = \frac{\partial}{\partial T}\left(\frac{a}{T}\right) = \frac{1}{T}\frac{\partial a}{\partial T} - \frac{a}{T^2} \quad . \quad . \quad . \quad . \quad . \quad (5)$$

and

$$\frac{\partial b}{\partial \mathrm{H}} = \frac{\partial a}{\partial \mathrm{F}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (6)$$

The differential coefficients of a, b, and c cannot be easily determined experimentally; hence we eliminate them by combining (1) with (4), (2) with (5), and (3) with (6), and obtain

$$\frac{\partial x}{\partial \mathrm{T}} = \frac{b}{\mathrm{T}},$$

$$\frac{v}{4\pi} \frac{\partial \mathrm{B}}{\partial \mathrm{T}} = \frac{a}{\mathrm{T}},$$

and

$$\frac{v}{4\pi} \frac{\partial \mathrm{B}}{\partial \mathrm{F}} = \frac{\partial x}{\partial \mathrm{H}}.$$

The first of these is the well-known relation between the coefficient of linear expansion of a wire and the cooling effect produced on stretching it.

The second relation states that if the induction in a wire increases with temperature, a is positive, and consequently the temperature will fall if H is increased; if the induction diminishes with temperature, the temperature of the wire will rise if H is increased. This theorem has been derived before by Lord Kelvin,* and in quite another way. He states the result differently,—that a substance in which the magnetism diminishes with temperature when drawn gently away from a magnet experiences a cooling effect; a substance in which the magnetism increases with temperature when drawn gently away from a magnet experiences a heating effect. This cooling and heating effect is probably always masked by the irreversible heating due to Foucault currents in the wire, and to the viscous resistance to the motion of the molecular magnets.

The third relation states, that if the induction increases when the wire is stretched, the length of the wire increases when it is magnetised, and *vice versa.* The connection between magneto-striction and the effect of stress on magnetism has been worked out several times, amongst others by J. J. Thomson in his *Applications of Mathematics to Physics and Chemistry.* The result nearest mine is given by A. Heydweiller.† His equation is

$$\frac{\partial \mathrm{E}}{\partial \mathrm{H}} = -\frac{\mathrm{E}^2}{4\pi} \frac{\partial^2 \mathrm{B}}{\partial p^2},$$

E being Young's nodulus for the wire and p the stress per unit area of

* "On the Thermo-elastic, Thermo-magnetic, and Pyro-electric Properties of Matter," *Phil. Mag.* (5) **5** (1878), p. 4.

† "Zur Theorie der magneto-elastischen Wechselbeziehungen," *Ann. d. Phys.* (4) **12** (1903), p. 602.

cross-section. My equation can be put in this form. Heydweiller starts with another term in the expression for the external work done on the wire, namely, $HBdv/4\pi$, but he makes approximations afterwards, which are equivalent to neglecting this term. Heydweiller's formula has been proved experimentally by H. Rensing,* and has been attacked by R. Gans.†

This paper will apply to the case of a twisted wire if F denote the twisting couple and x the angle of twist of the lower end.

* "Über magneto-elastische Wechselbeziehungen in para-magnetischen Substanzen," *Ann. d. Phys.* (4) **14** (1904), p. 363.

† "Magneto-striktion ferromagnetischer Körper," *Ann. d. Phys.* (4) **13** (1904), p. 634. "Zur Heydweillerschen Kritik meiner Formeln betreffend Magneto-striktion ferromagnetischer Körper," *Ann. d. Phys.* (4) **14** (1904), p. 638.

(*Issued separately July* 28, 1910.)

XXXII.—Observations of the Earth-air Electric Current and the Atmospheric Potential Gradient at Edinburgh. By G. A. Carse, D.Sc., and D. MacOwan.

(Read January 10, 1910. MS. received May 31, 1910.)

THE portable gold-leaf electrometer designed by C. T. R. Wilson * gives a means of measuring the charge upon and current through a conductor exposed to the earth's field and maintained at zero potential. Further, Wilson † has shown (1) that the dissipation factor (the ratio of current per minute to the corresponding charge) is approximately the same for a surface of turf and the metal test-plate; and (2) how to deduce the corresponding charge and current per square centimetre on the neighbouring ground-level. Wilson's measurements were made chiefly in a country atmosphere (near Peebles), and we have made observations with a similar instrument in town air, in and near Edinburgh, to find out whether the dissipation factor is notably affected by the purity of the atmosphere as regards smoke, etc.

Apparatus and Method.

For a full description of the electrometer and the method of using it, the reader is referred to Wilson's papers (*loc. cit.*). The gold leaf, which

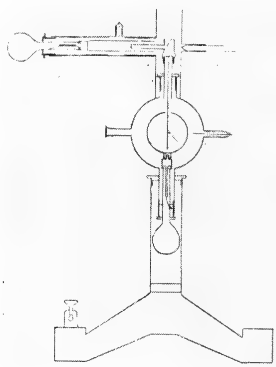


Fig. 1.

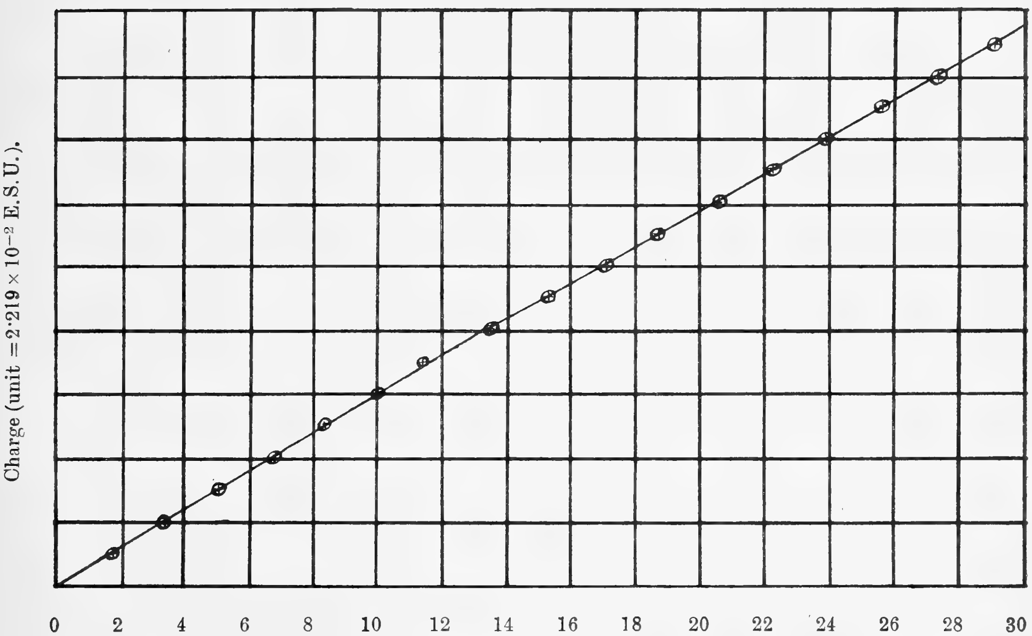
hangs within an inner case, kept at a constant potential by means of a quartz Leyden jar, has connected with it, by a vertical metal rod, a horizontal circular brass plate, the test-plate, 6·95 cms. in diameter, and this

* *Proc. Camb. Phil. Soc.*, xiii., 4, 184, 1905; *ibid.*, xiii., 6, 363, 1906.
† *Proc. Roy. Soc.*, A, lxxx., 537, 1908.

plate is surrounded by an earthed guard-ring, 16·8 cms. external diameter, the annular gap between being 0·23 cms. wide. A brass cover rests on the guard-ring and shields the test-plate; a horizontal metal rod connected to the vertical rod of the gold-leaf system is surrounded by a brass tube kept at a constant potential by another quartz Leyden jar; this system forms a cylindrical condenser—the compensator—the capacity of which can be varied by moving the brass tube parallel to its length.

The calibration curve, which gives the charge to be measured from the

CALIBRATION CURVE.



Compensator Reading.

Fig. 2.

compensator readings, was found to approximate very closely to a straight line, as in the case of Wilson's later apparatus. Fig. 2 shows a calibration curve, the abscissæ being compensator readings and the ordinates charges. The form of this curve is independent of the potential of the compensator tube, this potential being constant during the determination of the curve; and to allow for a change of potential, the ordinates have merely to be multiplied by a factor, viz. the ratio of the new potential to the old potential. This ratio can be obtained from the deflection of the gold leaf produced by a given motion of the compensator in the two cases. (If the compensator be moved inwards, say from 0 to 30, the gold-leaf system

being insulated, the charge on the compensator rod is proportional to the potential of the compensator tube. This charge is equal and opposite to that induced on the gold-leaf system, and is therefore proportional to the gold-leaf deflection, for the capacity of the leaf system in the final position is constant. Hence the ratio of the new potential to the old potential is the same as the ratio of the new deflection of the gold leaf to the old deflection.) This method is slightly different from that adopted by Wilson, but gives identical results, as is shown by the following data, the reduction factor being the ratio of the new deflection of the gold leaf to the deflection at standard potential. By altering the potential of the compensator the following series of factors were got:—

Wilson.	Authors.	Wilson.	Authors.
1·15	1·13	·99	·99
1·14	1·12	·99	·98
·93	·93	·86	·88
·93	·94	·86	·87
1·07	1·05	·93	·93
1·07	1·05		
·87	·89	·81	·83
·87	·89	·81	·82

In general, the sensitiveness of the gold leaf was from 2·5 to 3·0 microscope scale divisions per volt, and at the calibration one division of the compensator was equivalent to approximately $0·13 \times 10^{-2}$ E.S.U. of charge, so that, if readings be made to a tenth of a scale division, charges could be estimated to within $0·2 \times 10^{-3}$ E.S.U., *i.e.* $0·5 \times 10^{-5}$ E.S.U. per sq. cm. of test-plate.

When the electrometer was used for open-air observations it was placed on the wooden box in which the instrument was kept when not in use, and thus the test-plate was at a standard height (63 cm.) above the ground during the observation. To a terminal attached to the base of the instrument an earth-wire was connected. The method of observation and order of operations were the same as adopted by Mr Wilson; a full description is given in his paper (p. 538).

Reduction of Charge and Current through the Test-Plate to neighbouring Ground Values.

To determine the factor by which the charge and current per sq. cm. of the test-plate had to be multiplied to deduce the corresponding values for the neighbouring ground a large wooden test-plate was used. This test-plate was square, had an area of 966 sq. cm., and was surrounded by a

guard-ring separated from it by a gap 0·75 cm. wide. The surface of this test-plate was within 5 cms. of the ground; it was supported by insulating rods of sealing-wax, and could be connected to the test-plate of the electrometer by a shielded wire. Alternate observations were made of the charge on the small test-plate and on the large, care being taken that the operations were carried through quickly enough to prevent any gain of charge, due to atmospheric leak, from affecting the results. Observations for this purpose were taken only on days on which the earth's electric field was found to be comparatively steady: the average value of the ratio of the charge on the large test-plate to that on the small was found to be 6·50, this being the mean of twenty-nine comparisons. As the effective area of the small test-plate is 40·4 sq. cms., and that of the large one is 966 sq. cms., the surface density of the electrification on the small test-plate to that on the large was $\frac{966}{40 \cdot 4} \times \frac{1}{6 \cdot 50} = 3 \cdot 7$. Wilson, who devised and used this method, got the value 4·2, the dimensions and height of his instrument not being the same as ours.

Constancy of the Dissipation Factor for Different Surfaces and Different Heights.

Currents as measured directly by the large test-plate did not give so consistent results as with the small test-plate; but if the dissipation factor is constant throughout a height of 65 cms. (the height of the instrument above the ground), the currents can be at once deduced from those through the small test-plate. To test this, observations were taken alternately on the roof of the Physical Laboratory and on the grass in front of the Laboratory with the small test-plate, the difference of height being about 50 feet. The average dissipation factors for nine measurements of each did not vary by more than 10 per cent. of their mean; and as the difference of height is about twenty times the height of test-plate from the ground, it seems legitimate to conclude that the variation of dissipation in a height of 65 cms. would, if it exists at all, not affect the results.

Wilson has shown that the dissipation factor under given conditions is the same whether the surface be the metal of the test-plate or turf, or a cylindrical conductor of the same shape as the turf.

We have made further observations on this point in town atmosphere, in which, as will be shown later, the mobility of the ions is much less than in country air. We obtained our results by means of an artificial field. The instrument was placed on a table under a sheet of tinned iron, which was insulated and kept charged to a definite constant potential of about

100 volts by means of small accumulators. The field, about the same magnitude as the earth's normal field, was thus very steady, and not liable to fluctuations as when the earth's field was used. The dissipation factors for the test-plate and turf did not differ by more than 5 per cent. More detailed experiments are being made on this point.

The following tables give the results of the observations:—

Table I.

Date.	Time.	Number of Observations.	Charge per sq. cm. E.S.U.	Current per sq. cm. E.S.U.	Dissipation per cent. per min.	Potential gradient volts per metre.
1909.						
May 24	1.42–3.12 p.m.	13	$55{\cdot}1 \times 10^{-5}$	$\cdot 34 \times 10^{-5}$	·66	207
„ 26	12.29–2.11 „	5	43·0 „	·27 „	·72	163
„ 28	3.25–3.39 „	3	24·2 „	·26 „	1·01	92
June 1	2.45–3.15 „	5	55·6 „	·48 „	·89	210
„ 1	4.01 „	1	39·0 „	·12 „	·30	147
„ 3	2.40–2.52 „	2	60·5 „	·74 „	1·23	228
	Means		46·2 „	·37 „	·80	175

Table II.

May 26	2.33–2.54 p.m.	3	$41{\cdot}3 \times 10^{-5}$	$\cdot 23 \times 10^{-5}$	·56	156
June 1	3.22–3.57 „	5	50·3 „	·40 „	·82	189
„ 2	2.33–3.08 „	4	28·9 „	·00 „	·00	109
„ 3	1.57–2.36 „	5	63·5 „	·57 „	·91	239
„ 3	2.57 3.08 „	2	78·9 „	·89 „	1·14	298
„ 4	2.48–3.51 „	3	33·9 „	·27 „	·77	128
„ 7	2.11–3.06 „	3	36·3 „	·32 „	·83	137
„ 15	2.32–3.47 „	3	47·2 „	·58 „	1·21	178
	Means		47·5 „	·41 „	·79	179

Table III.

June 17	2.51–4.16 p.m.	12	$64{\cdot}4 \times 10^{-5}$	$\cdot 83 \times 10^{-5}$	1·29	241
„ 17	6.13–6.46 „	5	48·6 „	·92 „	1·92	184
	Means		56·5 „	·87 „	1·60	212

Table IV.

June 1	9.39–10.21 p.m.	3	$95{\cdot}2 \times 10^{-5}$	$1{\cdot}47 \times 10^{-5}$	1·54	359
„ 8	9.00–10.59 „	16	100·2 „	3·59 „	3·58	377
„ 15	9.17–10.49 „	11	88·9 „	2·68 „	3·04	335
	Means		94·8 „	2·58 „	2·72	357

Table I. contains the observations made on the roof of the Physical Laboratory, reduced to corresponding values at ground-level. Table II., the results of similar observations made on the grass in front of the Laboratory. The results in Tables III. and IV. were got from observations made respectively at Waverley Park, Newington, and the Blackford Hills, in the vicinity of the Royal Observatory. The values of the charges, etc. are the means for the group of observations denoted by the first three columns, while the numbers at the end of each table are the averages of the column.

The tables show a gradual increase in the dissipation factor as the distance from the central part of the town increases. The much smaller values of the dissipation factor in town air as compared with country air are probably due to the loading up of the ions by their becoming attached to dust particles, etc.

After thunderstorms we have observed the reversal of charge and of the direction of the field (this, of course, being well known in connection with the ordinary measurements with the water-dropper). Wilson has also noticed this on several occasions, as well as sudden jumps in the field each time flashes take place.

If the conductivity of the air be estimated from the measurements of current and potential gradient a value of about 2×10^{-5} E.S.U. is got, which is in agreement with the results of Pollock,* who, working in Sydney, has estimated the conductivity of the air there as of the order of 10^{-5} E.S.U., while Wilson's and Gerdren's values are about 10^{-4} E.S.U. for country air.

It is possible from the measurements of the charge to obtain values which give the order of the potential and charge of the surface of the earth.

Taking the average potential gradient as 190 volts per metre (the weighted mean of Wilson's observations and those in this paper), we have charge of the earth $= 4\pi r^2 \sigma (r = 6360 \times 10^5 \text{ cms.}) = -2{\cdot}56 \times 10^{15}$ E.S.U., and potential $= -4{\cdot}03 \times 10^6$ E.S.U., or $-1{\cdot}21 \times 10^9$ volts.

For the electrometer used in this investigation we are indebted to a grant from the Carnegie Trust, and for subsidiary pieces of apparatus to the Tait Memorial Fund; and we have to thank Mr C. T. R. Wilson, F.R.S., and Professor MacGregor, F.R.S., for kindly criticism.

Physical Laboratory,
University of Edinburgh.

* Australian Assoc. for the Adv. of Science Address, sect. A, p. 40, 1909.

(*Issued separately July* 28, 1910.)

XXXIII.—On Continuous, and Stable, Isothermal Change of State.

By Professor W. Peddie.

(MS. received June 3, 1910. Read June 6, 1910.)

1. THE three physical states of a substance, solid, liquid, and gaseous, are representable, as is well known, by means of three functions of pressure, volume, and temperature; say

$$f_1(p, v, t), f_2(p, v, t), f_3(p, v, t).$$

In the case of water-substance, James Thomson represented these functions as surfaces of his geometrical (p, v, t) model.

No attempt has been made to group these three functions in a single form $F(p, v, t)$, though various equations combining f_2 and f_3, either empirically or as deductions from theory, have been given; first, and notably, that of Van der Waals. In Thomson's model the process of gradual isothermal and isopiestic passage from one state to another is indicated by motion of the representative point along a line parallel to the axis of volume. This gradual passage, which represents the change occurring normally in the processes of solidification, liquefaction, or vaporisation, does not represent a strictly continuous physical process. The continuous change of volume occurs because the associated discontinuous change of molecular arrangement takes place in gradual instalments. A sudden change of density occurs on an instantaneously infinitesimal volume scale. It is now recognised that this normal process of passage from one state to another is dependent on the presence of suitable nuclei. In the absence of suitable nuclei, abnormal extension of the surfaces f_2 and f_3, at temperatures above the triple-point, is made evident by the phenomena of super-heated or infra-pressed water and of super-pressed or infra-heated steam. So also, abnormal extension of the surfaces f_1, f_2, and f_3, at temperatures below the triple-point, is made evident, in the case of f_2, by the existence of infra-pressed or infra-heated water; in the case of f_1, by the existence (?) of infra-pressed ice under suitable conditions analogous to those of Berthelot's experiment, or (?) of super-pressed nucleus-free ice; in the case of f_3, by the existence of infra-heated or super-pressed vapour.

2. Those cylindrical surfaces, which, in the (p, v, t) model, represent normal conditions during change of state, do not therefore correspond to the

true conditions which subsist in the passage, if it be presumed possible, of a constantly *homogeneous* substance from one molecular state to another. Here the word homogeneous is to be understood in the only sense in which it can be employed with reference to a molecularly constituted medium. If there be even a single effective nucleus present, and even if that nucleus be a portion of the same substance in a different state of average molecular aggregation from the remainder, the complex is, in this sense, non-homogeneous.

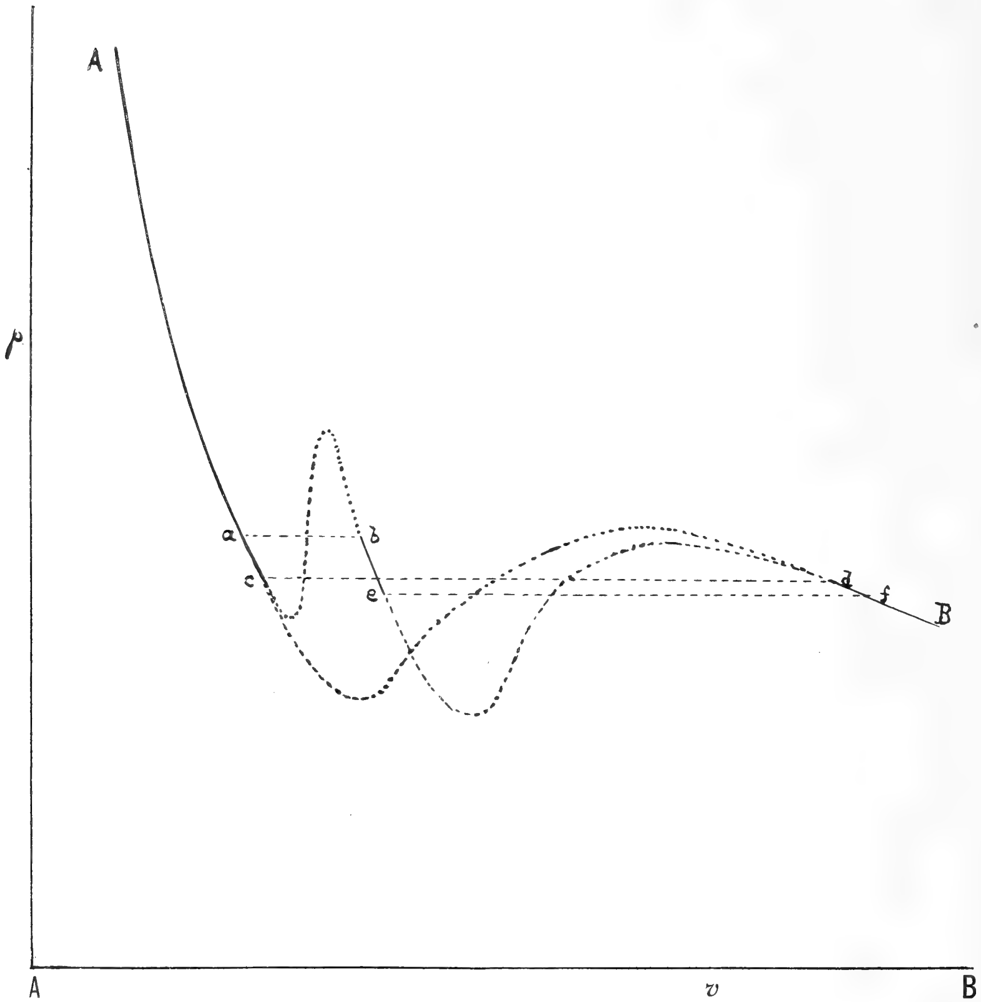
James Thomson's suggestion that the true surface, in the liquid-vapour region, passes, in continuation of its normal liquid portion, to the smaller pressure side of the cylindrical surface, and passes, in continuation of its normal vapour portion, to the greater pressure side of the cylindrical surface, both extensions finally curving right round and joining each other along a line of inflection, secured geometrical continuity, but, as Maxwell remarked, did not secure physical continuity, for there is necessarily physical instability in the intermediate region where pressure and volume increase or decrease together. Van der Waal's and other equations of state give a theoretical deduction of Thomson's form of representation.

The existence of physical instability would, on the molecular theory, be representable as the result of molecular repulsion increasing with average molecular distance; and the instability could be prevented, by external control, from manifesting itself in an explosion. The substance might be enclosed in a vessel provided with a screw plunger working with sufficient frictional resistance to prevent its rotation under the largest axial pressure supplied by the substance in the process of transformation. Just sufficient external force applied to the plunger would be followed by outward motion of the plunger, which in consequence would be removed from the required external action, so that its motion would cease as instantaneously as might be desired. If the external work performed by the substance during the expansion in this region were greater than the loss of internal energy, latent heat would have to be supplied. Thus, apart from difficulties arising from possible nucleation, the whole course of one of Thomson's isothermals might be traced in such a case.

3. Thomson's considerations are as applicable to the passage of a substance from the solid state to the liquid state, and from the solid state to the state of vapour, as to the passage between the liquid and the vapour states. Hence, below the triple-point—at which normally (*i.e.* with nucleation) all three states coexist stably—we have to recognise the existence of three regions of instability on any one isothermal.

As the point characteristic of the state of the substance moves along an

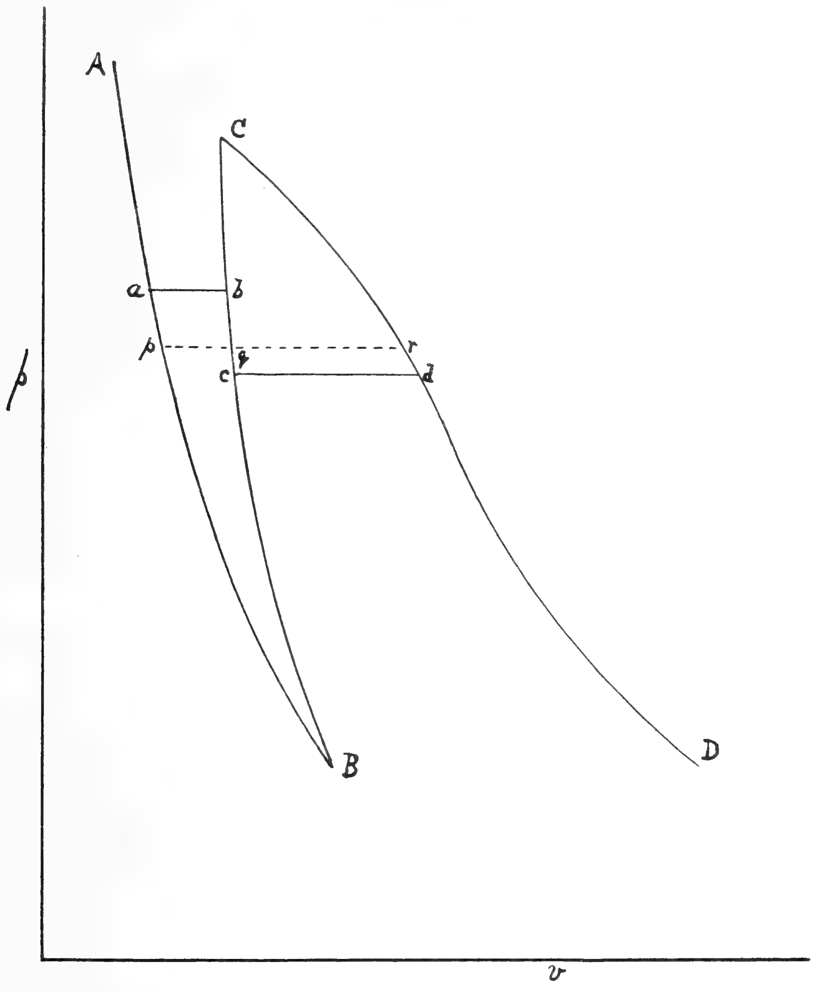
isothermal A B, the temperature being supposed to be below the triple-point, between A and a the isothermal is unique. At the point a, corresponding to the normal freezing-point pressure, the isothermal splits. The normal path is $a\,b\,e\,f$B. Between a and b solidification occurs; between b and e the solid expands. At e the solid evaporates, the normal sublimation point



having been reached; at f evaporation is complete, and from f to B the vapour expands uniquely. But, at a, in the absence of nucleation for solidification, the liquid course is continued to c, where the normal vaporisation pressure of the liquid is reached. Vaporisation then proceeds till d, a point on the vapour region of the isothermal, is reached; and the two courses rejoin at f.

In the case of water-substance, the experiments of Ramsay and Young,

which show that, at a pressure a little below the triple-point pressure, water evaporates at a lower temperature than ice, demonstrate the existence of the two courses between *a* and *f*. No ambiguity comes in at these points. Thus, in passing from B to *f* and onwards, the course is to *e* if solidification nuclei are present; it is to *d* if solidification nuclei are absent, while



liquefaction nuclei are present. But, if we postulate Thomson's waved form of isothermal between *a* and *b*, *c* and *d*, *e* and *f*, difficulty appears at the points *c* and *d*. The parts *a c* and *f d* are portions respectively of the waved isothermals joining *a* and *b*, *f* and *e*. From *d*, or beyond it, two waved isothermals must proceed respectively to the points *e* and *c*; from *c*, or beyond it, two waved isothermals must proceed respectively to the points *d* and *b*; and, in either case, the two isothermals correspond to the absence

of *all* nuclei. But then no physical distinction is left to account for the splitting of the isothermal.

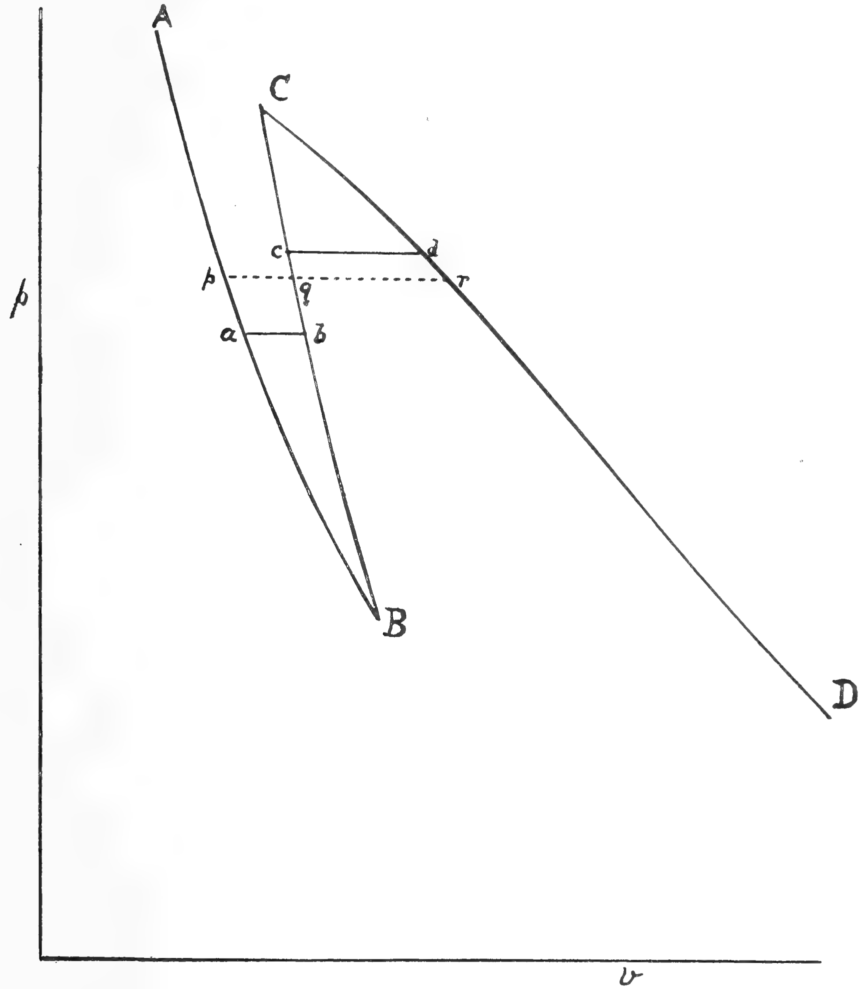
4. Physical, and even geometrical, continuity debars the adoption, above the triple-point, of a construction which is inapplicable below and at it. Therefore, if physical continuity of the three states of matter is to be regarded as a possibility, the following modification of Thomson's suggestion seems to be necessary.

Regard a complete isothermal as constituted of three portions A B, B C, C D, characteristic respectively of the liquid, solid, and vapour states. At the point B the liquid and solid states merge, without change of volume, but with change of molecular configuration. Similarly, merging of the solid and vapour states occurs at the point C. The diagram above refers to any substance such as water-substance below its triple-point; the diagram below refers to a substance of that kind above the triple-point. Change of molecular configuration, such as is here supposed to take place at the points B and C, is analogous to the change (now well known through Kelvin's discussion of Madan's observations) which occurs in crystals of chlorate of potash at a high temperature. The normal isothermal proceeds by the path A $a\,b\,c\,d$ D, the positions of $a\,b$ and $c\,d$ being such that the areas a B b and c C d are equal. If, in the absence of nuclei for solidification, the pressure is reduced to a value less than that corresponding to the position $a\,b$, but greater than that corresponding to the position $c\,d$, the production or introduction of effective nuclei will give rise to explosive solidification. If, subsequently, the pressure on the ice is reduced to a value less than that corresponding to the position $c\,d$, the presence of effective nuclei would cause explosive evaporation. On the other hand, if, in the liquid, nuclei for evaporation are present while nuclei for solidification are absent or ineffective, the normal isothermal will be A $p\,q\,r$ D, determined by the condition that the areas p B q and q C r are equal.

At the triple-point b and c coincide, ice water and steam being in mutual equilibrium. Above the triple-point, normal solidification takes place at a less pressure than that of normal vaporisation. Consequently any solid produced in the manner indicated by, say, the course $a\,b$ at once evaporates if vapour nuclei be present. That is to say, in the presence of effective nuclei for vaporisation, ice cannot exist above the triple-point. So, if ice always effectively provides nucleation for vaporisation, we have an explanation of the impossibility of superheating ice. It would be of interest to test the point farther by an attempt to superheat ice which has first, if possible, been super-pressed at a temperature immediately below the triple point.

Should vapour nuclei be present in the liquid, the normal path, and in that case also the only path, is by way of $p\,q\,r$.

5. The length of the range B C becomes smaller and smaller as the temperature rises above the triple-point, and it ultimately vanishes at the liquid-vapour critical temperature.



The lowering of the freezing-point, and the elevation of the sublimation-point, by increase of pressure, indicates that, below the triple-point, the length of the range B C increases as the temperature is lowered. Hence, if the view here indicated be correct, the existence of either an ice-water or an ice-vapour critical temperature, *of the same nature as the water-vapour critical temperature,* is not to be considered probable. Yet it may not be impossible that, at a sufficiently low temperature, by fusion of, say, the

liquid and solid states into a single state, latent heat may vanish. On the other hand, Tammann has shown that, at a temperature of -22° C. and under a pressure of over 2000 atmospheres, ice tends to assume two other crystalline forms; so that multiplication of states takes place.

It is to be noted that there is no region of instability on the complete isothermals corresponding to the postulated condition of entire absence of nuclei. To realise the condition of homogeneity referred to in section 2, it must be assumed that the molecules of the containing vessel exert appropriate forces upon the molecules of the substance.

To present the case of a three-state substance which expands in the process of liquefaction, and volatilises, below its triple-point, without the possibility of formation of liquid, it is only necessary to interchange the diagrams and the terms liquid and solid, etc.

6. The following reasoning establishes the law of equality of areas given above.

Except in so far as rapidity is concerned, the normal process of change of state takes place independently of the *amount* of nucleus present. Therefore the physical state of the substance is independent of the presence or absence of nuclei. Thus the same terminal conditions subsist whether we pass from p to r by the path $p\,q\,r$, or by the path $p\,\mathrm{B}\,q\,\mathrm{C}\,r$; and the amounts of work performed are identical, so that the areas $p\,\mathrm{B}\,q$ and $q\,\mathrm{C}\,r$ are equal. Another alternative path is $p\,a\,b\,q\,c\,d\,r$. Therefore $p\,q\,b\,a = q\,c\,d\,r$, and $a\,\mathrm{B}\,b = c\,\mathrm{C}\,d$.

(*Issued separately July* 28, 1910.)

XXXIV.—The Significance of the Correlation Coefficient when applied to Mendelian Distributions. By John Brownlee, M.D., D.Sc.

(MS. received February 22, 1910. Read January 24, 1910.)

1. At the present moment there is much discussion regarding the means by which properties are hereditarily transmitted from a parent organism to its offspring, and of the extent to which the Mendelian theory is capable of accounting for the facts. In this note it is not proposed to discuss the general question but to investigate the conditions under which the theory of correlation may be applied to Mendelian groupings. Two important papers on this subject have already been published: one by Professor Pearson, entitled "A Generalized Theory of Mendelian Inheritance";* the other, which is largely a criticism of this, by Professor Udny Yule.† In Professor Pearson's paper the results produced when two organisms with any number of pairs of different zygotes mate indiscriminately are fully considered. He finds that such a population once established is stable, and he then deduces the parental and fraternal correlation coefficients. He finds that the parental correlations are independent of the number of zygotes, and also that the coefficients are considerably inferior in value to the numbers actually found by observation. Professor Yule, in criticism, says that the observed value of the coefficients can be obtained if a certain amount of weight is given to the effect of the hybrid and recessive elements, and he gives a formula in which this result is exhibited.

2. Professor Yule's criticism suggests that if the Mendelian theory is true, great care will be required in interpreting the meaning of a correlation coefficient, and the purpose of this paper is to investigate how far values of the latter can be taken as representations of real relationships. As Professor Pearson has shown that the simplest Mendelian formula has the same regression as the more complex, it is unnecessary for me to repeat his mathematical proofs, the case of the mating of two organisms differing in one particular giving the information required.

3. Professor Yule has pointed out there are several varieties of correlation possible on a Mendelian basis. The chief, however, are, (1) where the hybrid has properties of its own differentiating it from either of its parents;

* *Royal Soc. Trans.*, 1903, p. 53.

† "On the Theory of Inheritance of Quantitatively Compound Character on the Basis of Mendel's Laws," by G. Udny Yule. Report of Conference on Genetics, published by Royal Horticultural Society of London.

and (2) where the dominant includes the hybrid. It is obvious that the correlation between parent and offspring will be much greater in the former case than in the latter. This argument will be made clearer if the elementary Mendelian formula is examined. In the first place, consider a population consisting of two pure races. Let them be denoted by (a, a) and (b, b) respectively and let (a, b) be the hybrid between them. Then the whole population may be expressed by a parentage of both sexes each represented by

$$x^2 \text{ (a, a)} + 2xy \text{ (a, b)} + y^2 \text{ (b, b)},$$

where x^2, $2xy$ and y^2 denote the numbers respectively of each type. If mating is random and fertility equal, we have offspring in the following proportions:—

x^2 (a, a)	mating with	x^2 (a, a)	gives	x^4 (a, a)
,,	,,	$2xy$ (a, b)	,,	x^3y (a, a) + x^3y (a, b)
,,	,,	y^2 (b, b)	,,	x^2y^2 (a, b)
$2xy$ (a, b)	,,	x^2 (a, a)	,,	x^3y (a, a) + x^3y (a, b)
,,	,,	$2xy$ (a, b)	,,	x^2y^2 (a, a) + $2x^2y^2$ (a, b) + y^2x^2 (b, b)
,,	,,	y^2 (b, b)	,,	xy^3 (a, b) + xy^3 (b, b)
y^2 (b, b)	,,	x^2 (a, a)	,,	x^2y^2 (a, b)
,,	,,	$2xy$ (a, b)	,,	xy^3 (a, b) + xy^3 (b, b)
,,	,,	y^2 (b, b)	,,	y^4 (b, b)

Adding together and arranging the terms, we have the population of offspring given by

$$x^2(x+y)^2 \text{ (a, a)}, \quad 2xy(x+y)^2 \text{ (a, b)}, \quad y^2(x+y)^2 \text{ (b, b)},$$

or the numbers of the offspring are in the same proportions as those of the parents; that is, the population is stable. Stability, then, depends on the number of the hybrid being equal to twice the geometric mean of the number of the pure races. It is also easily shown that even though these proportions are not originally present they at once appear.

4. When these figures are arranged so as to show the correlation from parent to child the following table is formed:—

Number of Parents of each Type.

Number of Offspring of each Type.	(a, a).	(a, b).	(b, b).
(a, a) . .	x^4+x^3y	$x^3y+x^2y^2$	
(a, b) . .	$x^3y+x^2y^2$	$x^3y+2x^2y^2+xy^3$	$x^2y^2+xy^3$
(b, b) . .	...	$x^2y^2+xy^3$	xy^3+y^4

Dividing by the common factor $x+y$ this becomes

PARENTS.

Offspring.	(a, a).	(a, b).	(b, b).
(a, a) .	x^3	x^2y	
(a, b) .	x^2y	$xy(x+y)$	xy^2
(b, b) .	...	xy^2	y^3

In this table the regression is linear, and therefore the correlation between parent and offspring may be determined by the product method and is given by

$$r = \cdot 5.$$

This shows that in a stable population the correlation is independent of the relative proportions of purer races. Now in ascertaining the correlation when the hybrid can be distinguished from the dominant the process given above is correct, but when the hybrid has no points of special distinction and must therefore be included in the dominant, the table is condensed to the following:—

PARENTS.

Offspring.	(a, a)+(a, b).	(b, b).
(a, a)+(a, b) .	$x^3+3x^2y+xy^2$	xy^2
(b, b) . .	xy^2	y^3

Here the regression is linear as shown by Professor Pearson, so that by the product method

$$r = \frac{y}{x+2y},$$

or,

$$= \cdot 333 \text{ when } x = y.$$

5. By repeating the above process the correlation of offspring with remoter ancestors can be easily evaluated. The first hypothesis, namely, that the hybrid is independent of the dominant, leads to correlation of ·5, ·25, ·125, etc., or, in other words, they are there given by Galton's Law of Ancestral Inheritance.* On the second hypothesis, the one investigated by

* Professor Pearson, *Royal Soc. Trans.*, vol. cxcv. p. 119, Table IX., "Exclusive Inheritance."

Professor Pearson, the same correlation coefficients are represented by $\frac{1}{3}$, $\frac{1}{6}$, $\frac{1}{12}$, $\frac{1}{24}$, etc. The well-known correlations found by observation have no obvious relation to either of these sets of figures, and if Mendel's law is proved to be efficient, some means of reconciling theory and observation must be found. In the subsequent pages the various factors which influence correlation will be considered under different heads.

Influence of the Different Methods of Calculating Correlation Coefficients on the Values Deduced if Mendelian Principles hold.

6. When the typical correlation table for parent and offspring given by Mendelian theory is considered it is evident that it shows several properties. If, say, the population consist of

(a, a), (a, b), (b, b),

then it may be tabulated in two ways:

Pure (a, a) containing two a elements;
Hybrid (a, b) „ one a element;
Pure (b, b) „ no a element;

or if the hybrid (a, b) resemble (a, a) in appearance we have (a, a)+(a, b) not having a pair of b zygotes and (b, b) possessing a pair of b zygotes. Both these forms have linear regression, and in consequence the product method of determining correlation is valid. The case already given may be repeated. Taking x equal to y the correlation of parent and offspring reduces to the following simple form:—

Parent.

Offspring.	(a, a).	(a, b).	(b, b).	Totals.
(a, a) .	1	1	...	2
(a, b) .	1	2	1	4
(b, b) .	...	1	1	2
Totals .	2	4	2	8

This table shows obvious symmetry, has evidently linear regression, and gives a correlation coefficient between parent and offspring of $r = \cdot 5$. But if the table is further condensed, that is, if (a, a) and (a, b) are considered as one class we have instead:—

PARENT.

Offspring.	(a, a)+(a, b).	(b, b).	Totals.
(a, a)+(a, b)	5	1	6
(b, b) . .	1	1	2
Totals .	6	2	8

Here again the regression is linear, and as the result we have

$$\cdot 333.$$

So far all is clear. In the last case, however, the distribution is markedly skew, and while the product method is applicable it is only applicable because the regression is linear.

7. It is therefore specially important to consider what happens when other methods of obtaining the correlation are employed. The chief of these is the fourfold division method. In a Mendelian instance such as this, the fourfold table seems specially applicable, but it assumes normality of distribution so that the fourfold table should give a higher correlation than $r = \cdot 3333$. As a matter of fact it does. The equation for determining r is

$$\cdot 62035 = r + \cdot 22747 r^2 + \cdot 04951 r^3 + \cdot 12279 r^4 + \cdot 001898 r^5 + \ldots$$

which gives

$$r = \cdot 53.$$

That is to say, the correlation is even higher than that obtained when the hybrid is distinguishable from the dominant, and in applying the fourfold method we have returned to or even gone beyond the uncondensed table. The higher coefficients are likewise increased and the series becomes

	Parental.	Grand-parental.	Great-grandparental.	Great-great-grandparental.
	·53,	·29,	·15,	and ·073
as against				
	·5,	·25,	·125,	and ·063.

8. If the simple Mendelian table be again considered, and if for the moment the distinguishing character of the hybrid and the dominant be assumed somewhat indefinite, we can make several tentative divisions, either bisecting the hybrid or dividing it into such divisions that one-fourth resembles the recessive as follows:—

Parent.

M. Offspring.	(a, a).	(a, b).		(b, b).
(a, a) .	4	2	2	
(a, b) .	2	2	2	2
	2	2	2	2
(b, b) .		2	2	4

Parent.

N. Offspring.	(a, a).	(a, b).		(b, b).
(a, a) .	4	3	1	
(a, b) .	3	5	1	3
	1	1	1	1
(b, b) .		3	1	4

giving fourfold distributions,

Parent.

M. Offspring.	
10	6
6	10

Parent.

N. Offspring.	
15	5
5	7

leading to correlations

$$\text{M.} \quad r = \cdot 441,$$
$$\text{N.} \quad r = \cdot 501,$$

when calculated by the fourfold method. Thus, again, Mendelian principles do not lead to low correlations but to figures approximately equal to those found by observation.

9. When more complex formulæ are taken the result is nearly the same. Supposing that instead of one pair of zygotes the parents possess two or three, that is, we have

Dominant.	*Recessive.*
Father.	Mother.
(a, a)	(b, b)
(c, c)	(d, d)
(e, e)	(f, f)

and let mating be random, then the correlation table in the case of two pairs of zygotes becomes

Parents.

Offspring.	Two Pairs of Dominants.	One Pair of Dominants.	No Dominants.
Two pairs .	25	10	1
One pair .	10	12	2
None .	1	2	1

admitting of two fourfold divisions, namely :—

B.	
25	11
11	17

C.	
57	3
3	1

The former of these gives

$$r = \cdot 45,$$

and the latter

$$r = \cdot 45,$$

both values much in excess of the ·333 given by the product method. When the three pairs are involved we have :—

Parent.

Offspring.	Three Pairs.	Two Pairs.	One Pair.	None.
Three pairs	125	75	15	1
Two pairs .	75	105	33	3
One pair .	15	33	21	3
None . .	1	3	3	1

This form is capable of three different fourfold divisions, namely :—

A.	
125	91
91	205

B.	
380	52
52	28

C.	
497	7
7	1

Giving

A. $r = \cdot 42$,
B. $r = \cdot 42$,
C. $r = \cdot 45$.

10. It is evident that when two and three pairs of zygotes are condensed we do not go straight back to the normal distribution. The reason of this is that the normal surface obtained when the elements are considered separately, represents something different from the surface which is condensed into the last tables.

If the parents be

a, a c, c	and	b, b d, d

then the offspring having two elements from the same parents are—

P.	Q.	R.
a, a d, d	c, c b, b	a, b c, d

which represent different things according as dominance exists or not; for if dominance exist R is included among those having apparently two pairs of dominant zygotes, while if the hybrid is distinct it is grouped with P and Q as containing two units from the same parent.

11. In addition to the methods just given Professor Pearson has also discovered two methods of determining correlation by means of what he calls contingency. It is not necessary to go fully into this part of the question. The manner in which the results given by these methods differ from those just considered is illustrated in the subjoined table. They are not in general suitable for simple Mendelian cases, as they depend for success on the number of divisions being much more numerous than these tables give.

Table showing the Correlation Coefficients calculated by different Methods where one, two, or three Dominant Zygotes occur in one Parent and a like Number of Recessive in the other.

	Product Method.	Mean Square Contingency.	Mean Contingency.	Fourfold Table.		
				A.*	B.*	C.*
One zygote . .	·333	·32	·37	...	...	·5
Two zygotes . .	·333	·33	·41	...	·46	·46
Three zygotes .	·333	·32	·39	·42	·42	·45

* See par. 9.

Results of Assortive Mating.

12. With the same notation as just used the most general form of correlation under a Mendelian system for assortive mating between husband and wife, if the standard deviation of each is equal, is the following:—

Husbands.

Wives.	(a, a).	(a, b).	(b, b).	Totals.
(a, a) .	m	$2r$	n	$m+n+2r$
(a, b) .	$2r$	$4p$	$2r$	$4(r+p)$
(b, b) .	n	$2r$	m	$m+2r+n$
Totals.	$m+2r+n$	$4(r+p)$	$m+2r+n$	

When the hybrid is distinct from the dominant the value of the correlation coefficient depends only on the value of m, n, or r, though in the case when the hybrid is not distinct the value of p exercises an influence on the result. In a typical simple Mendelian distribution of the population $m+n$ will be equal to $2r$ and p to r. Those values, however, do not give an immediately stable population, the standard deviation of the offspring being higher than that of the parents. This population, however, quickly tends to stability. On the other hand, if the population is immediately stable it is easily seen that p must be equal to n, for the first generation gives a parentage and offspring as below:—

PARENT.

Offspring.	(a, a).	(a, b).	(b, b).	Totals.
(a, a) .	$m+r$	$r+p$	...	$m+2r+p$
(a, b) .	$r+n$	$2(r+p)$	$r+n$	$4r+2p+2n$
(b, b) .	...	$(r+p)$	$m+r$	$m+2r+p$
Totals .	$m+2r+n$	$4(r+p)$	$m+2r+n$	$2m+2n+8r+4p$

and as the total is the same whether the addition is made by columns or by rows, the sum of each row must be equal to the sum of the corresponding column if the standard deviation remains the same.

Or,

$$m+2r+p=m+2r+n,$$

which requires that n shall be equal to p.

13. In the first place, the varieties of the correlation coefficients when $m+n=2p$ will be considered. In this case, changing the letters for convenience, the initial correlation table between husband and wife may be taken to be:—

HUSBANDS.

Wives.	(a, a).	(a, b).	(b, b).	Totals.
(a, a) .	$n-a$	n	a	$2n$
(a, b) .	n	$2n$	n	$4n$
(b, b) .	a	n	$n-a$	$2n$
Totals.	$2n$	$4n$	$2n$	$8n$

. . . . (a)

If the hybrid is distinct from the dominant the correlation of husband and wife is given by—

$$r = \cdot 5 - \frac{a}{n}.$$

If the dominant include the hybrid, then the table condenses to—

HUSBANDS.

Wives.	(a, a)+(a, b).	(b, b).	Totals.
(a, a)+(a, b) .	$5n-a$	$n+a$	$6n$
(b, b). . .	$n+a$	$n-a$	$2n$
Totals . .	$6n$	$2n$	$8n$

giving a correlation

$$r = \tfrac{2}{3}\left(\cdot 5 - \frac{a}{n}\right).$$

14. If the parentage be as in (a) the correlation table for parent and offspring is—

PARENT.

Offspring.	(a, a).	(a, b).	(b, b).	Totals.
(a, a) .	$\frac{3}{2}n-a$	n	...	$\frac{5}{2}n-a$
(a, b) .	$\frac{1}{2}n+a$	$2n$	$\frac{1}{2}n+a$	$3n+2a$
(b, b) .	...	n	$\frac{3}{2}n-a$	$\frac{5}{2}n-a$
Totals .	$2n$	$4n$	$2n$	$8n$

giving a correlation—

$$r_{\text{f.o.}}{}^{*} = \frac{3n-2a}{\sqrt{(4n \cdot 5n - 2a)}},$$

reducing if $a=0$ to $r=\cdot 5$, *i.e.* there is no assortive mating, or to

$$r_{\text{f.o.}} = \cdot 596 \text{ if } r_{\text{f.m.}} = \cdot 25.$$

15. The population given by the parentage (a) is evidently represented by offspring in the proportion

$$\tfrac{5}{2}(n-a)\ (\text{a, a}) + (3n+2a)\ (\text{a, b}) + \tfrac{5}{2}(n-a)\ (\text{b,b}),$$

* $r_{\text{f.o.}}$ signifies correlation of father and offspring.
$r_{\text{f.m.}}$ " " " " mother.

which has a higher standard deviation than the parentage, being equal in the latter case to ·5 and in the former to

$$\frac{5n-2a}{8n}, \text{ or, if } a=\tfrac{1}{4}n, \text{ to } \cdot 5625.$$

This latter value is not, however, constant with such mating, but increases gradually up to a limit.

16. In addition to the correlation coefficients the contingency coefficients have also been calculated in some instances to show the degree of correspondence of the two. It is seen that for the parental correlations they fall short of the former, but approach them closely when they arrive at great-grandparental correlations. Three sets of figures have been calculated for each case.

Case 1. That when there is no assortive mating.

Case 2. That when there is assortive mating with equal fertility and population not immediately stable.

Case 3. That when there is assortive mating with equal fertility and an immediately stable population.

17. Tables of Parental, etc., Correlations based on Different Hypotheses.

i. The hybrid separate :—

	No Assortive Mating.		Assortive Mating. $r=\cdot 25$.		Assortive Mating: Immediately Stable Population. $r=\cdot 125$.	$r=\cdot 25$.
	Product Method.	Contingency Method.	Product Method.	Contingency Method.	Product Method.	Product Method.
Parental	·5	·487	·589	·576	·563	·625
Grandparental . . .	·25	·242	·366	·343	·316	·391
Great-grandparental .	·125	·124	·234	·223	·178	·244
Grt.-grt.-grandparental .	·0625	·0624	·143	·142	·100	·153

ii. The dominant including the hybrid :—

	No Assortive Mating.	Assortive Mating. $r=\cdot 25$.	Assortive Mating: Immediately Stable Population. $r=\cdot 1875$.
Parental	·3333	·495	·548
Grandparental . . .	·1667	·307	·351
Great-grandparental .	·0833	·203	·236
Grt.-grt.-grandparental .	·0417	·141	·161

It is to be noted that column 2 in Table ii. gives almost exactly the figures found by observation and would thus appear a possible expression of the facts, though it is more probably a mere coincidence, as will be shown later.

18. Two more cases of importance remain to be considered: that where like mates unlike, and that where the dominant includes the hybrid. Taking that where like mates unlike and reversing the mating given in par. 13, we have :—

HUSBANDS.

Wives.	(a, a).	(a, b).	(b, b).
(a, a) .	a	n	$n-a$
(a, b) .	n	$2n$	n
(b, b) .	$n-a$	n	a

If the population of offspring be then found and the correlation calculated we find that—

$$r = \frac{1+2\frac{a}{n}}{2\left(3+2\frac{a}{n}\right)^{\frac{1}{2}}}.$$

TABLE OF VALUES. (*Hybrid distinct.*)

Value of $\frac{a}{n}$.	Correlation, Husband and Wife.	Correlation, Parent and Offspring.
·000	− ·500	·289
·125	− ·375	·347
·250	− ·250	·401
·375	− ·125	·452
·500	0	·500
·675	·125	·546
·750	·250	·593
·875	·375	·631
1·000	·500	·671

This table also gives the effect of assortive mating when it is positive as well as negative.

19. When the dominant includes the hybrid and the assortive mating is confined to the mixture we have then a correlation table as the following:—

HUSBANDS.

Wives.	(a, a).	(a, b).	(b, b).
(a, a) .	m	$2m$	n
(a, b) .	$2m$	$4m$	$2n$
(b, b) .	n	$2n$	m

which gives the parent and offspring table:—

PARENT.

Offspring.	(a, a).	(a, b).	(b, b).
(a, a) .	$2m$	$2m$	...
(a, b) .	$m+n$	$3m+n$	$2n$
(b, b) .	...	$m+n$	$m+n$

reducing to—

PARENT.

Offspring.	(a, a)+(a, b).	(b, b).
(a, a)+(a, b)	$8m+2n$	$2n$
(b, b) . .	$m+n$	$n+m$

or

PARENT.

Offspring.	(a, a)+(a, b).	(b, b).
(a, a)+(a, b)	$8+2\alpha$	2α
(b, b) .	$1+\alpha$	$1+\alpha$

$$\text{if} \quad \alpha = \frac{n}{m}.$$

Table of Values of the Correlation of Parent and Offspring for Different Values of $\frac{n}{m}$ by Fourfold Table Method.

Values of a.	Assortive Mating.	Parent–Offspring Correlation.
·500	·454	·666
·667	·287	·621
·750	...	·593
1·000	·000	·539
1·5	– ·315	·454
2	– ·525	·397

20. The values of the grandparental coefficients can likewise be evaluated, but the labour is somewhat greater than in the previous sections, and does not seem to promise any results beyond what can be surmised from the previous argument. In this case a moderate degree of assortive mating in the parents has apparently little effect on the correlation coefficients.

21. In general it is to be noted that a large variety of different values of the correlation coefficients arises on different hypotheses, and also that the correlation of parent and offspring differs greatly according to the kind of assortive mating of the parents, so that the value of the coefficient of assortive mating gives very little guide to the value of correlation between parent and offspring. It is also to be noted that the successive heredity correlation coefficients are not in an exact geometrical progression.

Effect of Parental Selection on the Correlation Coefficient.

22. The effect of parental selection has been investigated by Professor Pearson on the basis of the normal curve of error. On this basis it is shown that the higher the parental selection the lower the correlation coefficients. This, however, does not seem to follow on a Mendelian mechanism. Three cases occur on this basis which require to be considered separately: (1) Where the dominant is present in excess or defect; (2) where the hybrid is present in excess or defect; (3) where the recessive is present in excess or defect. These are very easily evaluated.

The correlation tables here, however, are different from those which go before. Regression is not linear, so that the product method does not give an exact but only an approximate value of the correlation coefficient.

23. Case I.—Let m (a, a)+2 (a, b)+(b, b) be the population of the selected parent and p {(a, a)+2 (a, b)+(b, b)} of the non-selected parent.

These are equal if $m+3=4p$; but p may be neglected as occurring in every term and therefore not affecting the result.

The correlation table for the selected parents and offspring, if mating be random, is then the following :—

SELECTED PARENTS.

Offspring.	(a, a).	(a, b).	(b, b).	Totals.
(a, a) .	m	1	...	$m+1$
(a, b) .	m	2	1	$m+3$
(b, b) .	...	1	1	2
Totals .	$2m$	4	2	$2m+6$

This gives

$$r_{\text{s.o.}}{}^{*} = \sqrt{\frac{6m+2}{m^2+14m+17}}$$

if the hybrid be distinct, or to

$$r'_{\text{s.o.}} = \frac{m+1}{2(m+2)}$$

if the dominant include the hybrid;
reducing if

$$m=1 \text{ to } r=\cdot 5,$$

and to

$$r'=\cdot 333,$$

respectively, as before seen to be the case.

The correlation table for the non-selected parent and the offspring is :—

NON-SELECTED PARENT.

Offspring.	(a, a).	(a, b).	(b, b).	Totals.
(a, a) .	$m+1$	$m+1$	...	$2m+2$
(a, b) .	2	$m+3$	$m+1$	$2m+6$
(b, b) .	...	2	2	4
Totals .	$m+3$	$2m+4$	$m+3$	$4m+12$

* $r_{\text{s.o.}}$ signifies the correlation of the selected parent and offspring.
$r_{\text{n.o.}}$ " " " non-selected parent and offspring.

This gives

$$r_{\text{n.o.}} = \frac{m+3}{\sqrt{\{2(m^2+14m+17)\}}}$$

if the hydrid be distinct;

$$r'_{\text{n.o.}} = \frac{1}{\sqrt{(3 \cdot m+2)}}$$

if the dominant includes the hybrid;
reducing if

$$m = 1 \text{ to } r = \cdot 5,$$

and

$$r' = \cdot 333.$$

24. CASE II.—In like manner, if the parentage be such that hybrid is in excess or defect we have, if the population of selected parents be

$$(\text{a, a}) + 2m, \ (\text{a, b}) + (\text{b, b}),$$

and the population of non-selected parents

$$p\{(\text{a, a}) + 2, \ (\text{a, b}) + (\text{b, b})\},$$

$$r_{\text{s.o.}} = \frac{1}{\sqrt{2(m+1)}}$$

if the hybrid be distinct;

$$r'_{\text{s.o.}} = \frac{1}{3(2m+1)}$$

if the hybrid be included in dominant.

The correlations in this case of the non-selected parents and offsprings are constant and identical with those where there is no selection, namely, $r = \cdot 5$ and $r' = \cdot 333$, for the correlation table for the non-selected parents when written out is as follows:—

NON-SELECTED PARENT.

Offspring.	(a, a).	(a, b).	(b, b)
(a, a) . .	$m+1$	$m+1$	...
(a, b) . .	$m+1$	$2m+2$	$m+1$
(b, b) .	...	$m+1$	$m+1$

and $m+1$ being a factor throughout, the result is not affected.

25. CASE III.—If the recessive be in excess or defect we take again,

the population of the selected parent, (a, a)+2, (a, b)+m (b, b),

and " " non-selected parent, $p\{$(a, a)+2, (a, b)+(b, b)$\}$.

From this the correlations, if the hybrid is distinct, are obviously the same as in Case I., but if the hybrid be included in dominant then we have—

$$r'_{\text{s.o.}} = \sqrt{\frac{4m}{3(m+1)(m+5)}}$$

$$r'_{\text{n.o.}} = \frac{(m+1)^{\frac{1}{2}}}{\sqrt{3}(m+5)^{\frac{1}{2}}}.$$

26. The values of the correlation coefficients on these bases as m varies are given in the following tables.

TABLE I.—CORRELATION OF PARENTS (SELECTED AND NON-SELECTED) AND OFFSPRING WHERE THE HYBRID IS DISTINCT FROM THE DOMINANT.

m.	Dominant or Recessive in Excess or Defect.		Hybrid in Excess or Defect.	
	$r_{\text{s.o.}}=\sqrt{\dfrac{6m+2}{m^2+14m+17}}$.	$r_{\text{n.o.}}=\dfrac{2(m+3)}{2(m^2+14m+17)}$.	$r_{\text{s.o.}}=\dfrac{\sqrt{2}}{2(m+1)}$.	$r_{\text{n.o.}}$ ·5.
·0	·343	·515	·702	·5
·25	·413	·507	·632	·5
·5	·454	·503	·577	·5
·75	·481	·501	·534	·5
1·00	·500	·500	·500	·5
1·5	·523	·502	·447	·5
2·0	·536	·505	·408	·5
2·5	·540	·510	·378	·5
3·0	·542	·516	·342	·5
4	·541	·525	·316	·5
5	·534	·534	·289	·5
6	·526	·544	·267	·5
∞	0	·702	0	

TABLE II.—CORRELATION OF PARENTS (SELECTED AND NON-SELECTED) AND OFFSPRING WHERE THE HYBRID IS INCLUDED IN THE DOMINANT.

m.	Dominant in Excess or Defect.		Hybrid in Excess or Defect.		Recessive in Excess or Defect.	
	$r'_{\text{s.o.}}$	$r'_{\text{n.o.}}$	$r'_{\text{s.o.}}$	$r'_{\text{n.o.}}$	$r'_{\text{s.o.}}$	$r'_{\text{n.o.}}$
	$r'_{\text{s.o.}}=\dfrac{m+1}{2(m+2)}$.	$\dfrac{1}{\sqrt{3}(m+2)^{\frac{1}{2}}}$.	$\dfrac{1}{\sqrt{3}(2m+1)^{\frac{1}{2}}}$.	·333	$\sqrt{\dfrac{4m}{3(m+1)(m+5)}}$.	$\dfrac{(m+1)^{\frac{1}{2}}}{\sqrt{3}(m+5)^{\frac{1}{2}}}$.
·0	·250	·408	·578	·333	·0	·258
·25	·278	·385	·471	·333	·225	·281
·50	·300	·365	·408	·333	·284	·301
·75	·318	·348	·365	·333	·319	·320
1·00	·333	·333	·333	·333	·333	·333
1·50	·357	·308	·289	·333	·350	·357
2·00	·375	·289	·258	·333	·356	·378
2·50	·388	·273	·235	·333	·356	·394
3·00	·400	·257	·218	·333	·353	·408
4·00	·411	·236	·192	·333	·344	·430
5·00	·429	·218	·179	·333	·333	·447
6·00	·437	·204	·160	·333	·236	·460
∞	·5	0	0	...	0	·577

27. Considering the values of the correlation coefficients in these tables, we see that uni-parental selection except when large makes little

difference in the correlation. Selection may raise or lower the correlation. In some cases there is a maximum and in others a minimum, these points being in general not far distant from the points of normal Mendelian distribution of the population. Selective mating is not, then, likely to interfere with the correlation coefficients to any appreciable extent except when the selection is stringent.

28. There are a few other cases which demand attention, some of which will be referred to when the actual figures are discussed, while some others are added in this place.

29. Case (A).—If both parents be equally selected and if the parentage is given being

$$\begin{array}{ccc} m\ (\mathrm{a,\ a}) & 2\ (\mathrm{a,\ b}) & (\mathrm{b,\ b}) \\ m\ (\mathrm{a,\ a}) & 2\ (\mathrm{a,\ b}) & (\mathrm{b,\ b}), \end{array}$$

we have as the correlation of either parent and offspring,

$$=\tfrac{1}{2}\sqrt{\frac{3m+1}{2(m+1)}}$$

when the hybrid is distinct.

Table of Values.

m.	r.	m.	r.
0	·353	1·5	·522
·5	·456	2	·577
1·0	·500	∞	·612

30. Case (B).—If the hybrid be present in normal numbers but the recessive present in defect and the dominant in corresponding excess. In other words, both parental populations consist of

$$(1+m)\ (\mathrm{a,\ a}) \qquad 2\ (\mathrm{a,\ b}) \qquad (1-m)\ (\mathrm{b,\ b}).$$

This gives a correlation coefficient when the hybrid is distinct of

$$r=\sqrt{\frac{2-m^2}{2(4-m^2)}},$$

m being always less than unity.

Table of Values.

m.	r.	m.	r.
0	·500	·6	·474
·2	·498	·8	·450
·4	·490		

31. Case (C).—Let the race be made up of such a population that a part only of the hybrid assumes dominant characters, that is, let it consist of parental populations of $(1+m)$ (apparently dominant), $(2-m)$ (hybrid), (1) (recessive), and let it mate indiscriminately.

CASE (a).—Let the hybrid offspring be distinguishable at birth, developing the resemblance to the dominant later, a condition frequently seen. The correlation table is as follows:—

PARENT.

Offspring.	(a, a).	(a, b).	(b, b).
(a, a) .	$2+m$	$2-m$	...
(a, b) .	$2+2m$	$4-2m$	2
(b, b) .	m	$2-m$	2

which gives a correlation coefficient between parents and offspring at birth of the latter,

$$r = \frac{\sqrt{2}}{(8+4m-m^2)^{\frac{1}{2}}}.$$

TABLE OF VALUES.

$m=0$	$r=\cdot500$	$m=1$	$r=\cdot426$
$m=\cdot5$	$r=\cdot453$	$m=1\cdot5$	$r=\cdot412$

32. CASE (b).—Let a normal population (a, a), 2 (a, a), (a, b), mate at random, and let dominance appear among the offspring later. The normal correlation table,

PARENT.

Offspring.	(a, a).	(a, b).	(b, b).
(a, a) .	2	2	...
(a, b) .	2	4	2
(b, b) .	...	2	2

then becomes—

PARENT.

Offspring.	(a, a).	(a, b)	(b, b).
(a, a) .	$2+m$	$2+2m$	m
(a, b) .	$2-m$	$4-2m$	$2-m$
(b, b) .	...	2	2

giving the same correlation coefficient as before, namely,

$$r = \frac{\sqrt{2}}{(8 + 4m - m^2)^{\frac{1}{2}}}.$$

With the increase of m the correlation becomes less. The values of r are given under Case (a).

Correlation Coefficients when more than two Races Mix.

33. So far, a mixture of two races alone has been considered. Many stocks of cattle, etc., are supposed to be derived from more than two, so that a brief consideration of how this affects the correlation values is necessary. With the same notation let the original races be—

(a, a) (b, b) (c, c).

Then the stable population with random mating is as before,

(a, a) + (b, b) + (c, c) + 2 (a, b) + 2 (a, c) + 2 (b, c).

A correlation table is then easily written down and is as follows:—

Parent.

Offspring.	(a, a).	(a, b).	(b, b).	(b, c).	(c, c).	(c, a).
(a, a) .	3	3				3
(a, b) .	3	6	3	3		3
(b, b) .		3	3	3		
(b, c) .		3	3	6	3	3
(c, c) .				3	3	3
(c, a) .	3	3		3	3	6

To evaluate the correlation the product method hitherto used is inapplicable, and the method of contingency must be employed. In the first place, on the supposition that all hybrids are distinct, we have $r = \cdot 597$, which is considerably higher than the value $r = \cdot 487$, found by contingency when only two types of parent are considered.

Secondly—

34. If a be dominant over b, b over c, and c over a (indicated in table by dotted lines), the coefficient when estimated by mean square contingency falls in value to ·425. This case is very suitable for a fourfold division,

and if a and b be gathered against c, allowing for dominance, the correlation coefficient rises to a value of $r = \cdot 51$, and when the mean contingency is used to $r = \cdot 56$.

Thirdly—

35. If b and c are both dominant over a and the hybrid (b, c) is distinct, the correlation becomes ·460.

Thus in all cases we have a higher figure than in the case where only two types intermingle. As Professor Pearson has shown, the figures in the latter case are quite independent of the number of zygotes, and the like will probably hold here.

Table showing the Correlation between Parent and Offspring in two and three Races.

	Two Races.		Three Races.	
	Correlation.	Contingency.	Contingency.	Fourfold Division.
Hybrid distinct . . .	·500	·487	·597	
Hybrid included in Dominant	·333	·316	·425	·51

The same effects will also be produced in this case by assortive mating and parental selection as in the previous cases.

Fraternal Correlation.

36. The question of fraternal correlation remains to be considered. As we have seen, uni-parental selection does not in general affect seriously the values of the correlation coefficients. Assortive mating is more powerful. The effect of the latter on fraternal correlation can be estimated as follows.

Consider a parentage of the following arrangement :—

Husbands.

Wives.	(a, a).	(a, b).	(b, b).
(a, a) .	3	4	1
(a, b) .	4	8	4
(b, b) .	1	4	3

This gives a correlation of ·25 between husbands and wives. With Professor Pearson let the average families be $4n$, and we get a family grouping as follows :—

Children.

Number of Times each Fraternal Group occurs.		(a, a).	(a, b).	(b, b).
Father (a, a) . .	3	$4n$	...	
	4	$2n$	$2n$	
	1	...	$4n$	
Father (a, b) . .	4	$2n$	$2n$	
	8	n	$2n$	n
	4	...	$2n$	$2n$
Father (b, b) . .	1	...	$4n$	
	4	...	$2n$	$2n$
	3	...	...	$4n$

On re-arranging we have each group occurring as in the table.

Brethren.

Number of Times a Group occurs.	(a, a).	(a, b).	(b, b).
3	$4n$		
8	$2n$	$2n$	
2	...	$4n$	
8	n	$2n$	n
8	...	$2n$	$2n$
3	...	...	$4n$

So that we can write the correlation table for brothers as follows:—

First Brother.

Second Brother.	(a, a).	(a, b).	(b, b).
(a, a) .	$3{\cdot}4n(4n-1)+8{\cdot}2n(2n-1)$ $+8n(n-1)$	$8(4n^2+2n^2)$	$8n^2$
(a, b) .	$8(4n^2+2n^2)$	$2{\cdot}4n(4n-1)$ $+24{\cdot}2n(2n-1)$	$8(4n^2+2n^2)$
(b, b) .	$8n^2$	$8(4n^2+2n^2)$	$3{\cdot}4n(4n-1)+8{\cdot}2n(2n-1)$ $8n(n-1)$

Or dividing by $4n$,

FIRST BROTHER.

Second Brother.	(a, a).	(a, b).	(b, b).
(a, a) . .	$22n-9$	$12n$	$2n$
(a, b) . .	$12n$	$32n-14$	$12n$
(b, b) . .	$2n$	$12n$	$22n-9$

This gives a correlation as below :—

Size of Family.		Assortive Mating $r_{\text{f.m.}} = \cdot 25$.	No Assortive Mating.
4	$n=1$	·407	·333
8	$n=2$	·508	·428
16	$n=3$	·515	·454
∞	$n=\infty$	·555	·500

That is, if the hybrid be distinct from the dominant, and an assortive mating of the parents equivalent to ·25 is assumed, the correlation coefficients quickly approach the figures given by observation.

37. Taking the dominant to include the hybrid we require a different parental grouping to give the necessary correlation, namely :—

HUSBANDS.

Wives.	(a, a).	(a, b).	(b, b).
(a, a) . .	7	8	1
(a, b) . .	8	16	8
(b, b) . .	1	8	7

This has a correlation of $= \frac{2}{3}(\cdot 5 - \frac{1}{8}) = \cdot 25$ when the dominant includes the hybrid. Proceeding as before, we obtain the table of fraternal correlation :—

FIRST BROTHER.

Second Brother.	(a, a).	(a, b).	(b, b).
(a, a) . .	$(48n-19)$	$24n$	$4n$
(a, b) . .	$24n$	$56n-26$	$24n$
(b, b) . .	$4n$	$24n$	$48n-19$

Or condensing,

First Brother.

Second Brother.	(a, a)+(a, b).	(b, b).
(a, a)+(a, b)	$152n-45$	$28n$
(b, b) . .	$28n$	$48n-19$

Which gives the correlation coefficients as in the following table :—

Size of Family.	$n=$	Correlation Co-efficients with Assortive Mating. $r=\cdot 25$.	Correlation as calculated by Prof. Pearson with no Assortive Mating.
4	1	·317	
8	2	·401	·333
16	3	·429	·364
∞	∞	·476	·407

The fraternal correlation is not therefore increased so much by assortive mating as the parental-offspring correlation is. The resulting figure is still in defect of observation.

38. The same process may be applied to ascertain the correlation coefficients when three races mix.

If a standard population is taken and the method just outlined applied we get the following correlation table :—

First Brother.

Second brother.	(a, a).	(a, b).	(b, b).	(b, c).	(c, c).	(c, a).
(a, a) . .	$16n-9$	$8n$	n	$2n$	n	$8n$
(a, b) . .	$8n$	$36n-18$	$8n$	$9n$	$2n$	$9n$
(b, b) .	n	$8n$	$16n-9$	$8n$	n	$2n$
(b, c) . .	$2n$	$9n$	$8n$	$36n-18$	$8n$	$9n$
(c, c) . .	n	$2n$	n	$8n$	$16n-9$	$8n$
(c, a) . .	$8n$	$9n$	$2n$	$9n$	$8n$	$36n-18$

Then if $n=1$ the contingency coefficient is $r=\cdot 449$, and when $n=2$, $r=\cdot 569$, much higher values, which will be further increased if assortive mating

exists in addition; and even when reduced by the inclusion of the hybrid with the dominant they must approach those given by observation.

39. The effect of parental selection on fraternal correlation remains to be considered. Referring to the parentages before given with reference to the correlation of offspring and parent, the two chief cases are given.

Case I. Let the parentage on both sides be m (a, a) + 2, (a, b) + (b, b), and let the pure zygotes (a, a) be in excess or defect; and,

Case II. Let the parentage on both sides be (a, a) + $2m$, (a, b) + (b, b), and let the hybrid (a, b) be in excess or defect.

Then if $n = 2$, *i.e.* if the family be 8 on an average, we have the fraternal correlation as in the accompanying table:—

Fraternal Correlation. (*Hybrid distinct.*)

Value of m.	Case I.	Case II.
·5	·333	·514
1·0	·428	·428
2·0	·523	·347
3·0	·572	·314

40. Thus, such selection as that when the dominant is in excess or the hybrid is in defect tends to raise the correlation, while the opposite condition tends to lower it. If both conditions exist, and if the parentage be such that the dominant is twice as numerous and the hybrid half as numerous as in the stable population, we have $r = ·70$ when hybrid is distinct and $r = ·40$ (product method) when dominant includes the hybrid.

Consideration of Actual Cases.

We have seen that many different factors affect the value of the correlation coefficients. What effect these have practically can only be estimated in a few cases. Professor Pearson has considered three cases of colour inheritance, namely:—

1. Coat colour in horses.*
2. Coat colour in cattle.†
3. Coat colour in greyhounds.‡

Each of these cases will be briefly discussed and the divergences of value in the correlation coefficients explained as far as possible on the basis of what has gone before.

* *Roy. Soc. Trans.*, vol. cxcv. p. 92. *Biometrika*, vol. i. p. 361; vol. ii. p. 230 *et seq.*

† *Biometrika*, vol. iii. p. 245 *et seq.*

‡ *Ibid.*, vol. iv. p. 427 *et seq.*

Coat Colour in Horses.

This case may well be considered first, as the data are large and probably accurate. Stud books giving the colour and pedigree of the horse have been in existence for many years, while the value of the animals and the great interest which exists in breeding combine to give the facts authority.

To find the correlation Professor Pearson has divided the parents and offspring into groups of Bay and Darker, and Chestnut and Lighter, and calculated the coefficients by the fourfold method; the coefficients as determined by him are as follows:—

Inheritance of Coat Colour in Horses.

Parental	·5216
Grandparental . . .	·2976
Great-grandparental . .	·1922
Great-great-grandparental .	·1469

Now brown and bay seem both dominant over chestnut and white,* at least to all intents and purposes. Chestnut with chestnut breeds true, and brown or bay mating with chestnut breeds in the first instance dark. The relations of brown and bay do not concern us, being both dominant. The number of pale horses not chestnut is so small that it may be neglected as not affecting the result to any appreciable extent. The proportion of these colours present is roughly that of three dark horses to one chestnut, though it must be borne in mind that this has nothing directly to do with Mendelism, but represents simply the proportions which find favour at present among those who breed horses.

It is worth while reproducing the fourfold tables. That of parent and offspring is as follows †:—

	Bay or Darker.	Chestnut or Lighter.	Totals.
Bay or darker . .	631	125	756
Chestnut or lighter .	147	147	294
Totals . . .	778	272	1000

This table at once reminds us of that already found from Mendel's theory, namely (pars. 6 and 7):—

* Bateson, *Mendel's Principles of Heredity*, p. 124.

† *Roy. Soc. Trans.*, vol. clxxv. p. 35.

PARENT.

Offspring.	Dominant.	Recessive.	Totals.
Dominant .	5	1	6
Recessive .	1	1	2
Totals .	6	2	8

which when evaluated by the fourfold method gives $r = {\cdot}53$ as the correlation. As a matter of fact the table just quoted gives $r = {\cdot}54$.

If the highest ancestral coefficient is now examined we find some difference. The table for great-great-grandparental inheritance *—

GREAT-GREAT-GRANDPARENTS.

Offspring.	Bay and Darker.	Chestnut and Lighter.	Totals.
Bay and darker . .	497	252	749
Chestnut and lighter .	130	99	229
Totals . . .	627	351	978

is marked by the presence of a great excess in chestnut horses. As before shown (par. 25),† this tends to raise the correlation of parent and offspring. The effect of this, however, on succeeding generations may be here inquired into. In the case in point we have approximately one-third of the parentage recessive. The remaining two-thirds may be divided in two ways: it may be taken as of pure Mendelian composition, that is, we have one case of pure dominant and two of hybrid dominant; on the other hand, considering that the pure horse may be a better animal than the hybrid, and therefore more likely to be chosen for breeding purposes, we may assume that the number of pure and of hybrid dominants is equal. The parentages on this hypothesis will then be:—

2 (a, a) 4 (a, b) 3 (b, b) (A.)

and

2 (a, a) 2 (a, b) 2 (b, b) (B.)

The former (A) will probably give the dominant in defect and the latter (B) in excess, so that some value between the results obtained on these two hypotheses may be taken as true.

* *Biometrika*, vol. ii. p. 255.

† *Cf.* also par. 4.

The first generation of parentage (A) mating freely gives offspring in the following proportions:—

PARENT.

Offspring.	(a, a).	(a, b).	(b, b).	Totals.	
(a, a) .	8	8	...	16	
(a, b) .	10	18	12	40	(a).
(b, b) .	...	10	15	25	
Totals .	18, or 2×9	36, or 4×9	27, or 3×9	81	

Which shows that the hybrid offspring are in number twice the geometric mean of the pure races as should be (par. 3). To obtain the next generation with a like parentage an increase in the number of the pure races is required, so that the last table becomes:—

PARENTS.

Offspring.	(a, a).	(a, b).	(b, b).	
(a, a) .	10	10	...	20 (2×10)
(a, b) .	10	18	12	40 (4×10)
(b, b) .	...	12	18	30 (3×10)

Let these mate freely and we have for the correlation table of grandparents and grand-offspring the following distribution:—

GRANDPARENTS.

Grand-offspring.	(a, a).	(a, b).	(b, b).	Totals.	
(a, a) .	300	380	120	800	
(a, b) .	475	895	630	$2000\{\sqrt[2]{(800 \times 1250)}\}$	(b).
(b, b) .	125	525	600	1250	
Totals .	900	1800	1350		

The correlations in these two cases are respectively (*a*) $r = \cdot 578$, and (*b*) $r = \cdot 330$.* This process may be continued indefinitely, and likewise results may be obtained on the second hypothesis.

If the excess of recessive be maintained for four generations the correlations are as follows:—

	A.	B.
Parental	·578	·638
Grandparental . . .	·330	·451
Great-grandparental . .	·200	·309
Great-great-grandparental .	·110	·216

But the heredity has not been quite this. The proportion of dark and light horses in each generation when means of each parentage are taken has altered in the following manner:—

	Dark.	Light.	Total.
Great-great-grandparents .	641	359	1000
Great-grandparents . .	664	336	1000
Grandparents	712	288	1000
Parents	728	272	1000

So that for two generations the proportion of recessive is one-third and above, and for the last two approaching the ratio of one-quarter, though, as before remarked, this is not of a Mendelian origin. If we calculate, then, the correlations for the great-great-grandparents on the hypothesis that the recessive is equal to one-third of the total for two generations and to one-quarter of the total during the next two generations, and for the grandparents that on the hypothesis that the recessive numbers one-third for

* These and the subsequent correlations have been obtained by the fourfold method though not by the full process. They have been calculated by the formula,

$$r = \sin \frac{\pi}{2} \frac{2}{\sqrt{1+k^2}} \text{ when } k^2 = \frac{4abcd\mathrm{N}^2}{(ad-bc)^2(a+d)(b+c)}$$

and where the fourfold division is

a	b
c	d

This formula gives results very near the truth. When those coefficients, previously calculated in this paper by the full method, were checked by the method here referred to, the result has been so close that in the present instance where many coefficients are required the extra labour of calculation has not seemed necessary.

two generations and one-quarter thereafter, the following correlations are obtained.

	A. One Generation ·33 and two ·25 Recessive.	A. Two Generations ·33 and two ·25 Recessive.	B. One Generation ·33 and two ·25 Recessive.	B. Two Generations ·33 and two ·25 Recessive.
Parental	·578	·578	·638	·638
Grandparental	·315 (·299) *	·340	·351	·451
Great-grandparental . .	·163 (·159) *	·171	·185	·241
Great-great-grandparental .	...	·089	...	·124

It is thus seen that high ancestral coefficients may arise simply from the kind of mating, and when it is noted that even in recent years chestnut horses are present in excess of one-quarter it will be seen that the values given in this table should be exceeded. In fact, the whole is capable of explanation as a result of Mendelism and of method of calculation. In addition to the effects ascertained assortive mating must be considered. If this consists of an excess of like mating like it will raise the correlation (par. 13). Such is the probable mating, and so it is not necessary to assume that the ancestral coefficients are high because of the nature of inheritance; a simple zygote formula is quite sufficient to explain the facts.

Coat Colour in Cattle.

In considering the value of the coefficients of inheritance in coat colour among cattle it is first necessary to see how far the coat changes can be expressed by a Mendelian law. In this instance we have the dominant group apparently divided into three classes: (1) red, (2) red with a little white, and (3) red and white, all of which seem for present purposes the same. In the accompanying table all the matings are given on the assumption that the red class is uniform.

The red class when mated with the white give in general roan, so that in this case the hybrid is distinct. If we represent red by (R, R), white by (W, W), roan will be (R, W). Considering further the mating of red and white we get out of 135 cases 128 roan calves, while the remaining 7 are red. Such a result might be expected on a Mendelian basis. All reds cannot be alike, nor all whites. There must be some variation among them;

* According to the method in which the population of parents is adjusted.

TABLE OF PARENTS AND OFFSPRING.
Shorthorn Cattle.

Mating of Parents.	Totals.	Colour of Offspring.				
		Red.	Roan.	White.	Number of White to be expected.	
Red with Red . .	440	413	27	0	·44	(1)
„ Roan . .	1046	521	521	4	16	(2)
„ White .	135	7	128	0	1·45	(3)
Roan with Roan .	514	152	278	84	app. 142	(4)
„ White .	74	3	47	24	app. 48	(5)
White with White .	3	...	...	3	...	(6)

it is almost inconceivable that any zygote could divide so as to result in two absolutely equal zygotes, so that the fact that some red calves are found does not mean that a zygote mechanism is impossible. But if red sometimes dominates white, a small percentage of the reds must be (R, W) in constitution. When red mates with red we have out of 440 matings 413 red and 27 roan calves. Making this a basis of calculation and taking the red cattle consisting of a (R, R), and h (R, W), when h represents the mixed zygotes among the red animals, we have a (R, R)$+h$ (R, W), all apparently red, mating at random. The resulting calves will be—

$$a^2(\mathrm{R, R}) + ah(\mathrm{R, R}) + ah(\mathrm{R, W}) + \frac{h^2}{4}(\mathrm{R, R}) + \frac{h^2}{2}(\mathrm{R, W}) + \frac{h^2}{4}(\mathrm{W, W}),$$

or,

$$\left(a+\frac{h}{2}\right)^2(\mathrm{R, R}) + h\left(a+\frac{h}{2}\right)(\mathrm{R, W}) + \frac{h^2}{4}(\mathrm{W, W}),$$

which gives

$$\left(a+\frac{h}{2}\right)^2 = 413 \quad . \quad . \quad . \qquad (1)$$

and

$$h\left(a+\frac{h}{2}\right) = 27 \quad . \quad . \qquad (2)$$

from (1)

$$a+\frac{h}{2} = 20 \cdot 32,$$

from (2)

$$h = 1 \cdot 33.$$

So that in this mating $\frac{h^2}{4}$ or ·44 white calves might be expected.

Further

$$\frac{h}{a} = \cdot 0627.$$

Returning now to the mating of red and white we have the red parentage

$$a\ (R, R) + h\ (R, W);$$

and the white parentage

$$(a+h)\ (W, W)$$

should give

$$\tfrac{1}{2}h(a+h)\ (W, W).$$

In this case $(a+h)^2 = 135$, so that 1·45 white cattle should occur. When red mates with roan in like manner about sixteen white calves should occur though only four are found.

These figures are, of course, based on the first group, and if all the groups were given equal weight the number of white to be expected in groups two and three would be less, but then likewise also the numbers of roan in group one.

Two more matings require to be considered, roan and roan, and roan and white. In both these cases it is to be noted that only about half the white turns up which might be expected. This is in line with what has been observed as regards expected whites. It is not necessarily against Mendelism. Many extracted races are comparatively sterile, and if such be proved with regard to white shorthorns it would explain not only the defect in expected whites but also their unpopularity from a breeder's point of view.

Apart altogether from refined theories the general aspect can be explained roughly on a Mendelian basis, and if it is so then the correlation coefficients may be calculated on the principles already enunciated. As the hybrid is distinct the correlation should be (par. 6) ·5. Several factors, however, lower this; the dominant is greatly in excess and the recessive in defect (par. 30). This makes a marked difference. Also the recessive only appears in half the number expected when roan and roan, etc., are mated; this also lowers the correlation. For let the population be 2 (a, a), 4 (a, b), 2 (b, b), and let the recessive only appear only in half numbers, and we get a correlation table:—

Parents.

Offspring.	(a, a).	(a, b).	(b, b).
(a, a) .	8	8	
(a, b) .	8	16	8
(b, b) .	...	4	4

Which gives $r = ·454$ instead of $r = ·5$.

Again the mating is unusual. The table of sires and dams for an offspring of colts is as follows:—

SIRES.

Dams.	Red.	Roan.	White.
Red .	197 (217)	277 (276)	45 (27)
Roan .	221 (210)	271 (268)	12 (26)
White .	34 (26)	28 (33)	0 (3)

Alongside the actual figures are placed within brackets those required by random mating. It is seen at once that red matings with white or dominant with recessive are much more numerous than required by chance, a further cause of low correlation (par. 17).

The values of the coefficient may now be considered. By the product method $r = \cdot 363$. By the fourfold table when normality is assumed $r = \cdot 46$. If, however, the parentage is taken first, the expected Mendelian population of offspring calculated and the correlation evaluated, we find the aberrance of type among the offspring (the absence of sufficient whites) has lowered the correlation to some extent. A typical offspring for the parentage gives $r = \cdot 383$. Professor Pearson by the contingency method gets $r = \cdot 40$, not greatly in excess of ·363, and probably arrived at because he has made the calculation with the red group divided into three classes, and with this increase of division the contingency may be expected to give higher figures. In using the contingency method it is clearly not legitimate to break up one class without breaking up others, especially if one class, as seems here, is arbitrarily divided.

One point remains to be considered: What is the correlation between parent and offspring among the different divisions of the dominants, as these, though of the same strain, have considerable variation of colour?

The table for sires and colts is as follows:—

SIRES.

Colts.	Red.	Red with little White.	Red and White.
Red	95	27	13
Red with little White .	14	6	6
Red and White . .	8	4	11

Calculated by the product method, though, this does not seem specially applicable here; $r = \cdot 337$. By the fourfold division $r = \cdot 393$, if the reds be taken on the one hand and the reds with white on the other. The parentages, however, are very unequal. If each be raised to 100 the correlation falls to $r = \cdot 269$. So that even among the dominant class there is a considerable hereditary influence. On what basis it is to be explained there are not sufficient facts to indicate. There must be great variation in the zygote constitution and a certain amount of dominance in the dominant class, but to what it amounts would require much investigation.

Colour Inheritance in Greyhounds.

(*Biometrika*, vol. iii. p. 245. Barrington and Pearson.)

The colour of greyhounds is somewhat complex; the classes used by Barrington and Pearson are: (1) red, (2) brindle, (3) white, (4) fawn, (5) pure black, and (6) mixed black. The exact relationship of these colours is not easily seen from the nature of the offspring. None are clearly dominant, and the hybrid must be largely separated from both dominant and recessive. The fanciers seem to derive the present stock from a mixture of at least three races, red, black, and white, and thus on a Mendelian mechanism the correlation coefficients should be high. The parental assortive mating obtained by the mean square contingency method is about $r = \cdot 20$, but its nature is unknown, so that its effect on raising or lowering the correlation coefficient cannot be estimated. The actual correlations obtained by the mean square contingency are as follows:—

Correlation between Parent and Offspring.

	Unselected Offspring.	Offspring selected for Record.
Sire and Dog . .	·512	·474
Sire and Bitch . .	·579	·404
Dam and Dog . .	·505	·485
Dam and Bitch. .	·532	·499
Mean . .	·532	·466

Here two classes of correlation are given: one for the whole litters taken at birth and the second for the offspring selected for record. The fall in the correlation is noticeable. The value in the first case is not far from

that given in Case I. (par. 33). The second is nearer that given in Cases II. or III.

That is, the height of the first can be explained on the ground that three races mix with the production of distinct hybrids, that of the second on the ground that dominance of some sort manifests itself with growth, an explanation as possible as that of the authors who attribute the fall in the correlation coefficient to the selection of puppies. The fraternal correlations are also high, being $r = \cdot 676$ for brethren of same litter and $r = \cdot 559$ for the selected record, both much higher than the Mendelian formulæ given in par. 38 warrants. A cause of such high coefficients has been shown in par. 40, where it is noted that if three races mix fraternal correlation is raised.

(*Issued separately August* 1, 1910.)

XXXV.—The Morphology of the Manus in *Platanista gangetica*, the Dolphin of the Ganges. By Sir Wm. Turner, K.C.B., D.C.L., F.R.S., President of the Society. (With Four Plates.)

(Read July 4, 1910. MS. received July 5, 1910.)

In July 1909 I communicated to the Society* a description of the skeleton of Sowerby's whale, and discussed the morphology of the Manus in the Ziphioids, Mesoplodon and Hyperoodon, and in the Delphinidæ. In this paper I propose to consider the morphology of the manus in the pentadactylous Gangetic dolphin, the type example of the Platanistidæ.

The most complete account of the anatomy of Platanista is contained in the chapter on the Cetacea in the important "Anatomical and Zoological Researches" by the late Dr John Anderson.† As director of the Indian Museum, Calcutta, he collected and studied both the soft parts and the skeleton in a number of specimens of both sexes. He analysed the constitution of the manus, and assigned to the carpal bones names corresponding with those used in descriptive human anatomy.

The material at my disposal for study consisted of two adult skeletons, a male and a female, presented some years ago by Dr Anderson to the Anatomical Museum of the University of Edinburgh; the skeleton of both hands from an adult male skeleton presented by Dr Alcock to Professor M'Intosh of St Andrews University and the body of a young female, about 3 feet long, in Professor M'Intosh's collection since 1883,‡ both of which, with his customary courtesy, he has allowed me to examine and describe; the flippers of a young male presented to me this year by Dr Nelson Annandale, Director of the Indian Museum, Calcutta, along with the tail and half the head. Altogether I have examined ten hands of Platanista, a number which has enabled a range of variation to be studied, and a conclusion to be drawn as to the morphology of the carpus.

The external characters of the flipper were well seen in Dr Annandale's specimen. It was flattened on both surfaces, and the outlines of the digits could be seen subjacent to the integument. Its length was 19·5 cm. ($7\frac{1}{2}$ inches), and the limb was short in relation to its breadth 13·5 cm.

* *Proc. Roy. Soc. Edin.*, vol. xxix., part vii., p. 687.

† Vol. i., text, pp. 417 *e.s.*, and vol. ii., plate xxxi., *e.s.*, London, 1878.

‡ This specimen had been sent from Calcutta as an exhibit for the London Fisheries Exhibition.

($5\frac{1}{4}$ inches). The free border was truncated and festooned owing to slight projections produced by the tips of digits ii to v. The radial or anterior border 24·3 cm. ($9\frac{1}{2}$ inches) was convex, and the relatively short pollex was parallel and close to it. The ulnar or posterior border 15·5 cm. ($6\frac{1}{4}$ inches) long was almost straight (fig. 1).

The young St Andrews Platanista measured in a straight line from the tip of the snout to the notch in the tail 89·5 cm. ($35\frac{1}{4}$ inches): from the tip to the angle of the mouth 20 cm. and to the base of the flipper 31·5 cm. Its girth round the head at the blowholes was 40·5 cm. and at the umbilicus 52 cm. Dr Anderson in his elaborate memoir gave in Table I. the measurements of eight specimens: the longest, a female, measured 99·12 inches (252 cm.), the skull and spine of which were together 90·85 inches (231 cm.)* In the same table the length of a fœtus was given as 27·75 inches, that of a young male 56 inches, and a young female 59 inches. In dimensions the St Andrews young animal was $7\frac{1}{2}$ inches longer than the fœtus measured by Anderson, and the relative length of its head and body to that of the longest Platanista in his Table I. was as 35 to 99 inches. The jaws in the St Andrews specimen had a formidable armature of simple acuminate teeth, which, after projecting through the gum, varied in length from 20 mm. near the tip of the jaws, to only 3 mm. near the back of the dentary arcade; but it was obvious from slight projections in the gums that the most posterior teeth had not yet erupted; no hairs projected from the skin of the tips of the jaws.† No trace of an umbilical cord was seen, and the specimen was obviously beyond the fœtal stage.

The flipper in the St Andrews young specimen in its general form resembled that above described, though on a smaller scale. Its length from the base to the truncated free border at the tip of digit ii was 15 cm. (5·9 inches); along the radial convex border 16·5 cm.; along the ulnar straight border 11·5 cm.; whilst the greatest breadth was 8 cm. (fig. 3). The tegumentary structures were so thin that the outlines of the bones of the fore arm and hand could be readily seen without a dissection.

Radiograms of the flippers of Dr Annandale's and Professor M'Intosh's specimens were taken for me by Mr E. J. Henderson of the Anatomical Museum, and illustrations have been produced from a selection of them. The extent of ossification and the relative position of the bones could therefore be efficiently studied in their natural undisturbed position. The hands

* He stated on p. 432 that the length of an aged female was $9\frac{1}{2}$ feet, whilst that of his largest male with a mature skeleton was scarcely 7 feet.

† The fœtus in utero figured by Anderson in plate xxxi. showed columnar ridges on the gums, marking the outlines of the teeth which had not cut the gums; also a few scattered hairs projecting from the skin of the tip of the jaws.

in the adults had been mounted as natural skeletons and the bones had retained their normal relations.* The carpal articulations of the radius were with the radiale, intermedium, and apparently with the os centrale; those of the ulna were with the intermedium, with carpalia 4+5, slightly with carpale 3, and also with metacarpal v.

Anderson stated that the carpal bones in Platanista were subject to variations in number, even in the opposite limbs of the same individual. Whilst six were generally seen, they might in some animals be reduced to three, owing to amalgamation of certain carpal bones with each other or even with the ulna. Kükenthal, who subsequently studied the carpus of this species in skeletons in the museums in London and Leiden, also recognised differences in number, and, whilst agreeing with Anderson's interpretation of some of the bones, differed from him in regard to others.†

The study of the ten hands now under consideration has confirmed the statements made by my predecessors in regard to variations in the number of carpal bones and to the occasional want of symmetry in the two limbs. The problem to be solved, notwithstanding the variations, is to determine which carpal bones are present in the hand of any specimen of Platanista under observation, and also to specify those which are absent.

In attempting to ascertain their morphology I have followed the plan pursued in my previous memoir on the Ziphioids and the Delphinidæ, and I have regarded the manus of Hyperoodon as providing the necessary key.‡ In Hyperoodon the proximal carpal row, *procarpus*, consisted of three bones, radiale, intermedium and ulnare: the distal row in the most complete specimens had five bones, now named carpalia 1 to 5 in their order from the pollex to the minimus, each of which was associated with the metacarpal of a numerically corresponding digit. An ossified pisiform might also be present at the ulnar border, and in some specimens an os centrale also.

In the skeletons of the adult Platanista the carpal bones, flattened on the dorsal and palmar surfaces, were usually polygonal in outline, and the margins formed borders of articulation with adjacent bones, the distance between them consisting of a narrow interval which represented the inter-

* The humerus of the adult St Andrews specimen showed an interesting pathological condition: the head, with the adjoining part of the shaft, was hollowed into a large cavity, and the articular surface, as well as that of the glenoid cavity, was roughened with bony nodules; possibly the animal had been shot early in its life and a bullet had lodged in the bone and hollowed out the cavity, from which doubtless pus had freely discharged.

† "Die Hand der Cetaceen," *Vergleich. anat. Entwick. Untersuch. an Wahltieren*, Zweiter Theil, 1893, Jena.

In Eschricht's specimen six carpal bones apparently were present, but no details were given. See Wallich's translation, *Ann. and Mag. Nat. Hist.*, March 1852.

‡ See my memoir on Sowerby's whale, *Journ. Anat. and Phys.*, vol. xx., Oct. 1885.

mediate joint (figs. 4, 5). The epiphyses of the bones of the fore arm, metacarpus and phalanges were fused with their respective shafts. In the two younger animals the carpal ossific nodules were imbedded in cartilage often of some thickness, so that the bony nodules did not directly articulate with each other. No appearance of ossifying epiphyses could be seen in connection with the bones of the fore arm, the metacarpus and the phalanges, but the proximal epiphysis of the humerus was ossified but not fused with its shaft.

The intermedium, the largest bone in the carpus, was present in all my specimens, and measured in one adult 30 by 26 mm. It was opposite the interosseous interval in the fore arm, and had a large articulation both with radius and ulna, less so with radiale, carpale 2, carpale 3, and with a bone which I regard as an os centrale. The ulnare was not present as a separate bone in any specimen. Owing to variations in the carpus at the radial margin the bones in that region required special attention. As a rule an elongated bone, 30 mm. by 13 mm. in one adult, lay close to this margin, which articulated proximally with the radius and distally with the metacarpal of the pollex (fig. 4). It was marked by a notch at its ulnar border, which indicated that it represented two bones, the radiale being the proximal and carpale 1 the distal element; it will be convenient to name the conjoined bone radiale-carpale (fig. 1, scheme 2). In the right carpus of one specimen, however, the radiale and carpale 1 were distinct bones, separated from each other by intervening cartilage, and this condition not only enabled me to determine the correct interpretation of the conjoined nature of the radiale-carpale bone (fig. 2, scheme 1), but also decided the question of the presence of an os centrale.

Variations existed in the number of the distal carpalia. In the right carpus above referred to carpale 1 was a separate bone and articulated with M i, radiale, carpale 2 and the os centrale. In all the specimens carpale 2 and carpale 3 were separate bones, carpale 2 articulating with and belonging to M ii; carpale 3 to M iii. On the ulnar side of carpale 3 was a distal carpal, usually the largest bone in the distal row, measuring in one adult 20 by 21 mm.: it articulated by its disto-ulnar border almost equally with M iv and M v, by its proximal border with the ulna and by the radial border with carpale 3. By articulating approximately in equal proportions with M iv and M v it probably represented carpale 4 and carpale 5 fused together at an early stage of development, though it is possible that carpale 5 might not have been differentiated even as a cartilage, and that carpale 4 had grown laterally to provide a carpal articulation for M v. As the ulnare did not exist as a separate bone it may perhaps be represented potentially in the

conjoined carpalia 4+5, or indeed in the carpal extremity of the ulna itself. It should be noted that in one of the adults the carpal end of the ulna was fused with the conjoined carpalia 4+5, and the relatively large bone produced had a complex composition (fig. 4).

In all the specimens a bone was intercalated between the radiale and carpale 1 on the one side, and the intermedium and carpale 2 on the other. It articulated with these bones, though in two adult hands it had fused with the radiale element of the conjoined radiale-carpale bone (fig. 5). From its position and relation it was evidently an os centrale (figures). From its occurrence in each carpus the centrale is to be regarded as a constant bone in the manus of Platanista. An ossified pisiform was not present in any of the specimens. In the carpus of the younger animals unossified cartilage was present at the ulnar margin, but a pisiform cartilaginous element did not seem to be differentiated.

The largest number of bones present in any of my specimens was seven, viz. intermedium, radiale, carpalia 1, 2, 3, carpalia 4+5 fused into one bone and os centrale. Kükenthal also saw in the Leiden Museum a carpus with seven bones and in the Museum of the London College of Surgeons one with eight bones, due to the presence of two centralia. Six, however, is the prevailing number in the majority of my specimens. Reduction in number was due either to fusion between bones of the proximal and distal rows, as when radiale was fused with carpale 1, to constitute the radiale-carpal bone, a condition which was frequent; to fusion between bones in the same row, carpale 4 with carpale 5, constant; fusion between os centrale and radiale, of which I had two specimens (fig. 5); fusion between carpalia 4+5 and ulna (fig. 4). The absence of ulnare and pisiform was probably due to their non-development even in the cartilaginous stage of the carpus.

I differ from Anderson in regarding the bone which he called radiale as an os centrale, and in this respect I concur with Kükenthal. Anderson described only three bones in the carpus of one of his specimens, which reduction, to employ the nomenclature used in this memoir, was due to fusion of radiale-carpale with os centrale, intermedium with carpale 3, ulna with conjoined carpalia 4+5; carpale 2 therefore was the only carpal element which had not fused.*

As a rule the carpal type in Platanista from the radial margin to the intermedium agreed with Hyperoodon, except in the tendency for the radiale to fuse with carpale 1 and the more constant presence of an os

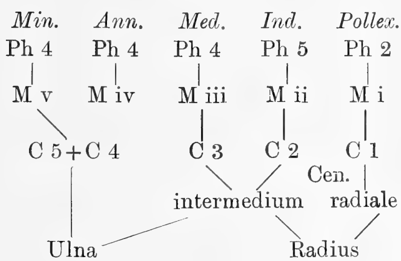
* The names given by Anderson to the carpal bones differed from those used in the text. The radiale-carpale is his trapezium, the os centrale his radiale, carpale 2 the trapezoid, carpale 3 the magnum, carpalia 4+5 the cuneiform.

centrale. At the ulnar margin, again, the absence of ulnare and pisiform, as well as the fusion of carpale 4+5, showed a marked difference from the customary arrangement in Hyperoodon.

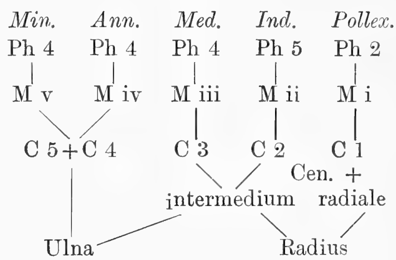
In all my specimens the metacarpals were distinct bones. M i of the pollex articulated with carpale 1 and possessed two phalanges: M ii with carpale 2 had five phalanges: M iii with carpale 3 had four phalanges: M iv with carpalia 4+5 had four phalanges: M v, the minimus, with carpalia 5+4 as well as with ulna had also four phalanges. In the youngest specimen the terminal phalanx was not ossified.

The following schemata show the modifications in my specimens. Scheme 1, corresponding with fig. 2, represented the condition which approached most closely to Hyperoodon, for in it carpale 1 was a distinct bone from the radiale. Scheme 2, figs. 1 and 3, was the most usual arrangement; radiale and carpale 1 were fused together. Scheme 3, illustrated by fig. 5, showed the fusion of the os centrale with the radiale-carpale. Scheme 4, illustrated by fig. 4, showed fusion of ulna with conjoined carpalia 4 and 5, and of carpale 1 with radiale.

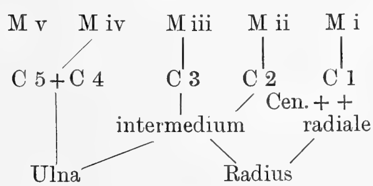
Scheme 1.



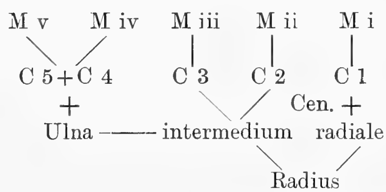
Scheme 2.



Scheme 3.



Scheme 4.



It is customary to place the genus Inia and possibly Pontoporia in the family Platanistidæ, though I cannot speak from personal observation of the constitution of the carpus in these genera. Sir Wm. Flower, in his memoir on the skeleton of a young *Inia geoffrensis*,* figured and named the bones in the carpus in accordance with the nomenclature in man. He spoke of scapho-trapezium, lunar, cuneiform, unciform and magno-trapezoid;

* *Trans. Zool. Soc. Lond.*, p. 105, pl. 25, Nov. 22, 1866.

also a bone which projected from the ulnar border of the carpus, possibly a pisiform. He figured at the radial border two distinct bones separated by intermediate cartilage which were probably radiale (scaphoid) and carpale 1 (trapezium); the lunare was undoubtedly the intermedium: it is difficult to say whether his cuneiform was ulnare or carpalia 4 + 5; for though it articulated with the ulna its distal border was almost equally divided between metacarpals iv and v; most likely therefore it was the conjoined carpalia 4 + 5 (unciform) and corresponded with the arrangement in Platanista; if this be the correct interpretation, the ulnare (cuneiform) was absent. Flower's magno-trapezoid was, I believe, carpale 3, for it was associated with M iii, whilst a bone, which he has numbered 5, was without doubt carpale 2, for it obviously belonged to M ii. According to this interpretation the carpus of Inia consisted of radiale and intermedium in the proximal row, the ulnare being absent: of carpalia 1, 2, 3 as separate bones and carpalia 4 + 5 conjoined in the distal row. It would correspond up to this point with the arrangement in Platanista, Scheme 1, but differed otherwise in having a pisiform and in not having an os centrale.

EXPLANATION OF FIGURES.

Fig. 1, scheme 1. Radiogram of right flipper of young *Platanista gangetica*, Dr Annandale's specimen, showing radiale-carpale bone. Reduced about $\frac{1}{3}$rd.

Fig. 2, scheme 2. Radiogram of left flipper of the same animal; the radiale and carpale 1 are distinct bones. Reduced about $\frac{1}{3}$rd.

Fig. 3. Radiogram of the flipper of the young specimen in Professor M'Intosh's collection in St Andrews. Natural size.

Fig. 4, scheme 3. Drawing of the carpus of an adult in the Anatomical Museum of the University of Edinburgh; fusion of radiale with carpale 1 and of conjoined carpalia 4 + 5 with ulna. Reduced.

Fig. 5, scheme 4. Drawing of the carpus of an adult in Professor M'Intosh's collection; fusion of radiale with carpale 1 and with os centrale. Reduced.

The radiograms were taken by Mr Ernest J. Henderson, Assistant in the Anatomical Museum, and the drawings were from nature by Mr J. T. Murray.

H. humerus; R. radius; U. ulna; *r.* radiale; *in.* intermedium; *cen.* os centrale; C 1, C 2, C 3 carpalia 1, 2, 3; C 4 + 5 carpalia 4 + 5 coalesced; P. pollex; M i, ii, iii, iv, v, the corresponding metacarpals.

(*Issued separately August* 1, 1910.)

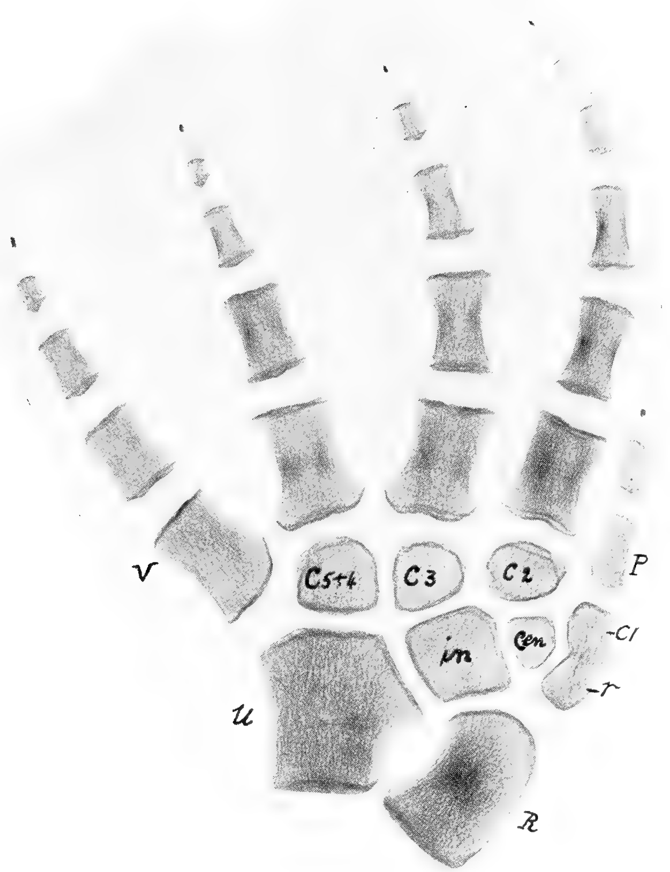


Fig. 1.—Radiogram of flipper of Dr Annandale's specimen.

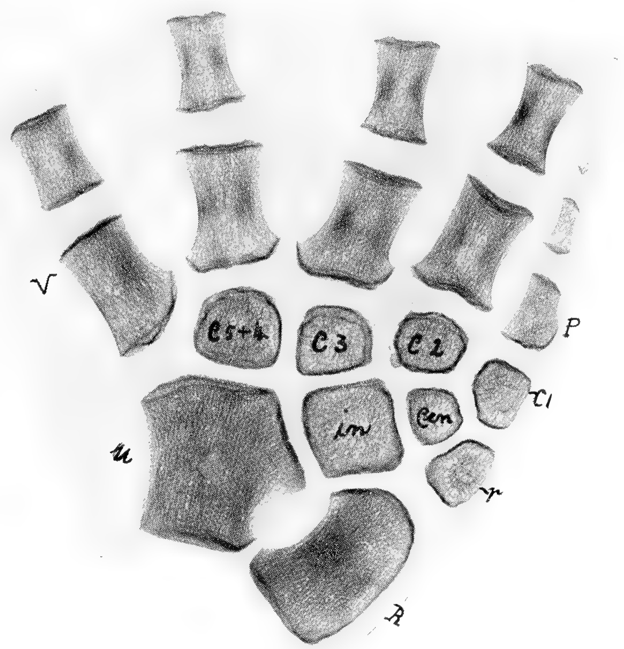


FIG. 2.—Radiogram of opposite flipper of Dr Annandale's specimen.

Fig. 3.—Radiogram of flipper of young specimen in St Andrews Museum.

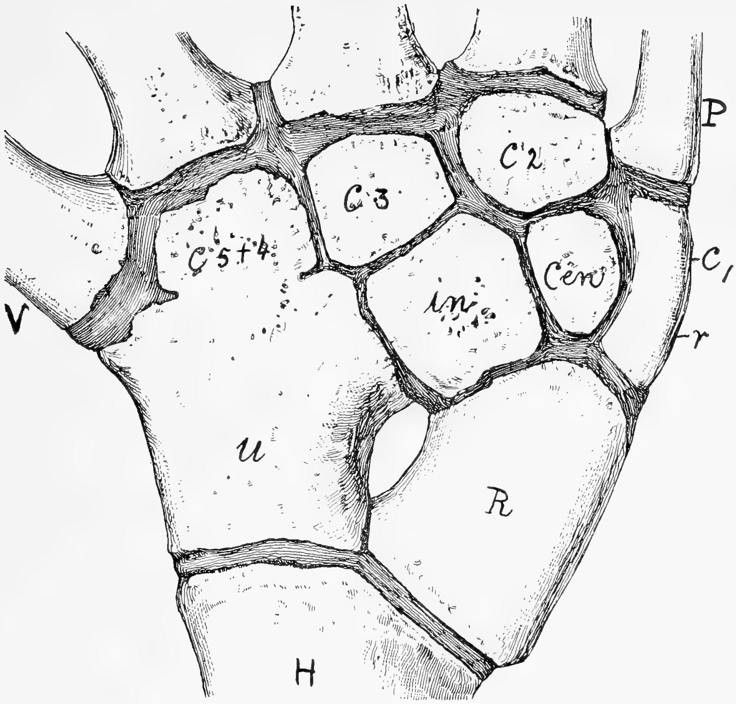


FIG. 4.—Drawing of carpus of adult *Platanista gangetica.*

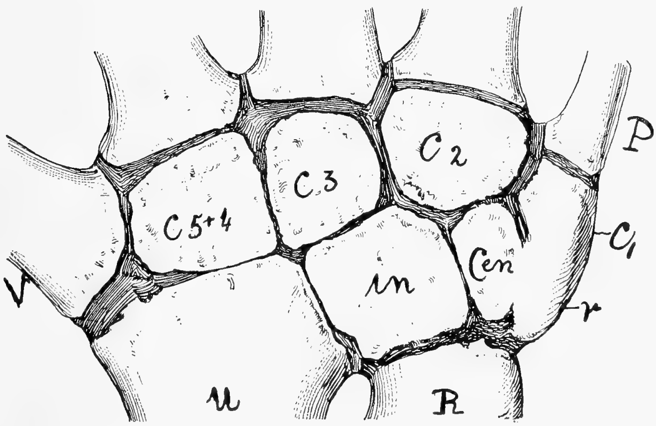


FIG. 5.—Drawing of carpus of adult *Platanista gangetica.*

XXXVI.—Specific Volume of Solutions of Tetra-propyl-ammonium Chloride. By J. W. M'David, B.Sc., Carnegie Research Scholar. *Communicated by* Professor James Walker.

(MS. received June 7, 1910. Read July 4, 1910.)

During the investigation of the viscosities of aqueous solutions of tetra-propyl-ammonium chloride, Taylor and Moore * noticed that the density of a concentrated solution was less than that of a dilute solution at the same temperature. The following investigation, undertaken at the suggestion of Dr W. W. Taylor, shows how the density depends on the concentration and temperature of the solution.

Tetra-propyl-ammonium chloride was prepared from tetra-propyl-ammonium iodide. Silver chloride was added to an aqueous solution of the iodide and the mixture allowed to stand in the dark for some time. The tetra-propyl-ammonium chloride solution which was formed was then filtered off from the solid silver iodide and evaporated to dryness in a vacuum desiccator. The solid so obtained was a white crystalline substance which was very deliquescent. It was tested and found to be free from iodide.

A number of solutions of various concentrations were then made up. By titrating with a standard solution of silver nitrate the concentration of each solution was obtained. Its density was next determined at four different temperatures, viz. 0° C., 25° C., 35° C., and 56° C., by means of a Sprengel pyknometer. The density of each solution was deduced from the formula

$$d_{t^\circ} = \frac{W'D}{W} - \frac{\cdot 0012\,(W' - W)}{W}.$$

The values for the density of water were taken from Landolt and Börnstein's Tables, pages 37 and 38.

The concentrations of the various solutions and their respective densities at each temperature are as follows :—

Concentration gms. per 100 gms. Solution.	Density.			
	0°.	25°.	35°.	56°.
0·0	·9999	·9971	·9942	·9853
0·93	·9997	·9968	·9938	·9850
1·73	·9995	·99645	·9933	·98465
3·12	·99915	·99595	·9928	·9839
6·87	·9991	·9952	·99195	·9826
9·75	·99895	·99445	·9908	·98155
10·90	·9992	·99465	·9910	·98155
15·69	·99965	·9940	·9901	·9798
23·98	1·0030	·99465	·9902	·9786
27·70	1·0055	·9959	·9909	·9792

* *Proc. Roy. Soc. Edin.*, xxviii. p. 470 (1908).

From the above table it is seen that at all temperatures the density of a dilute solution is less than the density of water at the same temperature. As the concentration increases the density gradually diminishes up to a certain point, depending on the temperature, and then begins to increase. By drawing four isothermals to represent the connection between the concentration and density of the solutions (fig. 1), it will be noticed that in each case the curve has a minimum, and that the minimum point moves away from the origin as the temperature increases. For example, the concentration of the solution of minimum density at 0° C. is about 7 per cent., at

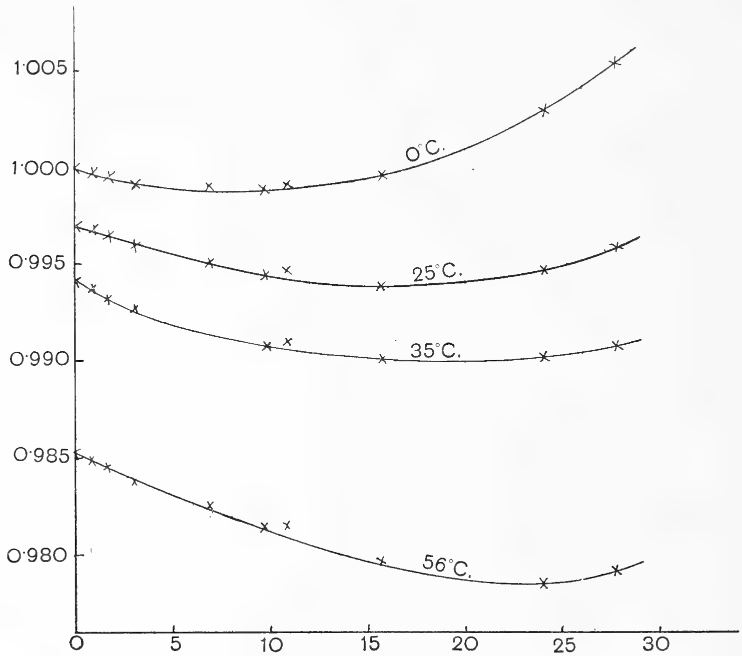


Fig. 1.—Concentration.

25° C. it is 16·5 per cent., at 35° C. 19 per cent., while at 56° C. it is approximately 22·5 per cent. It will be noticed that it is possible to obtain two solutions of different concentrations which have the same density. Thus at 25° C. a solution whose concentration is 3·12 per cent. has the same density as a 27·7 per cent. solution at that temperature. If now these two solutions are mixed, the resulting solution would have a smaller density. Similar results are obtained at the other temperatures.

Generally the density of a solution of a solid increases considerably as the concentration increases. It has been found, however, that the density of a solution of ammonium chloride * increases very slowly with the

* Landolt and Börnstein's Tables.

concentration. A 5 per cent. solution has a density of 1·058, while the density of a 25 per cent. solution is 1·0730. In the case of tetrethyl-ammonium chloride * the increase of density with concentration is even smaller, 4·14 per cent. and 16·30 per cent. solutions having densities of ·9971 and ·9994 respectively, while in the present case of tetra-propyl-ammonium chloride under certain conditions the density actually decreases as the concentration increases.

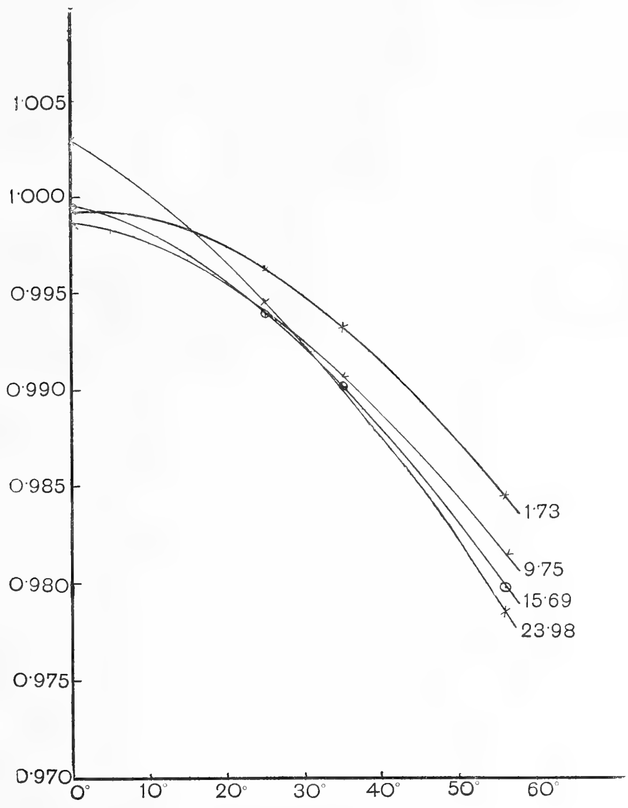


Fig. 2.—Temperature.

The curves in fig. 2 show the connection between density and temperature for solutions whose concentrations are constant.

Since the density of a dilute solution is less than that of water, it is evident that there must be expansion on solution. In order to find out whether the volume of a solution was greater than the combined volumes of solid and water, it was necessary to determine the density of the solid. To do this the method of floating was employed, the density being

* Taylor and Moore, *loc. cit.*

determined at two temperatures, viz. 2° C. and 13° C. The liquids used were bromonaphthalene and toluene. A mixture of these liquids was placed in a small test-tube and a few small crystals of tetra-propyl-ammonium chloride were introduced. The density of the liquid was adjusted by adding a few drops of bromonaphthalene or toluene until the crystals remained suspended in the liquid. The density of the mixture was then determined at 13° C.: this was found by means of the Westphal Balance, while at 2° C. the density of the liquid was determined by means of a pyknometer.

The following results were obtained :—

	3° C.	13° C.
	1·0329	1·0290
	1·0330	1·0295
	1·0344	1·0300
Average	1·0334	1·0296

The values 1·0341 and 1·0255 were taken as the density of the solid at 0° C. and 25° C. respectively, these values being obtained by extrapolation.

In order to determine the expansion on solution, it was necessary to determine the volume of unit mass of the various solutions, the volumes which the water in that unit mass would occupy at the same temperature, and also the volume of the solid in unit mass of each solution.

If d be the density of the solution at $t°$ C., $\frac{1}{d}$ = volume 1 gm. solution at $t°$ C. Suppose now that c is the concentration in gms. of solid per 100 gms. solution, and that Δ is the density of water at $t°$ C., then $\frac{1-c/100}{\Delta}$ is the volume that the water in 1 gm. of solution would occupy if it were free.

If δ be the density of the solid, then $\frac{1}{\delta} \times \frac{c}{100}$ is the volume of solid in 1 gm. of solution.

The expansion on solution will be given by the formula

$$\frac{1}{d} - \left\{ \frac{1-c/100}{\Delta} + \frac{c/100}{\delta} \right\}.$$

The following table gives the results calculated for five of the foregoing solutions :—

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Cells, Liver,—Intra-cellular Canaliculi in.
E. A. Schäfer. Proc. Roy. Soc. Edin., vol. , 1902, pp. .

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PROCEEDINGS

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SESSION 1909–10.

Part VII.] VOL. XXX. [Pp. 519–631.

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[*Continued on page* iii *of Cover*.

Concentration gms. per 100 gms. Solution.	Temperature $t°$ C.	Density gms. per c.c.	Vol. of 1 gm. Solution at $t°$ V_1.	Vol. of Water in 1 gm. Solution.	Vol. of Solid in 1 gm. Solution.	Vol. of Water + Vol. of Solid in 1 gm. Solution V_2.	Expansion on Solution $V_1 - V_2$.
1·73	0	·9995	1·0005	·9828	·0167	·9995	·0010
"	25	·9964	1·0036	·9856	·0169	1·0025	·0011
10·90	0	·9992	1·0008	·8911	·1054	·9965	·0043
"	25	·99465	1·0054	·8936	·1063	·9999	·0055
15·69	0	·99965	1·0004	·8432	·1517	·9949	·0055
"	25	·9940	1·0060	·8456	·1530	·9986	·0074
23·98	0	1·0030	·9970	·7603	·2319	·9922	·0048
"	25	·99465	1·0054	·7624	·2338	·9962	·0092
27·70	0	1·0055	·9945	·7231	·2679	·9910	·0035
"	25	·9959	1·0041	·7250	·2701	·9951	·0090

The figures in the last column of the table show that in all cases there is expansion on solution, *i.e.* the volume of solution is greater than the combined volumes of the solid and water in that solution. It will also be noticed that at 0° C. the expansion at first increases with concentration and

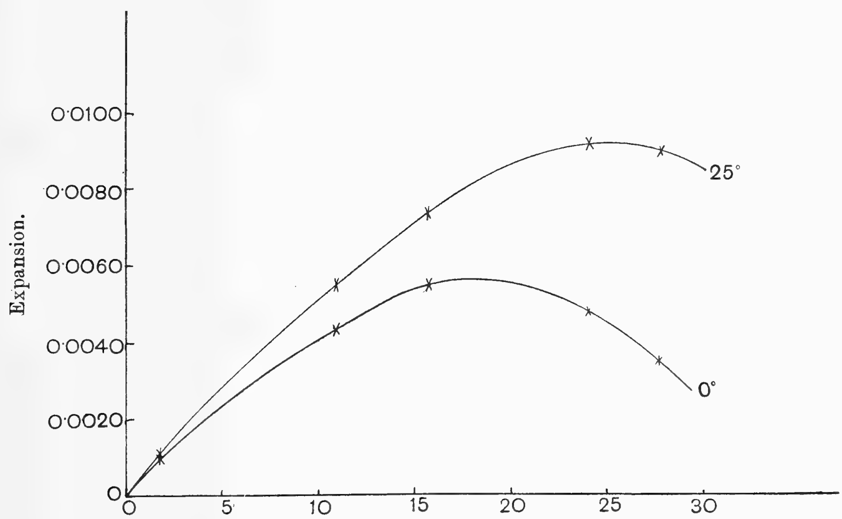


Fig. 3.—Concentration.

then decreases. Generally when a solid dissolves in water there is a slight contraction, *i.e.* the volume of the solution is greater than the volume of water in that solution, but less than the combined volumes of water and solid. There are, however, a few exceptions, of which the present case is one. Braun * mentions the chloride, bromide, and iodide of ammonium,

* *Zeit. f. physik. Chem.*, i. p. 259 (1887).

tartaric acid, and magnesium chloride hexahydrate as being solids which give expansion on solution.

On the other hand, Carse * found that in the case of dilute solutions of certain hydroxides the volume of solution was less than the volume of water it contained.

The curves in fig. 3 show the connection between the expansion on solution and the concentration at the two temperatures 0° C. and 25° C. At 0° C. a solution whose concentration is about 18·5 per cent. has the maximum expansion. At 25° C. a 26 per cent. solution evidently has maximum expansion. It will also be observed that the expansion increases considerably as the temperature is raised.

Similar results were calculated for the temperatures 35° C. and 56° C., but, owing to no allowance being made for the expansion of the solid, their accuracy could not be guaranteed, hence they have been omitted.

Chemistry Department,
University of Edinburgh.

* *Proc. Roy. Soc. Edin.*, xxv. p. 286 (1904).

(*Issued separately September* 7, 1910.)

Proceedings of the Royal Society of Edinburgh,
Session 1909–10.

ERRATUM.

The order of this and the following paper by Messrs Smith and Menzies has accidentally been inverted. The second (as printed) should be read first.

XXXVII.—The Vapour Pressures of Mercury. By Alexander Smith and Alan W. C. Menzies.

(MS. received June 6, 1910. Read July 4, 1910.)

An examination of the existing data of the vapour pressures of mercury shows that at the higher temperatures a most astonishing lack of precision characterises all the measurements. The existing tables, especially above 300°, are therefore completely untrustworthy, a fact which might be guessed from the disagreement between the values they give (see table below). Regnault made his measurements with the thermometer bulb completely immersed in 50 kilos of the violently "bumping" metal. Hence his results are much too high. Ramsay and Young's well-known values were based on observations at four points. The two lowest of these were original: one was the boiling-point, taken from discordant data of Regnault's; and one was a measurement of the pressure at the boiling-point of sulphur. For the last, Regnault's temperature was taken, and Callendar and Griffiths have shown that the value is 4° too high—a change which alters the vapour pressure of mercury at 445° by about 140 mm. Young (*J. Chem. Soc.*, **59** (1891), 629) has modified the table to take account of Callendar and Griffiths' work. But he assumes that an unscreened bulb in an unjacketed bath of sulphur vapour has the same temperature as was attained by Callendar and Griffiths with most careful jacketing. His temperature at 445° is therefore still about 2° too high. Gebhardt (*Berichte deutsch. physik. Gesell.*, **7** (1905), 184), with no mention of precautions or corrections in respect to temperature or pressure readings, publishes data which, in view of our results, are probably on the whole nearly 1° too low. The average deviation of a single observation from a smooth curve is 1·2°. Cailletet, Colardeau, and Rivière (*Comptes Rendus*, **130** (1900), 1585) measure the vapour pressures from 400° up to 880°, without stating what values they assumed for the fundamental points in their temperature scale.

In the present work the isoteniscope, platinum resistance thermometer and gauge described in the preceding paper were employed. Forty-three measurements between 250° and 435° were made. An accuracy of ±0·1° was aimed at. The results are represented by the following Kirchoff-Rankine formula:—

$$\log p = 9{\cdot}9073436 - 3276{\cdot}628/\theta - 0{\cdot}6519904 \log \theta.$$

The consistency of the results was tested by calculating, with the aid of this formula, the temperatures corresponding to the observed pressures. The calculated temperatures, which, as thus derived, lie on a smooth curve, were then compared with those observed. In thirty cases the deviations range from 0·00° to 0·05°, in eight cases from 0·06° to 0·1°, and in only five cases do they exceed 0·1°. The greatest deviation (0·23°) is undoubtedly due to an accidental error in measurement. The average deviation of a single observation, including the erroneous one, is only 0·05°, and well within the proposed limit of accuracy.

The temperatures are thermodynamic, with the boiling-point of sulphur assumed to be 445° exactly. If this point, as seems likely, should finally be found to be 444·9°, adjustment can be made accordingly.

The following table shows the smoothed values of Regnault (Reg.), Ramsay and Young (R & Y), Young (Y), Gebhardt (G), Cailletet, Colardeau, and Rivière (C C R), and of the authors (S & M). In the last case the values above 435° are extrapolated. Laby's (*Phil. Mag.* [6], **16** (1908), 789) recalculated values are also given. The boiling-point given by our formula is 356·95°±0·1°, on the scale before mentioned.

Comparative Table of Rounded Results.

Temp.	Reg.	R & Y.	Y.	G.	Laby.	S & M.
255	...	85·0	...	...	86·2	84·45
260	96·7	96·7	96·5	100·0	97·8	95·94
270	123·0	123·9	124·0	120·0	124·8	123·02
280	155·2	157·4	157·8	158·8	158·4	156·29
290	194·5	198·0	198·9	199·5	199·3	196·81
300	242·2	246·8	248·6	249·0	248·6	245·85
310	299·7	304·9	308·0	309·0	307·7	304·69
320	368·7	373·7	378·5	...	378·1	374·82
330	450·9	454·4	461·7	...	461·3	457·85
340	548·4	548·6	559·1	...	559·1	555·54
350	663·2	658·0	672·5	...	673·3	669·77
360	797·7	784·3	803·7	...	805·9	802·62
370	954·7	930·3	954·7	...	959·2	956·25
380	1140	1096	1228	C C R	1135	1133·0
390	1347	1284	1325	...	1337	1335·4
400	1588	1496	1549	1596	1566	1566·1
410	1864	1734	1801	...	1826	1827·6
420	2178	2000	2085	...	2119	2123·4
430	2533	2299	2403	...	2446	2456·0
435	...	2459	2572	...	2628	2637·5
440	2934	2629	2757	...	2817	*2828·8*
445	...	2808	2939	...	3018	*3031·5*
450	3384	2996	3150	3230	3229	*3245·0*

(*Issued separately September* 7, 1910.)

XXXVIII.—A Static Method for Determining the Vapour Pressures of Solids and Liquids. By Alexander Smith and Alan W. C. Menzies.

(MS. received June 7, 1910. Read July 4, 1910.)

THE apparatus to be described was developed from the submerged bulblet apparatus, a description of which was recently published in these *Proceedings* (vol. xxx. p. 437).

The Isoteniscope, Bath, and Stirrer.—The substance is placed in the spherical bulb (20 mm. in diameter). The confining fluid, which, when a liquid is being investigated, is the same material as the substance, occupies the lower part of the U-tube (fig. 1). The whole instrument, which we have called the *Static Isoteniscope*, is 24 cm. long. It is suspended, along with the thermometer and mechanical stirrer, in a tall 2-litre beaker, containing a suitable bath liquid. When the substance is a solid, then the U-tube is charged with mercury, melted paraffin, a fusible alloy, or a molten salt or mixture of salts. To secure steadiness and equal distribution of temperature, a cylindrical glass screen (cut from a broken beaker) surrounds the bath, and a very active stirring arrangement is used.

Measurement of Temperature and Pressure.—In the work to be described, a platinum resistance thermometer, adjusted and standardised so as to give the absolute temperature with an error of less than $\pm 0^{\circ}{\cdot}01$ below 100°, and of less than $\pm 0^{\circ}{\cdot}1$ up to 450°, was employed. The fixed points were 0°, 100°, and the boiling-point of sulphur (taken to be 445°), and the thermodynamic scale was used. The fixed points were redetermined at frequent intervals during the work.

The gauge was read by means of a hairline ruled on a strip of glass travelling on a cursor. A mirror behind the gauge eliminated the effects of parallax. For the reduction of the readings to mercury height at 0°, five thermometers, suspended at intervals beside the gauge, were employed. Correction was made for the coefficient of expansion of the previously calibrated scale (a steel tape), and for the local value of the gravity constant.

The Manipulation.—Fig. 2 shows diagrammatically the arrangement of the rest of the apparatus. The isoteniscope and gauge were connected with a large iron bottle. This, in turn, could be put into communication

either with the atmosphere, with a vacuum reservoir and water pump, or with a pressure reservoir and pump.

The manipulation may be understood from an example. When the vapour pressures of water below one atmosphere were being measured, the temperature of the bath was first brought to the desired point. The exit to the vacuum bottle and pump was opened, and the pressure in the iron

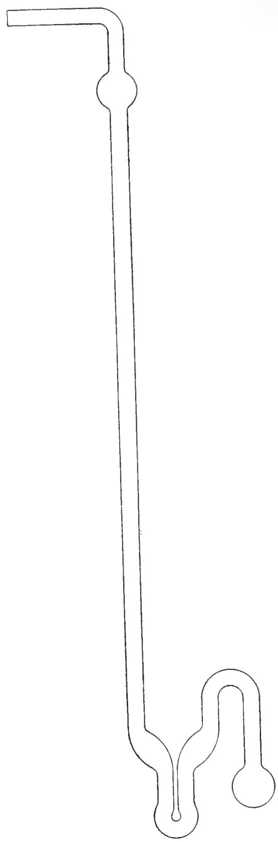


Fig. 1.

bottle was reduced until the water in the bulb boiled. This exit was then closed. When the air and dissolved gases had been expelled, the stopcock leading to the atmosphere was opened. Simultaneously, the rubber tube attached to this stopcock was grasped with the fingers at two places, and air was admitted in small portions until the levels of the water in the U-tube were identical. The stopcocks on the exit of the atmosphere, as well as that on the gauge, were then closed, the gauge was read, and the temperature reading was confirmed. The whole process of boiling out and adjusting was then repeated, to make sure that all gases had been expelled from the

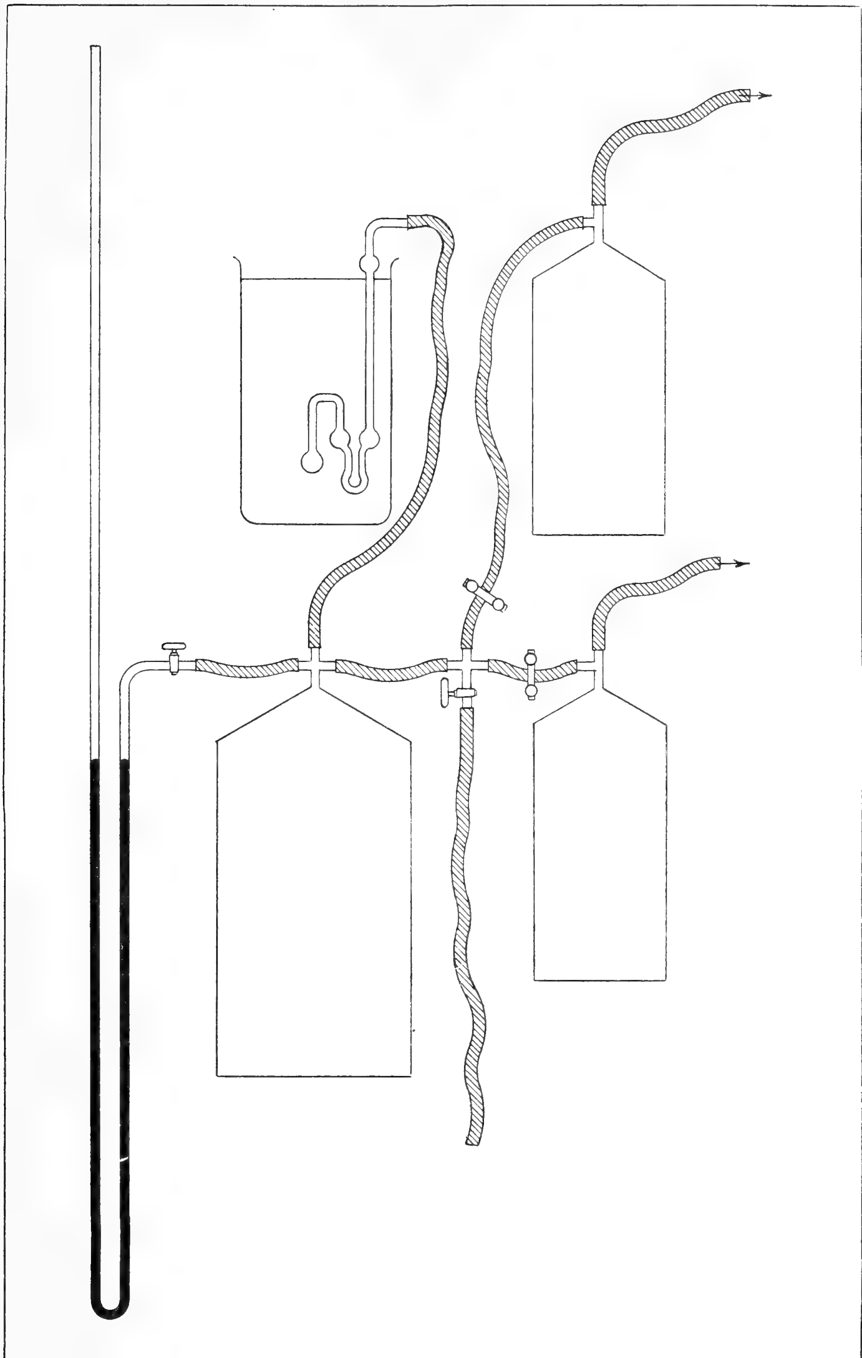


Fig. 2.

water. When constant values were reached, the temperature of the bath was raised and a new observation taken.

Criticism of the Method.—It will be seen that the isoteniscope is free from the sources of error which affect the older forms of apparatus for determining vapour pressures. For example:—

1. The isoteniscope substitutes for mercury a confining fluid of low specific gravity, and thus eliminates the error in levelling involved in some static methods.

2. The apparatus dethrones mercury from its position as sole confining fluid, and substitutes a wide choice of liquids. In consequence, all errors due to the volatility of a foreign confining fluid, and to solubility of the substance in or interaction of the substance with such a confining fluid, disappear entirely.

3. By being adapted to use in a liquid bath (with violent stirring), the apparatus reduces to a minimum all errors due to unsteadiness of the temperature, and to inequality in the temperatures of the substance, its vapour, and the thermometer.

4. By providing a simple method for expelling air bubbles and adhering and dissolved gases, and for repeating the expulsion until the success of the operation is demonstrated, the isoteniscope removes a source of error (and of total uncertainty as to the amount of that error) which destroys all confidence in the exactness of the results obtained by the older static methods.

5. By permitting the making of an extended series of observations without using more than half of the sample, the apparatus obviates an error due to concentration of involatile impurities in the residue which affects some dynamic methods

6. By allowing the expulsion of accumulated gas immediately before a reading, the apparatus makes possible accurate measurements with many substances which decompose slowly to give a permanent gas. By the older static method, accurate measurement in such cases was necessarily impossible.

Aside from this avoidance of certain sources of error, the apparatus possesses some noteworthy advantages:—

1. A small amount of material suffices.

2. The filling of the apparatus, and the manipulations involved in a measurement, are incomparably simpler than in any other static method.

3. The correction for dilatation of the bulb of the mercury thermometer, which some dynamic methods involve, is avoided.

4. The method permits measurements with all reasonably stable liquids, no matter how active, chemically, they may be.

5. The method is applicable to all solids, provided a non-interacting, non-solvent, non-volatile confining liquid can be found. When such a liquid cannot be found, a dynamic method, to be described in the next paper, may be used.

The Vapour Pressures of Water.—The ease with which water may be purified, the relatively slight solubility of gases in water as compared with other liquids, the almost normal behaviour which water alone showed when studied by Tammann (*Wied. Ann.*, **32**, 683) and by Wüllner and Grotrian (*Wied. Ann.*, **11**, 545), and the relatively good agreement amongst the previous measurements of its vapour pressures, indicate water as the only suitable substance for testing the merits of the method. No other substance combines all these qualities.

The measurements were made, by design, close to temperatures the values of which should be represented by whole numbers. The differences for one-tenth of a degree given by Eckholm were then used in making the small adjustments required to reduce each observation to the nearest whole number of degrees. The results are given in the table under "S. and M. Observations." The pressures in this column, therefore, are not smoothed results, but are essentially actual observations, subject to the irregularities which individual observations usually show. This enables them to afford a rigorous test of the method, while the adjustment facilitates comparison with the data obtained by other methods.

TABLE.

t.	S. and M. Observations.		H. and H.	Differences H. and H. – S. and M. Observations.	
	No. of Observations.	*p.*		Mm.	Degrees.
50	3	92·27	92·30	+0·03	−0·006
51	4	97·03	96 99	−0·04	+0·009
55	1	117·87	117·85	−0·02	+0·004
60	2	149·13	149·19	+0·06	−0 010
65	3	187·19	187·36	+0·17	−0·019
70	3	233·44	235·53	+0·09	−0·009
75	1	288·78	289·0	+0·22	−0·018
80	1	354·90	355·1	+0·20	−0·014
85	3	433·54	433·5	−0·04	+0·002
89	5	505·87	506·1	+0·23	−0·011
90	1	525·94	525·8	−0·14	−0·007

For comparison, we give under H. and H. the values recently found by Holborn and Henning in the Reichsanstalt. These are smoothed results, and

are undoubtedly the most accurate data on this subject. The algebraic sum of all the temperature differences, divided by the whole number of our observations (27), gives the mean divergence from H. and H.'s curve of a curve drawn through our observations. This mean divergence is $-0{\cdot}0063^\circ$, which is well within the limit of error assigned to our measurement of absolute temperature ($\pm 0{\cdot}01^\circ$). This corresponds to a mean pressure deviation of $+0{\cdot}08$ mm.

Comparison with other Methods.—We may now institute a comparison between the results obtained by our method and those obtained by other methods, using Holborn and Henning's values as the standard of comparison.

There are only two sets of static determinations in the region 50–90°. The mean divergence of Magnus' results in this region is $+0{\cdot}81$ mm. (or $-0{\cdot}07^\circ$), and that of Batelli's, $+2{\cdot}3$ mm. These are respectively ten and thirty times as great as our divergence. Ramsay and Young's static observations began at 120°, and the mean deviation of their values between 120° and 150° from those of H. and H. is $-0{\cdot}1^\circ$, or of the same order as that of Magnus.

There are three sets of observations by dynamic methods in this region, one of which is that of H. and H. Regnault's mean divergence is $+0{\cdot}4$ mm. (or $-0{\cdot}04^\circ$) and Wiebe's $-0{\cdot}25$ mm., respectively five and three times as great as ours.

The deviations of the recalculated values are as follows: Brock $+0{\cdot}27$ mm., Thieson $-0{\cdot}11$ mm., Eckholm $-0{\cdot}33$ mm.

Assuming the correctness of Holborn and Henning's values, this comparison shows that the present method, in addition to the advantages already enumerated, possesses also that of giving accurate results. The degree of self-consistency of the observations made by this method may be seen by inspection of the table, and a more detailed study of this point will be given in the following paper, in connection with another example.

(*Issued separately September* 7, 1910.)

XXXIX.—Did the Tail of Halley's Comet affect the Earth's Atmosphere? By Dr John Aitken, F.R.S.

Part I.

(MS. received June 21, 1910. Read July 18, 1910.)

The return of Halley's Comet in May of this year gave rise to much speculation as to its possible effects on the earth. As it was expected that the earth would pass through the tail of the comet when the comet passed between us and the sun, many observations were arranged for in order to see if the tail, whatever it was composed of, had any effect on the earth or on its atmosphere. If the tail was composed of matter in any form, gaseous, or fine solid or liquid particles, then it seemed possible to get some evidence of its presence in the atmosphere; or if the tail was composed of electrons, then these would disturb the electrical condition of the atmosphere, and also the magnetic condition of the earth.

The part of this inquiry which specially appealed to me was the investigation of the question of the possibility of the tail being composed of minute solid or liquid particles. If the earth passed through the tail of the comet and the tail was composed of fine dust, then we would expect an increase in the number of dust particles in our atmosphere, which we might expect to find in its lower layers some time after the passage. As there are few observers in this particular part of meteorology, I made arrangements for conducting dust observations. If the observations were to be made with the dust-counter, it was obviously necessary that they be made in some isolated part of the country, so as to get free from artificial sources of dust, and also in a district of which we have already some information as to the number of particles under different conditions, as without this knowledge it would be impossible to come to any conclusion from merely isolated observations. It was suggested indeed by someone that the dust-counter might be used along with other means of observation in balloon ascents; but as there are no previous records of observations of dust made in balloons in the upper air, it is evident any such observations would be useless, as there is nothing with which to compare them.

For the purpose of this investigation the north-west Highlands was selected, because the air in that district is but little contaminated by local pollution, and especially because I have on previous occasions made many

hundreds of dust observations in that district, and have records with which to compare those taken after the passage of the comet. Kingairloch, where the previous observations were made, not being available, I selected Morar in Inverness-shire, situated 25 miles to the north-west of Kingairloch. Morar is fairly free from local pollution, there being only a few crofters' houses near, and the smoke from their fires can be easily avoided, though at first, owing to some of them being hidden among low hills, they gave trouble, causing the number of particles observed to be irregular; but by changing the position of observation cross-wind ways, one could easily get clear of their effects.

Along with dust observations I made arrangements for taking observations on the electrical condition of the atmosphere, because it was thought that it might possibly be affected by the tail of the comet. Observations on sunset colours were also to be made, and recorded by means of colour photographs, as it was thought that if there was an increase of dust after the passage of the comet the sunset colouring would probably be finer.

Dust Observations.

I arrived at Morar on the 13th of May to get things in working order and get some readings of the number of dust particles before the passage of the comet on the morning of the 19th. The weather conditions during part of the time the observations were made were not very favourable for the investigation, owing to the absence of any well-developed circulation, with the usual result, under these conditions, of high numbers being observed. On the 13th, when the observations began, the circulation was dominated by a depression situated over the English Channel. This caused easterly winds over our area, and as the air was drawn in from the Continent the number of particles was high, varying from 1250 per c.c. to 3000 per c.c. On the 14th the circulation was more northerly, and the number fell slightly, remaining under 2000 per c.c. On the 15th the number was still about the same as on previous day, though the local wind had changed to S.E., while the weather report of the Meteorological Office shows the general circulation to have been N.E. On the 16th the local wind was still S.E., though the general circulation was N.E., and the number of particles fell to about 1000. On the 17th and 18th, though the local wind was still S.E., the general circulation was slightly north of east, and the numbers fell to from 500 to 750, showing that the local circulation was dominated by the general circulation, as these figures are low for true S.E. winds in that district. On the 19th and 20th the general circulation began to draw

a little more to the south, and the numbers began to rise to from 1000 to 2700 per c.c. On the 21st there was no well-marked general circulation, and the local wind was from the west and light, and the numbers fell to from 341 to 1750. On the 22nd the general circulation was still weak and the numbers were slightly higher. On the 23rd, 24th, and 25th the general circulation was feeble and irregular, and the number of particles varied, being as low as 626 on the 23rd, rose to 5500 on the 24th, and remained between 3000 and 4000 on the 25th. On the 26th the wind changed to S.S.W. in the morning, but veered to N.W. in the evening, and the number of particles fell as low as 203 per c.c. During the next day the numbers, under the influence of the west wind, varied from 70 to 560. During the 28th, 29th, and 30th the numbers varied from 217 to 1500, the general circulation and local winds being westerly.

The question now is,—How do these figures compare with the previous observations made at Kingairloch? East winds at Kingairloch have been observed with as many as 3000, 4000, and even 5000 particles per c.c., and S.E. winds have also been observed with similar numbers, so that the 2750 observed in S.E. winds at Morar on the 20th is only what might be expected. There is nothing unusual in the figures for the 21st, 22nd, and 23rd. But when we come to examine the figures for the 24th and 25th, which are much higher than previously observed at Kingairloch in W. and N.W. winds, we are presented with a puzzle which may be interpreted in different ways, according to the view one feels inclined to adopt. It may be comet's dust or it may be due to the meteorological conditions. During these two days the centre of an irregular and feeble anticyclone was situated over Morar, and there was no general circulation. The meteorological charts show light winds blowing in different directions over a limited area, and it has been shown in previous communications that these are the conditions in which large numbers of particles may be observed, as local impurities tend to increase under these conditions. This explanation, however, does not seem very satisfactory, because in that district local pollution is very small, and, further, the centre of the anticyclone on the 24th was situated partly over the very north-west of Scotland, but mostly over the sea to the north; so that, under the conditions, local pollution hardly seems sufficient to account for the high numbers.

On the other hand, if we wish to blame the comet for these high numbers, then we can point out that it would be just in such a position that the comet's dust could be first observed, as it would reach the earth's surface quickest in the descending current in the centre of an anticyclone, as the movement of these fine dust particles is more determined by air currents

than by their power of falling through the air. After the 25th the number of particles became low, under the influence of a S.W. wind; and though the wind returned to the W. and N.W. on the succeeding days, the high numbers did not return, the meteorological conditions having changed, the centre of the anticyclone having moved away to S.W., and a general circulation from the N.W., under the influence of a cyclone to the east, had become well established, and all the dust observations were similar to those obtained at Kingairloch under the same conditions.

It may be as well here to give a word of caution as to the comparison which we can draw between the figures obtained at Kingairloch and at Morar in N.W. winds. It has already been shown that there are certain very abnormal numbers obtained at Kingairloch when the wind was from the N.W., the numbers going up to 9000 per c.c., and even to 12,000 per c.c.; and there was this peculiarity in those Kingairloch numbers, that they were only obtained in sunshine. With N.W. winds the numbers were uniformly low on clouded days, and were low on the mornings of sunny days, but they gradually rose under the influence of the sunshine, falling low again at night. Further, there was another peculiarity in these Kingairloch observations—there was no haze corresponding to the high number of particles; the air retained the usual clearness of the N.W. winds. Now the high numbers observed in N.W. winds at Morar were also observed in sunshine, but, unlike the Kingairloch air, it was densely hazed. The islands of Rum and Eigg, distant respectively 16 and 12 miles, were quite invisible, and the hills two or three miles away were thickly hazed. The abnormal high readings obtained at Kingairloch in N.W. winds still remain a mystery, and they have no relation and throw no light whatever on the high numbers obtained at Morar in air from the same direction. From this point of view, the high numbers at Morar seem to point to the tail of the comet as a possible cause, but, from the complication of the evidence, perhaps "Not proven" is the only verdict that can be safely given.

Observations on Atmospheric Electricity.

For the purpose of studying the electrical condition of the atmosphere, I was only provided with apparatus of the simplest construction, as all I intended was to take eye observations of the sign and the potential of the electrification. The apparatus consisted of a common leaf electroscope and one of Glew's Radium Collectors. The collector was hung from the end of a fishing-rod which projected from a second-story window looking S.S.E., the collector being connected with the electroscope by means of a fine wire

passing through the open window. No attempt was made to get absolute measurements, though a rough approximation might be yet obtained if required. Observations were made at frequent intervals all day, and the angle of divergence of the leaf noted if the electrification was weak, and the number of discharges per minute if it was strong. The sign of the electrification was also at the same time ascertained.

When the observations began on the 13th of May the electrification was weak and the usual steady positive of fine weather. The fine weather conditions remained on till the 18th with but little variation. Sometimes the electrification was only just strong enough to enable its sign to be determined, and sometimes the divergence of the leaf might amount to 45 or 50 degrees, but it was never observed strong enough to discharge. The electrification on the 18th was the same as previous days, but on making observations after 10 p.m. it was noticed that the electrification had now changed and was negative, but weak. This change in the electrification at once awakened one's interest, as it took place within a few hours of the time the earth was expected to pass through the tail of the comet. One naturally asked, could this be the advance skirmishers of the electron host which some authorities thought might constitute the comet's tail? If one had only taken time to think, it would have become apparent that this could hardly be the case, because even supposing the comet's tail was composed of electrons, and these had passed into our atmosphere, they could hardly be expected to penetrate through it and reach the earth. However, one's interest being aroused, I watched the instrument for some time, and was rewarded by seeing the electrification rapidly increase, till at last I saw it discharge for the first time. It then quickened the rate of its discharge till it was discharging at the rate of six times a minute, the electrification being − all the time. It may be as well to state that the weather, which from the beginning of the observations had been very warm and nearly cloudless, had on the afternoon of the 18th shown signs of a change. Clouds formed in the afternoon, and the sky was quite clouded two hours before the change in the electrification took place.

This change in the electrification so near the time of the earth's passage through the tail of the comet seemed to promise some interesting results to follow, but alas! on returning to the electroscope next morning at 4 a.m., while we were still supposed to be moving through the tail of the comet, the electrification was found to have returned to its usual settled fair-weather condition of feeble +, and it remained feeble till the evening, when it gave a display of + discharges. On the morning of the 20th the potential was low and +, but a short thunderstorm before 5 p.m. changed the conditions.

After the storm was past, the electroscope discharged − electricity at the rate of 2 per min., ten minutes later it was + and discharging ten times per minute, at 7.30 it was − and discharging twenty times per minute. The electrification changed again at 8.40 to + and discharging eight times per minute. During the rest of the time the observations were made there was nothing calling for remark. The electrification remained steady +, and generally weak, only occasionally making slow discharges.

Sunset Colours.

There is nothing to report in this part of the investigation. No brightly coloured sunsets were observed either before or after the 19th. If anything they were finer before, but this may have been due to the condition of the clouds. The only thing worth recording was the appearance of a sun pillar on the evenings of the 23rd and 24th, shortly after the sunset. Though that phenomenon took place about the time the high dust-readings were observed, yet it probably had no connection with the comet, as it is often seen under ordinary conditions. I was fortunate in securing a colour photograph on a Lumière plate of the sun pillar on the 23rd, but failed with the one on the 24th, which was not so distinct, and my lack of experience did not guide me to the correct exposure of the plate to catch the faint colour.

It must be admitted that the result of this investigation has been most unsatisfactory. There is not sufficient evidence to convict the comet of having in any way affected our atmosphere, and I think all of us who have been searching either for solid or gaseous parts of the tail have to admit that our visitor has not bequeathed a lock of its tail to any one of its admirers.

The large amount of dust observed in the air on the 24th and 25th, and the doubtful source of it, have suggested that it might be worth while to continue dust observations for some time. If the tail of the comet has added any dust to our atmosphere, it will by this time be getting pretty well mixed up with the air at all elevations, and will thus enable me to make observations at Falkirk in the method described in *Proc. Roy. Soc. Edin.*, vol. xx.

TABLE I.—THE NUMBER OF DUST PARTICLES IN THE AIR AT MORAR IN MAY 1910.

Date.	Hour.	Number of Particles.	Wind Direction and Force.	Temperature.	Wet-Bulb Depression.	Transparency of Air.
13	12.50	1250	S.E. ·5	62·5	6	Thickish haze.
"	2.40	1625	"	65	5	"
"	6.30	3000	"	61·5	6·8	"
14	9.30	1850	E. ·5	59	8·5	"
"	12	1900	N. ·5	61	8	"
"	3.30	1950	"	62·5	9·5	"
"	6.45	1800	E. ·5	59·5	8·5	"
15	10	1850	S.E. ·5	60	5	"
"	1	1700	"	66	7	"
"	7	2250	"	62	6	"
16	9.30	950	S.E. 1	61	6·5	"
"	2.15	975	"	...	...	"
"	7	1250	"	57	5	"
17	10	650	"	57	5	Medium.
"	1	550	"	61	6·5	"
"	8.30	600	"	51·5	2·5	"
18	9.30	520	"	57·5	5·3	"
"	12	475	"	60	6·5	"
"	3.15	700	"	61·5	6·5	"
"	8.30	750	S.E. ·2	57·5	4·5	"
19	9	1125	S.S.E. 2	62	9	Clear.
"	12	1075	S.E. 1	66	11	Medium.
"	4.30	825	"	66	11	"
"	6.30	1100	E.S.E. ·5	61	8	"
"	9.15	1450	E. ·5	60	7	Thick.
20	9.15	950	E. 1	66	8	Medium.
"	11	1175	S.E. 1	70	10	"
"	5.15	2750	S.S.E. 1	68·5	8	Thick.
"	6.45	1275	S.E	...	...	"
21	9.30	341	W. ·5	54	2	Thick.
"	3.30	1750	"	63	6	Medium.
"	6	675	N.W. 1	57	4	Clear.
"	6.30	625	"	55	3·5	"
22	10.30	1850	N.W. ·5	59	5	"
"	7	1850	N. 1	55·5	4·3	"
23	10	625	N.E. 1·5	56	2·7	Medium.
"	1	755	"	58	4·3	"
"	3.30	1750	N.E. 2	60	5·5	"
"	6.45	850	"	56	3·2	"
24	9.30	525	E. ·5	60	5	"
"	4.30	5500	N.N.E. 1	65·5	7·5	"
"	7	2800	N. 1	60	6	"
25	9.15	3900	N.W. ·5	57·5	3·7	Very thick.
"	2	2800	W. ·5	...	...	"
"	2.30	3100	"	...	...	"
"	6.30	3300	N.W. ·5	58	5·5	"
26	9.30	287	S.S.W. 1	52·5	·5	Raining.
"	11.15	350	S.W.	52	...	"
"	1	203	W.	54·5	·5	"
"	4	257	N.W. 1	57	2	Medium.
"	8.30	875	W.N.W. 1	...	...	"

Table I.—*continued.*

Date.	Hour.	Number of Particles.	Wind Direction and Force.	Tempera-ture.	Wet Bulb Depression.	Trans-parency of Air.
27	9.30	70	W. ·2	53	0·2	Thick.
”	12	189	”	55	0·2	Air clear
”	1.30	70	”	”	”	when not
”	3.30	500	W.S.W. ·5	53		raining.
”	6.30	238	S.S.W. 1	52·5	0·3	
28	9.45	217	W.N.W. 1	55	2·5	Clear.
”	3	1200	W. 1	56	5	”
”	6.30	1250	W.N.W. 1	52·5	5·5	”
29	9.30	217	S. 2	52	0	Raining. Thick.
”	11.30	1250	S.W. 2	52	0	”
”	12.30	1500	W.S.W. 2	”	”	”
”	6.30	950	W. 2	50	4	Medium.
”	7.30	950	W. 2	50	4	”
30	9.30	675	W. 3	46	0	Raining.
”	10.30	1250	”	46	1	Showers.
”	12.30	1400	”	”	”	”

Part II.

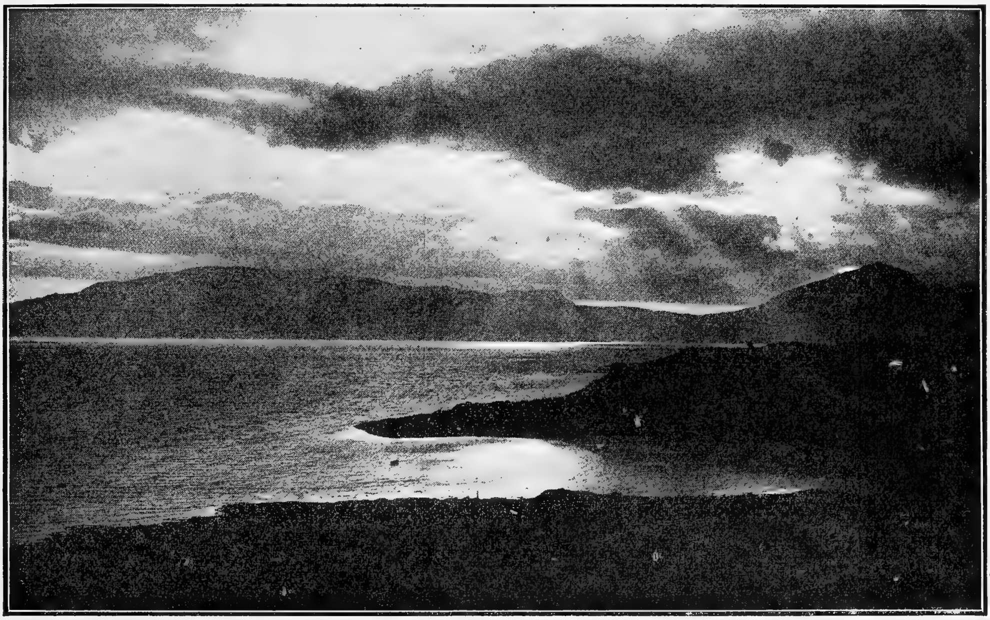
(MS. received July 29, 1910. Read July 18, 1910.)

After the observations described in the first part of this investigation were concluded, another series was made on the haze in the atmosphere at Falkirk. But before describing this part of the work we will consider some further observations conducted in the Highlands with the dust-counter. For this second series Appin was selected, because it is near Kingairloch, being situated on the opposite side of Loch Linnhe in an east-south-east direction, at a distance of 6½ miles. Appin was found to be not so suitable as Morar or Kingairloch for work of this kind, as there are more houses in the neighbourhood, and more time is taken up with going to places free from local pollution.

The Appin observations were begun on the 29th June and continued to the 11th July. When the observations began the wind was W.S.W., but by the evening it veered to W.N.W., and it remained in the N.W. quadrant till the evening of the 7th, when it veered to a little east of north. The observations, accordingly, were taken in northerly winds, which were uniformly found to be very free from dust at Kingairloch, except on the days of abnormal readings when the numbers went high; on these occasions

the wind was accompanied by sunshine, and, though the numbers were high, the air remained clear.

From Table No. II. it will be seen that from the 29th June to the 4th July the number of particles remained low—once as low as 140 per c.c., but generally from 200 to 300 per c.c., and occasionally going up to nearly 1000. On the 5th it was too calm to get trustworthy results. On the 6th a change took place; the numbers in the morning were low, but rose after midday and remained high all afternoon, though very variable, being as low as 1000 and as high as 5000 per c.c. These high numbers were probably



not due to sunshine, as the sky was nine-tenths clouded most of the day; and, further, the high numbers did not fall at night as the sunshine high numbers at Kingairloch did, but remained over 3000 at the evening reading. To make certain of the observations on this day, sixteen different tests were made, each consisting of the usual five or ten tests to get the mean. The tests were made all through the day, some at sea-level and some on a hill 600 feet high, and all gave high readings. Further, in the evening there was an opportunity for getting a photographic record of the state of the air. Ordinary photographs of mountains are of no use for recording haze, because an under-exposed photograph would show little haze on a mountain, while in a fully-exposed one it might be invisible. In the photograph it will be seen that the sun's rays where they shone through the openings in

the clouds revealed the dusty condition of the air, which shows that the high numbers observed on this day were not entirely due to the sunshine effect but to real dust. Then, again, the limit of visibility was only 40 miles, which is decidedly low for north-westerly winds.

On the 7th the numbers were very variable; but as there was much sunshine, the high readings were probably of the abnormal sunshine kind. This is supported by the fact that the numbers, which had been over 5000 during the day, fell to 375 in the evening, and also by the fact that the limit of visibility had increased to 70 miles. The observations made on the 8th were similar to those of the previous day, and the conditions were the same except that the wind had veered slightly. The numbers were low, 336 per c.c., in the morning, but, under the influence of the sun in a cloudless sky, soon began to increase, and rose to 6000, falling a good deal in the evening. The high numbers on this day were evidently due to sunshine, as the sky all day was cloudless. This conclusion is confirmed by the observations on the haze, the limit of visibility having increased to 100 miles. Every effort was made on this occasion, as on the 6th, to check all observations, sixteen different tests being made at different positions, at sea-level and at different heights up to 600 feet.

On the 9th the conditions remained much the same. The sky was cloudless, and the wind N.N.E. but stronger. No low numbers were observed on this day, neither morning nor evening, the readings being always over 3000 and rising to 6000. This rise would be due to sunshine effect, but probably morning and evening readings were not greatly affected by sunshine, as the limit of visibility had decreased from 100 to 40 miles. On the 10th the conditions were but little changed. It was still cloudless, but the wind had fallen and become variable. The numbers were never low, and the air was of only medium clearness. On the morning of the 11th the sky was still cloudless, but the wind had changed to W. The humidity had greatly increased, and, as the numbers were still high, the air thickened and the limit of visibility was reduced to 15 miles.

On looking at the figures for Appin during the thirteen days one naturally asks, Why did the numbers at the beginning of the period vary so much from those at the end? Throughout the first half of the time the numbers were low, while in the last half they were high, though the wind was coming all the time from nearly the same northerly direction. An examination of the weather charts shows that during the first half the circulation over the place of observation was cyclonic, while during the last half it was anticyclonic, and that the nearer the centre of the anticyclone came to the place of observation the higher the numbers were. It has been

shown in the first part of this paper that this is exactly what happened when the Morar observations were made: air clear and dust-free in cyclonic areas, and thick and dusty when near the centre of an anticyclone.

It will be noticed from Table No. II. that during the first half of the period, while the dust particles were few, the air was clear, though the wet-bulb depression was slight, being frequently under 4 degrees; while during the last half, while the number of particles was high, the air was hazed, though the wet-bulb depression was generally high, being sometimes over 10 degrees. That is, the haze was greater, though the hazing effect of the dust was greatly decreased by its extreme dryness; and it will be further noticed that it was only on the last day of these observations, while the number of particles was still high, that the air became thick owing to the decrease in the wet-bulb depression to 4 degrees. These results all confirm the conclusions recorded in previous papers on the effect of humidity in increasing the hazing effect of dust.

Haze.

At the end of Part I. of this communication it is stated that it would be possible for me to continue the observation on dust in our atmosphere by observing the amount of haze at Falkirk. In a paper read before this Society in January 1893 it is pointed out that it was possible to make observations on dust without a dust-counter, and there is given the result of some 200 observations taken prior to the reading of the paper. As the first paper may be inaccessible to many, I shall here recapitulate the method of working.

In previous communications giving the results of observations made with the dust-counter it was shown that the thickness of a haze depends on the number of dust particles and on the humidity of the air at the time. The greater the number of particles, the thicker the haze; and the dryer the air with the same number, the clearer it is. It is shown roughly that if the air is four times drier, as measured by the wet-bulb depression, it is four times clearer. It results from these dust-counting observations that if we know the amount of haze and the wet-bulb depression we can get a rough approximation to the number of particles present. One result of the first series of observations on haze was to show that the air at Falkirk is much more densely hazed when it comes from the thickly populated parts of the country than when it comes from a direction where it is thinly populated. Air coming to Falkirk from all directions save from the N.W. quadrant comes from thickly populated parts, and it was shown that the

air coming from some of these directions was ten times more hazed than the air from the N.W. quadrant. From this it is evident we can only compare the air on the different days when the wind blows from the same direction and is of the same humidity.

In these observations the things noted were: the temperature and wet-bulb depression, the direction and force of the wind, and the limit of visibility—that is, the most distant object, such as a hill, that was visible at the time. When the air was so clear that the most distant hill visible from the point of observation was quite distinct, then the haze on it was estimated. For instance, the most distant hill used in these observations is Ben Ledi, distant 25 miles. If this hill was just visible, the limit would be entered as 25 miles; but if the hill was quite distinct, then the haze on it was estimated. If it appeared to be one-half obscured by the haze, the limit would be 50 miles.

The first observations on haze were begun at Falkirk in June 1891, and continued to the end of 1892. Some 200 observations were made during that time, and the results are given in vol. xx., *Proc. Roy. Soc. Edin.* These observations were continued during 1893 and to near the end of 1894, and also during August 1896. Over 200 observations were made during this second period, but have not been published. They are now worked up and incorporated with the figures given in the first paper. Some of the observations were rejected owing to too high humidity or to there being too little wind, as high humidities and feeble circulation make observations of this kind unreliable.

In working out the results of these haze observations, instead of giving all the readings the means are given in Table No. III. and the lowest and highest readings in Table No. IV. In preparing the observations for the tables the first thing to be done was to arrange them according to the direction of the wind at the time, all those taken when the wind was south being together, and all the others in a similar manner. Then the observations in each direction of wind were again separated according to the wet-bulb depression at the time, all those in which the wet-bulb depression was 2 degrees being together, then those in which it was 3 and 4 degrees, and so on. When this was done, the mean limit of visibility of each lot was calculated. All these means are entered in Table No. III. There is also a parallel column in this table giving the number of observations of which the limit is the mean. It will be noticed that for all the winds the limit of visibility increases as the wet-bulb depression increases, and that all winds from N. to W. are very clear, while those coming from all other directions are more or less hazed; those having an

easterly component are the worst, being at all degrees of humidity only about one-tenth as clear as the north-westerlys.

Table No. IV. shows how variable the limit is in the different observations in winds from the same direction and with the same humidity. Part of this variability is due to the difficulty of estimating haze. For instance, if the sun is in front and near the line of sight the haze is much denser than when the sun is behind. Then the background of the distant hill has a great effect when estimating the haze. If backed by shaded clouds, which I consider to be standard condition, it seems much more hazed than when backed by a clear bright sky or a setting sun, against which it gets silhouetted; and estimates made under these conditions are not trustworthy, or must be allowed for. Another thing which causes difficulty is the want of homogeneity in the air. I have stood on a hill about five miles to the west of Falkirk when a W.S.W. wind was blowing. Looking towards the south the limit of visibility was only a mile or two, while looking towards the north-west the air was very clear, limit being about 200 miles. At the time referred to I happened to stand on the dividing line, which is here very sharp, between the populated and unpopulated areas, and along this line the wind was blowing. All south of this line is a busy manufacturing area, and all north of west is pasture and moorland. Supposing, now, that the W.S.W. wind then blowing changed to S.W., this would bring some miles of the thick air between me and Ben Ledi, while beyond that thick air there would be pure air, and in entering the observation it would go down under S.W. air, while the estimate would be made only partly in that air and mostly in purer air. Now, from this it is evident that all estimates of haze should be done either up or down wind, and when possible with the back to the sun. But as there are few situations giving an all-round view to a great distance, we have to be content with and make the most of what is within our range. Difficulty in estimating haze is also caused by the air at low level not always having the same transparency as that at the level of the top of the mountain.

It was thought that if the tail of the comet had any effect on our atmosphere, either by increasing the dusty contents or otherwise, we might expect to find some evidence of it in the haze in the lower air some time after the passage of the comet. I therefore began taking observations on haze at Falkirk after my return from Morar. These observations were interrupted while I was observing at Appin, but were started again on my return and continued till the 24th July. They have been treated in the same way as the earlier ones, and the mean

limits of visibility for the different winds at Falkirk at the different wet-bulb depressions will be found in Table No. V., while the lowest and highest readings are given in Table No. VI.

An examination of Tables No. III. and No. V. shows that there has been a great increase in the haze in certain winds since the early observations, and it will be noticed that the greatest increase is in the easterly winds; it is also marked in the south-westerlys, while the north-westerlys are not much affected. For comparison there are arranged in Table No. VII. the limits of visibility of the first series of observations and those of this year for E.N.E. and E. winds. In Tables No. III. and No. V. there is a considerable number of observations for each wet-bulb depression for these directions of wind, and the results are thus the more reliable. In the E.N.E. observation, when the wet-bulb depression was under 5 degrees, the limit is much less this year than before; but in the observations made when the wet-bulb depression was over 5 degrees the limit is greater than in the first series. I shall refer to this later. The observations taken in east wind this year show that it was about five times more hazed at all wet-bulb depressions, except those taken when the depression was very great, when the increase in the haze was more than three times that of the first observations.

Further, an examination of the individual observations shows that the lowest limit observed in these winds in the first observations was 3 miles in E.N.E. winds and 4 miles in E. winds, while the figures for this year show that a limit of $1\frac{1}{2}$ miles was observed on five occasions, and that limits of 2 and 3 miles were frequent in dry air.

Turning again to the weather charts, let us see if the circulation in our atmosphere throws any light on these observations. When the haze readings began this year on the 15th June an anticyclone was situated over our area, where it remained till the 21st. During these days the limit of visibility was always exceptionally low, $1\frac{1}{2}$ miles being frequent even in dry air. On the 21st a feeble cyclonic circulation came in from the west, but was not well developed till the 25th. During the first part of this circulation the limit rose slightly, but after the 25th, when the circulation was well established, the limit extended, and the air remained clear till the 29th of June, when the observations closed.

When the second series of observations began on my return from Appin on the 11th July, the circulation was again anticyclonic and the haze dense, but not so many readings of $1\frac{1}{2}$ miles were noted as during the June anticyclone. The circulation remained anticyclonic till the 17th, but the centre of the anticyclone had receded from our area, and the limit of visibility had begun to increase.

I shall now make a few remarks on some of the figures shown in Table No. VII. The E.N.E. winds in that table are more transparent than those of the previous years, when the wet-bulb depression was high; while all other easterly winds are less transparent than before. From the 11th to the 13th July the air was densely hazed, and during this period we were situated near the centre of an anticyclone. Afterwards the centre of the anticyclone moved away towards Iceland, and we were till the 19th situated between an anticyclone and a cyclone, so that during this time we were out of the anticyclonic air, and it was then that all the E.N.E. observations giving high transparencies were obtained. Further, the weather charts show that the general circulation was not E.N.E. on these days, but only slightly east of north. This northerly wind when it arrives at Falkirk is turned into a more easterly wind by striking the southern slopes of the Forth valley, which gives it an increase in its easterly component. These observations in E.N.E. winds ought therefore to have had their place among the more northerly winds, and in that position would show decreased and not increased transparency.

A word of caution as to any conclusions we may draw from these dust and haze observations. It must be remembered that it is about fifteen years since the early observations with the dust-counter and on haze were made, so that we are comparing the state of the air now with what it was fifteen years ago; and it may be contended that the increase in haze is due to the increased consumption of coal. It is very doubtful if the better combustion now more general in our furnaces will have done anything to balance the effect of the increased consumption, because perfect combustion gives rise to many nuclei, while the smoky products soon fall out of the air. The increased consumption of coal does not, however, seem to explain the condition of the air, because the increase, so far as I have been able to ascertain, only amounts to one-fourth during the last fifteen years. It is probable, however, that the consumption of coal on the Continent has increased to a greater extent than in this country. In addition to their own increased output our coal export has been nearly doubled during the last fifteen years; and as our east winds come to us laden with Continental impurities, making them always thickly hazed, the increased Continental consumption may have some effect. It may also be as well to remember that during this year there have been exceptionally extensive forest fires in America, and the products from these may have been carried to the upper parts of our atmosphere.

It will be noticed that the abnormally high numbers of dust-particles were observed at Morar only when an anticyclone was over the place of

observation, and that the same thing happened at Appin. Under cyclonic conditions the numbers were much as previously observed, but with the approach of an anticyclone the figures became high even in air coming from what are usually pure directions. The observations on haze at Falkirk also point to the same conclusion. All the abnormally densely-hazed days occurred while the circulation was anticyclonic; and the nearer the centre of the anticyclone was to the place, the denser was the haze. These facts all point one way, namely, that there was something in the descending air of the anticyclones which increased the number of dust-particles in the pure areas and greatly increased the haze in the polluted areas. Now, one must not rush to the conclusion that both these results indicate one thing, namely, that they are due to the dust contents of the anticyclonic air; because the increased dust and haze in the pure areas, if added to the air of our polluted districts, would have but little or no effect on its transparency. If, then, the increased haze is due to the comet, it must have had, in addition to its dust, some hazing quality, with which we are not acquainted, which, when added to dusty air, increases its hazing effect.

If the change in our atmosphere which these observations demonstrate be due to the tail of the comet, then one would have expected that the dust, or emanation, or whatever it may be supposed to be, would have had some effect on the upper part of our atmosphere immediately after the passage; but, so far as I am aware, no such effects have been recorded. So far as my observations go, there has been no increase in the colour of the sunsets. Both when at Morar and Appin I was most favourably situated for observing sunsets, but never saw any good colouring. Before the passage of the comet the sunsets at Morar were finer-coloured than I have seen since. I took some colour-photographs of sunsets before the passage, but have not seen a sunset since worth exposing a plate on. So far as my observations go, instead of the colours being finer they seem to be quite the reverse. At Appin cloudy evenings gave no fine colouring, and on some evenings the sun set in perfectly cloudless skies, but the north-west horizon never showed more than a poor and rather dirty yellow, and not so much colour as was visible in the south-east sky. None of the evenings showed as much colour as was seen under similar conditions at Morar before the passage, when the records were taken in colour-photographs.

Thunderstorms.

The great number of thunderstorms of unusual violence which have occurred in this country and all over the Continent, and probably else-

where, since the passage of the comet, seems to suggest some connection between the two. No electrical effects have been traced to the comet, but may not the great amount of dust in our atmosphere account for them? Leaving alone the question as to whether the electric charge is on the gaseous molecules or on the dust, there is a point on which I think most will agree, and that is, that thunderstorms always take place in densely-hazed air; at least, summer thunderstorms do. I do not remember ever seeing one take place in clear air, but that may have been from want of observation or lack of opportunity, as very few thunderstorms take place in the Falkirk district. But I have noticed at the Italian lakes, where they are frequent, that if a thunderstorm comes and does not clear the air, it is sure to return on the following day. It seemed to me that on these occasions the first storm only cleared the upper air, the next day it cleared it lower down, and perhaps on the third day it cleared it to lake level; and when it did that, it did not return the following day.

Conclusion.

These observations show that while our atmosphere has been much more hazed during June and July of this year than it was fifteen years ago, there is nothing to prove that the tail of the comet was the cause of this increase in dust and haze. The only evidence against it is, that all the increased dust and haze were connected in some way with a change in the upper air, as it was only while observing in anticyclonic circulation that the increase was markedly greater; but there is no evidence to show whether or no the change in the upper air was due to the comet. If it was not due to the comet, then it would seem to be indicated that our atmosphere is rapidly getting more impure from local pollution. The fact, however, that the increase in dust and haze was almost entirely observed in anticylonic areas seems to negative this supposition, as local pollution is most likely to affect cyclonic areas. Whatever the cause of the increase may be, one thing is very evident, and that is our great ignorance of everything connected with dust and haze; and it does seem desirable that they should be subjected to more systematic observation in the future than they have been in the past. Dust, like water vapour, is an ever-present constituent of our atmosphere; like it, is present in ever-varying quantity; and, along with it, plays an important part in meteorological phenomena.

[Table

Table II.—The Number of Dust Particles in the Air at Appin in June and July 1910.

Date.	Hour.	Number of Particles.	Wind Direction and Force.	Temperature.	Wet-Bulb Depression.	Cloud Amount.	Transparency of Air.
29	6.45	688	W.S.W. 1	50·5	5	$\frac{1}{1}$	Raining
"	9	266	W.N.W. 1	...	...	"	Very clear
30	8.45	357	"	51	2	"	"
"	11.30	322	"	55	3·8	$\frac{9}{10}$	"
"	3	154	N.W. 2	54	4	"	"
"	6	189	"	52·5	3·5	"	"
1	8.45	315	W.N.W. ·2	52·5	4	"	"
"	12	322	N.W. 1	52	2	$\frac{1}{1}$	"
"	2.30	329	W. ·5	55	5	"	"
"	5	280	S.W. 1	53	3·5	"	"
2	8.45	329	N. ·5	54	3	$\frac{3}{4}$	"
"	11.30	791	W.N.W. 1	55·5	4·5	$\frac{1}{1}$	"
"	2	485	"	56·5	5·5	"	"
"	5.30	252	N.W. 1	54	4	"	"
3	8.30	350	N. ·5	52	3	"	"
"	1	725	N.W. 1	59·5	5·5	$\frac{1}{2}$	"
"	3	950	"	60·5	6·5	$\frac{1}{4}$	" 150
"	6	942	"	61	7	$\frac{1}{10}$	Clear 100
"	8.45	315	W.N.W. ·5	56·5	5·5	0	"
4	8.30	161	"	54·5	3·5	$\frac{1}{10}$	"
"	11.30	245	W.N.W. 1	...	...	$\frac{1}{2}$	"
"	12.15	245	"	56·5	3·5	"	"
"	5.30	625	"	57	4	$\frac{1}{10}$	"
"	8.30	140	"	53	3	$\frac{1}{1}$	"
5	8.45	273	S.W. ·2	54·4	3·4	"	...
"	5.15	1550 ?	N.N.E. ·1	53·5	1	"	Very thick
"	8.30	1225 ?	Calm	53	·5	"	"
6	8.30	140	N.W. ·5	55	2·5	"	Very clear
"	11	224	N.W. 1	58	5	$\frac{9}{10}$	"
"	1	900	W.N.W. 1	57	3·5	"	Medium
"	2.45	1135	W.N.W. 2	...	...	...	Thickish
"	4.10	4900	W.N.W. 3	60	5	$\frac{3}{4}$	" 40
"	6	3350	W.N.W. 2	56	4	$\frac{1}{2}$	"
"	8.30	3400	"	54·5	4	$\frac{3}{4}$	"
7	8.30	500	N.N.W. 1	56·5	5·5	$\frac{1}{2}$	Clear
"	10	1000	"	58·5	7	$\frac{3}{4}$	"
"	12	1900	"	...	...	...	" 70
"	2.30	3200	N.N.W. 2	62·5	9	$\frac{1}{2}$	"
"	7.30	875	N. ·5	59·5	7	$\frac{1}{3}$	"
"	8.45	394	N.N.E. ·5	57	6·5	$\frac{1}{4}$	"
8	8.30	336	N.N.E. 1	55·5	5·5	0	"
"	10.30	1900	"	...	...	"	"
"	12.5	3600	N.N.E. 2	...	...	"	"
"	1	6000	"	63	9	"	"
"	4	4800	"	67·5	10·5	"	"
"	8.30	1700	"	58	6	"	Medium
9	8.30	3750	"	54·5	5	"	" 60
"	12.30	4000	"	65	9	"	"
"	3.15	3400	"	72·5	13·5	"	" 40
"	6	3900	"	70	12	"	" 40
"	9.30	3700	N.N.E. ·5	62	8	"	"

Table II.—*continued.*

Date.	Hour.	Number of Particles.	Wind Direction and Force.	Temperature.	Wet-Bulb Depression.	Cloud Amount.	Transparency of Air.
10	8.45	3175	N.E. ·5	60	8	0	Medium 40
"	10	3400	N.E. 1	66	10	"	"
"	1	2300	N.E. ·5	74	16	"	"
"	7	3400	W.S.W. ·5	67	8	"	"
"	9.30	2700	N.E. 2	60·5	6	"	...
11	8.45	2100	Calm	58·5	6	"	Thick
"	10.30	3600	W. ·2	60	4	"	" 15
"	11	3580	W. ·5	60·5	4	"	"

Table III.—Showing the Mean Limit of Visibility of the Air in Miles at Falkirk for Winds from Different Directions, and for each increase of 2° in the Wet-Bulb Depression, from 1891 to 1896.

Direction of Wind.	2°.		3° and 4°.		5° and 6°.		7° and over.	
	Number of Observations.	Limit of Visibility.	Number of Observations.	Limit of Visibility.	Number of Observations.	Limit of Visibility.	Number of Observations.	Limit of Visibility.
N.	...	...	1	80	5	189	7	182
N.N.E.	...	...	3	47	...	...	2	37
N.E.	3	32	5	33	3	33	4	54
E.N.E.	6	6	7	21	3	16	6	19
E.	5	8	14	12	15	19	20	19·5
E.S.E.	...	...	...	...	...	...	...	...
S.E.	...	...	3	14	2	22	1	15
S.S.E.	...	...	3	12	1	12	2	15
S.	4	8	8	15	8	20	6	25
S.S.W.	5	6	12	12	7	13	3	38
S.W.	10	6	26	13	19	13	14	19
W.S.W.	3	14	8	22	13	31	13	42
W.	6	50	21	110	23	110	8	156
W.N.W.	...	...	2	75	3	190	8	188
N.W.	...	...	4	142	10	195	7	200
N.N.W.	...	...	...	...	2	175	...	...

Table IV.—Showing the Lowest and Highest Limits of Visibility observed at Falkirk for Winds from the Principal Directions, and at Different Degrees of Dryness, from 1891 to 1896.

Direction of Wind.	Extreme Limits of Visibility in Miles for Different Wet-Bulb Depressions.			
	2°.	3° and 4°.	5° and 6°.	7° and over.
N.	...	40 to 80	120 to 250	100 to 250
N.E.	3 to 40	5 „ 50	25 „ 50	12 „ 100
E.	4 „ 15	5 „ 25	6 „ 50	10 „ 50
S.E.	...	12 „ 17	15 „ 30	...
S.	6 to 10	10 „ 24	12 „ 30	15 to 50
S W.	2 „ 12	5 „ 20	6 „ 25	8 „ 60
W.	20 „ 60	20 „ 250	50 „ 250	50 „ 200
N.W.	...	30 „ 250	50 „ 250	100 „ 250

Table V.—Showing the Mean Limit of Visibility of the Air at Falkirk for Winds from the Different Directions, and for each increase of 2° in the Wet-Bulb Depression, for June and July 1910.

Direction of Wind.	2°.		3° and 4°.		5° and 6°.		7° and over.	
	Number of Observations.	Limit of Visibility.	Number of Observations.	Limit of Visibility.	Number of Observations.	Limit of Visibility.	Number of Observations.	Limit of Visibility.
N.	...	...	...	...	...	...	1	150
N.N.E.	...	...	...	...	1	60	2	40
N.E.	...	...	1	2·5	2	3·25	...	...
E.N.E.	1	1·5	5	10	6	19	2	30
E.	2	1·5	7	2·5	8	3·5	10	6
E.S.E.	...	...	...	...	...	...	...	...
S.E.	...	...	1	15	...	...	...	...
S.S.E.	...	...	1	2	1	4	...	...
S.	...	...	...	...	2	3·5	1	15
S.S.W.	...	...	...	...	...	...	...	...
S.W.	...	...	...	...	6	10	1	6
W.S.W.	...	...	1	15	3	16	2	15
W.	...	...	1	100	2	45	3	77
W.N.W.	...	...	...	...	1	150	...	...
N.W.	...	...	...	...	1	125	1	125
N.N.W.	...	...	...	...	...	...	1	100

Table VI.—Showing the Lowest and Highest Limits of Visibility observed at Falkirk for Winds from the Principal Directions, and at Different Degrees of Dryness, for June and July 1910.

Direction of Wind.	Extreme Limits of Visibility in Miles for the Different Wet-Bulb Depressions.			
	2°.	3° and 4°.	5° and 6°.	7° and over.
N.	...	...	...	30 to 150
N.E.	...	$2\frac{1}{2}$ to 20	$2\frac{1}{2}$ to 30	30 „ 30
E.	$1\frac{1}{2}$ to $1\frac{1}{2}$	$1\frac{1}{2}$ „ 4	2 „ 6	2 „ 10
S.E.	...	2 „ 15	...	...
S.	...	...	3 „ 4	...
S.W.	...	...	2·5 „ 25	4 to 25
W.	...	...	15 „ 150	40 „ 125
N.W.	...	...	...	100 „ 125

Table VII.—Showing the Mean Limit of Visibility in Miles for Winds from the E.N.E. and E. during the Years 1891, 1896, and June and July 1910.

Year.	Direction of Wind.	Mean Limits of Visibility in Miles.			
		2°.	3° and 4°.	5° and 6°.	7° and over.
1891–1896	E.N.E.	6	21	16	19
1910	„	**1·5**	**10**	**19**	**30**
1891–1896	E.	8	12	19	19·5
1910	„	**1·5**	**2·5**	**3·5**	**6**

Note added 1st August.

While the observations made during the last two months show an increase in the haze in our atmosphere, and prove that the increase was mostly found during anticyclonic circulation, it may be asked, Was not the air in the early observations more hazed in anticyclonic than in cyclonic areas? To test this point, all the early observations taken in anticyclonic circulation were arranged in tables, in the same way as all the observations are arranged in Table No. III. The result is that the tables are practically alike for all winds at the different wet-bulb depressions. In some cases the figure is a little over and in others a little under; showing that the anticyclonic air

was not more hazed during the early observations than the cyclonic. Further, in the early observations the air was only thickly hazed when the wet-bulb depression was slight, under 4 degrees; whereas this year there were observations with very low limits of visibility when the wet-bulb depression was as much as from 6 to 12 degrees.

(*Issued separately September* 7, 1910.)

XL.—A Stereoscopic Illusion. By Professor **Francis Gibson Baily**, M.A.

(Read February 7, 1910. MS. received March 14, 1910.)

If the finger be held vertically in front of the eyes and a distant object be looked at, it is well known that two images of the finger will be seen, quite transparent if the two images do not overlap, and opaque only at overlapping parts. In place of the finger, use a thin rod at a distance of some 6 or more feet, and focus on another vertical rod at a distance of 30 feet or more, so that the second rod is seen between the two images of the first. The apparent position of the second rod will be found to be distinctly nearer than its real position.

In fig. 1, $A_1 A_2$ are the observer's eyes, B is the first rod and C the

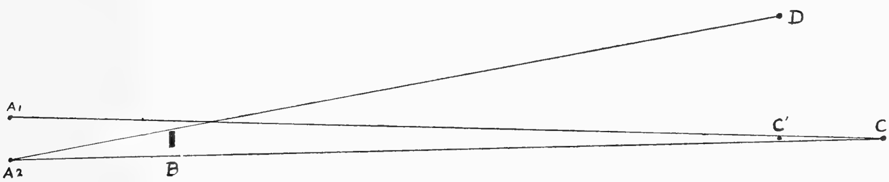


Fig. 1.

second. Each eye sees C without hindrance, and the observer sees a transparent image of B on each side. C then appears to be at C′.

To observe this, B is preferably not strongly lighted, or the tendency to focus on B may be too great. C must be well away from its background, so as to stand alone with the background out of focus. The ground on which C stands must be hidden, so as to prevent any other knowledge of its position. For if the actual situation on the ground is seen, the visible proof of its position will mentally outweigh the indication given by the stereoscopic effect.

The apparent position of C may be determined by setting up a similar rod D near to the line A C, sufficiently on one side to be visible to both eyes. D is then moved backwards and forwards, until it is judged to be the same distance from the observer as C appears to be. The diagram has been drawn to correspond with the following actual figures, the vertical scale being exaggerated to twenty times the horizontal:—$A_1 A_2$, $2\frac{5}{8}$ inches; B, 1 inch broad; A C, 100 feet; and C D, 1 foot.

Experiments were made as follows :—The observer stood back in a room overlooking a level lawn with a background of trees. In the window was the rod B. The window-sill cut off the view of the ground. The rod C was set up on the lawn, and the rod D was moved backwards and forwards by an assistant according to signals, until the correct position was found. The distance C C′ was then measured. It will be noticed that the observer had no means of knowing the actual positions of C and D until the reading was completed. In another set of readings the rod D was fixed on a trolley and pulled to and fro by an endless cord round a pulley at the end of the lawn.

The position of C was varied, the greatest distance from B being 110 feet, for beyond that distance the background came into focus and confused the reading. Two or three readings were taken at each distance, D being displaced each time, and good agreement was usually obtained when the conditions were favourable, as described below, the variation seldom exceeding three or four inches.

It was found that readings were most consistent when the distance C D was kept small. This of course facilitated the comparison of distance. But it was also found that the distance C C′ was increased, the largest values being obtained when the image of B came close to the edges of C, and when D was immediately on the other side of one of the B images, or even if B partially overlapped C and D. The apparent distance between the two B images did not make any difference to C C′, if it was not much larger than the angle subtended by C. But if considerably larger it caused irregularity in the reading, and greater difficulty in determining the correct position. Probably in this case the eyes were trying to see normally, and it is well known that if the eyes are receiving two impressions simultaneously, the mental effect is apt to alternate between the two competing appearances.

The judgment was better if C and D were dark or quiet in colour. White posts were found to be somewhat difficult to place stereoscopically, even without any disturbing images. The posts were the dull greyish brown of weather-stained wood, and excellent demonstrations were also obtained with the trunks of young trees.

The distance A B did not affect the results, over a range from 9 to 19 feet, except that more accurate readings were obtained if A B was chosen so as to allow C′ and D to be close together. And, as stated above, if A B was short when B C was very long, the clear space between the edges of the B images and C became too great. This seemed more disturbing on a dull day than on a bright sunny day.

Fig. 2 shows three sets of readings under different conditions. Line 1, shown with dots, was taken on a bright day with C D kept small. Line 2, marked with crosses, was taken at a later date with a different background (leaves off the trees) on a dull day, but with C D kept small as before. Line 3, with circles, was taken on the same day as line 2, but C D was made large. The lowest point of all, at 20 feet, was taken in the house by artificial light, carefully arranged so as to obtain very accurate results.

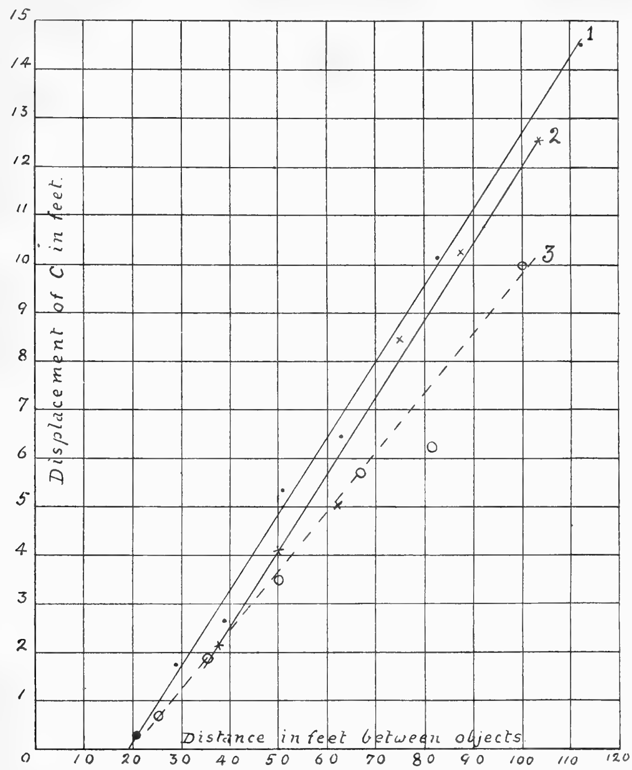


Fig. 2.

The relation over this range of distance is clearly a close approximation to a linear function starting from a distance of about 20 feet. Before making quantitative measurements the author had tried to observe the phenomenon in a room, but without success. No movement of C could be detected. The subsequent quantitative determination showed that below a distance of 20 feet no effect would be visible.

It will be noted that the values in line 3 are distinctly lowered by the increased distance between C′ and D. A large number of readings were taken with irregular values of C D before its effect was identified, and the

results were consistently low, but very irregular and uncertain in determination, repeat readings not agreeing well.

The image C′ appears to come up to its final position somewhat slowly, the best results being obtained when the gaze is focussed on the space between C′ and D, so as to have both equally in vision. Moving the eyes from one to the other did not give such conclusive impressions.

The author has asked several people to observe the phenomenon, and they have found little difficulty in qualitative observation, though some have found trouble in keeping the focus on the distant object. Quantitative work seems more difficult, and will doubtless depend on the stereoscopic perception of the observer. The author had previously been tested for this power, and was found to have a perception above the average; some people, on the other hand, are markedly deficient, and any defect of sight, such as would cause a person to attend to the impressions of one eye more than the other, would probably prevent the phenomenon being seen. Some power of control over the direction and focus of the gaze is essential, and this is not possessed equally by all people.

It is obvious that the effect is physiological or psychological, and is not due to diffraction at the edge of the nearer rod, or any phenomenon external to the eye. Its simple proportionality above 20 feet distance and its complete cessation below 20 feet are remarkable; but these figures refer only to the author's eyesight, and may be entirely personal. He does not suggest any theory in explanation, but brings it forward in case it should be of interest to physiologists and psychologists.

The phenomenon is probably another instance of the various optical illusions, more or less of a stereoscopic nature, which are well known and are described in textbooks on physiological optics. But its simplicity and adaptability to numerical measurement, its freedom from pictorial deception, and the large scale of the dimensions involved may render it suggestive in the theory of stereoscopy. This particular cause of illusion seems not to have been noticed before, even qualitatively.

(*Issued separately October* 12, 1910.)

XLI.—The Efficiency of Metallic Filament Lamps. By R. A. Houstoun, M.A., D.Sc., Ph.D., Lecturer in Physical Optics in the University of Glasgow. *Communicated by* Professor A. Gray, F.R.S.

(MS. received April 25, 1910. Read November 7, 1910.)

Practical men are accustomed to measure the efficiency of a glow-lamp in watts per candle or, in other words, by the quantity of energy which must be supplied to it for every mean spherical candle-power of light it gives. The energy supplied is measured by an ammeter and voltmeter, and the mean spherical candle-power is obtained on the photometric bench by comparing the light given in different directions with that of some standard source. The watts per candle together with the life and initial cost of the lamp determine its commercial value.

On the other hand, we may regard the lamp as an energy-transforming device, and measure its efficiency by the percentage of electrical energy it transforms into light. Let Q = total electrical energy consumed per second, R the total energy radiated per second, and L the total luminous energy radiated per second. Then we make the following definitions:—

$$\frac{L}{Q} = \text{luminous efficiency,}$$

$$\frac{L}{R} = \text{radiant efficiency of the lamp.}$$

R is somewhat less than Q, as all the energy is not lost by radiation, but some by conduction and convection. This proportion is, however, so small that it is usually neglected. Dr C. V. Drysdale * states that he found the convection loss to be not more than 2 or 3 per cent. of the total energy consumed. W. Wedding † finds it to be very much larger; but, on account of the wide difference between his results and those of other investigators, it is impossible to avoid the suspicion that he makes an error somewhere. It should be stated, though, that there is nothing in his book itself to justify this suspicion.

For the purpose of defining L, it is usual to take 760 $\mu\mu$ as the upper

* Charles V. Drysdale, "On Luminous Efficiency and the Mechanical Equivalent of Light," *Proc. Roy. Soc.*, A, 80 (1907–08), p. 19.

† W. Wedding, *Über den Wirkungsgrad und die praktische Bedeutung der gebräuchlichsten Lichtquellen*; published by R. Oldenbourg, Munich and Berlin, 1905.

limit of the visible spectrum. The lower limit is immaterial; there is so little energy in the violet and ultraviolet spectrum of glow-lamps. The luminous efficiency does not measure the commercial value of a lamp, because the eye is very much more sensitive to green than to red light, and it is possible for one lamp to have much more energy in its spectrum than a second one but yet to have that energy so disadvantageously distributed that the second produces a stronger effect on the eye. The luminous efficiency is of more theoretical interest. It tells us how much of our energy is wasted in dark heat and how far we can with advantage hope to improve our lamps.

The object of this paper is to describe some measurements made on the radiant efficiency of carbon, osmium, tantalum, and tungsten glow-lamps.

An old method of measuring the efficiency was the calorimetric one. The lamp was placed in a glass calorimeter and then in a similar copper one, and from the difference in the rise of temperature, with various corrections, the percentage of light radiated could be calculated. The method is in principle bad; the whole energy and the dark heat are measured, and the light is obtained as the difference of two much larger quantities, both subject to considerable error of observation. Another method was to measure the total radiation by a thermopile. A water filter was then placed in front of the thermopile and the radiation that penetrated through the filter measured. The latter was supposed to be light, and if we made a correction for the light absorbed in, or reflected by, the filter—which could easily be determined photometrically—the radiant efficiency could be obtained. The objection to this method is that the water filter does not absorb all the heat. A solution of alum is no better than water, although popularly supposed to be.* Hence corrections were made for the dark heat transmitted by the water by means of solutions of iodine in carbon disulphide, which were supposed to absorb all the light, and let the heat pass. The utter untrustworthiness of all such methods has, however, been demonstrated by E. L. Nichols and W. W. Coblentz.† They have shown that they give values much too large.

The older methods being thus discredited, there remain two modern methods. One consists in determining the energy radiated for different

* It has been shown recently by R. A. Houstoun and J. Logie, *Phys. Zs.*, xi. p. 672, that an aqueous solution of ferrous ammonium sulphate absorbs the dark heat much more effectively than water alone while allowing the light to pass. Nevertheless, it is not satisfactory enough as a means of separating the dark heat from the visible radiation.

† "On Methods of Measuring Radiant Efficiencies," *Physical Review*, xvii., 1903, p. 267.

wave-lengths by means of the spectroscope and thermopile or bolometer and plotting it as a function of the wave-length. Then if an ordinate be erected at 760 $\mu\mu$, the radiant efficiency is the area of the curve on the side of this ordinate towards the shorter wave-lengths divided by the area of the whole curve. G. W. Stewart * has determined the radiant efficiency of the acetylene flame in this way. The other method, which has been introduced by K. Ångström,† is not such a simple one. It has been applied with modifications to the case of the carbon glow-lamp by C. E. Mendenhall,‡ who obtained 2·8 as the radiant efficiency of the latter at its normal brilliancy. This, I think, is the only previous determination of the radiant efficiency of a glow-lamp to which exception cannot be taken on the point of method.

The method I have applied is, as far as I am aware, an original one, though it almost follows from the article of Nichols and Coblentz. It seems preferable to both the above methods. The radiation was first measured in the usual way by a thermopile and galvanometer with and without a 1 cm. thick water filter, and the percentage of total radiation transmitted through the filter determined. In this case the radiation from the whole lamp was received by the thermopile. A straight part of the filament of average brightness was then focussed by an achromatic glass lens on the slit of a spectrometer with glass prism which was furnished with a Rubens linear thermopile in place of the cross wires. The same water filter was placed in front of the slit and an energy curve of the filament taken through the filter. For the region of the spectrum for which water transmits, the absorption of light in the glass and the loss of light by reflection at the glass surfaces may be taken as independent of the colour. If the prismatic energy curve thus determined be taken and an ordinate set up at the point 760 $\mu\mu$, the area on the side of shorter wave-lengths divided by the whole area gives the fraction of the radiation transmitted by the filter, which is light. It is only necessary now to find what fraction of the incident light is transmitted by the water cell. This was done by measuring the transmission coefficient of the latter when filled with water with a spectrophotometer for four different points in the spectrum. The value found was 0·84. It is somewhat low, as the glass sides of the cell absorbed rather more than usual; it did not not vary with the colour.

* "The Spectral Energy Curve of the Acetylene Flame," *Physical Review*, xvi., 1903, p. 123.

† "Energy in the Visible Spectrum of the Hefner Standard," *Physical Review*, xvii., 1903, p. 302.

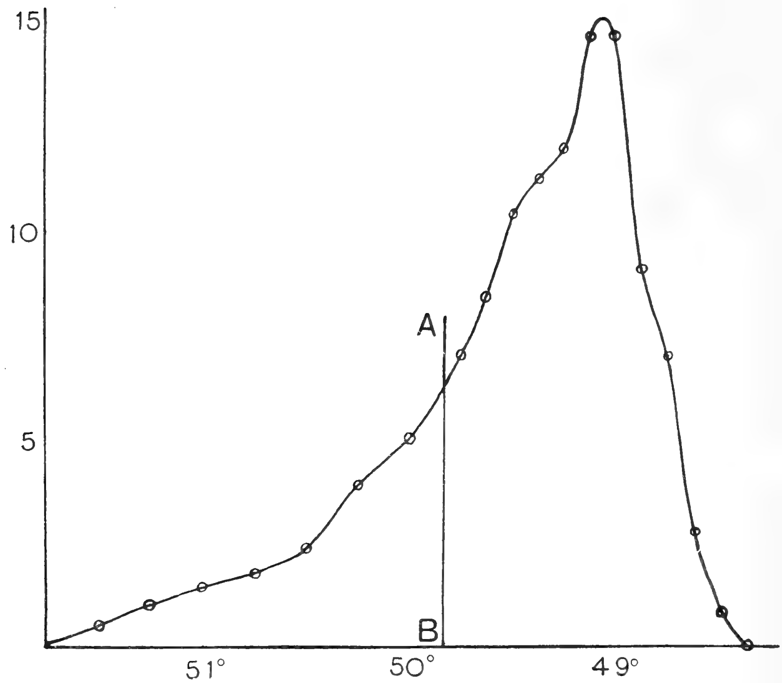
‡ "On the Luminous Efficiency of the Carbon Filament," *Physical Review*, xx., 1905, p. 160.

An example will make the method clear. When a tantalum lamp was run at its marked voltage, 125 volts, the fraction of the total radiation transmitted by the water cell was 0·222. The energy curve is given below, the depression to the left of the top being due to an absorption band of water.

The fraction of the area on the left of AB is 0·251. Hence the radiant efficiency is

$$\frac{\cdot 251 \times \cdot 222}{\cdot 84} = 6\cdot 7 \text{ per cent.}$$

In comparing the energy-curve method with Ångström's method,



Nichols and Coblentz give the preference to the latter on account of the errors introduced in reducing the prismatic energy curves to normal energy curves. In finding the ratio of the areas it is, however, not necessary that the abscissæ should be wave-lengths. If s denote the spectrometer reading and h the galvanometer throw, to change to the normal spectrum we multiply h by $\frac{ds}{d\lambda}$. The normal energy spectrum is $h\frac{ds}{d\lambda}$ plotted as a function of λ. Its area is given by $\int h\frac{ds}{d\lambda}d\lambda$, which is the same as $\int hds$.

The thermopiles employed were of the ordinary Rubens linear type. Their resistances were about four ohms each. The telescope of the spectrometer was carefully wrapped round with cotton wadding. The field-glass of the eyepiece was blackened except for one small opening by means of which the thermopile could be adjusted on the sodium line. The instrument was calibrated partly by bringing the thermopile into superposition with lines in the visible spectrum and partly by plotting the absorption curve of a very thin water film and using the maxima, which have been determined by Aschkinass * to be at $1·500\ \mu$ and $1·956\ \mu$, and the minimum between them at $1·708\ \mu$.

The galvanometer was a Du Bois-Rubens iron-clad one. Its total resistance was 10·5 ohms, and when the light suspension system was used the sensitiveness was $3\ 10^{-10}$ amps. per half mm. at one metre distance for a period of 5 secs. For greater sensitiveness the field was asymmetric. This sensitiveness was not all required. When measuring such small currents, of course, the keys and binding screws in the circuit must be made of copper alone; otherwise there are always thermo-electric forces at the junctions. The instrument was found impervious to magnetic disturbance. But it was not slung in a vibrationless suspension, as the makers recommend, and whenever there was a strong south-west wind, observations had to be discontinued owing to the vibration of the laboratory.

Current was supplied from the laboratory secondary battery. This gave voltages up to 260. For the readings at 270 volts the lighting circuit was used in series with fifteen accumulators, and the voltage kept constant by varying a resistance in the circuit. In order to measure the e.m.f. between the terminals of the glow-lamp, the latter was shunted by means of a Kelvin centi-ampere balance and a resistance, the total resistance of the shunt being 3226·8 ohms. By measuring the current through the shunt, the voltage between the terminals of the lamp was obtained. The total current through the lamp and shunt was obtained from an ammeter, and the current through the lamp obtained by subtracting the current through the shunt. The current balance was one of a set of three belonging to the laboratory, which had been tested by copper electrolysis some years previously, and as they all agree yet within the error of observation, it could be assumed to be perfectly correct. The ammeter was checked by the current balances.

* E. Aschkinass, "Über das Absorptionsspectrum des flüssigen Wassers und über die Durchlässigkeit der Augenmedien für rothe und ultrarothe Strahlen," *Wied. Ann.*, lv. 1895, p. 401.

The following table gives some results obtained:—

RADIANT EFFICIENCIES IN PER CENT.

Voltage.	Carbon (1).	Carbon (2).	Tungsten (3).	Tungsten (4).
116	0·12	...	...	2·48
184	1·40	0·72	4·96	5·98
216	1·97	1·48	7·42	9·02
250	3·42	2·42	7·60	9·39
270	3·95	3·43	8·06	9·59

Voltage.	Osmium (5).
30	2·17
45	3·84
55	6·52

Voltage.	Tungsten (6).	Tungsten (7).	Tantalum (8).	Tantalum (9).
75	1·61	1·69	2·06	3·00
125	6·61	6·43	6·66	6·35
146·5	9·04	8·83		
150	...	...	7·70	9·25

All the lamps with the exception of the osmium are well-known makes much in use at present. The osmium lamp, which was invented by Auer von Welsbach, was the first metallic filament lamp to be a commercial success. It is not made now, and some trouble was experienced in getting one. The marked voltage of (1), (2), (3), and (4) was 250 volts; of (5), 50 volts; of (6) and (7), 130 volts; and of (8) and (9), 125 volts. The lamps (1) and (2) were of different makes, the filament in (2) having a metallic lustre; (3) and (4) were the same make, (6) and (7) the same make, and (8) and (9) the same make.

The figures in the table are probably accurate to 5 or 6 per cent. The difference in the behaviour of the 250-volt and 130-volt tungsten lamps I believe to be genuine and to be probably due to a different method of manufacture. In order to investigate this point thoroughly, however, it would be necessary to get special lamps made with one straight filament in each.

If we calculate the radiant efficiencies of the different filaments at their marked voltage and take the mean, we obtain:—

	Radiant efficiency.	Watts per candle.
Carbon . . .	2·9	3·5
Osmium . . .	5·2	1·5
Tantalum . . .	6·5	1·7
Tungsten . . .	7·5	1·0

For purposes of comparison I have taken the figures in the second column from a paper by H. Hirst.* He gives them as the values generally accepted for these lamps, though I think 1·2 watts per candle more accurate for tungsten. My value, 2·9, agrees well with Mendenhall's, 2·8, for carbon; but in order to speak with absolute certainty of the relative merit of the different metals, it would be necessary to experiment with a greater number of lamps.

The measurements described in this article were carried out in the Physical Laboratory of the University of Glasgow with the assistance of a grant from the Carnegie Trust for the Universities of Scotland.

* "Recent Progress in Tungsten Metallic Filament Lamps," *Journ. of El. Eng.*, xli., 1908, p. 636.

(*Issued separately December* 16, 1910.)

XLII.—On the Precipitation of Soluble Chlorides by Hydrochloric Acid. By John Gibson, Ph.D., and R. B. Denison, D.Sc., Ph.D.

(Read June 22, 1908. MS. received August 20, 1910.)

In a communication made to this Society on December 6, 1897, a tendency towards maximum specific electrolytic conductivity was shown to be a characteristic of many chemical reactions.* This tendency is very marked in reactions in which strong acids play a part. Thus, to take a well-known example, chlorine decomposes water under the influence of light with production of hydrogen chloride and oxygen, but as the hydrogen chloride accumulates in the solution, the decomposition of the water is more and more retarded. This photo-chemical reaction is a reversible one, for in the case of highly concentrated solutions the hydrogen chloride is oxidised to water and free chlorine.

The equation is usually written—

$$4HCl + O_2 \rightleftharpoons 2H_2O + 2Cl_2 \quad . \quad . \quad . \quad . \quad . \quad (1)$$

but it may also be written—

$$(x+4)HCl + yH_2O + O_2 \rightleftharpoons xHCl + (y+2)H_2O + 2Cl_2 \quad . \quad . \quad (2)$$

It has been found by one of us that the equilibrium point of this reaction corresponds closely with the point of maximum specific conductivity. According to Kohlrausch, this maximum is reached when the hydrochloric acid contains about 18·25 per cent. HCl, so that the conductivity falls when either water or hydrogen chloride is added to a solution having this concentration. In the case of solutions of hydrogen chloride containing 19 per cent. HCl along with a little free chlorine, a trace of free chlorine was found to persist even when the solution was exposed to sunlight. On the other hand, in solutions containing 17 per cent. HCl or less, the hydrogen chloride is not oxidised. On the contrary, a trace of free chlorine soon disappears.

For the purpose of our discussion, it is important to note that equations (1) and (2) read from left to right imply a dilution, but from right to left a concentration of the hydrochloric acid. The hydrochloric acid may, therefore, be said to tend towards maximum specific conductivity, for the reaction always proceeds so that the conductivity approaches the maximum either by increasing or by lessening the concentration of the hydrogen chloride in the solution.

* *Proc. Roy. Soc. Edin.*, vol. xxii. p. 33.

The precipitation of certain soluble chlorides from aqueous solution on addition of concentrated hydrochloric acid is of special interest from this point of view. Not all soluble chlorides can be precipitated in this way; thus the extremely soluble chlorides of lithium and cæsium are not precipitated by hydrochloric acid. The chlorides of the other alkali metals may be precipitated by hydrochloric acid under certain conditions.

Let it be assumed as an hypothesis that in the precipitations to be considered a tendency towards maximum specific conductivity is the determining factor:—Then a solution of hydrochloric acid should tend to abstract water when its concentration is greater than that corresponding to maximum conductivity, because its conductivity is increased by dilution. Consequently, we are led to expect such solutions to precipitate sodium chloride from its saturated aqueous solution, for in a saturated solution of a salt, which crystallises without water of crystallisation, we may regard the whole of the water in the solution as taken up in dissolving the salt. On the other hand, according to the same hypothesis, whenever the concentration of the hydrochloric acid falls below that corresponding to maximum conductivity, the tendency to abstract water should cease, because further dilution diminishes the specific conductivity. We, therefore, should not expect such dilute solutions of hydrochloric acid to precipitate sodium chloride from its saturated solution; on the contrary, we should regard such solutions as containing some water available for the solution of an additional quantity of solid salt.

Looking at the matter from the present standpoint of the dissociation theory, the precipitation might be ascribed to a decrease in the dissociation of the salt, and to the consequent increase in the concentration of the undissociated molecules. We should, from this point of view, expect precipitation to occur only when the concentration of the chlorine ions in the solution of hydrochloric acid is greater than the concentration of the chlorine ions in the saturated solution to which it is added.

Unfortunately, the applicability of this principle to concentrated solutions cannot be tested in a satisfactory manner, because the dissociation theory in its present form does not enable us to calculate the ionic concentrations of concentrated solutions with sufficient accuracy, more especially in the case of strong electrolytes.

For the purposes of a very rough comparison, the laws established for dilute solutions may be assumed to be applicable to highly concentrated solutions such as those we are considering.

Table I., column VII., gives the ionic concentrations calculated on the assumption that even for saturated solutions of the chlorides of ammonium,

potassium, rubidium, and sodium, the degree of dissociation is equal to $\frac{\mu}{\mu_\infty}$; and further, that this holds also for the solution of hydrochloric acid of the concentration having maximum conductivity.

To judge by the ionic concentrations calculated in this way, we should expect that a solution slightly more concentrated than that having maximum conductivity would precipitate sodium chloride from its saturated solution, but we should not expect it to precipitate any of the other salts from their saturated solutions.

If we take the hypothesis of a tendency towards maximum specific conductivity as a guide, we are led to expect that such a solution of hydrochloric acid would precipitate all four salts from their saturated solutions.

The first trials were made by simply mixing the respective solutions in test-tubes at room temperature. The final experimental tests were conducted as follows:—

To ensure saturation, the salt solutions contained in glass-stoppered bottles were shaken for some hours along with a large excess of the solid salt. Some of the solution was then drawn out by a pipette through a plug of cotton-wool, and at once transferred to another stoppered vessel. The clear portion of saturated solution thus obtained was then mixed with from 3 per cent. to 5 per cent. of its volume of the solution of hydrochloric acid to be tested. From first to last the temperature of the several solutions was kept at 18° C. to within one-hundredth of a degree.

TABLE I.

I.	II.	III.	IV.	V.	VI.	VII.	VIII.
Electrolyte.	Concentration in mols./litre $=c$.	Specific conductivity $K_{18°}$.	Molecular conductivity $\mu=\frac{K}{\eta}$.	μ_∞.	Degree of dissociation $\alpha=\frac{\mu}{\mu_\infty}$.	$c\alpha$.*	$\frac{c\alpha}{2{\cdot}02}$.†
NaCl . .	5·44	0·217	39·9	110·3	0·36	1·96	0·96
KCl . .	3·98	0·333	83·7	131·2	0·64	2·55	1·26
RbCl . .	5·88	0·442	75·2	133·5	0·56	3·29	1·63
NH_4Cl .	5·67	0·420	74·2	130·1	0·57	3·23	1·60
Max. HCl	5·46	0·765	140·1	384·0	0·37	2·02	1·00

* $c\alpha$ = calculated ionic concentrations.

† $\frac{c\alpha}{2{\cdot}02}$ = calculated ionic concentrations relatively to HCl max.

Many experiments were made with the saturated solutions of each of the four salts. The results arrived at may be summed up as follows:—

1. In no case was precipitation observed when the concentration of the hydrochloric acid added did not exceed 17 per cent. HCl.

2. When the concentration was 19 per cent. HCl or higher, precipitation was observed invariably.

3. The mean of a number of observations gave 18·5 per cent. HCl. or 5·5 mols. per litre as the lowest concentration of HCl which caused precipitation.

In order, however, to make sure that we were not dealing with something of the nature of a coincidence dependent on the temperature being in the neighbourhood of 18° C., the experiments were repeated with solutions of potassium chloride and sodium chloride saturated at 0° C. This lowering of the temperature involves a very considerable decrease in the solubility of the potassium chloride, and a comparatively slight decrease in the solubility of the sodium chloride. There seems, however, reason to suppose that the fall in temperature may not involve so great a relative change in the ionic concentrations of the saturated solutions of these two salts as one might infer from the great difference between their temperature coefficients of solubility, for the temperature coefficient of conductivity is greater for sodium chloride than for potassium chloride. Be this as it may, the hypothesis was found to be as applicable at 0° C. as at 18° C. Precipitation was only observed on adding acid solutions containing at least 18·0 per cent. HCl, and it was invariably well marked when the concentration was as high as 19 per cent. HCl. According to as yet unpublished determinations by one of us, the maximum specific conductivity at 0° C. occurs at a slightly higher concentration than at 18° C. This change is very slight, and does not affect the present argument.

If a greater ionic concentration in the acid than in the saturated salt solution be really a necessary condition for precipitation, as the dissociation theory seems to demand, then the experiments described above clearly prove that the calculated ionic concentrations given in col. VII. of Table I. cannot be even approximately correct.

The maximum acid, however, has probably a concentration of chlorine ions at least equal to that of the saturated salt solutions. Granting that this is so, it does not explain why the saturated salt solutions, which are all precipitated by hydrochloric acid of the same concentration, are in no case precipitated by a solution containing less hydrogen chloride. Moreover, at equal molecular concentration up to 4 normal and at 18° C., the specific conductivities of potassium chloride and ammonium chloride are almost identical. At 18° C. the solution of potassium chloride is saturated at a concentration of 4 normal, while that

of ammonium chloride is not saturated until a concentration of 5·67 normal has been reached. Curves showing the variation of specific conductivity with concentration for these two isomorphous salts are nearly coincident up to a concentration of 4 normal, and beyond this the curve for ammonium chloride appears almost as a direct continuation of that for potassium chloride, the specific conductivity rising steadily ($K_{18} = 0·333$) at 4 normal to ($K_{18^\circ} = 0·420$) at 5·67 normal. It is difficult to see how this can be brought into agreement with the suggestion of equal concentration of chlorine ions in these two saturated solutions.

Unquestionably, the total ionic concentration is greater in the case of the saturated solution of ammonium chloride, and the only way that equal chlorine ion concentration can be imagined is by assuming that above 4 normal a considerable complex formation takes place. On this assumption, however, the complex ions would have to have nearly the same mobility as simple chlorine ions, judging by the close similarity between the two curves.

A similar precipitation of certain nitrates by nitric acid of greater concentration than that corresponding to maximum specific conductivity has been observed, but there are certain facts in connection therewith which cause us to leave this subject for a further communication. Sulphates behave quite differently, requiring a much higher concentration of acid before salt separates out. This is doubtless due to the formation of acid salts.

Two questions arise out of our discussion: "Can we predict from current theoretical considerations which chlorides should be precipitated by hydrochloric acid of concentration higher than that of maximum conductivity?" and "Why is it that the minimum concentration of acid causing precipitation is that of the acid with maximum specific conductivity?"

To the latter question current theories give as yet no really satisfactory explanation, and we can only expect one when we have more experimental facts regarding the properties of this maximum acid and of saturated solutions. As regards the former question we may consider the matter as follows:—

When a salt dissolves, it may (*a*) dissolve with a molecular weight higher than that given by the simplest formula for the salt; as we generally say, the salt molecules may associate in solution. There may be, however, no process of association at all. It is doubtful whether the phrase molecular weight of a solid has any definite meaning. The molecules in solution may be many times the size represented by their simplest molecular weight (but still dissociation of complex molecules into simpler ones may have taken place on solution, assuming a high molecular weight in the solid form). This dissociation would be different in different solvents,

and thus we should get the solid appearing to associate in one solvent and not in another.

(*b*) The solid may dissolve with the molecular weight corresponding to its simplest formula.

(*c*) The undissociated solid may split up more or less completely into ions, as is generally the case when inorganic salts dissolve in water.

(*d*) There may be complex formation between the "simpler ions" and the molecules, *e.g.* formation of complex ions.

(*e*) In all these cases there almost certainly is some kind of combination with the molecules of the solvent.

All these processes may take place simultaneously when solution occurs, just as a given chemical process may consist of a number of simultaneous reactions which are very difficult to detect. The different processes involved in solution may proceed to a different extent in different cases; and moreover, in the case of any one salt and solvent, the different processes above mentioned may severally become more predominant at different concentrations, so that at different concentrations we may have very different chemical systems.

Now the ionic theory, as applied to precipitation reactions such as we have been considering, treats of nothing more complex than the following equilibrium:—

$$\text{Solid salt} \rightleftarrows \text{undissociated salt in solution} \rightleftarrows \text{ions.}$$

There still remain complex formation and hydration to be taken into account. In dilute solution complex formation has been very fully studied by the application of the law of mass action to the ionic theory. In concentrated solutions this fails us, so for the moment we will eliminate complex formation as much as possible by confining our attention to those salts which are known to undergo but little complex formation, *i.e.* the salts which give ions of greatest electro-affinity. We have then in solution the equilibria—

$$\text{Solid salt} \rightleftarrows \text{undissociated salt in solution} \rightleftarrows \text{ions} \rightleftarrows \text{hydrated ions,}$$

with possibly also

$$\text{undissociated salt} \rightleftarrows \text{hydrated undissociated salt;}$$

but, as Abegg * points out, the equilibrium which predominates in the case of those salts which give ions of strongest electro-affinity is probably—

$$\text{ions} \rightleftarrows \text{hydrated ions.}$$

* *Z. für anorg. Ch.*, xxxix. 359, 1904.

This hydration equilibrium may be the first disturbed when concentrated hydrochloric acid is added to saturated solutions of those chlorides considered in this paper, with the result that salt is precipitated. Considered in this way, there is qualitatively nothing in the modern theory of solution against the views which we have expressed earlier in the paper regarding this precipitation. The problem cannot be solved quantitatively with the aid of the law of mass action until we are better acquainted with the above hydration equilibria.

In the more intricate cases where complex ions are formed we have in a saturated solution the equilibria—

Solid salt $\rightleftarrows$ undissociated salt, hydrated or not $\rightleftarrows$ ions, hydrated or not

complex ions, hydrated or not.

The addition of concentrated hydrochloric acid to a saturated solution of a chloride which forms complex ions and crystallises with water of crystallisation would not necessarily precipitate the salt, owing to the variety of ways in which the equilibrium could readjust itself, or the tendency towards maximum specific conductivity be satisfied. It is thus not to be wondered at that maximum conductivity HCl does not precipitate LiCl, $MgCl_2$, $CaCl_2$, $SrCl_2$, $ZnCl_2$, etc., but it is interesting to note that the chloride of the alkaline earths which comes nearest to being so precipitated is the one in which complex formation is certainly least marked, namely, barium chloride.

We have endeavoured to explain according to current theory the facts of the precipitation of these chlorides by hydrochloric acid, and we find any explanation on the basis of the ionic theory very difficult, in so far as this theory involves the assumption of a concentration of chlorine ions in the saturated solutions of the chlorides in question equal to that in hydrochloric acid having maximum conductivity. So far as we can judge from the present data, this assumption does not seem tenable. On the other hand, the hypothesis of a tendency towards maximum specific conductivity has proved most useful, for by it the conditions necessary for these precipitations are correlated and were predicted.

Chemistry Department,
Heriot-Watt College.

(*Issued separately December* 23, 1910.)

OBITUARY NOTICE.

Daniel John Cunningham. By Professor C. G. Knott.

(MS. received 7th December 1910.)

Daniel John Cunningham was born at Crieff on April 15, 1850. He was a son of the manse, and his father, the Rev. John Cunningham, subsequently became Principal of St Mary's College, St Andrews University. His mother was descended from a brother of Captain Porteous, of the Porteous Riot fame.

Daniel Cunningham was first educated at a small private school started by the heads of six families, who engaged a teacher for their boys. Ere long, however, Crieff Academy was organised, and to it the younger scholars were transferred. Cunningham received the simple, strong education of which Scotland was proud in those days, an education which fitted a promising boy for any walk in life. His first intention was to enter on a business career, and with this object in view he spent a year or two in Glasgow. When, at the age of twenty, he decided to follow medicine, he began his university studies with a more matured experience of life than was possessed by the majority of his associates. His business training had taught him the value of method, and, although a little out of touch with recognised methods of book study, he very quickly showed himself to be a man of marked ability. He proved an excellent all-round student, and specially distinguished himself in the classes of anatomy, physiology, materia medica, and surgery. After his graduation as M.B. and C.M. in 1874, he commenced practice as a physician in Glasgow.

The bent of his mind was, however, towards scientific studies. This he had already indicated by publishing in 1873 a short paper in the *Journal of Anatomy and Physiology* while he was still an undergraduate. The paper was called "Observations on the Distribution of some of the Nerves of the Head and Neck," and will be found in vol. vii. of the *Journal*, pp. 94–97.

A much stronger evidence of his scientific leanings was given by his M.D. thesis on the Anatomy of the Cetacea, for which in 1876 he received a gold medal. The same year, on the invitation of Sir William (then Professor) Turner, Cunningham returned to Edinburgh University as

Demonstrator in Anatomy. As Senior Demonstrator he took an important share in the removal of the Anatomical Department from the old to the new buildings in 1880, and in arranging the rooms for teaching purposes. Here he spent six years of strenuous work and continuous study, fulfilling not only his University duties but also during the last four years lecturing on physiology in the Royal Veterinary College. Meanwhile, his love of research was evidenced by some dozen notes and papers communicated to the *Journal of Anatomy and Physiology* between 1873 and 1882, and still more by the publication in the last-named year of an elaborate *Challenger* Report. On the return of the *Challenger* Expedition in 1876, Sir Wyville Thomson, under the advice of Sir William Turner, placed in Cunningham's hands the marsupial animals collected during the voyage. In preparing the report on their anatomy he met a puzzling modification in the arrangement of the muscles of the hand and foot, which led him to investigate the myology of those parts of the limbs in mammalia generally. This memoir established his reputation as an anatomist of the first rank. Some of the results had been published in the *Journal of Anatomy and Physiology* during the preparation of the Report, and it is not surprising that in the spring of 1882 he was elected Professor of Anatomy at the Royal College of Surgeons in Dublin.

In October 1883 he was transferred to the Anatomy chair in Trinity College, Dublin; and during his twenty years' tenure of this office Cunningham became by the simple force of his character a leading spirit. Into all that made for efficiency in university teaching and research he threw himself with devoted and single-hearted energy. Professor A. F. Dixon, his successor in the Dublin chair, describes his master's influence and work in these words:—

"Cunningham came to Trinity College at a very critical period in the history of her Medical School. The old medical buildings had long been inadequate for the requirements of her students, and the authorities of the College had become alive to the fact, thanks largely to the energy and zeal of the late Rev. Dr Samuel Haughton, F.R.S., Senior Fellow of Trinity College. The old spirit which regarded the school of physic as something foreign to and outside the university was dying fast, and, like the wall which for so long a period had separated the Medical School from the rest of the College, was soon to completely disappear. It is needless to say that in every movement for the advancement of the School, and for the better housing and equipment of its departments, Cunningham took a leading part, and assisted Haughton ably and enthusiastically. These gifted and able men became fast friends, and to their combined efforts the School of

Trinity College owes much—probably more than even she herself recognises. . . . In the equipment of his own department it was his aim and ambition to make it a perfect machine for teaching and research, and, as it stands to-day, the Anatomy School is a monument to the genius and energy of Cunningham—his foresight and powers of organisation are to be recognised in almost every detail. . . .

"A great and inspiring teacher, no detail was too trivial and no labour too arduous where the interests of his students were concerned. His knowledge and advice were ever freely at the disposal of all who desired to consult him, and those who went to him received counsel, encouragement, and assistance for the carrying out of their ideas such as few men have it in their power to give. His approbation and approval were most generously given and most highly appreciated. We doubt if any man ever exercised a more powerful influence over his students than Cunningham did, and the keen interest which he took in their work and pleasures was repaid by an admiration, esteem, and affection such as are rarely bestowed upon any teacher."

Cunningham's best energies were now devoted to two great purposes—scientific research in anatomy and anthropology and increased efficiency in the teaching and training of medical students. These two purposes shaped his whole career; but, as is ever the case with men of high character and unselfish devotion, his energies overflowed into other channels partly conditioned by circumstance. Trinity College and its Medical School were no doubt foremost in his thoughts and affections; but he gave unstintedly of his very best to other important institutions, such as the Royal Irish Academy, the Royal Dublin Society, the Royal Zoological Society, and the Royal Veterinary Society. Not only was he a student of the anatomy and physiology of all types of animals—he was a lover of them for their own sakes. Second only to the interests of his students was the welfare of the animals in the gardens of the Zoological Society. From 1895 to 1902 he was honorary secretary to the Society, and under his enthusiastic supervision great advances were made in improving the housing of the animals and in making the gardens a beauty and a pride in the eyes of the people of Dublin. Of special value was his success in providing open air for monkeys and other animals whose native haunts were of much warmer and milder climate than prevails even in Ireland. The Haughton House for kangaroos and monkeys and the later Roberts House for the lions were among his creations. The plans for these houses were drawn up after careful consideration of what had been done for similar purposes both on the Continent and in America. Only those who worked with him had any

true conception of the time and thought Cunningham gave to the whole question. He became president of the Society during his last year in Dublin; and there is no doubt that, although in 1903 he felt it his duty to obey the call to Edinburgh, he was sorry to part with his wild pets of the "Zoo," and especially with the young lion cubs whom he had known from their birth in captivity.

In 1901 Cunningham wrote a pamphlet on the "Origin and Early History of the Royal Zoological Society of Ireland." This eminently readable tract is a good example of the author's skill as a searcher of records. He showed how curiously fortuitous the initiation of the Society was, and how much its final success was due to its catholicity of spirit and to the self-devotion of the early secretaries and presidents. This interest in the history of institutions was almost a passion with Cunningham, and many of his less technical addresses take the form of an historic sketch.

Keenly interested in horses and cattle, Cunningham was also for a number of years the honorary secretary and afterwards vice-president of the Royal Dublin Society, whose annual show is one of the outstanding events in Dublin, and indeed in all Ireland. In the same connection may be mentioned his services in helping to found the Royal Veterinary College.

A man of Cunningham's scientific eminence could not long escape the eye of the administrators of national affairs. Accordingly, we find him serving on the Viceregal Commission appointed to inquire into the condition of the Inland Fisheries of Ireland (1900), on the Royal Commission on the Care of the Sick and Wounded during the South African War (1900), and on the War Office Commission appointed to report on the Physical Standards required for Candidates for Commissions and for Recruits. More recently he was convener of the Committee appointed to look after the arrangements for the medical equipment of the Territorial force in the East of Scotland.

Professor Cunningham entered on what might be called the third great stage of his life in 1903, when he was invited to return to Edinburgh University and take up the duties of the chair of Anatomy, which his former master, Sir William Turner, had vacated on assuming the office of Principal. Here at once he stepped into the very heart of the scientific life of the city. Many of the leading physicians had been his associates and pupils in the early days, and the younger medical men knew him as a foremost anatomist and author. The University staff still contained a goodly number of his former colleagues, and one and all welcomed him back to the scenes of his early triumphs. It seemed but natural that he

should almost immediately become the Dean of the Faculty of Medicine and one of the honorary secretaries of the Royal Society of Edinburgh.

Cunningham began his professoriate duties in Edinburgh by an inaugural address in which, in characteristic fashion, he dipped into the records of the past. Rapidly tracing the rise of the true study of anatomy from its beginnings under Vesalius of Padua in 1537, he showed how the Edinburgh school began to take form in 1700, although it was not till 1720 that with Monro Primus the University School of Anatomy assumed a definite organisation. The great developments associated with the introduction of antiseptic surgery and the application of the Röntgen Rays were touched upon in a luminous manner, and the address ended with suggestive remarks on the relation between the great size of the brain of man and his erect attitude. The succeeding year, when acting as Promotor at the July graduation ceremony, Cunningham delivered an address on "The Evolution of the Graduation Ceremonial." The address contains the description of very curious customs and regulations in several of the oldest universities of Europe. Most of these have disappeared with the advance of the centuries, although the fairly complete mediæval ceremonial still survives in the universities of Spain and of Coimbra in Portugal. In a valuable appendix to the address proper the regulations of the ceremonial details of graduation as practised to-day in nearly twenty of these old universities are given in considerable detail.

As Dean of the Faculty of Medicine, Cunningham carried out a number of changes in the curriculum, his guiding principles being the efficiency of the teaching and the benefit of the student. One great feature of his method of teaching was the regular periodic intercourse between each student and himself or one of his assistants. Only in this way, he was convinced, could the student be tested as to the progress he was making.

As one of the honorary secretaries of the Royal Society of Edinburgh he was of invaluable service, not only on account of the advice he gave the Council on all matters of import, but also during the removal of the Society from its former rooms in Princes Street to its new home in George Street.

In December 1908 Cunningham's health became so unsatisfactory that he was relieved of his University duties for the session and ordered to Egypt for rest and sunshine. At first the change seemed beneficial, but the improvement did not continue. He returned to his home in Edinburgh in the month of May in a condition which gave little hope of recovery. From this condition he never rallied, but passed away on June 23, 1909, at the comparatively early age of fifty-nine.

These are the main facts in the life of a man whose scientific eminence was early recognised by the Fellowship of the Royal Society of London. The Universities of Dublin, Oxford, St Andrews, and Glasgow conferred on him their honorary degrees. He was President of the Anthropological Section of the British Association at the Glasgow meeting of 1901, delivering on that occasion a suggestive address on the influence of the brain in the development of the human race. He also served as President of the Royal Anthropological Institute (1908), of the Anatomical Society of Great Britain and Ireland (1895), and of the Royal Academy of Medicine of Ireland (1902).

Cunningham's scientific work is marked by accuracy, lucidity, and a great sanity of judgment. In both public and private life his human sympathy and beauty of character shone through all he undertook. To know him was to love him. Inspired with a high sense of the duties and responsibilities of the position he occupied, he brought into the wide world which formed his environment all the strong and delicate traits of mind and heart which go to the making of the highest type of civilised man.

Some of his scientific work has been touched on incidentally in the foregoing paragraphs. It remains to indicate in the following list of papers and addresses the character of the more important of these. For convenience of reference the writings are grouped according to the Journal or Transactions in which they were published.

A. "**Challenger**" **Reports.**

1. Reports on some points in the Anatomy of the Thylacine (*Th. cynocephalus*), Cuscus (*Phalangista maculata*), and Phascogale (*Ph. calura*), collected during the voyage of H.M.S. *Challenger* in the years 1873–1876; with an account of the Comparative Anatomy of the Intrinsic Muscles of the Mammalian Pes. 1882. 192 pages; 13 plates.

In the material supplied, in addition to eight specimens of the animal mentioned above, there were three specimens of *Dasyurus viverrimus*, the Tasmanian Devil, the anatomy of which was not discussed in the same detail as in the other less familiar species.

B. **Transactions of the Royal Irish Academy.**

2. The Lumbar Curve in Man and in the Apes. The second "Cunningham Memoir." 1886. 148 pages.

This is regarded as one of his most important papers. He showed how necessary it was to consider the influence of the intervertebral discs in the constitution of the curves, and how very erroneous conclusions might be drawn from a study of macerated skeletons where the discs had disappeared.

3. Surface Anatomy of the Primate Cerebrum. Seventh "Cunningham Memoir." 1892. 360 pages.

This great work embodied researches covering many years, some of which had been published already in shorter papers.

4. On the Brain and Eyeball of a Human Cyclopian Monster. 1891. Vol. xxix. pp. 101–126. In conjunction with Dr E. H. Bennett.

5. On the Skeleton of the Irish Giant Cornelius. 1891. Vol. xxix. pp. 553–612.

This forms part of Cunningham's extended studies in acromegaly and giantism, concerning which he advanced several important views.

C. **Proceedings of the Royal Irish Academy.**

6. On some Ossean Remains found at Old Connaught, Bray, County Dublin. 1894. Vol. iii. pp. 421–427.

7. On some Human Remains recently discovered near Lismore in the County of Waterford. In conjunction with Dr C. R. Browne. 1897. Vol. iv. pp. 552–558.

D. **Transactions of the Royal Society of Edinburgh.**

8. The Varying Form of the Stomach in Man and the Anthropoid Ape. 1906. Vol. xlv. pp. 9–49.

Here Cunningham discusses with lucidity and fair-mindedness the various observations made by many physiologists and anatomists, and the conclusions arrived at. He shows how the marked changes in the form of the stomach are closely associated with its functioning at the time, according as it is filling, or emptying, or empty.

9. The Evolution of the Eyebrow Region of the Forehead, with special reference to the excessive Supraorbital Development in the Neanderthal Race. 1908. Vol. xlvi. pp. 283–312.

From a general survey of the morphological characters of the eyebrow eminence in man and the apes, Cunningham doubts its value in determining specific differences between the Neanderthal and other races of mankind, in this respect disagreeing with Schwalbe.

E. **Proceedings of the Royal Society of Edinburgh.**

10. Cape Hunting Dogs (*Lycaon pictus*) in the Gardens of the Royal Zoological Society of Ireland. 1905. Vol. xxv. pp. 843–848.

11. Obituary Notice of Professor W. His. 1905. Vol. xxv. pp. 1235–1240.

12. Report on the Skulls appended to Dr Robert Munro's paper on a Human Skeleton with Prehistoric Objects found at Great Casterton, Rutland, etc. 1906. Vol. xxvi. pp. 293–309.

Also from time to time exhibits of slides and photographs.

F. Journal of the Royal Anthropological Institute of Great Britain and Ireland.

13. Account of Anthropometric Laboratory in Dublin founded by D. J. Cunningham and A. C. Haddon. 1892. Vol. xxi. pp. 35–39.

14. The Skull and some of the Bones of the Skeleton of Cornelius Magrath, the Irish Giant. 1892. Vol. xxi. pp. 40–41.

15. On the Microcephalic Brain. 1900. Vol. xxx. Given also at the B.A. Meeting at Bradford, 1900.

16. On the Sacral Index. 1900.

17. Right-handedness and Left-brainedness. The Huxley Memorial Lecture, delivered October 21, 1902. Vol. xxxii. pp. 272–296.

> Cunningham's conclusion was that an explanation of right-handedness had still to be found.

18. The Head of the Aboriginal Australian. 1907. Vol. xxxvii. pp. 47–57.

19. Anthropology in the Eighteenth Century. Presidential Address, 1908. Vol. xxxviii.

> The address contains a record and criticism of the genius and labours of Camper, White, Blumenbach, Pritchard, Lawrence, and others.

20. Deputation on proposed National Anthropometric Survey to the Prime Minister (Campbell-Bannerman). 1907. Vol. xxxvii. Cunningham was first spokesman.

G. Transactions of the Royal Dublin Society.

21. The Brain and Head of the Microcephalic Idiot. 1895. Vol. v.

22. Lantern Demonstration of the Development of the Convolutions and Fissures of the Human Brain. 1894. Vol. v.

23. The Cape Hunting Dogs in the Gardens of the Royal Zoological Society. 1897. Vol. vi.

24. The Seventh Cranial Nerve in the Orang. 1898. Vol. vi.

Also from time to time various lantern-slide exhibitions on the Cape Hunting Dog, Chimpanzee, Orang, etc.

H. Journal of Anatomy and Physiology.

25. Observations on the Distribution of some of the Nerves of the Head and Neck. 1873. Vol. vii. pp. 94–97.

26. Notes on the Broncho-œsophageal and Pleuro-œsophageal Muscles. 1876. Vol. x. pp. 320–323.

27. The Spinal Nervous System of the Porpoise and Dolphin. 1877. Vol. xi. pp. 209–228.

28. Note on a Connecting Twig between the Anterior Divisions of the First and Second Dorsal Nerves. 1878. Vol. xii. pp. 539–540.

29. Note on Hypertrophy of the Sympathetic Nervous System. 1878. Vol. xii. pp. 294–296.

30. The Nerves of the Fore-limb of the Thylacine and Cuscus. 1878. Vol. xii. pp. 427–433. A first instalment of his *Challenger* Report.

31. The Intrinsic Muscles of the Hand of the Thylacine, Cuscus, and Phascogale. 1878. Vol. xii. pp. 434–444. Also in *Challenger* Report.

32. The Intrinsic Muscles of the Mammalian Foot. 1879. Vol. xiii. pp. 1–16. See also *Challenger* Report.

33. Note on the Distribution of the Anterior Tibial Nerve on the Dorsum of the Foot. 1879. Vol. xiii. pp. 398–399.

34. A large Sub-Arachnoid Cyst involving the greater part of the Parietal Lobe of the Brain. 1879. Vol. xiii. pp. 508–517.

This paper contains one of the earliest descriptions of the condition now known as acromegaly.

35. The Nerves of the Hind-Limb of the Thylacine and Cuscus. 1881. Vol. xv. pp. 265–277. Also in *Challenger* Report.

36. The Relation of Nerve Supply to Muscle-homology. 1882. Vol. xvi. pp. 1–9.

37. The Development of the Suspensory Ligament of the Fetlock in the Fœtal Horse, Ox, Roe-deer, and Sambre-deer. 1884. Vol. xviii. pp. 1–12.

38. The Musculus Sternalis. 1884. Vol. xviii. pp. 208–210.

39. The Connection of the Os odontoideum with the Body of the Axis Vertebræ. 1886. Vol. xx. pp. 238–243.

40. The Neural Spines of the Cervical Vertebræ as a Race Character. 1886. Vol. xx. pp. 637–640.

41. The Musculus Sternalis. 1888. Vol. xxii. pp. 391–407.

42. The Proportion of Bone and Cartilage in the Lumbar Section of the Vertebral Column of the Ape and several Races of Men. 1890. Vol. xxiv. pp. 117–126.

43. The Occasional Eighth True Rib in Man and its Relation to Right-handedness. 1890. Vol. xxiv. pp. 127–129.

44. The Intraparietal Sulcus of the Brain. 1890. Vol. xxiv. pp. 136–155.

45. The Complete Fissures of the Human Cerebrum and their Significance in Connection with the Growth of the Hemisphere and the Appearance of the Occipital Lobe. 1890. Vol. xxiv. pp. 309–345.

46. The Fissure of Rolando. 1891. Vol. xxv. pp. 1–23.

47. The Value of Nerve Supply in the Determination of Muscular Homologies and Anomalies. 1891. Vol. xxv. pp. 31–40.

48. The Sylvian Fissure and the Island of Reil in the Primate Brain. 1891. Vol. xxv. pp. 286–291.

49. The Development of the Gyri and Sulci on the Surface of the Island of Reil of the Human Brain. 1891. Vol. xxv. pp. 338–348.

50. Delimitation of the Regions of the Abdomen. 1893. Vol. xxvii. pp. 257–274.

51. On the Form of the Spleen and the Kidneys. 1895. Vol. xxix. pp. 501–507.

52. The Rolandic and Calcarine Fissures—a Study of the Growing Cortex of the Cerebrum. 1897. Vol. xxxi. pp. 586–598.

53. The Insular District of the Cerebral Cortex in Man and in the Man-like Ape. 1898. Vol. xxxii. pp. 11–22.

54. The Significance of Anatomical Variations. 1899. Vol. xxxiii. pp. 1–9.

55. Supra-condyloid Process in the Child. 1899. Vol. xxxiii. pp. 357–358.

I. **Dublin Journal of Medical Science.**

56. Bologna: the Part which it has played in the History of Anatomy. 1888. Reprinted in *Die internationalen Monatschrift f. Anat. u. Phys.*, 1890, Bd. vii.

This paper was read before the Royal Academy of Medicine of Ireland, whose meetings and discussions were regularly reported in the *Dublin Journal of Medical Science.* Cunningham faithfully attended these meetings, frequently exhibiting anatomical models and communicating short notes, eighteen of which will be found chronicled, with brief reports, in the same *Journal* between the years 1883 and 1902. Several of these communications, which were not mere exhibits, were published more fully in the *Journal of Anatomy and Physiology.*

J. **British Association Reports.**

57. Address to the Anthropological Section. 1901.

K. **Books and Pamphlets.**

Cunningham's *Manual of Anatomy* (1889; last ed., in two volumes, 1907) was a natural development of his first modest *Students' Guide to Dissection* (in three parts, 1879–87), and is the best of its kind in the English language.

In 1888 he published along with Professor E. H. Bennett *The Topographical Anatomy of Congenital Inguinal Hernia.*

He was the editor and one of the principal writers of the *Text-book of Anatomy* (1902; 3rd ed., 1909), to which other pupils of Sir William Turner contributed as collaborators.

He was also for many years one of the editors of the *Journal of Anatomy and Physiology.*

The following addresses were printed in pamphlet form:—Inaugural Address delivered at the opening of the New Anatomical Theatre, Trinity College, Dublin, Nov. 2, 1885; The Origin and Early History of the Royal Zoological Society of Ireland, 1901; Introductory Address to the Class of Anatomy in the University of Edinburgh, Oct. 13, 1903; The Evolution of the Graduation Ceremonial—Address to Graduates in Medicine in Edinburgh University, July 1904.

APPENDIX.

CONTENTS.

PROCEEDINGS OF THE STATUTORY GENERAL MEETING.

The 127th Session.

At the Annual Statutory Meeting of the Royal Society of Edinburgh, held in the New Premises, 22–24 George Street, Monday, 25th October 1909.

Sir Wm. Turner, K.C.B., President, in the Chair,
the Minutes of last Statutory Meeting, 26th October 1908, were read, approved, and signed.

On the motion of Dr Knott, seconded by Dr Black, Dr Ferguson and Mr O'Connor, C.E., were appointed Scrutineers, and the ballot for the New Council commenced.

The Treasurer's Accounts for the past year were submitted. These, with the Auditor's Report, were read and approved.

The Scrutineers reported that the following New Council had been duly elected:—

Principal Sir Wm. Turner, K.C.B., D.C.L., F.R.S., President.
Ramsay H. Traquair, M.D., LL.D., F.R.S.,
Professor Crum Brown, M.D., LL.D., F.R.S.,
Professor J. C. Ewart, M.D., F.R.S.,
John Horne, LL.D., F.R.S., F.G.S.,
James Burgess, C.I.E., LL.D., M.R.A.S.,
Professor T. Hudson Beare, M.Inst. C.E.,
} Vice-Presidents.
Professor George Chrystal, LL.D., General Secretary.
Cargill G. Knott, D.Sc.,
Robert Kidston, LL.D., F.R.S., F.G.S.,
} Secretaries to Ordinary Meetings.
James Currie, M.A., Treasurer.
John S. Black, M.A., LL.D., Curator of Library and Museum.

COUNCILLORS.

Professor F. W. Dyson, M.A., F.R.S., Astronomer Royal for Scotland.
Professor D'Arcy W. Thompson, C.B.
O. Charnock Bradley, M.D., D.Sc.
Charles Tweedie, M.A., B.Sc.
Professor J. W. Gregory, D.Sc., F.R.S.
A. P. Laurie, M.A., D.Sc.
Professor Wm. Peddie, D.Sc.
Professor H. M. Macdonald, M.A., F.R.S.
Professor D. Noël Paton, M.D., B.Sc., F.R.C.P.E.
William S. Bruce, LL.D.
Professor F. G. Baily, M.A.
J. G. Bartholomew, LL.D., F.R.G.S.

On the motion of Dr Black, seconded by Professor Hudson Beare, thanks were voted to the Scrutineers.

On the motion of the Chairman, seconded by Dr James Burgess, the Auditors were thanked and reappointed.

J. Horne.

PROCEEDINGS OF THE ORDINARY MEETINGS,

Session 1909–1910.

The First Special Meeting of the Royal Society for the Session 1909–1910 was held in the Freemason's Hall, where Sir Wm. Turner, The President, delivered his Inaugural Address on Monday, 8th November 1909, at 8 o'clock.

After the Address a Reception was held at No. 24 George Street.

FIRST ORDINARY MEETING.

Monday, 22nd November 1909.

Sir William Turner, President, in the Chair.

The following Prizes were awarded and presented :—

1. The Makdougall-Brisbane Prize for the biennial period 1906–08 to D. T. Gwynne-Vaughan, M.A., F.L.S., for his papers (1st) "On the Fossil Osmundaceæ," and (2nd) "On the Origin of the adaxially-curved Leaf-trace in the Filicales," communicated by him conjointly with Dr R. Kidston.

Professor Gwynne-Vaughan holds a leading place among the plant-anatomists of the present time. This position is based, not only on the very fine series of memoirs on the fossil Osmundaceæ which have been presented to this Society, but also upon earlier work. He successfully investigated the polystelic condition in the stems of the Primulaceæ and the abnormalities in the Nymphæaceæ ; but the papers which have probably contributed most to his high reputation are the two on Solenostelic Ferns, published in the *Annals of Botany.* The effect of these has been to show how along phyletic lines the complicated condition seen in the cyatheaceous and polypodiaceous stems came into existence and to reduce these to terms of a clear theory of dictyostely.

He is an anatomist who has already achieved much and from whom much may still be expected.

2. The Gunning Victoria Jubilee Prize for the third quadrennial period 1904–08 to Professor George Chrystal, M.A., LL.D., for "A series of papers on 'Seiches,' including 'The Hydrodynamical Theory and Experimental Investigations of the Seiche Phenomena of certain Scottish Lakes.'"

Professor Chrystal's attention was drawn to the subject of Seiches by the work of Sir John Murray and his staff in the Survey of the Scottish Fresh-water Lakes. It was known that a seiche was a standing oscillation of a lake usually in the direction of its greatest length, but the periods and positions of the nodes of the harmonic components of the oscillation could only be calculated mathematically for the simple case of a lake of uniform depth. Professor Chrystal discovered what would have seemed prior to its discovery a very difficult problem, a mathematical treatment applicable to lakes of variable depth. The hydrodynamical theory which he established gives a close numerical accordance with the facts of observation.

The theory was compared with observations on Loch Ness, Loch Treig, Loch Tay, and especially on Loch Earn. In the course of these observations Professor Chrystal modified the sarasin limnograph, making it more sensitive for seiches of small amplitude, and also devised a new instrument, the stato-limnograph, in which a Richard barograph is adapted to measure very small fluctuations of level occurring in short periods of one minute or less.

These instruments present the rhythmical rise and fall of the level of a lake as a continuous curve. In some cases these curves are complicated and their analysis into the sum of a number of simple oscillations is a matter of difficulty. Professor Chrystal devised the method of "residuation" which is applicable to all numerical data in which regular but unknown variations are hidden.

Professor Chrystal has also considered the conditions under which seiches are formed and seiches of long duration maintained. Among other causes, seiches on Loch Earn are traced to the fall of rain on part of the lake and to abrupt changes of the barometer. It is also established that seiches of long duration are closely associated with barometric fluctuations of a quasi-periodic character.

By a combination of mathematical and experimental skill, Professor Chrystal has completely solved the kinematical problem of the Seiches of Loch Earn, and his investigations may well serve as a model for the examination of the seiches of other lakes. In addition he has made important advances in discovering the relationship between seiches and the attendant meteorological conditions.

The following Communications were read :—

1. A New Hydrate of Orthophosphoric Acid (Abstract). By Professor ALEX. SMITH and Professor A. W. C. MENZIES. *Proc.*, vol. xxx. pp. 63–64.

2. The Pharmacological Action of Harmaline. By JAMES A. GUNN, M.A., M.D., D.Sc. Communicated by Professor Sir THOMAS R. FRASER, M.D. *Trans.*, vol. xlvii. pp. 245–252.

3. Mendelism and Zygotic Segregation in the production of Anomalous Sex. I.—The Free-Martin. By Dr BERRY HART, F.R.C.P.E. (*With Lantern Illustrations.*) *Proc.*, vol. xxx. pp. 230–241.

4. The Theory of Orthogonants in the Historical Order of Development up to 1860. By Dr THOMAS MUIR. *Proc.*, vol. xxx. pp. 265–290.

The following Candidate for Fellowship was balloted for, and declared duly elected :— A. ANSTRUTHER LAWSON, D.Sc.

SECOND ORDINARY MEETING.

Monday, 6th December 1909.

Professor J. Cossar Ewart, Vice-President, in the Chair.

The following Communications were read :—

1. The Short Muscles of the Hand of the Agile Gibbon (*Hylobates agilis*), with comments on the Morphological Position and Function of the Short Muscles of the Hand of Man. By Dr D. C. L. FITZWILLIAMS. Communicated by Principal Sir WM. TURNER, K.C.B., D.C.L. *Proc.*, vol. xxx. pp. 202–218.

2. On Waves in a Dispersive Medium resulting from a Limited Initial Disturbance. By GEORGE GREEN, M.A., B.Sc. Communicated by Professor A. GRAY, F.R.S. *Proc.*, vol. xxx. pp. 242–253.

3. The Composition and Character of Oceanic Red Clay. By Dr W. A. CASPARI, B.Sc., F.I.C. Communicated by Sir JOHN MURRAY, K.C.B. *Proc.*, vol. xxx. pp. 183–201.

THIRD ORDINARY MEETING.

Monday, 20th December 1909.

Principal Sir William Turner, K.C.B., President, in the Chair.

The following Communications were read :—

1. The Aborigines of Tasmania. Part II.—The Skeleton. By Sir WM. TURNER, K.C.B., President. *Trans.*, vol. xlvii. pp. 411–454.

2. On the Structure and Affinities of Zygopteris Römeri (Solms). By W. T. GORDON, M.A., B.Sc., Falconer Fellow of Edinburgh University. Communicated by Professor JAMES GEIKIE, D.C.L., LL.D. *Trans.*

3. On the Fossil Osmundaceæ. Part IV. and Conclusion. By Professor GWYNNE-VAUGHAN and Dr R. KIDSTON, F.R.S. *Trans.*, vol. xlvii. pp. 455–477.

4. The Restoration of an Ancient Race of Horses. By Professor J. C. EWART, F.R.S. *Proc.* vol. xxx. pp. 291–311.

5. Borel's Integral and q-Series. By the Rev. F. H. JACKSON. Communicated by Professor CHRYSTAL. *Proc.*, vol. xxx. pp. 378–385.

The following Gentlemen were balloted for, and declared duly elected :—PERCY HALL GRIMSHAW, ALEXANDER FRASER, and Professor E. H. ARCHIBALD.

FOURTH ORDINARY MEETING.

Monday, 10th January 1910.

Professor Hudson Beare, Vice-President, in the Chair.

The following Communications were read :—

1. Current Measurements in Loch Garry. By E. M. WEDDERBURN, W.S. *Proc.*, vol. xxx. pp. 312–323.

2. Observations on some Spark-Gap Phenomena. By JOHN M'WHAN, M.A. Communicated by Professor A. GRAY. *Proc.*, vol. xxx. pp. 219–229.

3. Atmospheric Electric Potential Gradient and Earth-Air Current at Edinburgh. By Dr G. A. CARSE and D. MACOWAN. *Proc.*, vol. xxx. pp. 460–465.

4. Alcyonaria from the Cape of Good Hope. Part I. By Dr J. S. Thomson, F.L.S. *Trans.*, vol. xlvii. pp. 549–589.

5. Notes on Proposed Meteorological Instruments. By Professor J. T. Morrison. *Proc.*, vol. xxx. pp. 386–395.

The Roll was signed by Mr Alex. Fraser, who was duly admitted a Fellow of the Society.

FIFTH ORDINARY MEETING.

Monday, 24th January 1910.

Professor Cossar Ewart, F.R.S., Vice-President, in the Chair.

The following Communications were read:—

1. The Development of the Autonomic Nervous Mechanism of the Alimentary Canal of the Bird. By Williamina Abel, M.D. Communicated by Professor D. Noël Paton. *Proc.*, vol. xxx. pp. 327–347.

2. On a New Species of Cactogorgia. By J. J. Simpson, M.A., B.Sc. Communicated by Professor J. Arthur Thomson. *Proc.*, vol. xxx. pp. 324–326.

3. The Stimulatory Action of the Oösperm in the Uterus. By Dr James Oliver.

4. The Significance of the Correlation Coefficient when applied to Mendelian Distribution. By Dr J. Brownlee. *Proc.*, vol. xxx. pp. 473–507.

Dr James Oliver and Dr John Brownlee signed the Roll, and were duly admitted Fellows.

The following Candidates were balloted for, and duly declared elected:—David Gibb, M.A., B.Sc., Charles R. Gibson, Alexander Levy, F.R C.V.S., Dr Alexander Lauder, Wm. John Watson, M.A. (Aber.), B.A. (Oxon.), Professor Arthur Robinson, M.D., M.R.C.S.

SIXTH ORDINARY MEETING.

Monday, 7th February 1910.

Dr James Burgess, Vice-President, in the Chair.

The following Communications were read:—

1. Short-Tailed Domestic Sheep. By Professor J. Cossar Ewart, F.R.S.

2. Electro-motive Force of Cells with a single Salt and two Solvents. By Principal A. P. Laurie.

3. A Stereoscopic Optical Illusion. By Professor F. G. Baily. *Proc.*, vol. xxx. pp. 551–554.

4. A New Form of Respiratory Calorimeter for Physiological Purposes. By E. P. Cathcart, M.D., James Gray, D.Sc., and A. Black. Communicated by Professor Noël Paton.

5. The Theory of Persymmetric Determinants in the Historical Order of Development up to 1860. By Dr Thomas Muir. *Proc.*, vol. xxx. pp. 407–431.

6. The Theory of Bigradients in the Historical Order of Development up to 1860. By Dr Thomas Muir. *Proc.*, vol. xxx. pp. 396–406.

Charles R. Gibson and Dr Alexander Lauder signed the Roll, and were duly admitted Fellows.

SEVENTH ORDINARY MEETING.

Monday, 21st February 1910.

Dr John Horne, F.R.S., Vice-President, in the Chair.

The following Address was delivered:—

The Scientific Work of the British Antarctic Expedition of 1907–9. By James Murray, F.R.S.E., Biologist to the Expedition.

The following Candidates were balloted for, and duly declared elected Fellows of the Society:—Professor D. T. Gwynne-Vaughan, F.L.S., Professor Swale Vincent, Professor John Miller.

EIGHTH ORDINARY MEETING.

Monday, 7th March 1910.

Sir William Turner, K.C.B., F.R.S., President, in the Chair.

Lantern Demonstration on:—

The Place in Nature of the Tasmanian Aboriginal: His Relation to the Anthropoid Ape (*Pithecanthropus erectus*), Primitive and Modern Man. By Professor R. J. A. Berry, of the University of Melbourne.

Mr P. Grimshaw, Dr A. A. Lawson, and Professor A. Robinson signed the Roll, and were duly admitted Fellows of the Society.

NINTH ORDINARY MEETING.

Monday, 21st March 1910.

Dr R. H. Traquair, F.R.S., Vice-President, in the Chair.

The following Communications were read:—

1. A New Species of "Rotating Sector" for varying at will the Intensity of a Beam of Light. (*The Apparatus was exhibited.*) By Dr J. R. Milne. (*Lantern Illustrations.*) *Proc.*, vol. xxxi.
2. A Flicker Spectro-Photometer. (*The Apparatus was exhibited.*) By Dr J. R. Milne. (*Lantern Illustrations.*)
3. A Chemical Investigation into the Nature of the Clay Substance in the Glenboig Fireclay. By D. P. M'Donald, M.A., B.Sc. Communicated by Professor J. W. Gregory. *Proc.*, vol. xxx. pp. 374–377.
4. On the Chemical Classification of Igneous Rocks. By Dr Warth. Communicated by Professor James Geikie.
5. Contributions to the Chemistry of Submarine Glauconite. By Dr W. A. Caspari. Communicated by Sir John Murray, K.C.B. *Proc.*, vol. xxx. pp. 364–373.

William J. Watson, M.A., B.A., and Professor Gwynne-Vaughan signed the Roll, and were duly admitted Fellows of the Society.

Lewis Bennett Barclay, C.E., was balloted for and declared duly elected a Fellow of the Society.

TENTH ORDINARY MEETING.

Monday, 2nd May 1910.

Professor T. Hudson Beare, Vice-President, in the Chair.

The following Address was delivered:—

The Dynamics of Molecular Diffusion in Fluids, with Extension to Suspended Particles. By Sir Joseph Larmor, M.A., F.R.S., Lucasian Professor of Mathematics, St John's College, Cambridge; Secretary Royal Society of London.

The Chairman read a Short Obituary Notice of the late Lord M'Laren.

A Vote of Thanks to Sir Joseph Larmor for his Address was moved by Professor Gray and seconded by Dr C. G. Knott.

ELEVENTH ORDINARY MEETING.

Monday, 16th May 1910.

Emeritus-Professor Crum Brown, F.R.S., Vice-President, in the Chair.

The following Communications were read:—

1. Equilibrium in the Ternary System: Water, Potassium Carbonate, Potassium Ethyl Dipropylmalonate. By J. W. M'David, B.Sc., Carnegie Research Scholar. Communicated by Professor James Walker. *Proc.*, vol. xxx. pp. 440–447.
2. A Method for Determining Boiling-Points under Constant Conditions. (Abstract.) By Professor Alex. Smith and A. W. C. Menzies, U.S.A. Communicated by Professor James Walker. *Proc.*, vol. xxx. pp. 432–435.
3. A Common Thermometric Error in the Determination of Boiling-Points under Reduced Pressure. (Abstract.) By Professor Alex. Smith and A. W. C. Menzies, U.S.A. Communicated by Professor James Walker. *Proc.*, vol. xxx. p. 436.
4. A Simple Dynamic Method for Determining Vapour Pressures. (Abstract.) By Professor Alex. Smith and A. W. C. Menzies, U.S.A. Communicated by Professor James Walker. *Proc.*, vol. xxx. pp. 437–439.
5. The Mathematical Theory of Random Migration, and Epidemic Distribution. By Dr John Brownlee. *Proc.*, vol. xxxi.
6. The Inheritance of Complex Growth Forms, such as Stature, on Mendel's Theory. By Dr John Brownlee. *Proc.*, vol. xxxi.
7. Craniological Observations on the Lengths, Breadths, and Heights of 100 Australian Aboriginal Crania. By Dr A. W. D. Robertson. Communicated by Professor R. J. A. Berry. *Proc.*, vol. xxxi. pp. 1–16.

8. A Biometrical Study of the Relative Degree of Purity of Race of the Tasmanian, Australian, and Papuan. By Professor R. J. A. BERRY, Dr A. W. D. ROBERTSON, and K. S. CROSS, M.Sc. *Proc.*, vol. xxxi. pp. 17–40.

The COUNCIL'S Address to the King was read by the SECRETARY.

The following Candidates were balloted for, and declared duly elected:—BRUCE M'GREGOR GRAY, C.E., DAVID CARNEGIE, M.Inst.C.E., etc., ROBERT SOMERVILLE, B.Sc., ALISTER THOMAS MACKENZIE, M.A., M.D., MUNGO M'CALLUM FAIRGRIEVE, M.A., WILLIAM WILLIAMSON, WILLIAM FRASER HUME, D.Sc., CHARLES EDWARD GREEN.

TWELFTH ORDINARY MEETING.

Monday, 6th June 1910.

Professor T. Hudson Beare, Vice-President, in the Chair.

The following Communications were read:—

1. On Two Relations in Magnetism. By Dr R. A. HOUSTON. Communicated by Professor A. GRAY. *Proc.*, vol. xxx. pp. 457–459.
2. On a New Method of Differentiating between Overlapping Orders in Mapping Grating Spectra. By ALEXANDER D. ROSS, M.A., B.Sc. *Proc.*, vol. xxx. pp. 448–456.
3. The Variation of Young's Modulus under an Electric Current. Pt. III. By Dr H. WALKER. Communicated by Professor J. G. MACGREGOR (*With Lantern Illustrations.*) *Proc.*, vol. xxxi.
4. On Continuous and Stable Isothermal Change of State. By Professor W. PEDDIE. *Proc.*, vol. xxx. pp. 466–472.

The following Gentlemen signed the Roll, and were duly admitted Fellows of the Society:—LEWIS BENNETT BARCLAY, C.E., Dr A. T. MACKENZIE, MUNGO M'CALLUM FAIRGRIEVE, M.A., and WILLIAM WILLIAMSON.

THIRTEENTH ORDINARY MEETING.

Monday, 20th June 1910.

Professor J. Cossar Ewart, F.R.S., Vice-President, in the Chair.

At the request of the Council, the following Address was delivered:—

The Fishes found in the Wealden Strata of Bernissart in Belgium in Association with the well-known *Iguanodon* Remains. By Dr R. H. TRAQUAIR, F.R.S. (*With Lantern Illustrations.*)

The following Communication was read:—

On the Magnetism of Copper-Manganese-Tin Alloys, under varying Thermal Treatment. By A. D. ROSS, M.A., B.Sc., and R. C. GRAY, M.A. *Proc.*, vol. xxxi. p. 85.

Mr CHARLES E. GREEN signed the Roll, and was duly admitted a Fellow of the Society.

The following Gentlemen were balloted for, and duly declared elected Fellows of the Society:—JOSEPH STRICKLAND GOODALL, M.B. (Lond.), THOMAS STEPHENSON, F.C.S., HUGH ALLAN MACEWAN, M.B., Ch.B. (Lond. and Camb.).

FOURTEENTH ORDINARY MEETING.

Monday, 4th July 1910.

Sir William Turner, K.C.B., President, in the Chair.

The following Communications were read:—

1. Morphology of the Manus in Platanista gangetica, the Dolphin of the Ganges. By the PRESIDENT. *Proc.*, vol. xxx. pp. 508–514.
2. A Static Method for Determining the Vapour Pressures of Solids and Liquids. By Professor ALEX. SMITH and A. W. C. MENZIES. *Proc.*, vol. xxx. pp. 523–528.
3. The Vapour Pressures of Mercury. By Professor ALEX. SMITH and A. W. C. MENZIES. *Proc.*, vol. xxx. pp. 521–522.
4. Specific Volume of Solutions of Tetrapropyl-ammonium Chloride. By J. W. M'DAVID, B.Sc. Communicated by Professor JAMES WALKER. *Proc.*, vol. xxx. pp. 515–520.

5. The Development of the Germ Cells in the Mammalian Ovary, with Special Reference to the Early Phase of Maturation. By A. LOUISE M'ILROY, M.D., D.Sc. Communicated by Professor D. NOËL PATON. *Proc.*, vol. xxxi.

6. The Theory of Wronskians, Recurrents, and all the other less common Special Forms of Determinants up to 1860. By Dr THOMAS MUIR. *Proc.*, vol. xxxi.

Mr DAVID CARNEGIE and Dr J. S. GOODALL signed the Roll, and were duly admitted Fellows of the Society.

The President read the List of three British and nine Foreign Gentlemen, proposed by the Council for Honorary Membership, to be elected by ballot at the Ordinary Meeting of 7th November 1910.

FIFTEENTH ORDINARY MEETING.

Monday, 18th July 1910.

Dr R. H. Traquair, F.R.S., Vice-President, in the Chair.

The following Prizes were awarded and presented:—

1. The Neill Prize for the biennial period 1907–08, 1908–09, to FRANCIS J. LEWIS, M.Sc., F.L.S., for his papers in the Society's *Transactions* "On the Plant Remains in the Scottish Peat Mosses."

This research was begun in 1904, and has been carried on for several years with the aid of grants from the Royal Society. The object was to make a systematic investigation of the stratification of the deep peat deposits and the plants contained in them in Scotland with the view of ascertaining the changes in climate during post-glacial time. Most generous help was given to the research by Dr Horne, F.R.S., who freely placed at the disposal of Mr Lewis information, in possession of the Geological Survey in Scotland, regarding the position and extent of many of the most promising areas for investigation.

Thirty-five separate districts have now been investigated and described, ranging from the south coast of Scotland to the Shetland Isles, and from Aberdeenshire to the Outer Hebrides. Mr Lewis has shown that these deposits are distinctly divided into horizontal beds characterised by great differences in the vegetation forming the peat. At the base lies, what he has termed the First Arctic Bed, indicating a period when the lower limit of an arctic vegetation reached sea-level in Shetland. This is followed by a Lower Forest Bed and Lower Peat Bog, overlain by a Second Arctic Bed, an Upper Peat Bog, and Upper Forest Zone.

The salient feature of the stratification is the fact that after the first arctic bed there have been two distinct forest periods separated by an interval during which an arctic vegetation spread over the ground formerly occupied by forest. Mr Lewis maintains that the existence of the first arctic bed at the base of the peat shows that it began its growth during the later phases of the glacial period, while the sequence of the overlying beds strongly supports the view that several distinct oscillations of climate occurred after the disappearance of the last ice-sheet.

Mr Lewis has been the first to place our knowledge of the plants in Scottish peat mosses on a true scientific basis, and it is to be hoped that others will emulate his enthusiasm in pursuing this research.

2. The Keith Prize for the biennial period 1907–08, 1908–09, to WHEELTON HIND, M.D., B.S., F.R.C.S., F.G.S., for a paper published in the *Transactions* of the Society "On the Lamellibranch and Gasteropod Fauna found in the Millstone Grit of Scotland."

The fossils forming the subject of this research were placed in the hands of Dr Hind for determination by the Geological Survey. They were found by Mr Tait in certain marine bands in the basal portion of the Millstone Grit, charged with lamellibranchs, brachiopods, and gasteropods, and associated with Lower Carboniferous species of plants. They have been collected from the counties of Mid-Lothian, West-Lothian, Lanark, and Stirling, their horizon being not far below the line which has been drawn between the Upper and Lower Carboniferous Floras in accordance with the determinations of Dr Kidston.

The remarkable feature of this research is the recognition in the Scottish collection of a lamellibranch fauna of which quite 50 per cent. of the species are new to Europe, and which closely resembles the lamellibranch fauna of the Coal Measures of Nebraska and Illinois of North America. The most striking member of the fauna is the shell *Prothyris elegans* (Meek), this being the first occurrence of the genus in the Carboniferous rocks of Great Britain. Dr Hind's researches show that it is impossible to distinguish any characters sufficient to separate the Scottish and American species from each other.

He has also shown that the gasteropods in this collection bear a strong relation to those of the North American fauna, several species being regarded as identical with those figured and described from the Coal Measures of Nebraska. He has also noted that the brachiopods belong to a late period of Carboniferous time.

Although the Keith Prize has been awarded to Dr Hind for this special research, we must not

forget that he has been engaged for many years in the study of Palæozoic Lamellibranchiata and is recognised as a leading authority on the subject. Among his more important contributions to the literature of this branch of palæontology it may be sufficient to mention his monograph on the British Carboniferous Lamellibranchiata, published by the Palæontographical Society (1896-1905), and his memoir on "The Lamellibranchs of the Silurian Rocks of Girvan," based on the fossils gathered by that enthusiastic collector Mrs Gray, and communicated to this Society.

The following Communications were read :—

1. Plant Remains in the Scottish Peat Mosses. Part IV. By FRANCIS J. LEWIS, M.Sc., F.L.S. Communicated by Dr JOHN HORNE, F.R.S. (*With Lantern Illustrations.*) *Trans.*, vol. xlvii.

2. On the Validity of the Mendelian Theory. By Dr D. BERRY HART. (*With Lantern Illustrations.*)

3. Did the Tail of Halley's Comet affect the Earth's Atmosphere? By Dr JOHN AITKEN, F.R.S. *Proc.*, vol. xxx. pp. 529–550.

4. Sir DAVID GILL exhibited some photographs of Halley's Comet taken in Egypt and in South Africa.

Mr ROBERT SOMERVILLE signed the Roll, and was duly admitted a Fellow of the Society.

The following Candidates were balloted for, and declared duly elected :—Rev. ROBERT SIBBALD CALDERWOOD, Professor JAMES MACKINNON, Dr GABRIEL W. LEE, LOUDON MACQUEEN DOUGLAS.

LAWS OF THE SOCIETY,

As revised 26th October 1908.

[By the Charter of the Society (printed in the *Transactions*, vol. vi. p. 5), the Laws cannot be altered, except at a Meeting held one month after that at which the Motion for alteration shall have been proposed.]

I.

Title.

THE ROYAL SOCIETY OF EDINBURGH shall consist of Ordinary and Honorary Fellows.

II.

The fees of Ordinary Fellows residing in Scotland.

Every Ordinary Fellow, within three months after his election, shall pay Two Guineas as the fee of admission, and Three Guineas as his contribution for the Session in which he has been elected; and annually at the commencement of every Session, Three Guineas into the hands of the Treasurer. This annual contribution shall continue for ten years after his admission, and it shall be limited to Two Guineas for fifteen years thereafter.* Fellows may compound for these contributions on such terms as the Council may from time to time fix.

III.

Payment to cease after 25 years.

All Fellows who shall have paid Twenty-five years' annual contribution shall be exempted from further payment.

IV.

Fees of Non-Resident Ordinary Fellows.

The fees of admission of an Ordinary Non-Resident Fellow shall be £26, 5s., payable on his admission; and in case of any Non-Resident Fellow coming to reside at any time in Scotland, he shall, during each year of his residence, pay the usual annual contribution of £3, 3s., payable by each Resident Fellow; but after payment of such annual contribution for eight years, he shall be exempt from any further payment. In the case of any Resident Fellow ceasing to reside in Scotland, and wishing to continue a Fellow of the Society, it shall be in the power of the Council to determine on what terms, in the circumstances of each case, the privilege of remaining a Fellow of the Society shall be continued to such Fellow while out of Scotland.

Case of Fellows becoming Non-Resident.

* A modification of this rule, in certain cases, was agreed to at a Meeting of the Society held on the 3rd January 1831.

At the Meeting of the Society, on the 5th January 1857, when the reduction of the Contributions from £3, 3s. to £2, 2s., from the 11th to the 25th year of membership, was adopted, it was resolved that the existing Members shall share in this reduction, so far as regards their future annual Contributions.

V.

Defaulters.

Members failing to pay their contributions for three successive years (due application having been made to them by the Treasurer) shall be reported to the Council, and, if they see fit, shall be declared from that period to be no longer Fellows, and the legal means for recovering such arrears shall be employed.

VI.

Privileges of Ordinary Fellows.

None but Ordinary Fellows shall bear any office in the Society, or vote in the choice of Fellows or Office-Bearers, or interfere in the patrimonial interests of the Society.

VII.

Numbers unlimited.

The number of Ordinary Fellows shall be unlimited.

VIII.

Fellows entitled to Transactions and Proceedings.

All Ordinary Fellows of the Society who are not in arrear of their Annual Contributions shall be entitled to receive, gratis, copies of the parts of the Transactions of the Society which shall be published subsequent to their admission, upon application, either personally or by an authorised agent, to the Librarian, provided they apply for them within five years of the date of publication of such parts.

Copies of the parts of the Proceedings shall be distributed to all Fellows of the Society, by post or otherwise, as soon as may be convenient after publication.

IX.

Mode of Recommending Ordinary Fellows.

Candidates for admission as Ordinary Fellows shall make an application in writing, and shall produce along with it a certificate of recommendation to the purport below,* signed by at least *four* Ordinary Fellows, two of whom shall certify their recommendation from personal knowledge. This recommendation shall be delivered to the Secretary, and by him laid before the Council, and shall be exhibited publicly in the Society's rooms for one month, after which it shall be considered by the Council. If the Candidate be approved by the Council, notice of the day fixed for the election shall be given in the circulars of at least two Ordinary Meetings of the Society.

X.

Honorary Fellows, British and Foreign.

Honorary Fellows shall not be subject to any contribution. This class shall consist of persons eminently distinguished for science or literature. Its number shall not exceed Fifty-six, of whom Twenty may be British subjects, and Thirty-six may be subjects of foreign states.

* "A. B., a gentleman well versed in science (*or Polite Literature, as the case may be*), being "to our knowledge desirous of becoming a Fellow of the Royal Society of Edinburgh, we hereby "recommend him as deserving of that honour, and as likely to prove a useful and valuable "Member."

XI.

Personages of Royal Blood may be elected Honorary Fellows, without regard to the limitation of numbers specified in Law X. **Royal Personages.**

XII.

Honorary Fellows may be proposed by the Council, or by a recommendation (in the form given below*) subscribed by three Ordinary Fellows; and in case the Council shall decline to bring this recommendation before the Society, it shall be competent for the proposers to bring the same before a General Meeting. The election shall be by ballot, after the proposal has been communicated *viva voce* from the Chair at one Meeting, and printed in the circulars for Two Ordinary Meetings of the Society, previous to the day of election. **Recommendation of Honorary Fellows.** **Mode of election.**

XIII.

The election of Ordinary Fellows shall take place only at one Afternoon Ordinary Meeting of each month during the Session. The election shall be by ballot, and shall be determined by a majority of at least two-thirds of the votes, provided Twenty-four Fellows be present and vote. **Election of Ordinary Fellows.**

XIV.

The Ordinary Meetings shall be held on the first and third Mondays of each month from November to March, and from May to July, inclusive; with the exception that when there are five Mondays in January, the Meetings for that month shall be held on its second and fourth Mondays. Regular Minutes shall be kept of the proceedings, and the Secretaries shall do the duty alternately, or according to such agreement as they may find it convenient to make. **Ordinary Meetings.**

XV.

The Society shall from time to time publish its Transactions and Proceedings. For this purpose the Council shall select and arrange the papers which they shall deem it expedient to publish in the Transactions of the Society, and shall superintend the printing of the same. **The Transactions.**

XVI.

The Transactions shall be published in parts or *Fasciculi* at the close of each Session, and the expense shall be defrayed by the Society. **How Published.**

* We hereby recommend ______________________________
for the distinction of being made an Honorary Fellow of this Society, declaring that each of us from our own knowledge of his services to (*Literature or Science, as the case may be*) believe him to be worthy of that honour.
(To be signed by three Ordinary Fellows.)

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To the President and Council of the Royal Society of Edinburgh.

XVII.

The Council.

That there shall be formed a Council, consisting—First, of such gentlemen as may have filled the office of President; and Secondly, of the following to be annually elected, viz.:—a President, Six Vice-Presidents (two at least of whom shall be Resident), Twelve Ordinary Fellows as Councillors, a General Secretary, Two Secretaries to the Ordinary Meetings, a Treasurer, and a Curator of the Museum and Library.

The Council shall have power to regulate the private business of the Society. At any Meeting of the Council the Chairman shall have a casting as well as a deliberative vote.

XVIII.

Retiring Councillors.

Four Councillors shall go out annually, to be taken according to the order in which they stand on the list of the Council.

XIX.

Election of Office-Bearers

An Extraordinary Meeting for the election of Office-Bearers shall be held annually on the fourth Monday of October, or on such other lawful day in October as the Council may fix, and each Session of the Society shall be held to begin at the date of the said Extraordinary Meeting.

XX.

Special Meetings; how called.

Special Meetings of the Society may be called by the Secretary, by direction of the Council; or on a requisition signed by six or more Ordinary Fellows. Notice of not less than two days must be given of such Meetings.

XXI.

Treasurer's Duties.

The Treasurer shall receive and disburse the money belonging to the Society, granting the necessary receipts, and collecting the money when due.

He shall keep regular accounts of all the cash received and expended, which shall be made up and balanced annually; and at the Extraordinary Meeting in October, he shall present the accounts for the preceding year, duly audited. At this Meeting, the Treasurer shall also lay before the Council a list of all arrears due above two years, and the Council shall thereupon give such directions as they may deem necessary for recovery thereof.

XXII.

Auditor.

At the Extraordinary Meeting in October, a professional accountant shall be chosen to audit the Treasurer's accounts for that year, and to give the necessary discharge of his intromissions.

XXIII.

General Secretary's Duties.

The General Secretary shall keep Minutes of the Extraordinary Meetings of the Society, and of the Meetings of the Council, in two distinct books. He shall, under the direction of the Council, conduct the correspondence of the Society, and superintend its publications. For these purposes he shall, when necessary, employ a clerk, to be paid by the Society.

XXIV.

The Secretaries to the Ordinary Meetings shall keep a regular Minute-book, in which a full account of the proceedings of these Meetings shall be entered; they shall specify all the Donations received, and furnish a list of them, and of the Donors' names, to the Curator of the Library and Museum; they shall likewise furnish the Treasurer with notes of all admissions of Ordinary Fellows. They shall assist the General Secretary in superintending the publications, and in his absence shall take his duty.

Secretaries to Ordinary Meetings.

XXV.

The Curator of the Museum and Library shall have the custody and charge of all the Books, Manuscripts, objects of Natural History, Scientific Productions, and other articles of a similar description belonging to the Society; he shall take an account of these when received, and keep a regular catalogue of the whole, which shall lie in the hall, for the inspection of the Fellows.

Curator of Museum and Library.

XXVI.

All articles of the above description shall be open to the inspection of the Fellows at the Hall of the Society, at such times and under such regulations as the Council from time to time shall appoint.

Use of Museum and Library.

XXVII.

A Register shall be kept, in which the names of the Fellows shall be enrolled at their admission, with the date.

Register Book.

XXVIII.

If, in the opinion of the Council of the Society, the conduct of any Fellow is unbecoming the position of a Member of a learned Society, or is injurious to the character and interests of this Society, the Council may request such Fellow to resign; and, if he fail to do so within one month of such request being addressed to him, the Council shall call a General Meeting of the Fellows of the Society to consider the matter; and, if a majority of the Fellows present at such Meeting agree to the expulsion of such Member, he shall be then and there expelled by the declaration of the Chairman of the said Meeting to that effect; and he shall thereafter cease to be a Fellow of the Society, and his name shall be erased from the Roll of Fellows, and he shall forfeit all right or claim in or to the property of the Society.

Power of Expulsion.

THE KEITH, MAKDOUGALL-BRISBANE, NEILL, AND GUNNING VICTORIA JUBILEE PRIZES.

The above Prizes will be awarded by the Council in the following manner:—

I. KEITH PRIZE.

The KEITH PRIZE, consisting of a Gold Medal and from £40 to £50 in Money, will be awarded in the Session 1911–1912 for the "best communication on a scientific subject, communicated,* in the first instance, to the Royal Society during the Sessions 1909–1910 and 1910–1911." Preference will be given to a paper containing a discovery.

II. MAKDOUGALL-BRISBANE PRIZE.

This Prize is to be awarded biennially by the Council of the Royal Society of Edinburgh to such person, for such purposes, for such objects, and in such manner as shall appear to them the most conducive to the promotion of the interests of science; with the *proviso* that the Council shall not be compelled to award the Prize unless there shall be some individual engaged in scientific pursuit, or some paper written on a scientific subject, or some discovery in science made during the biennial period, of sufficient merit or importance in the opinion of the Council to be entitled to the Prize.

1. The Prize, consisting of a Gold Medal and a sum of Money, will be awarded at the commencement of the Session 1912–1913, for an Essay or Paper having reference to any branch of scientific inquiry, whether Material or Mental.

2. Competing Essays to be addressed to the Secretary of the Society, and transmitted not later than 8th July 1912.

3. The Competition is open to all men of science.

4. The Essays may be either anonymous or otherwise. In the former case, they must be distinguished by mottoes, with corresponding sealed billets, superscribed with the same motto, and containing the name of the Author.

5. The Council impose no restriction as to the length of the Essays, which may be, at the discretion of the Council, read at the Ordinary Meetings of the Society.

* For the purposes of this award the word "communicated" shall be understood to mean the date on which the manuscript of a paper is received in its final form for printing, as recorded by the General Secretary or other responsible official.

They wish also to leave the property and free disposal of the manuscripts to the Authors; a copy, however, being deposited in the Archives of the Society, unless the paper shall be published in the Transactions.

6. In awarding the Prize, the Council will also take into consideration any scientific papers presented * to the Society during the Sessions 1910–11, 1911–12, whether they may have been given in with a view to the prize or not.

III. NEILL PRIZE.

The Council of the Royal Society of Edinburgh having received the bequest of the late Dr Patrick Neill of the sum of £500, for the purpose of "the interest thereof being applied in furnishing a Medal or other reward every second or third year to any distinguished Scottish Naturalist, according as such Medal or reward shall be voted by the Council of the said Society," hereby intimate:

1. The Neill Prize, consisting of a Gold Medal and a sum of Money, will be awarded during the Session 1911–1912.

2. The Prize will be given for a Paper of distinguished merit, on a subject of Natural History, by a Scottish Naturalist, which shall have been presented * to the Society during the two years preceding the 23rd October 1911,—or failing presentation of a paper sufficiently meritorious, it will be awarded for a work or publication by some distinguished Scottish Naturalist, on some branch of Natural History, bearing date within five years of the time of award.

IV. GUNNING VICTORIA JUBILEE PRIZE.

This Prize, founded in the year 1887 by Dr R. H. Gunning, is to be awarded quadrennially by the Council of the Royal Society of Edinburgh, in recognition of original work in Physics, Chemistry, or Pure or Applied Mathematics.

Evidence of such work may be afforded either by a Paper presented to the Society, or by a Paper on one of the above subjects, or some discovery in them elsewhere communicated or made, which the Council may consider to be deserving of the Prize.

The Prize consists of a sum of money, and is open to men of science resident in or connected with Scotland. The first award was made in the year 1887.

In accordance with the wish of the Donor, the Council of the Society may on fit occasions award the Prize for work of a definite kind to be undertaken during the three succeeding years by a scientific man of recognised ability.

* For the purposes of this award the word "presented" shall be understood to mean the date on which the manuscript of a paper is received in its final form for printing, as recorded by the General Secretary or other responsible official.

AWARDS OF THE KEITH, MAKDOUGALL-BRISBANE, NEILL, AND GUNNING VICTORIA JUBILEE PRIZES.

I. KEITH PRIZE.

1st Biennial Period, 1827–29.—Dr Brewster, for his papers "on his Discovery of Two New Immiscible Fluids in the Cavities of certain Minerals," published in the Transactions of the Society.

2nd Biennial Period, 1829–31.—Dr Brewster, for his paper "on a New Analysis of Solar Light," published in the Transactions of the Society.

3rd Biennial Period, 1831–33.—Thomas Graham, Esq., for his paper "on the Law of the Diffusion of Gases," published in the Transactions of the Society.

4th Biennial Period, 1833–35.—Professor J. D. Forbes, for his paper "on the Refraction and Polarization of Heat," published in the Transactions of the Society.

5th Biennial Period, 1835–37.—John Scott Russell, Esq., for his researches "on Hydrodynamics," published in the Transactions of the Society.

6th Biennial Period, 1837–39.—Mr John Shaw, for his experiments "on the Development and Growth of the Salmon," published in the Transactions of the Society.

7th Biennial Period, 1839–41.—Not awarded.

8th Biennial Period, 1841–1843.—Professor James David Forbes, for his papers "on Glaciers," published in the Proceedings of the Society.

9th Biennial Period, 1843–45.—Not awarded.

10th Biennial Period, 1845–47.—General Sir Thomas Brisbane, Bart., for the Makerstoun Observations on Magnetic Phenomena, made at his expense, and published in the Transactions of the Society.

11th Biennial Period, 1847–49.—Not awarded.

12th Biennial Period, 1849–51.—Professor Kelland, for his papers "on General Differentiation, including his more recent Communication on a process of the Differential Calculus, and its application to the solution of certain Differential Equations," published in the Transactions of the Society.

13th Biennial Period, 1851–53.—W. J. Macquorn Rankine, Esq., for his series of papers "on the Mechanical Action of Heat," published in the Transactions of the Society.

14th Biennial Period, 1853–55.—Dr Thomas Anderson, for his papers "on the Crystalline Constituents of Opium, and on the Products of the Destructive Distillation of Animal Substances," published in the Transactions of the Society.

15th Biennial Period, 1855–57.—Professor Boole, for his Memoir "on the Application of the Theory of Probabilities to Questions of the Combination of Testimonies and Judgments," published in the Transactions of the Society.

16th Biennial Period, 1857–59.—Not awarded.

17th Biennial Period, 1859–61.—John Allan Broun, Esq., F.R.S., Director of the Trevandrum Observatory, for his papers "on the Horizontal Force of the Earth's Magnetism, on the Correction of the Bifilar Magnetometer, and on Terrestrial Magnetism generally," published in the Transactions of the Society.

18th Biennial Period, 1861–63.—Professor William Thomson, of the University of Glasgow, for his Communication "on some Kinematical and Dynamical Theorems."

19th Biennial Period, 1863–65.—Principal Forbes, St Andrews, for his "Experimental Inquiry into the Laws of Conduction of Heat in Iron Bars," published in the Transactions of the Society.

20th Biennial Period, 1865–67.—Professor C. Piazzi Smyth, for his paper "on Recent Measures at the Great Pyramid," published in the Transactions of the Society.

21st Biennial Period, 1867–69.—Professor P. G. Tait, for his paper "on the Rotation of a Rigid Body about a Fixed Point," published in the Transactions of the Society.

22ND BIENNIAL PERIOD, 1869–71.—Professor CLERK MAXWELL, for his paper "on Figures, Frames, and Diagrams of Forces," published in the Transactions of the Society.

23RD BIENNIAL PERIOD, 1871–73.—Professor P. G. TAIT, for his paper entitled "First Approximation to a Thermo-electric Diagram," published in the Transactions of the Society.

24TH BIENNIAL PERIOD, 1873–75.—Professor CRUM BROWN, for his Researches "on the Sense of Rotation, and on the Anatomical Relations of the Semicircular Canals of the Internal Ear."

25TH BIENNIAL PERIOD, 1875–77.—Professor M. FORSTER HEDDLE, for his papers "on the Rhombohedral Carbonates," and "on the Felspars of Scotland," published in the Transactions of the Society.

26TH BIENNIAL PERIOD, 1877–79.—Professor H. C. FLEEMING JENKIN, for his paper "on the Application of Graphic Methods to the Determination of the Efficiency of Machinery," published in the Transactions of the Society; Part II. having appeared in the volume for 1877–78.

27TH BIENNIAL PERIOD, 1879–81.—Professor GEORGE CHRYSTAL, for his paper "on the Differential Telephone," published in the Transactions of the Society.

28TH BIENNIAL PERIOD, 1881–83.—THOMAS MUIR, Esq., LL.D., for his "Researches into the Theory of Determinants and Continued Fractions," published in the Proceedings of the Society.

29TH BIENNIAL PERIOD, 1883–85.—JOHN AITKEN, Esq., for his paper "on the Formation of Small Clear Spaces in Dusty Air," and for previous papers on Atmospheric Phenomena, published in the Transactions of the Society.

30TH BIENNIAL PERIOD, 1885–87.—JOHN YOUNG BUCHANAN, Esq., for a series of communications, extending over several years, on subjects connected with Ocean Circulation, Compressibility of Glass, etc.; two of which, viz., "On Ice and Brines," and "On the Distribution of Temperature in the Antarctic Ocean," have been published in the Proceedings of the Society.

31ST BIENNIAL PERIOD, 1887–89.—Professor E. A. LETTS, for his papers on the Organic Compounds of Phosphorus, published in the Transactions of the Society.

32ND BIENNIAL PERIOD, 1889–91.—R. T. OMOND, Esq., for his contributions to Meteorological Science, many of which are contained in vol. xxxiv. of the Society's Transactions.

33RD BIENNIAL PERIOD, 1891–93.—Professor THOMAS R. FRASER, F.R.S., for his papers on *Strophanthus hispidus*, Strophanthin, and Strophanthidin, read to the Society in February and June 1889 and in December 1891, and printed in vols. xxxv., xxxvi., and xxxvii. of the Society's Transactions.

34TH BIENNIAL PERIOD, 1893–95.—Dr CARGILL G. KNOTT, for his papers on the Strains produced by Magnetism in Iron and in Nickel, which have appeared in the Transactions and Proceedings of the Society.

35TH BIENNIAL PERIOD, 1895–97.—Dr THOMAS MUIR, for his continued communications on Determinants and Allied Questions.

36TH BIENNIAL PERIOD, 1897–99.—Dr JAMES BURGESS, for his paper "on the Definite Integral $\frac{2}{\sqrt{\pi}}\int_0^t \epsilon^{-t^2}dt$, with extended Tables of Values," printed in vol. xxxix. of the Transactions of the Society.

37TH BIENNIAL PERIOD, 1899–1901.—Dr HUGH MARSHALL, for his discovery of the Persulphates, and for his Communications on the Properties and Reactions of these Salts, published in the Proceedings of the Society.

38TH BIENNIAL PERIOD, 1901–03.—Sir WILLIAM TURNER, K.C.B., LL.D., F.R.S., &c., for his memoirs entitled "A Contribution to the Craniology of the People of Scotland," published in the Transactions of the Society, and for his "Contributions to the Craniology of the People of the Empire of India," Parts I., II., likewise published in the Transactions of the Society.

39TH BIENNIAL PERIOD, 1903–05.—THOMAS H. BRYCE, M.A., M.D., for his two papers on "The Histology of the Blood of the Larva of *Lepidosiren paradoxa*," published in the Transactions of the Society within the period.

40TH BIENNIAL PERIOD, 1905–07.—ALEXANDER BRUCE, M.A., M.D., F.R.C.P.E., for his paper entitled "Distribution of the Cells in the Intermedio-Lateral Tract of the Spinal Cord," published in the Transactions of the Society within the period.

41ST BIENNIAL PERIOD, 1907–09.—WHEELTON HIND, M.D., B.S., F.R.C.S., F.G.S., for a paper published in the Transactions of the Society, "On the Lamellibranch and Gasteropod Fauna found in the Millstone Grit of Scotland."

II. MAKDOUGALL-BRISBANE PRIZE.

1st Biennial Period, 1859.—Sir Roderick Impey Murchison, on account of his Contributions to the Geology of Scotland.

2nd Biennial Period, 1860–62.—William Seller, M.D., F.R.C.P.E., for his "Memoir of the Life and Writings of Dr Robert Whytt," published in the Transactions of the Society.

3rd Biennial Period, 1862–64.—John Denis Macdonald, Esq., R.N., F.R.S., Surgeon of H.M.S. "Icarus," for his paper "on the Representative Relationships of the Fixed and Free Tunicata, regarded as Two Sub-classes of equivalent value; with some General Remarks on their Morphology," published in the Transactions of the Society.

4th Biennial Period, 1864–66.—Not awarded.

5th Biennial Period, 1866–68.—Dr Alexander Crum Brown and Dr Thomas Richard Fraser, for their conjoint paper "on the Connection between Chemical Constitution and Physiological Action," published in the Transactions of the Society.

6th Biennial Period, 1868–70.—Not awarded.

7th Biennial Period, 1870–72.—George James Allman, M.D., F.R.S., Emeritus Professor of Natural History, for his paper "on the Homological Relations of the Cœlenterata," published in the Transactions, which forms a leading chapter of his Monograph of Gymnoblastic or Tubularian Hydroids—since published.

8th Biennial Period, 1872–74.—Professor Lister, for his paper "on the Germ Theory of Putrefaction and the Fermentive Changes," communicated to the Society, 7th April 1873.

9th Biennial Period, 1874–76.—Alexander Buchan, A.M., for his paper "on the Diurnal Oscillation of the Barometer," published in the Transactions of the Society.

10th Biennial Period, 1876–78.—Professor Archibald Geikie, for his paper "on the Old Red Sandstone of Western Europe," published in the Transactions of the Society.

11th Biennial Period, 1878–80.—Professor Piazzi Smyth, Astronomer-Royal for Scotland, for his paper "on the Solar Spectrum in 1877–78, with some Practical Idea of its probable Temperature of Origination," published in the Transactions of the Society.

12th Biennial Period, 1880–82.—Professor James Geikie, for his "Contributions to the Geology of the North-West of Europe," including his paper "on the Geology of the Faroes," published in the Transactions of the Society.

13th Biennial Period, 1882–84.—Edward Sang, Esq., LL.D., for his paper "on the Need of Decimal Subdivisions in Astronomy and Navigation, and on Tables requisite therefor," and generally for his Recalculation of Logarithms both of Numbers and Trigonometrical Ratios,—the former communication being published in the Proceedings of the Society.

14th Biennial Period, 1884–86.—John Murray, Esq., LL.D., for his papers "On the Drainage Areas of Continents, and Ocean Deposits," "The Rainfall of the Globe, and Discharge of Rivers," "The Height of the Land and Depth of the Ocean," and "The Distribution of Temperature in the Scottish Lochs as affected by the Wind."

15th Biennial Period, 1886–88.—Archibald Geikie, Esq., LL.D., for numerous Communications, especially that entitled "History of Volcanic Action during the Tertiary Period in the British Isles," published in the Transactions of the Society.

16th Biennial Period, 1889–90.—Dr Ludwig Becker, for his paper on "The Solar Spectrum at Medium and Low Altitudes," printed in vol. xxxvi. Part I. of the Society's Transactions.

17th Biennial Period, 1890–92.—Hugh Robert Mill, Esq., D.Sc., for his papers on "The Physical Conditions of the Clyde Sea Area," Part I. being already published in vol. xxxvi. of the Society's Transactions.

18th Biennial Period, 1892–94.—Professor James Walker, D.Sc., Ph.D., for his work on Physical Chemistry, part of which has been published in the Proceedings of the Society, vol. xx. pp. 255–263. In making this award, the Council took into consideration the work done by Professor Walker along with Professor Crum Brown on the Electrolytic Synthesis of Dibasic Acids, published in the Transactions of the Society.

19th Biennial Period, 1894–96.—Professor John G. M'Kendrick, for numerous Physiological papers, especially in connection with Sound, many of which have appeared in the Society's publications.

20th Biennial Period, 1896–98.—Dr William Peddie, for his papers on the Torsional Rigidity of Wires.

21st Biennial Period, 1898–1900.—Dr Ramsay H. Traquair, for his paper entitled "Report on Fossil Fishes collected by the Geological Survey in the Upper Silurian Rocks of Scotland," printed in vol. xxxix. of the Transactions of the Society.

22ND BIENNIAL PERIOD, 1900–02.—Dr ARTHUR T. MASTERMAN, for his paper entitled "The Early Development of *Cribrella oculata* (Forbes), with remarks on Echinoderm Development," printed in vol. xl. of the Transactions of the Society.

23RD BIENNIAL PERIOD, 1902–04.—Mr JOHN DOUGALL, M.A., for his paper on "An Analytical Theory of the Equilibrium of an Isotropic Elastic Plate," published in vol. xli. of the Transactions of the Society.

24TH BIENNIAL PERIOD, 1904–06.—JACOB E. HALM, Ph.D., for his two papers entitled "Spectroscopic Observations of the Rotation of the Sun," and "Some Further Results obtained with the Spectroheliometer," and for other astronomical and mathematical papers published in the Transactions and Proceedings of the Society within the period.

25TH BIENNIAL PERIOD, 1906–08.—D. T. GWYNNE-VAUGHAN, M.A., F.L.S., for his papers, 1st, "On the Fossil Osmundaceæ," and 2nd, "On the Origin of the Adaxially-curved Leaf-trace in the Filicales," communicated by him conjointly with Dr R. Kidston.

III. THE NEILL PRIZE.

1ST TRIENNIAL PERIOD, 1856–59.—Dr W. LAUDER LINDSAY, for his paper "on the Spermogones and Pycnides of Filamentous, Fruticulose, and Foliaceous Lichens," published in the Transactions of the Society.

2ND TRIENNIAL PERIOD, 1859–61.—ROBERT KAYE GREVILLE, LL.D., for his Contributions to Scottish Natural History, more especially in the department of Cryptogamic Botany, including his recent papers on Diatomaceæ.

3RD TRIENNIAL PERIOD, 1862–65.—ANDREW CROMBIE RAMSAY, F.R.S., Professor of Geology in the Government School of Mines, and Local Director of the Geological Survey of Great Britain, for his various works and memoirs published during the last five years, in which he has applied the large experience acquired by him in the Direction of the arduous work of the Geological Survey of Great Britain to the elucidation of important questions bearing on Geological Science.

4TH TRIENNIAL PERIOD, 1865–68.—Dr WILLIAM CARMICHAEL M'INTOSH, for his paper "on the Structure of the British Nemerteans, and on some New British Annelids," published in the Transactions of the Society.

5TH TRIENNIAL PERIOD, 1868–71.—Professor WILLIAM TURNER, for his papers "on the Great Finner Whale; and on the Gravid Uterus, and the Arrangement of the Fœtal Membranes in the Cetacea," published in the Transactions of the Society.

6TH TRIENNIAL PERIOD, 1871–74.—CHARLES WILLIAM PEACH, Esq., for his Contributions to Scottish Zoology and Geology, and for his recent contributions to Fossil Botany.

7TH TRIENNIAL PERIOD, 1874–77.—Dr RAMSAY H. TRAQUAIR, for his paper "on the Structure and Affinities of *Tristichopterus alatus* (Egerton)," published in the Transactions of the Society, and also for his contributions to the Knowledge of the Structure of Recent and Fossil Fishes.

8TH TRIENNIAL PERIOD, 1877–80.—JOHN MURRAY, Esq., for his paper "on the Structure and Origin of Coral Reefs and Islands," published (in abstract) in the Proceedings of the Society.

9TH TRIENNIAL PERIOD, 1880–83.—Professor HERDMAN, for his papers "on the Tunicata," published in the Proceedings and Transactions of the Society.

10TH TRIENNIAL PERIOD, 1883–86.—B. N. PEACH, Esq., for his Contributions to the Geology and Palæontology of Scotland, published in the Transactions of the Society.

11TH TRIENNIAL PERIOD, 1886–89.—ROBERT KIDSTON, Esq., for his Researches in Fossil Botany, published in the Transactions of the Society.

12TH TRIENNIAL PERIOD, 1889–92.—JOHN HORNE, Esq., F.G.S., for his Investigations into the Geological Structure and Petrology of the North-West Highlands.

13TH TRIENNIAL PERIOD, 1892–95.—ROBERT IRVINE, Esq., for his papers on the Action of Organisms in the Secretion of Carbonate of Lime and Silica, and on the solution of these substances in Organic Juices. These are printed in the Society's Transactions and Proceedings.

14TH TRIENNIAL PERIOD, 1895–98.—Professor COSSAR EWART, for his recent Investigations connected with Telegony.

15TH TRIENNIAL PERIOD, 1898–1901.—Dr JOHN S. FLETT, for his papers entitled "The Old Red Sandstone of the Orkneys" and "The Trap Dykes of the Orkneys," printed in vol. xxxix. of the Transactions of the Society.

16TH TRIENNIAL PERIOD, 1901–04.—Professor J. GRAHAM KERR, M.A., for his Researches on *Lepidosiren paradoxa*, published in the Philosophical Transactions of the Royal Society, London.

17TH TRIENNIAL PERIOD, 1904–07.—FRANK J. COLE, B.Sc., for his paper entitled "A Monograph on the General Morphology of the Myxinoid Fishes, based on a study of Myxine," published in the Transactions of the Society, regard being also paid to Mr Cole's other valuable contributions to the Anatomy and Morphology of Fishes.

1ST BIENNIAL PERIOD, 1907–09.—FRANCIS J. LEWIS, M.Sc., F.L.S., for his papers in the Society's Transactions "On the Plant Remains of the Scottish Peat Mosses."

IV. GUNNING VICTORIA JUBILEE PRIZE.

1ST TRIENNIAL PERIOD, 1884–87.—Sir WILLIAM THOMSON, Pres. R.S.E., F.R.S., for a remarkable series of papers "on Hydrokinetics," especially on Waves and Vortices, which have been communicated to the Society.

2ND TRIENNIAL PERIOD, 1887–90.—Professor P. G. TAIT, Sec. R.S.E., for his work in connection with the "Challenger" Expedition, and his other Researches in Physical Science.

3RD TRIENNIAL PERIOD, 1890–93.—ALEXANDER BUCHAN, Esq., LL.D., for his varied, extensive, and extremely important Contributions to Meteorology, many of which have appeared in the Society's Publications.

4TH TRIENNIAL PERIOD, 1893–96.—JOHN AITKEN, Esq., for his brilliant Investigations in Physics, especially in connection with the Formation and Condensation of Aqueous Vapour.

1ST QUADRENNIAL PERIOD, 1896–1900.—Dr T. D. ANDERSON, for his discoveries of New and Variable Stars.

2ND QUADRENNIAL PERIOD, 1900–04.—Sir JAMES DEWAR, LL.D., D.C.L., F.R.S., etc., for his researches on the Liquefaction of Gases, extending over the last quarter of a century, and on the Chemical and Physical Properties of Substances at Low Temperatures: his earliest papers being published in the Transactions and Proceedings of the Society.

3RD QUADRENNIAL PERIOD, 1904–08.—Professor GEORGE CHRYSTAL, M.A., LL.D., for a series of papers on "Seiches," including "The Hydrodynamical Theory and Experimental Investigations of the Seiche Phenomena of Certain Scottish Lakes."

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October 1910.

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W. S. M'Cormick, M.A., LL.D.

ALPHABETICAL LIST OF THE ORDINARY FELLOWS OF THE SOCIETY,

Corrected to November 1910.

N.B.—*Those marked * are Annual Contributors.*

B. prefixed to a name indicates that the Fellow has received a Makdougall-Brisbane Medal.
K. „ „ „ Keith Medal.
N. „ „ „ Neill Medal.
V. J. „ „ „ the Gunning-Victoria Jubilee Prize.
C. „ „ „ contributed one or more Communications to the Society's TRANSACTIONS or PROCEEDINGS.

Date of Election.			
1898	C.	* Abercromby, the Hon. John, 62 Palmerston Place	
1898		Adami, Prof. J. G., M.A., M.D. Cantab., F.R.S., Professor of Pathology in M'Gill University, Montreal	
1896		* Affleck, Jas. Ormiston, M.D., LL.D., F.R.C.P.E., 38 Heriot Row	
1871		Agnew, Sir Stair, K.C.B., M.A., late Registrar-General for Scotland, 22 Buckingham Terrace	
1875	C. K. V. J.	Aitken, John, LL.D., F.R.S., Ardenlea, Falkirk	5
1895		* Alford, Robert Gervase, Memb. Inst. C.E., 1 Windmill Hill, Hampstead, London, N.W.	
1889		* Alison, John, M.A., Head Master, George Watson's College, Edinburgh	
1894		Allan, Francis John, M.D., C.M. Edin., M.O.H. City of Westminster, Westminster City Hall, Charing Cross Road, London	
1888	C.	* Allardice, R. E., M.A., Professor of Mathematics in Stanford University, Palo Alto, Santa Clara Co., California	
1878		Allchin, Sir William H., M.D., F.R.C.P.L., Senior Physician to the Westminster Hospital, 5 Chandos Street, Cavendish Square, London	10
1906		Anderson, Daniel E., M.D., B.A., B.Sc., Green Bank, Merton Lane, Highgate, London, N.	
1893		Anderson, J. Macvicar, Architect, 6 Stratton Street, London	
1883		Anderson, Sir Robert Rowand, LL.D., 16 Rutland Square	
1905		Anderson, William, F.G.S., Box 4303, Johannesburg, Transvaal, South Africa	
1905		* Anderson, William, M.A., Head Science Master, George Watson's College, Edinburgh, 29 Lutton Place	15
1903		Anderson-Berry, David, M.D., LL.D., F.R.S.L., M.R.A.S., F.S.A. (Scot.), Versailles, Highgate, London, N.	
1905		* Andrew, George, M.A., B.A., H.M.I.S., Glenhuntly, Hyndford Road, Lanark	
1881	C.	Anglin, A. H., M.A., LL.D., M.R.I.A., Professor of Mathematics, Queen's College, Cork	
1906		Appleton, Arthur Frederick, F.R.C.V.S., Lieut.-Col., Army Headquarters, Pretoria, S.A.	
1899		Appleyard, James R., Royal Technical Institute, Salford, Manchester	20
1893		* Archer, Walter E., 17 Sloan Court, London	
1910	C.	Archibald, E. H., B.Sc., Professor of Chemistry, Syracuse University, Syracuse, N.Y., U.S.A.	
1907		* Archibald, James, M.A., Head Master, St Bernard's School, 1 Leamington Terrace, Edinburgh	
1907		* Badre, Muhammad, Ph.D.	
1894		* Bailey, Frederick, Lieut.-Col. (*late*) R.E., 7 Drummond Place	25
1896	C.	* Baily, Francis Gibson, M.A., Professor of Applied Physics, Heriot-Watt College	

Date of Election.			
1877	C.	Balfour, I. Bayley, M.A., Sc.D., M.D., LL.D., F.R.S., F.L.S., King's Botanist in Scotland, Professor of Botany in the University of Edinburgh and Keeper of the Royal Botanic Gardens, Inverleith House	
1905	C.	Balfour-Browne, William Alexander Francis, M.A., Barrister-at-Law, Claremont, Holywood, Co. Down, Ireland	
1892		* Ballantyne, J. W., M.D., F.R.C.P.E., 19 Rothesay Terrace	
1902	C.	Bannerman, W. B., M.D., D.Sc., Lt.-Col., Indian Medical Service, Director, Bacteriological Laboratory, Parel, Bombay, India	30
1889		* Barbour, A. H. F., M.A., M.D., F.R.C.P.E., 4 Charlotte Square	
1886		Barclay, A. J. Gunion, M.A., 729 Great Western Road, Glasgow	
1872		Barclay, George, M.A., 17 Coates Crescent	
1883		Barclay, G. W. W., M.A., 91 Union Street, Aberdeen	
1910		* Barclay, Lewis Bennett, C.E., 16 Craiglockart Terrace, Edinburgh	35
1903		Bardswell, Noël Dean, M.D., M.R.C.P. Ed. and Lond., King Edward VII. Sanatorium, Midhurst	
1882	C.	Barnes, Henry, M.D., LL.D., 6 Portland Square, Carlisle	
1904		Barr, Sir James, M.D., F.R.C.P. Lond., 72 Rodney Street, Liverpool	
1874		Barrett, William F., F.R.S., M.R.I.A., late Professor of Physics, Royal College of Science, Dublin, Kingston, County Dublin	
1887		* Bartholomew, J. G., LL.D., F.R.G.S., The Geographical Institute, Dalkeith Road	40
1895	C.	Barton, Edwin H., D.Sc., A.M.I.E.E., Fellow Physical Society of London, Professor of Experimental Physics, University College, Nottingham	
1904		* Baxter, William Muirhead, St Colms, 21B Strathearn Road, Edinburgh	
1888		* Beare, Thomas Hudson, B.Sc., Memb. Inst. C.E., Professor of Engineering in the University of Edinburgh (Vice-President)	
1897	C.	* Beattie, John Carruthers, D.Sc., Professor of Physics, South African College, Cape Town	
1892		Beck, J. H. Meining, M.D., M.R.C.P.E., Rondebosch, Cape Town	45
1893	B. C.	* Becker, Ludwig, Ph.D., Regius Professor of Astronomy in the University of Glasgow, The Observatory, Glasgow	
1882	C.	Beddard, Frank E., M.A. Oxon., F.R.S., Prosector to the Zoological Society of London, Zoological Society's Gardens, Regent's Park, London	
1887		* Begg, Ferdinand Faithfull, Bartholomew House, London	
1886		Bell, A. Beatson, 17 Lansdowne Crescent	
1906		Bell, John Patrick Fair, F.Z.S., Fulforth, Witton Gilbert, Durham	50
1874		Bell, Joseph, M.D., F.R.C.S.E., 2 Melville Crescent	
1900		* Bennett, James Bower, Memb. Inst. C.E., 42 Frederick Street	
1887		* Bernard, J. Mackay, of Dunsinnan, B.Sc., Dunsinnan, Perth	
1893	C.	* Berry, George A., M.D., C.M., F.R.C.S., 31 Drumsheugh Gardens	
1897	C.	Berry, Richard J., M.D., F.R.C.S.E., Professor of Anatomy in the University of Melbourne, Victoria	55
1904		* Beveridge, Erskine, LL.D., St Leonards Hill, Dunfermline	
1880	C.	Birch, De Burgh, C.B., M.D., Professor of Physiology in the University of Leeds, 16 De Grey Terrace, Leeds	
1900		* Bisset, James, M.A., F.L.S., F.G.S., 35 Mortonhall Road, Edinburgh	
1907		* Black, Frederick Alexander, Solicitor, 59 Academy Street, Inverness	
1884		Black, John S., M.A., LL.D. (Curator of Library and Museum), 6 Oxford Terrace	60
1897		* Blaikie, Walter Biggar, The Loan, Colinton	
1904	C.	* Bles, Edward J., M.A., D.Sc., The Mill House, Iffley, Oxford	
1898	C.	* Blyth, Benjamin Hall, M.A., Memb. Inst. C.E., 17 Palmerston Place	
1894		* Bolton, Herbert, F.G.S., F.L.S., Curator of the Bristol Museum, Queen's Road, Bristol	
1884		Bond, Francis T., B.A., M.D., M.R.C.S., Gloucester	65
1872	C.	Bottomley, J. Thomson, M.A., D.Sc., LL.D., F.R.S., F.C.S., 13 University Gardens, Glasgow	
1886		Bower, Frederick O., M.A., D.Sc., F.R.S., F.L.S., Regius Professor of Botany in the University of Glasgow, 1 St John's Terrace, Hillhead, Glasgow (Vice-President)	
1884	C.	Bowman, Frederick Hungerford, D.Sc., F.C.S. (Lond. and Berl.), F.I.C., Assoc. Inst. C.E., Assoc. Inst. M.E., M.I.E.E., etc., 4 Albert Square, Manchester	
1901		Bradbury, J. B., M.D., Downing Professor of Medicine, University of Cambridge	
1903	C.	* Bradley, O. Charnock, M.D., D.Sc., Royal Veterinary College, Edinburgh	70
1886		Bramwell, Byrom, M.D., F.R.C.P.E., 23 Drumsheugh Gardens	

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1907		* Bramwell, Edwin, M.B., F.R.C.P.E., M.R.C.P. Lond., 23 Drumsheugh Gardens	
1895		Bright, Charles, Assoc. Memb. Inst. C.E., Memb. Inst. E.E., F.R.A.S., F.G.S., London, 26 Devonshire Terrace, Hyde Park, and Caxton House, Westminster, London, S.W.	
1893		Brock, G. Sandison, M.D., 6 Corso d'Italia, Rome, Italy	
1901	C.	* Brodie, W. Brodie, M.B., Thaxted, Essex	75
1907		Brown, Alexander, M.A., B.Sc., Professor of Applied Mathematics, South African College, Cape Town	
1864	C. K. B.	Brown, Alex. Crum, M.D., D.Sc., F.R.C.P.E., LL.D., F.R.S. (VICE-PRESIDENT), Emeritus Professor of Chemistry in the University of Edinburgh, 8 Belgrave Crescent	
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1883	C.	Brown, J. J. Graham, M.D., F.R.C.P.E., 3 Chester Street	
1885	C.	Brown, J. Macdonald, M.D., F.R.C.S., 2 Frognal, London, N.W.	80
1909	C.	* Brownlee, John, M.A., M.D., D.Sc., Ruchill Hospital, Bilsland Drive, Glasgow	
1883	K. C.	Bruce, Alexander, M.A., M.D., LL.D., F.R.C.P.E., 8 Ainslie Place	
1906		* Bruce, William Speirs, LL.D., Antarctica, Joppa, Midlothian	
1898	K. C.	* Bryce, T. H., M.A., M.D. (Edin.), 2 The University, Glasgow	
1870	C. K.	Buchanan, John Young, M.A., F.R.S., Christ's College, Cambridge	85
1882		Buchanan, T. R., M.A., M.P., 12 South Street, Park Lane, London, W.	
1887	C.	* Buist, J. B., M.D., F.R.C.P.E., 1 Clifton Terrace	
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1902		* Burgess, A. G., M.A., Mathematical Master, Edinburgh Ladies' College, 64 Strathearn Road	
1894	C. K.	* Burgess, James, C.I.E., LL.D., Hon. A.R.I.B.A., F.R.G.S., Hon. M. Imp. Russ. Archæol. Soc., and Amer. Or. Soc., M. Soc. Asiat. de Paris, M.R.A.S., *H.* Corr. M. Batavian Soc. of Arts and Sciences, and Berlin Soc. Anthrop., *H.* Assoc. Finno-Ugrian Soc. (VICE-PRESIDENT), 22 Seton Place	90
1902		* Burn, Rev. John Henry, B.D., The Parsonage, Ballater	
1887		* Burnet, John James, Architect, 18 University Avenue, Hillhead, Glasgow	
1888		* Burns, Rev. T., D.D., F.S.A. Scot., Minister of Lady Glenorchy's Parish Church, Croston Lodge, Chalmers Crescent	
1896		* Butters, J. W., M.A., B.Sc., Rector of Ardrossan Academy	
1887	C.	* Cadell, Henry Moubray, of Grange, B.Sc., Bo'ness	95
1897		* Caird, Robert, LL.D., Shipbuilder, Greenock	
1910		* Calderwood, Rev. Robert Sibbald, Minister of Cambuslang, The Manse, Cambuslang, Lanarkshire	
1893	C.	Calderwood, W. L., Inspector of Salmon Fisheries of Scotland, South Bank, Canaan Lane, Edinburgh	
1894		* Cameron, James Angus, M.D., Medical Officer of Health, Firhall, Nairn	
1905	C.	Cameron, John, M.D., D.Sc., M.R.C.S. Eng., Anatomy Department, Middlesex Hospital Medical School, London	100
1904		* Campbell, Charles Duff, 21 Montague Terrace, Inverleith Row	
1908		* Campbell, Lt.-Col. John, Westwood, Cupar, Fife	
1899	C.	* Carlier, Edmund W. W., M.D., B.Sc., Professor of Physiology in Mason College, Birmingham	
1910		Carnegie, David, Memb. Inst. C.E., Memb. Inst. Mech. E., Memb. I. S. Inst., Weston Lodge, Broomhall Place, Sheffield	
1905	C.	* Carse, George Alexander, M.A., D.Sc., Lecturer on Natural Philosophy, University of Edinburgh, 3 Middleby Street	105
1901		Carslaw, H. S., M.A., D.Sc., Professor of Mathematics in the University of Sydney, New South Wales	
1905		Carter, Joseph Henry, F.R.C.V.S., Rowley Hall, Burnley, Lancashire	
1898		* Carter, Wm. Allan, Memb. Inst. C.E., 32 Great King Street	
1898		Carus-Wilson, Cecil, F.R.G.S., F.G.S., 16 Waldegrave Park, Strawberry Hill, Middlesex	
1908		Cavanagh, Thomas Francis, M.D., 396 Eccleshall Road, Sheffield	110
1882		Cay, W. Dyce, Memb. Inst. C.E., 39 Victoria Street, Westminster, London	
1890		Charles, John J., M.A., M.D., C.M., late Professor of Anatomy and Physiology, Queen's College, Cork, 8 Clyde Road, Dublin	
1899		Chatham, James, Actuary, 7 Belgrave Crescent	
1874		Chiene, John, C.B., M.D., LL.D., F.R.C.S.E., Emeritus Professor of Surgery in the University of Edinburgh, Barnton Avenue, Davidson's Mains	
1880	C. K. V. J.	Chrystal, George, M.A., LL.D., Professor of Mathematics in the University of Edinburgh (GENERAL SECRETARY), 5 Belgrave Crescent	115

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1891		* Clark, John B., M.A., Head Master of Heriot's Hospital School, Lauriston, Garleffin, Craiglea Drive
1903		* Clarke, William Eagle, F.L.S., Keeper of the Natural History Collections in the Royal Scottish Museum, Edinburgh, 35 Braid Road
1909		Clayton, Thomas Morrison, M.D., D.Hy., B.Sc., D.P.H., Medical Officer of Health, Gateshead, 13 The Crescent, Gateshead-on-Tyne
1875		Clouston, T. S., M.D., LL.D., 26 Heriot Row
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1904		Coles, Alfred Charles, M.D., D.Sc., York House, Poole Road, Bournemouth, W.
1888	C.	Collie, John Norman, Ph.D., F.R.S., F.C.S., Professor of Organic Chemistry in the University College, Gower Street, London
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1909		* Comrie, Peter, M.A., B.Sc., Head Mathematical Master, Boroughmuir Junior Student Centre, 19 Craighouse Terrace
1886		Connan, Daniel M., M.A. 125
1872		Constable, Archibald, LL.D., 11 Thistle Street
1894		Cook, John, M.A., 30 Hermitage Gardens, late Principal, Central College, Bangalore, Director of Meteorology in Mysore, and Fellow, University of Madras, India
1891		* Cooper, Charles A., LL.D., 41 Drumsheugh Gardens
1905		* Corrie, David, F.C.S., Nobel's Explosives Company, Polmont Station
1908		Craig, James Ireland, M.A., B.A., Director of the Computation Office, Survey Department, Egypt, Mataria, Egypt 130
1875		Craig, William, M.D., F.R.C.S.E., Lecturer on Materia Medica to the College of Surgeons, 71 Bruntsfield Place
1907		* Cramer, William, Ph.D., Lecturer in Physiological Chemistry in the University of Edinburgh, Physiological Department, The University
1903		Crawford, Lawrence, M.A., D.Sc., Professor of Mathematics in the South African College, Cape Town
1887		* Crawford, William Caldwell, 1 Lockharton Gardens, Colinton Road
1870		Crichton-Browne, Sir Jas., M.D., LL.D., F.R.S., Lord Chancellor's Visitor and Vice-President of the Royal Institution of Great Britain, 72 Queen's Gate, and Royal Courts of Justice, Strand, London 135
1886		Croom, Sir John Halliday, M.D., F.R.C.P.E., Professor of Midwifery in the University of Edinburgh, Vice-President, Royal College of Surgeons, Edinburgh, 25 Charlotte Square
1898		* Cullen, Alexander, F.S.A. Scot., Millburn House, by Hamilton
1898		* Currie, James, M.A. Cantab. (Treasurer), Larkfield, Goldenacre
1904		* Cuthbertson, John, Secretary, West of Scotland Agricultural College, 6 Charles Street, Kilmarnock
1885		Daniell, Alfred, M.A., LL.B., D.Sc., Advocate, The Athenæum Club, Pall Mall, London 140
1884		Davy, R., F.R.C.S. Eng., Consulting Surgeon to Westminster Hospital, Burstone House, Bow, North Devon
1894		* Denny, Archibald, Cardross Park, Cardross, Dumbartonshire
1869	C. V. J.	Dewar, Sir James, M.A., LL.D., D.C.L., D.Sc. Dub., F.R.S., F.C.S., Jacksonian Professor of Natural and Experimental Philosophy in the University of Cambridge, and Fullerian Professor of Chemistry at the Royal Institution of Great Britain, London
1905		* Dewar, James Campbell, C.A., 27 Douglas Crescent
1906		* Dewar, Thomas William, M.D., F.R.C.P., Kincairn, Dunblane 145
1904		Dickinson, Walter George Burnett, F.R.C.V.S., Boston, Lincolnshire
1884		Dickson, the Right Hon. Charles Scott, K.C., LL.D., 22 Moray Place
1888	C.	* Dickson, Henry Newton, M.A., D.Sc., The Lawn, Upper Redlands Road, Reading
1876	C.	Dickson, J. D. Hamilton, M.A., Fellow and Tutor, St Peter's College, Cambridge
1885	C.	Dixon, James Main, M.A., Litt. Hum. Doctor, Professor of English, University of Southern California, Wesley Avenue, Los Angelos, California, United States 150
1897		* Dobbie, James Bell, F.Z.S., 12 South Inverleith Avenue
1904	C.	* Dobbie, James Johnston, M.A., D.Sc., LL.D., F.R.S., 4 Vicarage Gate, Kensington, London, W.
1881	C.	Dobbin, Leonard, Ph.D., Lecturer on Chemistry in the University of Edinburgh, 6 Wilton Road

Date of Election.		
1867	C.	Donaldson, Sir James, M.A., LL.D., Principal of the University of St Andrews, St Andrews
1896		* Donaldson, William, M.A., Viewpark House, Spylaw Road 155
1905		* Donaldson, Rev. Wm. Galloway, The Manse, Forfar
1882		Dott, David B., F.I.C., Memb. Pharm. Soc., Ravenslea, Musselburgh
1901		* Douglas, Carstairs Cumming, M.D., D.Sc., Professor of Medical Jurisprudence and Hygiene, Anderson's College, Glasgow, 2 Royal Crescent, Glasgow
1866		Douglas, David, 22 Drummond Place
1910		* Douglas, Loudon MacQueen, Author and Lecturer, 3 Lauder Road, Edinburgh 160
1908	C.	Drinkwater, Harry, M.D., M.R.C.S. (Eng.), Grosvenor Lodge, Wrexham, North Wales
1901		* Drinkwater, Thomas W., L.R.C.P.E., L.R.C.S.E., Chemical Laboratory, Surgeons' Hall
1878		Duncanson, J. J. Kirk, M.D., F.R.C.P.E., 22 Drumsheugh Gardens
1904		* Dunlop, William Brown, M.A., 7 Carlton Street
1903		* Dunstan, John, M.R.C.V.S., 1 Dean Terrace, Liskeard, Cornwall 165
1892	C.	Dunstan, M. J. R., M.A., F.I.C., F.C.S., Principal, South-Eastern Agricultural College, Wye, Kent
1899		* Duthie, George, M.A., Inspector-General of Education, Salisbury, Rhodesia
1906	C.	* Dyson, Frank Watson, M.A., F.R.S., Astronomer Royal, Royal Observatory, Greenwich
1893		Edington, Alexander, M.D., 20 Kilmaurs Road
1904		* Edwards, John, 4 Great Western Terrace, Kelvinside, Glasgow 170
1904		* Elder, William, M.D., F.R.C.P.E., 4 John's Place, Leith
1875		Elliot, Daniel G., American Museum of Natural History, Central Park West, New York City, New York, U.S.A.
1906	C.	* Ellis, David, D.Sc., Ph.D., Lecturer in Botany and Bacteriology, Glasgow and West of Scotland Technical College, Glasgow
1897	C.	* Erskine-Murray, James Robert, D.Sc., 77 Kingsfield Road, Watford, Herts
1884		Evans, William, F.F.A., 38 Morningside Park 175
1879	C. N.	Ewart, James Cossar, M.D., F.R.C.S.E., F.R.S., F.L.S., Regius Professor of Natural History, University of Edinburgh (VICE-PRESIDENT), Craigiebield, Penicuik, Midlothian
1902		* Ewen, J. T., B.Sc., Memb. Inst. Mech. E., H.M.I.S., 104 King's Gate, Aberdeen
1878	C.	Ewing, James Alfred, C.B., M.A., B.Sc., LL.D., Memb. Inst. C.E., F.R.S., Director of Naval Education, Admiralty, Froghole, Edenbridge, Kent
1900	C.	Eyre, John W. H., M.D., M.S. (Dunelm), D.P.H. (Camb.), Guy's Hospital (Bacteriological Department), London
1910		* Fairgrieve, Mungo M'Callum, M.A. (Glasg.), M.A. (Cambridge), Master at the Edinburgh Academy, 67 Great King Street, Edinburgh 180
1875		Fairley, Thomas, Lecturer on Chemistry, 8 Newton Grove, Leeds
1907	C.	Falconer, John Downie, M.A., D.Sc., F.G.S., Director, Mineral Survey of Northern Nigeria, The Limes, Little Berkhampstead, Hertford, and Imperial Institute, London
1888	C.	* Fawsitt, Charles A., 9 Foremount Terrace, Dowanhill, Glasgow
1883	C.	Felkin, Robert W., M.D., F.R.G.S., Fellow of the Anthropological Society of Berlin, 47 Bassett Road, North Kensington, London, W.
1899		* Fergus, Andrew Freeland, M.D., 22 Blythswood Square, Glasgow 185
1907		* Fergus, Edward Oswald, 12 Clairmont Gardens, Glasgow
1904		* Ferguson, James Haig, M.D., F.R.C.P.E., F.R.C.S.E., 7 Coates Crescent
1888		* Ferguson, John, M.A., LL.D., Professor of Chemistry in the University of Glasgow
1868	C.	Ferguson, Robert M., Ph.D., LL.D. (SOCIETY'S REPRESENTATIVE ON GEORGE HERIOT'S TRUST), 5 Douglas Gardens
1898		* Findlay, John R., M.A. Oxon., 27 Drumsheugh Gardens 190
1899		* Finlay, David W., B.A., M.D., LL.D., F.R.C.P., D.P.H., Professor of Medicine in the University of Aberdeen, Honorary Physician to His Majesty in Scotland, 2 Queen's Terrace, Aberdeen
1906		* Fleming, Robert Alexander, M.D., F.R.C.P.E., Assistant Physician, Royal Infirmary, 10 Chester Street
1900	C. N.	* Flett, John S., M.A., D.Sc., Geological Survey Office, 28 Jermyn Street, London
1880		Flint, Robert, D.D., Corresponding Member of the Institute of France, Corresponding Member of the Royal Academy of Sciences of Palermo, Emeritus Professor of Divinity in the University of Edinburgh, 5 Royal Terrace
1872	C.	Forbes, Professor George, M.A., Memb. Inst. C.E., Memb. Inst. E.E., F.R.S., F.R.A.S., 11 Little College Street, Westminster, S.W. 195
1904		Forbes, Norman Hay, F.R.C.S.E., L.R.C.P. Lond., M.R.C.S. Eng., Corres. Memb. Soc. d'Hydrologie médicale de Paris, Druminnor, Church Stretton, Salop

Date of Election.		
1892		* Ford, John Simpson, F.C.S., 4 Nile Grove
1910		* Fraser, Alexander, Actuary, 17 Eildon Street, Edinburgh
1858		Fraser, A. Campbell, Fellow of the British Academy, Hon. D.C.L. Oxford, LL.D., Litt. D., Emeritus Professor of Logic and Metaphysics in the University of Edinburgh, Gorton House, Hawthornden
1896		* Fraser, John, M.B., F.R.C.P.E., one of H.M. Commissioners in Lunacy for Scotland, 13 Heriot Row 200
1867	C. K. B.	Fraser, Sir Thomas R., M.D., LL.D., F.R.C.P.E., F.R.S., Professor of Materia Medica in the University of Edinburgh, Honorary Physician to the King in Scotland, 13 Drumsheugh Gardens
1891		* Fullarton, J. H., M.A., D.Sc., Brodick, Arran
1891		* Fulton, T. Wemyss, M.D., Scientific Superintendent, Scottish Fishery Board, 41 Queen's Road, Aberdeen
1907		* Galbraith, Alexander, Assistant Superintendent Engineer, Cunard Line, Liverpool, 93 Trinity Road, Bootle, Liverpool
1888	C.	* Galt, Alexander, D.Sc., Keeper of the Technological Department, Royal Scottish Museum, Edinburgh 205
1901		Ganguli, Sanjiban, M.A., Principal, Maharaja's College, and Director of Public Instruction, Jaipur State, Jaipur, India
1899		Gatehouse, T. E., Assoc. Memb. Inst. C.E., Memb. Inst. M.E., Memb. Inst. E.E., Tulse Hill Lodge, 100 Tulse Hill, London
1867		Gayner, Charles, M.D., F.L.S.
1900		Gayton, William, M.D., M.R.C.P.E., Ravensworth, Regent's Park Road, Finchley, London, N.
1909	C.	* Geddes, Auckland C., M.D., Professor of Anatomy, Royal College of Surgeons in Ireland, Dublin 210
1880	C.	Geddes, Patrick, Professor of Botany in University College, Dundee, and Lecturer on Zoology, Ramsay Garden, University Hall, Edinburgh
1861	C. B.	Geikie, Sir Archibald, K.C.B., D.C.L. Oxf., D.Sc. Camb. Dub., LL.D. St And., Glasg., Aberdeen, Edin., Ph.D. Upsala, Pres. R.S., Pres. G.S., Foreign Member of the Reale Accad. Lincei, Rome, of the National Acad. of the United States, of the Academies of Stockholm, Christiania, Göttingen, Corresponding Member of the Institute of France and of the Academies of Berlin, Vienna, Munich, Turin, Belgium, Philadelphia, New York, etc., Shepherd's Down, Haslemere, Surrey
1871	C. B.	Geikie, James, LL.D., D.C.L., F.R.S., F.G.S., Professor of Geology in the University of Edinburgh, Kilmorie, Colinton Road
1909		* Gentle, William, B.Sc., 2 Blackwood Crescent, Edinburgh
1910	C.	* Gibb, David, M.A., B.Sc., Lecturer in Mathematics, Edinburgh University, Sunnybank, Durie Street, Leven 215
1910		* Gibson, Charles Robert, Mansewood, by Pollokshaws, N.B.
1881	C.	Gibson, George Alexander, D.Sc., M.D., LL.D., F.R.C.P.E., 3 Drumsheugh Gardens
1890		* Gibson, George A., M.A., LL.D., Professor of Mathematics in the University of Glasgow, 10 The University, Glasgow
1877	C.	Gibson, John, Ph.D., Professor of Chemistry in the Heriot-Watt College, 16 Woodhall Terrace, Juniper Green
1900		Gilchrist, Douglas A., B.Sc., Professor of Agriculture and Rural Economy, Armstrong College, Newcastle-upon-Tyne 220
1880		Gilruth, George Ritchie, Surgeon, 53 Northumberland Street
1907		Gilruth, John Anderson, M.R.C.V.S., Professor, University, Melbourne, Australia
1909		* Gladstone, Hugh Steuart, M.A., M.B.O.U., F.Z.S., Capenoch, Thornhill, Dumfriesshire
1898		* Glaister, John, M.D., F.F.P.S. Glasgow, D.P.H. Camb., Professor of Forensic Medicine in the University of Glasgow, 3 Newton Place, Glasgow
1910		Goodall, Joseph Strickland, M.B. (Lond.), M.S.A. (Eng.), Lecturer on Physiology, Middlesex Hospital, London, Annandale Lodge, Vanbrugh Park, Blackheath, London, S.E. 225
1901		Goodwillie, James, M.A., B.Sc., Liberton, Edinburgh
1899		* Goodwin, Thomas S., M.B., C.M., F.C.S., 1 Heron Terrace, St Margaret's, Middlesex
1897		Gordon-Munn, John Gordon, M.D., Heigham Hall, Norwich
1891		* Graham, Richard D., 11 Strathearn Road
1898	C.	* Gray, Albert A., M.D., 14 Newton Terrace, Glasgow 230
1883		Gray, Andrew, M.A., LL.D., F.R.S., Professor of Natural Philosophy in the University of Glasgow
1910		Gray, Bruce M'Gregor, C.E., Assoc. Memb. Inst. C.E., Westbourne Grove, Selby, Yorkshire

Date of Election.			
1909	C.	* Gray, James Gordon, B.Sc., Lecturer in Physics in the University of Glasgow, 11 The University, Glasgow	
1910		* Green, Charles Edward, Publisher, Gracemount House, Liberton	
1886		Greenfield, W. S., M.D., F.R.C.P.E., Professor of General Pathology in the University of Edinburgh, 7 Heriot Row	235
1897		Greenlees, Thomas Duncan, M.D. Edin., Amana, Tulse Hill, London	
1905	C.	* Gregory, John Walter, D.Sc., F.R.S., Professor of Geology in the University of Glasgow, 4 Park Quadrant, Glasgow	
1906		Greig, Edward David Wilson, M.D., B.Sc., Captain, H.M.'s Indian Medical Service, Byculla Club, Bombay, India	
1905		Greig, Robert Blyth, F.Z.S., Fordyce Lecturer in Agriculture, University of Aberdeen, Torloisk, Cults, Aberdeenshire	
1910		* Grimshaw, Percy Hall, Assistant Keeper, Natural History Department, The Royal Scottish Museum, 49 Comiston Drive, Edinburgh	240
1899		* Guest, Edward Graham, M.A., B.Sc., 5 Newbattle Terrace	
1907	C.	* Gulliver, Gilbert Henry, B.Sc., A.M.I. Mech. E., Lecturer in Experimental Engineering in the University of Edinburgh, 10 Stanley Street, Portobello	
1888	C.	Guppy, Henry Brougham, M.B., Rosario, Salcombe, Devon	
1910	B. C.	Gwynne-Vaughan, D. T., F.L.S., Professor of Botany, Queen's University, Belfast, The Cottage, Balmoral, Belfast	
1905	B. C.	* Halm, Jacob E., Ph.D., Chief Assistant Astronomer, Royal Observatory, Cape Town, Cape of Good Hope	245
1899		Hamilton, Allan M'Lane, M.D., 44 East Twenty-ninth Street, New York	
1876	C.	Hannay, J. Ballantyne, Cove Castle, Loch Long	
1896		* Harris, David, Fellow of the Statistical Society, Lyncombe Rise, Prior Park Road, Bath	
1896	C.	* Harris, David Fraser, B.Sc. (Lond.), M.D., F.S.A. Scot., The Physiological Department, The University, Birmingham	
1888	C.	* Hart, D. Berry, M.D., F.R.C.P.E., 5 Randolph Cliff	250
1869		Hartley, Sir Charles A., K.C.M.G., Memb. Inst. C.E., 26 Pall Mall, London	
1877	C.	Hartley, W. N., D.Sc., F.R.S., F.I.C., Professor of Chemistry, Royal College of Science for Ireland, Dublin	
1881		Harvie-Brown, J. A., of Quarter, F.Z.S., Dunipace House, Larbert, Stirlingshire	
1880	C.	Haycraft, J. Berry, M.D., D.Sc., Professor of Physiology in the University College of South Wales and Monmouthshire, Cardiff	
1892	C.	* Heath, Thomas, B.A., Assistant Astronomer, Royal Observatory, Edinburgh	255
1893		Hehir, Patrick, M.D., F.R.C.S.E., M.R.C.S.L., L.R.C.P.E., Surgeon-Captain, Indian Medical Service, Principal Medical Officer, H.H. the Nizam's Army, Hyderabad, Deccan, India	
1890	C.	Helme, T. Arthur, M.D., M.R.C.P.L., M.R.C.S., 3 St Peter's Square, Manchester	
1900		Henderson, John, D.Sc., Assoc. Inst. E.E., Kinnoul, Warwick's Bench Road, Guildford, Surrey	
1908		* Henderson, William Dawson, M.A., B.Sc., Ph.D., Lecturer, Zoological Laboratories, University, Manchester	
1890	C.	Hepburn, David, M.D., Professor of Anatomy in the University College of South Wales and Monmouthshire, Cardiff	260
1881	C. N.	Herdman, W. A., D.Sc., F.R.S., Pres.L.S., Professor of Natural History in the University of Liverpool, Croxteth Lodge, Ullet Road, Liverpool	
1908		* Hewat, Archibald, F.F.A., F.I.A., 13 Eton Terrace	
1894		Hill, Alfred, M.D., M.R.C.S., F.I.C., Valentine Mount, Freshwater Bay, Isle of Wight	
1902		* Hinxman, Lionel W., B.A., Geological Survey Office, 33 George Square	
1904		Hobday, Frederick T. G., F.R.C.V.S., 165 Church Street, London, W.	265
1885		Hodgkinson, W. R., Ph.D., F.I.C., F.C.S., Professor of Chemistry and Physics at the Royal Military Academy and Royal Artillery College, Woolwich, 89 Shooter's Hill Road, Blackheath, Kent	
1881	C. N.	Horne, John, LL.D., F.R.S., F.G.S., Director of the Geological Survey of Scotland (VICE-PRESIDENT), 33 George Square, Edinburgh	
1896		Horne, J. Fletcher, M.D., F.R.C.S.E., The Poplars, Barnsley	
1904		* Horsburgh, Ellice Martin, M.A., B.Sc., Lecturer in Technical Mathematics, University of Edinburgh, 11 Granville Terrace	
1897		Houston, Alex. Cruikshanks, M.B., C.M., D.Sc., 19 Fairhazel Gardens, South Hampstead, London, N.W.	270
1893		Howden, Robert, M.A., M.B., C.M., Professor of Anatomy in the University of Durham, 14 Burdon Terrace, Newcastle-on-Tyne	
1899		Howie, W. Lamond, F.C.S., 26 Neville Court, Abbey Road, Regent's Park, London, N.W.	

Date of Election.		
1883	C.	Hoyle, William Evans, M.A., D.Sc., M.R.C.S., Crowland, Llandaff, Wales
1910		Hume, William Fraser, D.Sc. (Lond.), Director, Geological Survey of Egypt, Helwân, Egypt
1886		Hunt, Rev. H. G. Bonavia, Mus.D. Dub., Mus.B. Oxon., The Vicarage, Burgess Hill, Sussex 275
1887	C.	* Hunter, James, F.R.C.S.E., F.R.A.S., Rosetta, Liberton, Midlothian
1887	C.	* Hunter, William, M.D., M.R.C.P.L. and E., M.R.C.S., 54 Harley Street, London
1908		Hyslop, Theophilus Bulkeley, M.D., M.R.C.P.E., Senior Physician, Bethlem Royal Hospital, London, S.E.
1882	C.	Inglis, J. W., Memb. Inst. C.E., 26 Pitt Street
1906		* Innes, Alexander Taylor, LL.D., M.A., Advocate, 48 Morningside Park 280
1904	C.	Innes, R. T. A., Director, Government Observatory, Johannesburg, Transvaal
1904		* Ireland, Alexander Scott, S.S.C., 2 Buckingham Terrace
1875		Jack, William, M.A., LL.D., Professor of Mathematics in the University of Glasgow
1894		Jackson, Sir John, LL.D., 48 Belgrave Square, London
1889		* James, Alexander, M.D., F.R.C.P.E., 14 Randolph Crescent 285
1882		Jamieson, Prof. A., Memb. Inst. C.E., 16 Rosslyn Terrace, Kelvinside, Glasgow
1901		* Jardine, Robert, M.D., M.R.C.S. Eng., F.F.P. and S. Glas., 20 Royal Crescent, Glasgow
1906	C.	* Jehu, Thomas James, M.A., M.D., F.G.S., Lecturer in Geology, University of St Andrews, Strathmartine, Hepburn Gardens, St Andrews
1900		* Jerdan, David Smiles, M.A., D.Sc., Ph.D., Temora, Colinton, Midlothian
1895		Johnston, Lieut.-Col. Henry Halcro, C.B., R.A.M.S., D.Sc., M.D., F.L.S., Orphir House, Kirkwall, Orkney 290
1903	C.	* Johnston, Thomas Nicol, M.B., C.M., Pogbie, Upper Keith, East Lothian
1902		Johnstone, George, Lieut. R.N.R., late Marine Superintendent, British India Steam Navigation Co., 26 Comiston Drive
1874		Jones, Francis, M.Sc., Lecturer on Chemistry, Beaufort House, Alexandra Park, Manchester
1905		Jones, George William, M.A., B.Sc., LL.B., Scottish Tutorial Institute, Edinburgh and Glasgow, 25 North Bridge, Coraldene, Kirk Brae, Liberton 295
1888		Jones, John Alfred, Memb. Inst. C.E., Fellow of the University of Madras, Sanitary Engineer to the Government of Madras, c/o Messrs Parry & Co., 70 Gracechurch Street, London
1907		* Kemp, John, M.A., Head Master, High School, Kelso
1909		Kenwood, Henry Richard, M.B., Chadwick Professor of Hygiene in the University of London, 150 Bethune Road, Amherst Park, London, N.
1908		* Kerr, Andrew William, F.S.A. Scot., Royal Bank House, St Andrew Square
1903	C. N.	* Kerr, John Graham, M.A., Professor of Zoology in the University of Glasgow
1891		Kerr, Joshua Law, M.D., Biddenden Hall, Cranbrook, Kent 300
1908		Kidd, Walter Aubrey, M.D., F.Z.S., 12 Montpelier Row, Blackheath, London
1886	C. N.	Kidston, Robert, LL.D., F.R.S., F.G.S. (SECRETARY), 12 Clarendon Place, Stirling
1907		* King, Archibald, M.A., B.Sc., Rector of the Academy, Castle-Douglas, Hazeldene, Castle-Douglas, Kirkcudbrightshire
1877		King, Sir James, of Campsie, Bart., LL.D., 115 Wellington Street, Glasgow
1880		King, W. F., Lonend, Russell Place, Trinity 305
1883		Kinnear, the Right Hon. Lord, one of the Senators of the College of Justice, 2 Moray Place
1878		Kintore, the Right Hon. the Earl of, M.A. Cantab., LL.D., Cambridge, Aberdeen and Adelaide, Keith Hall, Inverurie, Aberdeenshire
1901		* Knight, the Rev. G. A. Frank, M.A., St Leonard's United Free Church, Perth
1907		* Knight, James, M.A., D.Sc., F.C.S., F.G.S., Head Master, St James' School, Glasgow, The Shieling, Uddingston, by Glasgow
1880	C. K.	Knott, C. G., D.Sc., Lecturer on Applied Mathematics in the University of Edinburgh (late Professor of Physics, Imperial University, Japan) (SECRETARY), 42 Upper Gray Street, Edinburgh 310
1886		Laing, Rev. George P., 17 Buckingham Terrace
1907		* Lanchester, William Forster, M.A., 19 Fernshaw Road, Chelsea, S.W.
1878	C.	Lang, P. R. Scott, M.A., B.Sc., Professor of Mathematics, University of St Andrews
1910		* Lauder, Alexander, D.Sc., Lecturer in Agricultural Chemistry, Edinburgh and East of Scotland College of Agriculture, 13 George Square, Edinburgh
1885	C.	Laurie, A. P., M.A., D.Sc., Principal of the Heriot-Watt College, Edinburgh 315
1894	C.	* Laurie, Malcolm, B.A., D.Sc., F.L.S., Royal College of Surgeons, Edinburgh

Date of Election			
1910		* Lawson, A. Anstruther, D.Sc., Lecturer in Botany, University of Glasgow, 16 Derby Crescent, Kelvinside, Glasgow	
1905		* Lawson, David, M.A., M.D., L.R.C.P. and S.E., Druimdarroch, Banchory, Kincardineshire	
1903		* Leighton, Gerald Rowley, M.D., Sunnyside, Russell Place	
1910	C.	* Lee, Gabriel W., Palæontologist, Geological Survey of Scotland, 33 George Square, Edinburgh	320
1874	C. K.	Letts, E. A., Ph.D., F.I.C., F.C.S., Professor of Chemistry, Queen's College, Belfast	
1910		Levie, Alexander, F.R.C.V.S., Veterinary Surgeon, Lecturer on Veterinary Science, 18 Park Road, Melton Mowbray, Leicestershire	
1905		* Lightbody, Forrest Hay, 56 Queen Street	
1889		* Lindsay, Rev. James, M.A., D.D., F.R.S.L., B.Sc., F.G.S., M.R.A.S., Corresponding Member of the Royal Academy of Sciences, Letters and Arts, of Padua, Associate of the Philosophical Society of Louvain, Annick Lodge, Irvine	
1870	C. B.	Lister, the Right Hon. Lord, O.M., P.C., M.D., F.R.C.S.L., F.R.C.S.E., LL.D., D.C.L., F.R.S., Foreign Associate of the Institute of France, Emeritus Professor of Clinical Surgery, King's College, Surgeon Extraordinary to the King, 12 Park Crescent, Portland Place, London	325
1903		Liston, William Glen, M.D., Captain, Indian Medical Service, c/o Grindlay, Groom & Co., Bombay, India	
1903		* Littlejohn, Henry Harvey, M.A., M.B., B.Sc., F.R.C.S.E., Professor of Forensic Medicine in the University of Edinburgh, 11 Rutland Street	
1898		* Lothian, Alexander Veitch, M.A., B.Sc., Glendoune, Manse Road, Bearsden, Glasgow	
1884		Low, George M., Actuary, 11 Moray Place	
1888		* Lowe, D. F., M.A., LL.D., late Head Master of Heriot's Hospital School, Lauriston, 19 George Square	330
1904		* Lowson, Charles Stewart, M.B., C.M., Captain, Indian Medical Service, c/o Messrs Thomas Cook & Son, Bombay, India.	
1900		Lusk, Graham, Ph.D., M.A., Professor of Physiology, University and Bellevue Medical College, N.Y.	
1894		* Mabbott, Walter John, M.A., Rector of County High School, Duns, Berwickshire	
1887		M'Aldowie, Alexander M., M.D., Glengarriff, Leckhampton, Cheltenham	
1907		MacAlister, Donald Alexander, A.R.S.M., F.G.S., 3 Motcomb Street, Belgrave Square, London	335
1891		Macallan, John, F.I.C., 3 Rutland Terrace, Clontarf, Dublin	
1888	C.	M'Arthur, John, F.C.S., 262 Trinity Road, Wandsworth, London, S.W.	
1883		M'Bride, P., M.D., F.R.C.P.E., 16 Chester Street	
1903		* M'Cormick, W. S., M.A., LL.D., 13 Douglas Crescent	
1899		* M'Cubbin, James, B.A., Rector of the Burgh Academy, Kilsyth	340
1905		* Macdonald, Hector Munro, M.A., F.R.S., Professor of Mathematics, University of Aberdeen, 52 College Bounds, Aberdeen	
1894		* Macdonald, James, Secretary of the Highland and Agricultural Society of Scotland, 2 Garscube Terrace	
1897	C.	* Macdonald, James A., M.A., B.Sc., H.M. Inspector of Schools, Glengarry, Dingwall	
1904		* Macdonald, John A., M.A., B.Sc., High School, Stellenbosch, Cape Colony	
1886		Macdonald, the Right Hon. Sir J. H. A., K.C.B., K.C., LL.D., F.R.S., M.I.E.E., Lord Justice-Clerk, and Lord President of the Second Division of the Court of Session, 15 Abercromby Place	345
1904		Macdonald, William, B.Sc., M.Sc., Agriculturist, Editor *Transvaal Agricultural Journal*, Department of Agriculture, Pretoria Club, Pretoria, Transvaal	
1886		Macdonald, William J., M.A., 15 Comiston Drive	
1901	C.	* MacDougal, R. Stewart, M.A., D.Sc., 9 Dryden Place	
1910		Macewen, Hugh Allan, M.B., Ch.B., D.P.H. (Lond. and Camb.), Assistant Medical Officer of Health for Cumberland, The Cottage, Stanwix, Carlisle	
1888	C.	* M'Fadyean, Sir John, M.B., B.Sc., LL.D., Principal, and Professor of Comparative Pathology in the Royal Veterinary College, Camden Town, London	350
1878	C.	Macfarlane, Alexander, M.A., D.Sc., LL.D., Lecturer in Physics in Lehigh University, Pennsylvania, Gowrie Grove, Chatham, Ontario, Canada	
1885	C.	Macfarlane, J. M., D.Sc., Professor of Botany and Director of the Botanic Garden, University of Pennsylvania, Philadelphia, Pennsylvania, U.S.A.	
1897		* MacGillivray, Angus, C.M., M.D., South Tay Street, Dundee	
1878		M'Gowan, George, F.I.C., Ph.D., 21 Montpelier Road, Ealing, Middlesex	

Date of Election.			
1886		MacGregor, the Very Rev. James, D.D., 3 Eton Terrace	355
1880	C.	MacGregor, James Gordon, M.A., D.Sc., LL.D., F.R.S., Professor of Natural Philosophy in the University of Edinburgh, 24 Dalrymple Crescent	
1903		* M'Intosh, D. C., M.A., B.Sc., 3 Glenisla Gardens	
1869	C. N.	M'Intosh, William Carmichael, M.D., LL.D., F.R.S., F.L.S., Professor of Natural History in the University of St Andrews, 2 Abbotsford Crescent, St Andrews	
1895	C.	* Macintyre, John, M.D., 179 Bath Street, Glasgow	
1882		Mackay, John Sturgeon, M.A., LL.D., late Mathematical Master in the Edinburgh Academy, 69 Northumberland Street	360
1873	C. B.	M'Kendrick, John G., M.D., F.R.C.P.E., LL.D., F.R.S., Emeritus Professor of Physiology in the University of Glasgow, Maxieburn, Stonehaven	
1900	C.	* M'Kendrick, John Souttar, M.D., F.F.P.S.G., 2 Buckingham Terrace, Glasgow	
1910	C.	* MacKenzie, Alister Thomas, M.A., M.D., D.P.H., Research Fellow of the University of Edinburgh, Alladale, Alness, Ross-shire	
1894		* Mackenzie, Robert, M.D., Napier, Nairn	
1898		** Mackenzie, W. Cossar, D.Sc., Alderston, Haddington	365
1904		* Mackenzie, W. Leslie, M.A., M.D., D.P.H., Medical Member of the Local Government Board for Scotland, 1 Stirling Road, Trinity	
1905		Mackenzie, William Colin, M.D., F.R.C.S., Demonstrator of Anatomy in the University of Melbourne, Elizabeth Street North, Melbourne, Victoria	
1910		* MacKinnon, James, M.A., Ph.D., Professor of Ecclesiastical History, Edinburgh University, 12 Lygon Road, Edinburgh	
1904		* Mackintosh, Donald James, M.V.O., M.B., Supt. Western Infirmary, Glasgow	
1869	C.	Maclagan, R. C., M.D., F.R.C.P.E., 5 Coates Crescent	370
1899		Maclean, Ewan John, M.D., M.R.C.P. Lond., 12 Park Place, Cardiff	
1888	C.	* Maclean, Magnus, M.A., D.Sc., Memb. Inst. E.E., Professor of Electrical Engineering in the Glasgow and West of Scotland Technical College, 51 Kerrsland Terrace, Hillhead, Glasgow	
1876		Macleod, Very Rev. Norman, D.D., 74 Murrayfield Gardens	
1876		Macmillan, John, M.A., D.Sc., M.B., C.M., F.R.C.P.E., F.R.C.S.E., 48 George Square	
1893		* M'Murtrie, Very Rev. John, M.A., D.D., 13 Inverleith Place	375
1906		* Macnair, Duncan Scott, Ph.D., B.Sc., H.M. Inspector of Schools, 67 Braid Avenue	
1907		* Macnair, Peter, Curator of the Natural History Collections in the Glasgow Museums, Kelvingrove Museum, Glasgow	
1898	C.	Mahâlanobis, S. C., B.Sc., Professor of Physiology, Presidency College, Calcutta, India	
1908		Mallik, Devendranath, B.A., B.Sc., Professor of Physics and Mathematics, Patna College, Bankipur, Bengal, India	
1880	C.	Marsden, R. Sydney, M.D., C.M., D.Sc., M.R.I.A., F.I.C, F.C.S., Rowallan House, Cearns Road, and Town Hall, Birkenhead	380
1909	C.	* Marshall, C. R., M.D., M.A., Professor of Materia Medica and Therapeutics, University of St Andrews, Amdreen, Newport, Fife	
1882	C.	Marshall, D. H., M.A., Professor of Physics in Queen's University and College, Kingston, Ontario, Canada	
1901	C.	* Marshall, F. H. A., M.A., D.Sc., Lecturer on Agricultural Physiology in the University of Cambridge, Christ's College, Cambridge	
1888	C. K.	* Marshall, Hugh, D.Sc., F.R.S., Professor of Chemistry in University College, Dundee	
1892		* Martin, Francis John, W.S., 17 Rothesay Place	385
1903		Martin, Nicholas Henry, F.L.S., F.C.S., Ravenswood, Low Fell, Gateshead	
1885	C.	Masson, Orme, D.Sc., F.R.S., Professor of Chemistry in the University of Melbourne	
1898	C. B.	* Masterman, Arthur Thomas, M.A., D.Sc., Inspector of Fisheries, Board of Agriculture, Whitehall, London	
1906		* Mathieson, Robert, F.C.S., Rillbank, Innerleithen	
1902		Matthews, Ernest Romney, Assoc. Memb. Inst. C.E., F.G.S., Bessemer Prizeman, Soc. Engineers, Bridlington, Yorkshire	390
1901	C.	* Menzies, Alan W. C., M.A., B.Sc., F.C.S., Kent Chemical Laboratory, University, Chicago, U.S.A.	
1888		* Methven, Cathcart W., Memb. Inst. C.E., F.R.I.B.A., Durban, Natal, S. Africa	
1902	C.	Metzler, William H., A.B., Ph.D., Corresponding Fellow of the Royal Society of Canada, Professor of Mathematics, Syracuse University, Syracuse, N.Y.	

Date of Election.			
1885	C B.	Mill, Hugh Robert, D.Sc., LL.D., 62 Camden Square, London	
1908		* Miller, Alexander Cameron, M.D., F.S.A. Scot., Craig Linnhe, Fort-William, Inverness-shire	395
1910		* Miller, John, M.A., D.Sc., Professor of Mathematics, Glasgow and West of Scotland Technical College, 2 Northbank Terrace, North Kelvinside, Glasgow	
1905		* Miller-Milne, C. H.. M.A., Rector, The High School, Arbroath, 8 Dalhousie Place, Arbroath	
1909		Mills, Bernard Langley, M.D., F.R.C.S.E., M.R.C.S.L., D.P.H., Lt.-Col. R.A.M.C., late Army Specialist in Hygiene, 84 Grange Crescent, Sharrow, Sheffield	
1905		* Milne, Archibald, M.A., B.Sc., Lecturer on Mathematics and Science, Edinburgh Provincial Training College, 108 Comiston Drive	
1904	C.	* Milne, James Robert, D.Sc., 11 Melville Crescent	400
1886		Milne, William, M.A., B.Sc., 70 Beechgrove Terrace, Aberdeen	
1899		* Milroy, T. H., M.D., B.Sc., Professor of Physiology in Queen's College, Belfast, Thornlee, Malone Park, Belfast	
1889	C.	Mitchell, A. Crichton, D.Sc., Director of Public Instruction in Travancore, India	
1897		* Mitchell, George Arthur, M.A., 9 Lowther Terrace, Kelvinside, Glasgow	
1900		* Mitchell, James, M.A., B.Sc., 4 Manse Street, Kilmarnock	405
1899		* Mitchell-Thomson, Sir Mitchell, Bart., 6 Charlotte Square	
1906	C.	Moffat, Rev. Alexander, M.A., B.Sc., Professor of Physical Science, Christian College, Madras, India	
1890	C.	Mond, R. L., M.A. Cantab., F.C.S., The Poplars, 20 Avenue Road, Regent's Park, London	
1887	C.	Moos, N. A. F., L.C.E., B.Sc., Professor of Physics, Elphinstone College, and Director of the Government Observatory, Colaba, Bombay	
1896		* Morgan, Alexander, M.A., D.Sc., Principal, Edinburgh Provincial Training College, 1 Midmar Gardens	410
1892	C.	Morrison, J. T., M.A., B.Sc., Professor of Physics and Chemistry, Victoria College, Stellenbosch, Cape Colony	
1901		Moses, O. St John, I.M.S., M.D., D.Sc., F.R.C.S., Captain, Professor of Medical Jurisprudence, 26 Park Street, Wellesley, Calcutta, India	
1892	C.	Mossman, Robert C., Superintendent of Publications, Argentine Meteorological Office, Cuyo 947, Buenos Ayres	
1874	C. K.	Muir, Thomas, C.M.G., M.A., LL.D., F.R.S., Superintendent-General of Education for Cape Colony, Education Office, Cape Town, and Mowbray Hall, Rosebank, Cape Colony	
1888	C.	* Muirhead, George, Commissioner to His Grace the Duke of Richmond and Gordon, K.G., Speybank, Fochabers	415
1907		Muirhead, James M. P., Bredisholm, Claremont, near Cape Town, Cape Colony	
1887		Mukhopâdhyay, Asûtosh, M.A., LL.D., F.R.A.S., M.R.I.A., Professor of Mathematics at the Indian Association for the Cultivation of Science, 77 Russa Road North, Bhowanipore, Calcutta	
1891	C.	* Munro. Robert, M.A., M.D., LL.D., Hon. Memb. R.I.A., Hon. Memb. Royal Society of Antiquaries of Ireland, Elmbank, Largs, Ayrshire	
1896		* Murray, Alfred A., M.A., LL.B., 20 Warriston Crescent	
1907	C.	* Murray, James, Woodhouse, Whitchurch Lane, Edgeware, Middlesex, England	420
1877	C. B. N.	Murray, Sir John, K.C.B., LL.D., D.C.L., Ph.D., D.Sc., F.R.S., Member of the Prussian Order *Pour le Mérite*, Director of the Challenger Expedition Publications. Office, Villa Medusa, Boswell Road. House, Challenger Lodge, Wardie, and United Service Club	
1907		* Musgrove, James, M.D., F.R.C.S. Edin. and Eng., Bute Professor of Anatomy, University of St Andrews, 56 South Street, St Andrews	
1887		Muter, John, M.A., F.C.S., South London Central Public Laboratory, 325 Kennington Road, London	
1902		Mylne, Rev. R. S., M.A., B.C.L. Oxford, F.S.A. Lond., Great Amwell, Herts	
1888		Napier, A. D. Leith, M.D., C.M., M.R.C.P.L., 28 Angas Street, Adelaide, S. Australia	425
1897		Nash, Alfred George, C.E., B.Sc., Engineer, Department of Public Works, Jamaica, Belretiro, Mandeville, Jamaica, W.I.	
1906		* Newington, Frank A., Memb. Inst. C.E., Memb. Inst. E.E., 4 Osborne Terrace	
1898		Newman, George, M.D., D.P.H. Cambridge, Lecturer on Preventive Medicine, St Bartholomew's Hospital, University of London: Dene, Hatch End, Middlesex	

Date of Election.			
1884		Nicholson, J. Shield, M.A., D.Sc., Professor of Political Economy in the University of Edinburgh, 3 Belford Park	
1880	C.	Nicol, W. W. J., M.A., D.Sc., 15 Blacket Place	430
1878		Norris, Richard, M.D., M.R.C.S. Eng., 3 Walsall Road, Birchfield, Birmingham	
1906		* O'Connor, Henry, C.E., Assoc. Memb. Inst. C.E., 1 Drummond Place	
1888		* Ogilvie, F. Grant, C.B., M.A., B.Sc., Principal Assistant Secretary for Science, Art, and Technology, Board of Education, South Kensington, London	
1888		* Oliphant, James, M.A., 11 Heathfield Park, Willesden, London	
1886		Oliver, James, M.D., F.L.S., Physician to the London Hospital for Women, 18 Gordon Square, London	435
1895	C.	Oliver, Sir Thomas, M.D., LL.D., F.R.C.P., Professor of Physiology in the University of Durham, 7 Ellison Place, Newcastle-upon-Tyne	
1884	C. K.	Omond, R. Traill, 3 Church Hill	
1908		Page, William Davidge, F.C.S., F.G.S., M. Inst. M.E., 10 Clifton Dale, York	
1905		Pallin, William Alfred, F.R.C.V.S., Captain in the Army Veterinary Department, c/o Messrs Holt & Co., 3 Whitehall Place, London	
1892		Parker, Thomas, Memb. Inst. C.E., Severn House, Iron Bridge, Salop	440
1901		* Paterson, David, F.C.S., Lea Bank, Rosslyn, Midlothian	
1886	C.	Paton, D. Noël, M.D., B.Sc., F.R.C.P.E., Professor of Physiology in the University of Glasgow, University, Glasgow	
1889		* Patrick, David, M.A., LL.D., c/o W. & R. Chambers, 339 High Street	
1892		* Paulin, Sir David, Actuary, 6 Forres Street	
1881	C. N.	Peach, Benjamin N., LL.D., F.R.S., F.G.S., late District Superintendent and Acting Palæontologist of the Geological Survey of Scotland, 72 Grange Loan	445
1907		* Pearce, John Thomson, B.A., B.Sc., School House, Tranent	
1904		* Peck, James Wallace, M.A., Clerk to Edinburgh School Board, School Board Offices, Castle Terrace	
1889		* Peck, William, F.R.A.S., Town's Astronomer, City Observatory, Calton Hill, Edinburgh	
1887	C. B.	* Peddie, Wm., D.Sc., Professor of Natural Philosophy in University College, Dundee, Rosemount, Forthill Road, Broughty Ferry	
1900		Penny, John, M.B., C.M., D.Sc., Great Broughton, near Cockermouth, Cumberland	450
1893		Perkin, Arthur George, F.R.S., 8 Montpellier Terrace, Hyde Park, Leeds	
1889		* Philip, R. W., M.A., M.D., F.R.C.P.E., 45 Charlotte Square	
1907	C.	Phillips, Charles E. S., Castle House, Shooter's Hill, Kent	
1905		* Pinkerton, Peter, M.A., D.Sc., Head Mathematical Master, George Watson's College, Edinburgh, 36 Morningside Grove	
1908	C.	* Pirie, James Hunter Harvey, B.Sc., M.D., M.R.C.P.E., 5 Castle Terrace	455
1906		Pitchford, Herbert Watkins, F.R.C.V.S., Bacteriologist and Analyst, Natal Government, The Laboratory, Pietermaritzburg, Natal	
1886		Pollock, Charles Frederick, M.D., F.R.C.S.E., 1 Buckingham Terrace, Hillhead, Glasgow	
1888		Prain, David, Lt.-Col., Indian Medical Service, M.A., M.B., LL.D., F.L.S., F.R.S., Hon. Memb. Soc. Lett. ed Arti d. Zelanti, Acireale; Corr. Memb. Pharm. Soc. Gt. Britain, etc.; Director, Royal Botanic Gardens, Kew (late Director, Botanical Survey of India, Calcutta), Botanic Gardens, Kew	
1902		* Preller, Charles Du Riche, M.A., Ph.D., Assoc. Memb. Inst. C.E., 61 Melville Street	
1892		* Pressland, Arthur, J., M.A. Camb., Edinburgh Academy	460
1875	C.	Prevost, E. W., Ph.D., Weston, Ross, Herefordshire	
1908		* Pringle, George Cossar, M.A., Rector of Peebles Burgh and County High School, Bloomfield, Peebles	
1885		Pullar, J. F., Rosebank, Perth	
1903		* Pullar, Laurence, The Lea, Bridge of Allan	
1880		Pullar, Sir Robert, LL.D., M.P. for the city of Perth, Tayside, Perth	465
1898		* Purves, John Archibald, D.Sc., 13 Albany Street	
1897		* Rainy, Harry, M.B., C.M., F.R.C.P. Ed., 16 Great Stuart Street	
1899		* Ramage, Alexander G., 8 Western Terrace, Murrayfield	
1884		Ramsay, E. Peirson, M.R.I.A., F.L.S., C.M.Z.S., F.R.G.S., F.G.S., Fellow of the Imperial and Royal Zoological and Botanical Society of Vienna, Curator of Australian Museum, Sydney, N.S.W.	
1891		* Rankine, John, M.A., LL.D., K.C., Professor of the Law of Scotland in the University of Edinburgh, 23 Ainslie Place	470

Date of Election.		
1904		Ratcliffe, Joseph Riley, M.B., C.M., c/o The Librarian, The University, Birmingham
1900		Raw, Nathan, M.D., 66 Rodney Street, Liverpool
1883	C.	Readman, J. B., D.Sc., F.C.S., Staffield Hall, Kirkoswald R.S.O., Cumberland
1889		Redwood, Sir Boverton, D.Sc. (Hon.), F.I.C., F.C.S., Assoc. Inst. C.E., Wadham Lodge, Wadham Gardens, London
1902		Rees-Roberts, John Vernon, M.D., D.Sc., D.P.H., Barrister-at-Law, National Liberal Club, Whitehall Place, London 475
1902		Reid, George Archdall O'Brien, M.B., C.M., 9 Victoria Road South, Southsea, Hants
1908	C.	* Rennie, John, D.Sc., Lecturer on Parasitology, and Assistant to the Professor of Natural History, University of Aberdeen, 60 Desswood Place, Aberdeen
1908		Richardson, Linsdall, F.G.S., F.L.S., Director, Cheltenham School of Science and Technology, 10 Oxford Parade, Cheltenham
1875		Richardson, Ralph, W.S., 10 Magdala Place
1906	C.	* Ritchie, William Thomas, M.D., F.R.C.P.E., 9 Atholl Place 480
1898	C.	Roberts, Alexander William, D.Sc., F.R.A.S., Lovedale, South Africa
1880		Roberts, D. Lloyd, M.D., F.R.C.P.L., 23 St John Street, Manchester
1900		* Robertson, Joseph M'Gregor, M.B., C.M., 26 Buckingham Terrace, Glasgow
1896		* Robertson, Robert, M.A., 25 Mansionhouse Road
1902	C.	* Robertson, Robert A., M.A., B.Sc., Lecturer on Botany in the University of St Andrews 485
1896	C.	* Robertson, W. G. Aitchison, D.Sc., M.D., F.R.C.P.E., 2 Mayfield Gardens
1910		* Robinson, Arthur, M.D., M.R.C.S., Professor of Anatomy, University of Edinburgh, 35 Coates Gardens, Edinburgh
1881		Rosebery, the Right Hon. the Earl of, K.G., K.T., LL.D., D.C.L., F.R.S., Dalmeny Park, Edinburgh
1909	C.	* Ross, Alex. David, M.A., B.Sc., Assistant to the Professor of Natural Philosophy in the University of Glasgow, 7 Queen's Terrace, Glasgow
1906		* Russell, Alexander Durie, B.Sc., Mathematical Master, Falkirk High School, Dunaura, Heugh Street, Falkirk 490
1902	C.	* Russell, James, 12 Argyll Place
1880		Russell, Sir James A., M.A., B.Sc., M.B., F.R.C.P.E., LL.D., Woodville, Canaan Lane
1904		Sachs, Edwin O., Architect, 7 Waterloo Place, Pall Mall, London, S.W.
1906		Saleeby, Caleb William, M.D., 13 Greville Place, London
1903		* Samuel, John S., 8 Park Avenue, Glasgow 495
1903		* Sarolea, Charles, Ph.D., D.Litt., Lecturer on French Language, Literature, and Romance Philology, University of Edinburgh, 21 Royal Terrace
1891		Sawyer, Sir James, Knt., M.D., F.R.C.P., F.S.A., J.P., Consulting Physician to the Queen's Hospital, 31 Temple Row, Birmingham
1900	C.	* Schäfer, Edward Albert, M.R.C.S., LL.D., F.R.S., Professor of Physiology in the University of Edinburgh
1885	C.	Scott, Alexander, M.A., D.Sc., F.R.S., The Davy-Faraday Research Laboratory of the Royal Institution, London
1880		Scott, J. H., M.B., C.M., M.R.C.S., Professor of Anatomy in the University of Otago, New Zealand 500
1905		Scougal, A. E., M.A., LL.D., H.M. Senior Chief Inspector of Schools and Inspector of Training Colleges, 1 Wester Coates Avenue
1902		Senn, Nicholas, M.D., LL.D., Professor of Surgery, Rush Medical College, Chicago, U.S.A.
1897		* Shepherd, John William, Carrickarden, Bearsden, Glasgow
1871		Simpson, Sir A. R., M.D., Emeritus Professor of Midwifery in the University of Edinburgh, 52 Queen Street
1908		* Simpson, George Freeland Barbour, M.D., F.R.C.P.E., F.R.C.S.E., 43 Manor Place 505
1900	C.	* Simpson, James Young, M.A., D.Sc., Professor of Natural Science in the New College, Edinburgh, 25 Chester Street
1900		Sinhjee, Sir Bhagvat, G.C.I.E., M.D., LL.D. Edin., H.H. the Thakur Sahib of Gondal, Gondal, Kathiawar, Bombay
1903		* Skinner, Robert Taylor, M.A., Governor and Head Master, Donaldson's Hospital, Edinburgh
1901		* Smart, Edward, B.A., B.Sc., Tillyloss, Tullylumb Terrace, Perth
1891	C.	* Smith, Alexander, B.Sc., Ph.D., Professor of General Chemistry, University of Chicago, Illinois, U.S. 510
1882	C.	Smith, C. Michie, B.Sc., F.R.A.S., Director of the Kodaikânal and Madras Observatories, The Observatory, Kodaikânal, South India

Date of Election.			
1885		Smith, George, F.C.S., 5 Rosehall Terrace, Falkirk	
1907	C.	Smith, William Ramsay, D.Sc., M.B., C.M., Permanent Head of the Health Department, South Australia, Winchester Street, East Adelaide, South Australia	
1880		Smith, William Robert, M.D., D.Sc., Barrister-at-Law, Professor of Forensic Medicine in King's College, 74 Great Russell Street, Bloomsbury Square, London	
1899		Snell, Ernest Hugh, M.D., B.Sc., D.P.H. Camb., Coventry	515
1880		Sollas, W. J., M.A., D.Sc., LL.D., F.R.S., late Fellow of St John's College, Cambridge, and Professor of Geology and Palæontology in the University of Oxford	
1910		* Somerville, Robert, B.Sc., Science Master, High School, Dunfermline, 38 Cameron Street, Dunfermline	
1889	C.	Somerville, Wm., M.A., D.Sc., D.Oec., Sibthorpian Professor of Rural Economy in the University of Oxford, 121 Banbury Road, Oxford	
1882		Sorley, James, F.I.A., F.F.A., C.A., 82 Onslow Gardens, London	
1896		* Spence, Frank, M.A., B.Sc., 25 Craiglea Drive	520
1874	C.	Sprague, T. B., M.A., LL.D., Actuary, 29 Buckingham Terrace	
1906		Squance, Thomas Coke, M.D., Physician and Pathologist in the Sunderland Infirmary, 15 Grange Crescent, Sunderland	
1891		* Stanfield, Richard, Professor of Mechanics and Engineering in the Heriot-Watt College	
1910		* Stephenson, Thomas, F.C.S., Editor of the *Prescriber*, Examiner to the Pharmaceutical Society, 9 Woodburn Terrace, Edinburgh	
1886	C.	Stevenson, Charles A., B.Sc., Memb. Inst. C.E., 28 Douglas Crescent	525
1884		Stevenson, David Alan, B.Sc., Memb. Inst. C.E., 84 George Street	
1888	C.	* Stewart, Charles Hunter, D.Sc., M.B., C.M., Professor of Public Health in the University of Edinburgh, Usher Institute of Public Health, Warrender Park Road	
1902		* Stockdale, Herbert Fitton, Director of the Glasgow and West of Scotland Technical College, Clairinch, Upper Helensburgh, Dumbartonshire	
1889		* Stockman, Ralph, M.D., F.R.C.P.E., Professor of Materia Medica and Therapeutics in the University of Glasgow	
1906		Story, Fraser, Lecturer in Forestry, University College, Bangor, North Wales	530
1907		* Strong, John, B.A., Rector of Montrose Academy, 11 Union Place, Montrose	
1903		Sutherland, David W., M.D., M.R.C.P. Lond., Captain, Indian Medical Service, Professor of Pathology and Materia Medica, Medical College, Lahore, India	
1896		* Sutherland, John Francis, M.D., Deputy Commissioner in Lunacy for Scotland, Scotsburn Road, Tain, Ross-shire	
1905		Swithinbank, Harold William, Denham Court, Denham, Bucks	
1885	C.	Symington, Johnson, M.D., F.R.C.S.E., F.R.S., Professor of Anatomy in Queen's College, Belfast	535
1904		* Tait, John W., B.Sc., Rector of Leith Academy, 18 Netherby Road, Leith	
1898	C.	Tait, William Archer, B.Sc., Memb. Inst. C.E., 38 George Square	
1895		Talmage, James Edward, D.Sc., Ph.D., F.R.M.S., F.G.S., Professor of Geology, University of Utah, Salt Lake City, Utah	
1890	C.	Tanakadate, Aikitu, Professor of Natural Philosophy in the Imperial University of Japan, Tokyo, Japan	
1870		Tatlock, Robert R., F.C.S., City Analyst's Office, 156 Bath Street, Glasgow	540
1899		* Taylor, James, M.A., Mathematical Master in the Edinburgh Academy, Edinburgh Academy	
1892		Thackwell, J. B., M.B., C.M.	
1885	C.	Thompson, D'Arcy W., C.B., B.A., F.L.S., Professor of Natural History in University College, Dundee	
1907		* Thompson, John Hannay, M. Inst. C.E., M. Inst. Mech. E., Engineer to the Dundee Harbour Trust, Earlville, Broughty Ferry	
1905		* Thoms, Alexander, 7 Playfair Terrace, St Andrews	545
1887		* Thomson, Andrew, M.A., D.Sc., F.I.C., Rector, Perth Academy, Ardenlea, Pitcullen, Perth	
1896		* Thomson, George Ritchie, M.B., C.M., Cumberland House, Von Brandis Square, Johannesburg, Transvaal	
1903		Thomson, George S., F.C.S., Ferma Albion, Marculesci, Roumania	
1906		* Thomson, Gilbert, C.E., 164 Bath Street, Glasgow	
1887	C.	* Thomson, J. Arthur, M.A., Regius Professor of Natural History in the University of Aberdeen	550
1906	C.	Thomson, James Stuart, F.L.S., Zoological Department, University, Manchester.	

Date of Election.		
1880		Thomson, John Millar, LL.D., F.R.S., Professor of Chemistry in King's College, London, 9 Campden Hill Gardens, London
1899		* Thomson, R. Tatlock, F.C.S., 156 Bath Street, Glasgow
1870		Thomson, Spencer C., Actuary, 10 Eglinton Crescent
1882		Thomson, Wm., M.A., B.Sc., LL.D., Registrar, University of the Cape of Good Hope, University Buildings, Cape Town 555
1876	C.	Thomson, William, Royal Institution, Manchester
1874	C. B. N.	Traquair, R. H., M.D., LL.D., F.R.S., F.G.S., late Keeper of the Natural History Collections in the Royal Scottish Museum, Edinburgh, The Bush, Colinton
1874		Tuke, Sir J. Batty, M.D., D.Sc., LL.D., F.R.C.P.E., M.P. for the Universities of Edinburgh and St Andrews, 20 Charlotte Square
1888		* Turnbull, Andrew H., Actuary, The Elms, Whitehouse Loan
1905		* Turner, Arthur Logan, M.D., F.R.C.S.E., 27 Walker Street 560
1906	C.	* Turner, Dawson F. D., B.A., M.D., F.R.C.P.E., M.R.C.P. Lond., Lecturer on Physics, Surgeon's Hall, and Physician in charge of Electrical Department, Royal Infirmary, Edinburgh, 37 George Square
1861	K. N. C.	Turner, Sir William, K.C.B., M.B., F.R.C.S.E., LL.D., D.C.L., D.Sc. Dub., F.R.S., Principal of the University of Edinburgh (PRESIDENT), 6 Eton Terrace
1895		Turton, Albert H., M.I.M.M., 18 Harrow Road, Bowenbrook, Birmingham
1898	C.	* Tweedie, Charles, M.A., B.Sc., Lecturer on Mathematics in the University of Edinburgh, 40 Gillespie Crescent
1889		Underhill, T. Edgar, M.D., F.R.C.S.E., Dunedin, Barnt Green, Worcestershire 565
1906		Vandenbergh, William J., Barrister-at-Law, S.S.C., F.R.S.L., F.R.M.S., 29–32 Exchange Buildings, Pirie Street, Adelaide, S. Australia
1910		Vincent, Swale, M.D. Lond, D.Sc. Edin., etc., Professor of Physiology, University of Manitoba, Winnipeg, Canada
1891	C. B.	* Walker, James, D.Sc., Ph.D., LL.D., F.R.S., Professor of Chemistry in the University of Edinburgh, 5 Wester Coates Road
1873	C.	Walker, Robert, M.A., LL.D., University, Aberdeen
1902		* Wallace, Alexander G., M.A., 56 Fonthill Road, Aberdeen 570
1886	C.	Wallace, R., F.L.S., Professor of Agriculture and Rural Economy in the University of Edinburgh
1898		Wallace, Wm., M.A., Belvedere, Alta, Canada
1891		* Walmsley, R. Mullineux, D.Sc., Principal of the Northampton Institute, Clerkenwell, London
1907		Waters, E. Wynston, Medical Officer, H.B.M. Administration, E. Africa, Malindi, British East Africa Protectorate, *via* Mombasa
1901	C.	* Waterston, David, M.A., M.D., F.R.C.S.E., Professor of Anatomy, King's College, London 575
1904		* Watson, Charles B. Boog, Huntly Lodge, 1 Napier Road
1900		* Watson, Thomas P., M.A., B.Sc., Principal, Keighley Institute, Keighley
1910		* Watson, William John, M.A. Aberdeen, B.A. Oxon., Rector of the Royal High School, Edinburgh, 17 Merchiston Avenue, Edinburgh
1907		* Watt, Andrew, M.A., Secretary to the Scottish Meteorological Society, 6 Woodburn Terrace
1896		Webster, John Clarence, B.A., M.D., F.R.C.P.E., Professor of Obstetrics and Gynæcology, Rush Medical College, Chicago, 706 Reliance Buildings, 100 State Street, Chicago 580
1907	C.	* Wedderburn, Ernest Maclagan, M.A., LL.B., 6 Succoth Gardens
1903	C.	* Wedderburn, J. H. Maclagan, M.A., D.Sc., 11 Alexander Street, Princeton, N.J., U.S.A.
1904		Wedderspoon, William Gibson, M.A., LL.D., Indian Educational Service, Senior Inspector of Schools, Burma, The Education Office, Rangoon, Burma
1896		Wenley, Robert Mark, M.A., D.Sc., D.Phil., Litt.D., LL.D., Professor of Philosophy in the University of Michigan, Ann Arbor, Michigan, U.S.A.
1909	C.	* Westergaard, Reginald Ludovic Andreas Emil, Lecturer in Technical Mycology, Heriot-Watt College, 6 Suffolk Road, Edinburgh 585
1896	C.	White, Philip J., M.B., Professor of Zoology in University College, Bangor, North Wales
1890		White, Sir William Henry, K.C.B., Memb. Inst. C.E., LL.D., F.R.S., late Assistant Controller of the Navy, and Director of Naval Construction, Cedarscroft, Putney Heath, London
1881		Whitehead, Walter, F.R.C.S.E., late Professor of Clinical Surgery, Owens College and Victoria University, Birchfield, Rusholme, Manchester
1894		Whymper, Edward, F.R.G.S., Holmwood, Waldegrave Road, Teddington, Middlesex

Date of Election.		
1879		Will, John Charles Ogilvie, of Newton of Pitfodels, M.D., 17 Bon-Accord Square, Aberdeen 590
1908		* Williamson, Henry Charles, M.A., D.Sc., Naturalist to the Fishery Board for Scotland, 28 Polmuir Road, Aberdeen
1910	C.	* Williamson, William, 4 Meadowbank Terrace, Edinburgh
1900		Wilson, Alfred C., F.C.S., Voewood Croft, Stockton-on-Tees
1879		Wilson, Andrew, Ph.D., F.L.S., Lecturer on Zoology and Comparative Anatomy, 110 Gilmore Place
1902		* Wilson, Charles T. R., M.A., F.R.S., Glencorse House, Peebles, and Sidney Sussex College, Cambridge 595
1895		Wilson-Barker, David, F.R.G.S., Captain-Superintendent Thames Nautical Training College, H.M.S. "Worcester," Greenhithe, Kent
1882		Wilson, George, M.A., M.D., LL.D., 7 Avon Place, Warwick
1891		* Wilson, John Hardie, D.Sc., University of St Andrews, 39 South Street, St Andrews
1902		Wilson, William Wright, F.R.C.S.E., M.R.C.S. Eng., Cottesbrook House, Acock's Green, Birmingham
1908		* Wood, Thomas, M.D., Eastwood, 182 Ferry Road, Bonnington, Leith 600
1886	C.	Woodhead, German Sims, M.D., F.R.C.P.E., Professor of Pathology in the University of Cambridge
1884		Woods, G. A., M.R.C.S., Eversleigh, 1 Newstead Road, Lee, Kent
1890		* Wright, Johnstone Christie, Northfield, Colinton
1896		* Wright, Robert Patrick, Professor of Agriculture, West of Scotland Agricultural College, 6 Blythswood Square, Glasgow
1882		Young, Frank W., F.C.S., H.M. Inspector of Science and Art Schools, 32 Buckingham Terrace, Botanic Gardens, Glasgow 605
1892		Young, George, Ph.D., "Bradda," Church Crescent, Church End, Finchley, London, N.
1896	C.	* Young, James Buchanan, M.B., D.Sc., Dalveen, Braeside, Liberton
1900		* Young, J. M'Lauchlan, F.R.C.V.S., Lecturer on Veterinary Hygiene, University of Aberdeen
1904		Young, R. B., M.A., B.Sc., Transvaal Technical Institute, Johannesburg, Transvaal

LIST OF HONORARY FELLOWS OF THE SOCIETY

At November 1910.

HIS MOST GRACIOUS MAJESTY THE KING.

FOREIGNERS (LIMITED TO THIRTY-SIX BY LAW X.).

Elected		
1897	E.-H. Amagat,	*Paris.*
1900	Arthur Auwers,	*Berlin.*
1900	Adolf Ritter von Baeyer,	*Munich.*
1905	Waldemar Chr. Brögger,	*Christiania.*
1905	Moritz Cantor,	*Heidelberg.*
1902	Jean Gaston Darboux,	*Paris.*
1910	Hugo de Vries,	*Amsterdam.*
1905	Paul Ehrlich,	*Frankfurt-a.-M.*
1908	Emil Fischer,	*Berlin.*
1910	F. A. Forel,	*Morges* (*Switzerland*).
1910	Karl F. von Goebel,	*Munich.*
1905	Paul Heinrich Groth,	*Munich.*
1888	Ernst Haeckel,	*Jena.*
1883	Julius Hann,	*Graz.*
1908	George William Hill,	*New York.*
1910	Jacobus Cornelius Kapteyn,	*Groningen.*
1897	Gabriel Lippmann,	*Paris.*
1895	Carl Menger,	*Vienna.*
1910	Élie Metchnikoff,	*Paris.*
1910	Albert Abraham Michelson,	*Chicago.*
1897	Fridtjof Nansen,	*Christiania.*
1908	Henry Fairfield Osborn,	*New York.*
1910	Wilhelm Ostwald,	*Leipzig.*
1908	Iwan P. Pawlov,	*St Petersburg.*
1895	Jules Henri Poincaré,	*Paris.*
1910	Frederick Ward Putnam,	*Cambridge* (*Mass.*).
1889	Georg Hermann Quincke,	*Heidelberg.*
1908	Gustaf Retzius,	*Stockholm.*
1908	Augusto Righi,	*Bologna.*
1905	Eduard Suess,	*Vienna.*
1908	Louis Joseph Troost,	*Paris.*
1905	Wilhelm Waldeyer,	*Berlin.*
1910	August F. L. Weismann,	*Freiburg* (*Baden*).
1905	Wilhelm Wundt,	*Leipzig.*
1897	Ferdinand Zirkel,	*Bonn am Rhein.*

Total, 35.

BRITISH SUBJECTS (LIMITED TO TWENTY BY LAW X.).

Elected		
1889	Sir Robert Stawell Ball, Kt., LL.D., F.R.S., M.R.I.A., Lowndean Professor of Astronomy in the University of Cambridge,	*Cambridge.*
1892	Colonel Alexander Ross Clarke, C.B., R.E., F.R.S.,	*Redhill, Surrey.*
1897	Sir George Howard Darwin, K.C.B., M.A., LL.D., F.R.S., Plumian Professor of Astronomy in the University of Cambridge,	*Cambridge.*
1900	David Ferrier, M.D., LL.D., F.R.S., Professor of Neuro-Pathology, King's College, London,	*London.*
1900	Andrew Russell Forsyth, D.Sc., F.R.S., Sadlerian Professor of Pure Mathematics in the University of Cambridge,	*Cambridge.*
1910	James George Frazer, D.C.L., LL.D., Litt.D., F.B.A., Fellow of Trinity College, Cambridge, Professor of Social Anthropology in the University of Liverpool,	*Liverpool.*
1892	Sir David Gill, K.C.B., LL.D., F.R.S., formerly His Majesty's Astronomer at the Cape of Good Hope,	*London.*
1895	Albert C. L. G. Günther, Ph.D., F.R.S.,	*London.*
1883	Sir Joseph Dalton Hooker, K.C.S.I., M.D., LL.D., D.C.L., F.R.S., Corresp. Mem. Inst. of France,	*London.*

1908	Sir Alexander B. W. Kennedy, LL.D., F.R.S., Past Pres. Inst. C.E.,	*London.*
1908	Sir Edwin Ray Lankester, K.C.B., LL.D., F.R.S.,	*London.*
1910	Sir Joseph Larmor, D.Sc., LL.D., D.C.L., F.R.S., Lucasian Professor of Mathematics in the University of Cambridge, Secretary of the Royal Society,	*Cambridge.*
1900	Archibald Liversidge, LL.D., F.R.S., Professor of Chemistry in the University of Sydney,	*Sydney.*
1908	Sir James A. H. Murray, LL.D., D.C.L., Editor of the Oxford English Dictionary,	*Oxford.*
1905	Sir William Ramsay, K.C.B., LL.D., F.R.S., Professor of Chemistry in the University College, London,	*London.*
1886	The Lord Rayleigh, D.C.L., LL.D., D.Sc. Dub., F.R.S., Corresp. Mem. Inst. of France,	*London.*
1908	Charles S. Sherrington, M.A., M.D., LL.D., F.R.S., Holt Professor of Physiology in the University of Liverpool,	*Liverpool.*
1905	Sir Joseph John Thomson, D.Sc., LL.D., F.R.S., Cavendish Professor of Experimental Physics, University of Cambridge,	*Cambridge.*
1900	Thomas Edward Thorpe, D.Sc., LL.D., F.R.S., Principal of the Government Laboratories, London,	*London.*
1910	Alfred Russel Wallace, O.M., LL.D., D.C.L., F.R.S.,	*Wimborne, Dorset.*

Total, 20.

ORDINARY FELLOWS OF THE SOCIETY ELECTED

During Session 1909–10.

(Arranged according to the date of their election.)

22nd November 1909.

A. Anstruther Lawson, D.Sc.

20th December 1909.

E. H. Archibald, B.Sc.
Alexander Fraser.
Percy Hall Grimshaw.

24th January 1910.

David Gibb, M.A., B.Sc.
Charles Robert Gibson.
Alexander Lauder, D.Sc.
Alexander Levie, F.R.C.V.S.
Arthur Robinson, M.D., M.R.C.S.
William John Watson, M.A., B.A.

21st February 1910.

D. T. Gwynne-Vaughan, F.L.S.
John Miller, M.A., D.Sc.
Swale Vincent, M.D., D.Sc.

21st March 1910.

Lewis Bennet Barclay, C.E.

16th May 1910.

David Carnegie, M.I.C.E.
Mungo M'Callum Fairgrieve, M.A.
Bruce M'Gregor Gray, C.E.
Charles Edward Green.
William Fraser Hume, D.Sc.
Alister Thomas Mackenzie, M.A., M.D.
Robert Somerville, B.Sc.
William Williamson.

20th June 1910.

Joseph Strickland Goodall, M.B., M.S.A.
Hugh Allan MacEwen, M.B., Ch.B.
Thomas Stephenson, F.C.S.

18th July 1910.

Rev. Robert Sibbald Calderwood.
Loudon MacQueen Douglas.
Gabriel W. Lee.
James MacKinnon, M.A., Ph.D.

ORDINARY FELLOWS DECEASED AND RESIGNED

During Session 1909–10.

DECEASED.

Lt.-Col. T. D. C. Barry, M.R.C.S.
The Hon. Lord M'Laren, LL.D., F.R.A.S.
Rev. J. G. MacPherson, M.A., Ph.D.
Prof. L. L. Rowland, M.A., M.D.
John Smith, M.D., F.R.C.S.E.
James Walker, M.I.C.E.
Rev. R. Boog Watson, B.A., LL.D.
John Bennett Carruthers, F.L.S.

RESIGNED.

R. M. Buchanan, M.B., F.F.P.S.G.
John C. M'Vail, M.D., LL.D. (Nov. '08.)
J. M. M. Munro, M.I.C.E
William Shield, M.I.C.E.
John Cockburn, F.R.A.S
Rev. John Kerr, M.A.
George R. M. Murray, F.R.S., F.L.S.
William C. Smith, K.C., M.A.
Thomas W. Stewart, M.A., B.Sc.
Prof. W. Stirling, D.Sc., M.D.
W. Owen Williams, F.R.V.S.

BRITISH HONORARY FELLOWS DECEASED.

Sir William Huggins, K.C.B.
Sir Charles Todd, K.C.M.G.

FOREIGN HONORARY FELLOWS DECEASED.

Alexander Agassiz.
Stanislao Cannizzaro.
Albert Gaudry.
Jules Janssen.
Friedrich W. Georg Kohlrausch.
Éleuthère Élie-N. Mascart.
Eduard Pflüger.
Giovanni V. Schiaparelli.

ABSTRACT

OF

THE ACCOUNTS OF JAMES CURRIE, ESQ.

As Treasurer of the Royal Society of Edinburgh.

SESSION 1909–1910.

I. ACCOUNT OF THE GENERAL FUND.

CHARGE.

1. Arrears of Contributions at 1st October 1909		£269 17 0
2. Contributions for present Session :—		
1. 160 Fellows at £2, 2s. each	£336 0 0	
130 Fellows at £3, 3s. each	409 10 0	
	£745 10 0	
Less included in payments in lieu of future contributions	2 2 0	
	£743 8 0	
2. Commutation Fee in lieu of Future Contributions of one Fellow	5 5 0	
3. Fees of Admission and Contributions of twenty new Resident Fellows at £5, 5s. each	105 0 0	
4. Fees of Admission of nine new Non-Resident Fellows at £26, 5s. each	236 5 0	
		1089 18 0
3. Interest received—		
Interest, less Tax £22, 17s. 8d.	£369 8 8	
Annuity from Edinburgh and District Water Trust, less Tax £3, 1s. 2d.	49 8 10	
		418 17 6
4. Transactions and Proceedings sold		112 5 6
5. Annual Grant from Government		600 0 0
6. Income Tax repaid for three years to April 1910		69 1 6
Amount of the Charge		**£2559 19 6**

DISCHARGE.

1. TAXES, INSURANCE, COAL AND LIGHTING :—		
Inhabited House Duty	£0 6 3	
Insurance	10 15 0	
Coal to 18th May 1910	22 14 2	
Gas to 5th May 1910	0 5 4	
Electric Light to 3rd May 1910	13 8 7	
Water	2 2 0	
		£49 11 4
2. SALARIES :—		
General Secretary	£100 0 0	
Librarian	85 0 0	
Assistant Librarian	25 6 8	
Office Keeper	86 14 0	
Treasurer's Clerk	25 0 0	
		322 0 8
Carry forward		£371 12 0

Brought forward			£371 12 0
3. Expenses of Transactions :—			
Neill & Co., Ltd., Printers		£209 5 10	
Do. for illustrations		18 7 0	
Do. (Ben Nevis) .	£102 18 11		
Less received from Royal Society, London, per the Meteorological Society of Scotland	50 0 0		
		52 18 11	
M'Farlane & Erskine, Lithographers		33 7 9	
Hislop & Day, Engravers		2 14 0	
Orrock & Son, Bookbinders		71 1 0	
Do. (Ben Nevis)		90 10 0	
A. H. Searle, London, Proportion of Account . . .		7 10 0	
			485 14 6
4. Expenses of Proceedings :—			
Neill & Co., Ltd., Printers		£548 13 3	
Do. (for illustrations)		50 0 9	
M'Farlane & Erskine, Lithographers		6 4 0	
Hislop & Day, Engravers		34 15 9	
			639 13 9
5. Books, Periodicals, Newspapers, etc. :—			
Otto Schulze & Co., Booksellers		£145 9 2	
James Thin, do.		53 1 9	
R. Grant & Son, do.		7 5 10	
Wm. Green & Sons, do.		0 15 6	
International Catalogue of Scientific Literature . . .		17 0 0	
Robertson & Scott, News Agents		5 6 9	
Egypt Exploration Funds Subscription		3 3 0	
Ray Society do.		1 1 0	
Palæontographical Society do.		1 1 0	
Journal de Conchyliologie		0 15 0	
Orrock & Son, Bookbinders		41 9 6	
			276 8 6
6. Expenses in Connection with Reception held on 8th November 1909 :—			
Neill & Co., Ltd., Printers		£6 8 6	
R. Blair & Son, Confectioners.		29 10 10	
D. N. Kennedy & Son, Joiners		6 7 0	
Rent of Freemasons' Hall		3 10 6	
Edward & Co., Electricians		1 3 0	
G. Waterston & Sons, Stationers		4 10 6	
Attendants, Extra Cleaning, Posts, etc.		9 1 0	
			60 11 4
7. Other Payments :—			
Neill & Co., Ltd., Printers		£62 18 0	
R. Blair & Son, Confectioners		29 19 0	
S. Duncan, Tailor (uniforms)		4 14 0	
Lantern Exhibitions, etc., at Lectures		10 10 0	
Lindsay, Jamieson & Haldane, C.A., Auditors . . .		6 6 0	
National Telephone Co., Ltd.		15 8 5	
A. Cowan & Sons, Ltd.		6 12 6	
G. Waterston & Sons		8 16 3	
J. & T. Scott.		4 2 3	
Petty Expenses, Postages, Carriage, etc.		56 10 8	
			205 17 1
8. Sums paid for new Premises and Furnishings at 22 George Street, Edinburgh		£28,526 16 0	
Deduct—Amount paid by Secretary for Scotland		28,000 0 0	
			526 16 0
9. Interest Paid on Borrowed Money :—			
Makerstoun Magnetic Meteorological Observation Fund . .		£3 14 1	
Makdougall-Brisbane Fund		2 6 6	
Union Bank of Scotland, Ltd.		2 18 10	
			8 19 5
10. Irrecoverable Arrears of Contributions written off per list			106 1 0
Carry forward			£2681 13 7

Brought forward . . .		£2681 13 7
11. ARREARS of CONTRIBUTIONS outstanding at 1st October 1910 :—		
Present Session, per list	£58 16 0	
Previous Sessions, per list	38 17 0	
		97 13 0
Amount of the Discharge . .		**£2779 6 7**

Amount of the Discharge	**£2779 6 7**
Amount of the Charge	**2559 19 6**
Excess of Payments over Receipts for 1909-1910 . .	**£219 7 1**
FLOATING BALANCE DUE BY THE SOCIETY at 1st October 1909 . . .	£628 8 2
Add Excess of Payments as above	219 7 1
Floating Balance due by the Society at 1st October 1910 . . .	**£847 15 3**
Being—	
Balance due to Union Bank of Scotland, Ltd., on Account Current	£628 4 6
Loan from the Makerstoun Magnetic Meteorological Observation Fund . .	219 10 9

II. ACCOUNT OF THE KEITH FUND

To 1st October 1910.

CHARGE.

1. BALANCE due by Union Bank of Scotland, Ltd., on Account Current at 1st October 1909		£56 9 9
2. INTEREST RECEIVED :—		
On £896, 19s. 1d. North British Railway Company 3 per cent. Debenture Stock for year to Whitsunday 1910, less Tax £1, 11s. 4d.	£25 6 10	
On £211, 4s. North British Railway Company 3 per cent. Lien Stock for year to Lammas 1910, less Tax 7s. 4d. . . .	5 19 4	
		31 6 2
3. INCOME TAX repaid for three years to April 1910		5 3 0
		£92 18 11

DISCHARGE.

1. Dr Wheelton Hind—Money portion of Prize for 1907–1909.	£47 2 7
2. Alex. Kirkwood & Son, Engravers, for Gold Medal	16 0 0
3. BALANCE due by Union Bank of Scotland, Ltd., on Account Current at 1st October 1910	29 16 4
	£92 18 11

III. ACCOUNT OF THE NEILL FUND

To 1*st October* 1910.

CHARGE.

1. BALANCE due by Union Bank of Scotland, Ltd., on Account Current at 1st October 1909	£45 14 3
2. INTEREST RECEIVED :—	
On £355 London, Chatham and Dover Railway $4\frac{1}{2}$ per cent. Arbitration Debenture Stock for year to 30th June 1910, less Tax 18s. 8d.	15 0 10
3. INCOME TAX repaid for three years to April 1910	2 10 0
	£63 5 1

DISCHARGE.

1. Mr Francis J. Lewis—Money portion of Prize 1907–1909		£14	1	7
2. Alex. Kirkwood & Son, Engravers, for Gold Medal		16	0	0
3. Balance due by Union Bank of Scotland, Ltd., on Account Current at 1st October 1910		33	3	6
		£63	5	1

IV. ACCOUNT OF THE MAKDOUGALL-BRISBANE FUND

To 1st October 1910.

CHARGE.

1. Balance due at 1st October 1909:—						
By Union Bank of Scotland, Ltd., on Deposit Receipt	£135	0	0			
By Union Bank of Scotland, Ltd., on Account Current	62	4	2			
				£197	4	2
2. Interest received on £365 Caledonian Railway Company 4 per cent. Consolidated Preference Stock No. 2 for year to 30th June 1910, less Tax 17s.	£13	15	0			
On Deposit Receipt with Union Bank of Scotland, Ltd.	0	12	5			
On £135, 12s. 5d. at Deposit Receipt Rates from 29th November 1909 to 1st October 1910 (transferred from General Fund)	2	6	6			
				16	13	11
3. Income Tax repaid for three years to April 1910				2	5	11
				£216	4	0

DISCHARGE.

1. D. T. Gwynne-Vaughan—Money Portion of Prize for 1906–1908	£14	0	0
2. Alex. Kirkwood & Son, Engravers, for Gold Medal	16	0	0
3. Balance due by Union Bank of Scotland, Ltd., on Account Current at 1st October 1910	186	4	0
	£216	4	0

V. ACCOUNT OF THE MAKERSTOUN MAGNETIC METEOROLOGICAL OBSERVATION FUND

To 1st October 1910.

CHARGE.

1. Balance due by Union Bank of Scotland, Ltd., on Deposit Receipt at 1st October 1909				£214	16	11
2. Interest Received:—						
On Deposit Receipt with Union Bank of Scotland, Ltd.	£0	19	9			
On £215, 16s. 8d. at Deposit Receipt Rates from 29th November 1909 to 1st October 1910 (transferred from General Fund)	3	14	1			
				4	13	10
				£219	10	9

DISCHARGE.

Nil.

Balance due by General Fund at 1st October 1910	£219	10	9

VI. ACCOUNT OF THE GUNNING-VICTORIA JUBILEE PRIZE FUND

To 1st October 1910.

(Instituted by Dr R. H. GUNNING of Edinburgh and Rio de Janeiro.)

CHARGE.

	£	s.	d.
1. BALANCE due by Union Bank of Scotland, Ltd., on Account Current at 1st October 1909	£131	6	3
2. INTEREST received on £1000 North British Railway Company Consolidated Lien Stock for year to Lammas 1910, less Tax £1, 15s.	28	5	0
3. INCOME TAX repaid for three years to April 1910	4	14	1
	£164	5	4

DISCHARGE.

	£	s.	d.
1. PROFESSOR CHRYSTAL—Prize for 1904-1908	£105	0	0
2. BALANCE due by Union Bank of Scotland, Ltd., on Account Current at 1st October 1910	59	5	4
	£164	5	4

STATE OF THE FUNDS BELONGING TO THE ROYAL SOCIETY OF EDINBURGH

As at 1st October 1910.

1. GENERAL FUND—

	£	s.	d.
1. £2090, 9s. 4d. three per cent. Lien Stock of the North British Railway Company at $81\frac{1}{4}$ per cent., the selling price at 1st October 1910	£1,698	10	1
2. £8519, 14s. 3d. three per cent. Debenture Stock of do. at $81\frac{7}{8}$ per cent., do.	6,975	10	4
3. £52, 10s. Annuity of the Edinburgh and District Water Trust, equivalent to £875 at 170 per cent., do.	1,487	10	0
4. £1811 four per cent. Debenture Stock of the Caledonian Railway Company at $109\frac{3}{4}$ per cent., do.	1,987	11	5
5. £35 four and a half per cent. Arbitration Debenture Stock of the London, Chatham and Dover Railway Company at $80\frac{3}{4}$ per cent., do.	28	5	3
6. Arrears of Contributions, as per preceding Abstract of Accounts	97	13	0
	£12,275	0	1
Deduct Floating Balance due by the Society, as per preceding Abstract of Accounts	847	15	3
AMOUNT	£11,427	4	10

Exclusive of Library, Museum, Pictures, etc., Furniture of the Society's Rooms at George Street, Edinburgh.

2. KEITH FUND—

	£	s.	d.
1. £896, 19s. 1d. three per cent. Debenture Stock of the North British Railway Company at $81\frac{7}{8}$ per cent., the selling price at 1st October 1910	£734	7	7
2. £211, 4s. three per cent. Lien Stock of do. at $81\frac{1}{4}$ per cent., do.	171	12	0
3. Balance due by Union Bank of Scotland, Ltd., on Account Current	29	16	4
AMOUNT	£935	15	11

3. NEILL FUND—

	£	s.	d.
1. £355 four and a half per cent. Arbitration Debenture Stock of the London, Chatham and Dover Railway Company at $80\frac{3}{4}$ per cent., the selling price at 1st October 1910	£286	13	3
2. Balance due by Union Bank of Scotland, Ltd., on Account Current	33	3	6
AMOUNT	£319	16	9

4. MAKDOUGALL-BRISBANE FUND—

1. £365 four per cent. Consolidated Preference Stock No. 2 of the Caledonian Railway Company at 102$\frac{3}{4}$ per cent., the selling price at 1st October 1910	£375	0	9
2. Balance due by Union Bank of Scotland, Ltd., on Account Current . .	186	4	0
AMOUNT	£561	4	9

5. MAKERSTOUN MAGNETIC METEOROLOGICAL OBSERVATION FUND—

Balance due by General Fund at 1st October 1910	£219	10	9

6. GUNNING-VICTORIA JUBILEE PRIZE FUND—Instituted by Dr Gunning of Edinburgh and Rio de Janeiro—

1. £1000 three per cent. Consolidated Lien Stock of the North British Railway Company at 81$\frac{1}{4}$ per cent., the selling price at 1st October 1910 . .	£812	10	0
2. Balance due by Union Bank of Scotland, Ltd., on Account Current . .	59	5	4
AMOUNT	£871	15	4

EDINBURGH, 14*th October* 1910.—We have examined the six preceding Accounts of the Treasurer of the Royal Society of Edinburgh for the Session 1909–1910, and have found them to be correct. The securities of the various Investments at 1st October 1910, as noted in the above Statement of Funds, have been exhibited to us.

LINDSAY, JAMIESON & HALDANE,
Auditors.

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MODEL INDEX.

Schäfer, E. A.—On the Existence within the Liver Cells of Channels which can be directly injected from the Blood-vessels. Proc. Roy. Soc. Edin., vol. , 1902, pp. .

Cells, Liver,—Intra-cellular Canaliculi in.
E. A. Schäfer. Proc. Roy. Soc. Edin., vol. , 1902, pp. .

Liver,—Injection within Cells of.
E. A. Schäfer. Proc. Roy. Soc. Edin., vol. , 1902, pp. .

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